Economics 792. Equilibrium dynamic marriage market

Andrew Shephard

Consider a two-period model of consumption and marriage, with limited commitment and marital search frictions. Females are of type $f \leq F$ and males are of type $m \leq M$. Associated with each type f (m) is an exogenous income y_f (y_m) that is constant across periods. There is no borrowing or saving. The measure of type f females in the economy is π_f with $\sum \pi_f = 1$, while the measure of type m males is π_m with $\sum \pi_m = 1$. The discount factor is β and is common to all types.

Individuals derive utility from private consumption c. If they are single, private consumption and private income are synonyms, with $u_i(c_i) = \log(c_i)$. If they are married, total family consumption must equal total family income, $c \equiv c_f + c_m = y_f + y_m \equiv y$. Let $s \equiv c_f/c$ denote f's share of consumption. In addition, couples are subject to a marital match quality $\theta \sim H$ which is common to both individuals, and is known prior to making any relationship decision. The utility flow of a married individual is therefore $u_i(c_i; \theta) = \log(c_i) + \theta$. Assume the match quality is independent across periods. Decisions in the household are efficient, and we use λ to denote the Pareto weight on female utility. To maintain symmetry, we will use $1 - \lambda$ for the male Pareto weight.

In each period, single individuals meet at most one potential spouse, and all individuals start period 1 as single. Individuals who are married at the beginning of period 2 can decide to divorce, but no remarriage is possible. In period 2, assume that all relationship formation decisions are made prior to the separation decisions. All newly matched individuals (in both periods) are subject to initial the Pareto weight $\lambda = \lambda_0$, which may be renegotiated.

1. Consider the final period. Married individuals enter this period with a Pareto weight λ, which was determined the previous period. The match type (f, m) is observed, as is the current marital shock θ. Ignoring the participation constraint, determine the optimal consumption share s.

2. Now solve for the threshold value of θ such that the marriage is preferred for the female *given* f, m, and λ. Call this function $\theta_f^*(y_f, y_m, \lambda)$. Similarly consider this problem from the perspective of the male and define the resultant function $\theta_m^*(y_f, y_m, \lambda)$.

3. Suppose that $\theta < \theta_f^*(y_f, y_m, \lambda)$ and $\theta \geq \theta_m^*(y_f, y_m, \lambda)$. That is, at the initial λ the participation constrain of f is violated. Solve for the Pareto weight $\lambda_f^*(y_f, y_m, \theta)$ that makes the participation constraint bind for f. Subject to making f indifferent, determine the lowest value of θ (the reservation match value) that is also consistent with the participation constraint of m being satisfied. Denote this as $\underline{\theta}(y_f, y_m)$.

4. We now want to determine the expected value for an individual entering period 2 in the (y_f, y_m, λ) state. The expectation is taken over the realisation of the

match quality (which we recall is independent across periods). Assuming that H defines a Logistic distribution (with location parameter 0, and scale parameter 1), write down an expression for the expected values. For both male and females, you should write expressions for when i) $\theta_f^*(y_f, y_m, \lambda) \geq \theta_m^*(y_f, y_m, \lambda)$; ii) $\theta_f^*(y_f, y_m, \lambda) < \theta_m^*(y_f, y_m, \lambda)$.

5. So far we have characterised the problem for period 2. Denote the expected value from period 2 (prior to the realisation of θ) to be $V_f(y_f, y_m, \lambda)$ for f (this is what we calculated above) and $V_m(y_f, y_m, \lambda)$ for m. In terms of these objects, write the expected value for both f and m if they do not marry in period 1, recalling that initial meetings are evaluated at $\lambda = \lambda_0$. That is, present an expression for the period 1 value of being single. Use p_f (p_m) to denote the equilibrium measure of single type-f females (type-m males) in period 2 (we solve for these later).

6. In period 1, a given f and m meet and observe their marital shock and decide whether to marry. The couple will face a reservation marital quality which we denote $\underline{\theta}_1(y_f, y_m)$. What conditions must this satisfy? In terms of this threshold amount, write down the expression that characterises p_f and p_m.

7. Now try and numerically solve for the equilibrium p_f and p_m. Assuming that incomes are given by $\mathbf{y}_f = [1, 2, 3]$ and $\mathbf{y}_m = [2, 3, 4]$, while the type distribution is $\boldsymbol{\pi}_f = \boldsymbol{\pi}_m = [1/3, 1/3, 1/3]$. The discount factor is $\beta = 0.9$ and the initial Pareto weight is $\lambda_0 = 1/2$. Your code should make use of your answers above. (When you need expressions for the opposite gender that we didn't explicitly derive, you can just use the symmetry of the problem.)