integration of multiple sources of information
outline

- information so far: evidence and prior
- A - multiple sensory sources
- B - when to integrate what? (hypothesis testing)
- C - self-conditioning
combine evidence and prior knowledge to solve perceptual “case”
hypothesis

“perception is our best guess as to what is in the world, given our current sensory evidence and our prior experience.”

helmholtz 1821-1894
Bayes' rule

Bayes' Theorem

Thomas Bayes (1702 - 1761)

Pierre-Simon de Laplace (1749-1827)

\[
P(H_i | D) = \frac{P(D | H_i) P(H_i)}{\sum_k P(D | H_k) P(H_k)}
\]

Prior

Likelihood

"sensory evidence"

Posterior

"best guess"

A : evidence (data)

B : variable to infer

thomas bayes 1702-1761
Bayes’ rule expanded

\[ P(B|A_1, A_2) = \frac{P(B)P(A_1, A_2|B)}{\sum_B P(B, A_1, A_2)} \]

Prior \hspace{1cm} Likelihood

Posterior

\( A_1 \): cue 1 (evidence)
\( A_2 \): cue 2 (evidence)
\( B \): variable to infer
simple example

- two coins; one fair, one fake (only heads)
- randomly pick one
- 1st flip: its “head” (cue 1)
- 2nd flip: its “head” (cue 2)
- what is the probability that it is the fake coin?

\[
P(B = \text{”fake”} | A_1 = \text{”head”}, A_2 = \text{”head”})
\]

\[
P(B | A_1, A_2) = \frac{P(B)P(A_1, A_2 | B)}{\sum_B P(B, A_1, A_2)} = \frac{(1/2)(1)(1)}{[(1/2)(1)(1) + (1/2)(1/2)(1/2)(1/2)]} = \frac{4}{5}
\]
testing the hypothesis: ernst/banks 2002

• are humans computing the correct (bayesian) probabilities when solving a perceptual “case”?
inference problem

- sometimes easy because structure of the generative model (i.e. number of sources) is known.
  \[ P(D|H, \theta) \rightarrow P(\theta|D, H) \rightarrow \hat{\theta} \]

- however, sometimes structure is unknown and must be inferred too.
  \[ P(D|H, \theta) \rightarrow P(\theta, H|D) \rightarrow \hat{\theta}, \hat{H} \]

- e.g. cue-integration: when to combine cues (same source) and when not (different sources)
integration versus segmentation
gestalt rules (here: “Prägnanz” [simplicity])
ventriloquism

**Good**

- $S$
- $x_A$
- $x_V$

**Bad**

- $S_A$
- $S_V$
- $x_A$
- $x_V$
We found that people have a modest prior estimating stimuli to be more likely to be central. Subjects have the tendency of indicating a direction that is straight ahead and the prior allows the model to show such behavior as well. The average probability of perceiving a common cause for visual and auditory stimuli is relatively low. This explained that the observed biases are small in comparison to the values predicted if subjects were certain that there is a common cause (Fig. 2e). In summary, the causal inference model provides precise predictions of the way people combine cues in an auditory-visual spatial localization task, and it does so better than earlier models.

kording et al. 2007
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relative bias in perceived position of sound
problem formulation - generative model

hierarchy
1
2

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inferring the number of sources (structure)

\[
P(C|m_A, m_V) = \frac{p(m_A, m_V|C)P(C)}{p(m_A, m_V)}
\]

\[
p(m_A, m_V|C = 1) = \int p(m_A, m_V|s)p(s)ds
\]

\[
= \int p(m_A|s)p(m_V|s)ds
\]

\[
p(m_A, m_V|C = 2) = \int \int p(m_A, m_V|s_A, s_V)p(s_A, s_V)ds
\]

\[
= \int p(m_A|s_A)p(s_A)ds_A \int p(m_V|s_V)p(s_V)ds_V
\]
reporting structure ("causal inference")

- Experimental paradigm
- Stimulus
- left
- right

kording et al. 2007
inferring the positions

- model averaging

\[ p(\hat{s}_A | m_A, m_V) = \sum_i p(\hat{s}_A | C_i, m_A, m_V) p(C_i | m_A, m_V) \]

\[ p(\hat{s}_A | s_A, s_V) = \int \int p(\hat{s}_A | m_A, m_V) p(m_A | s_A) p(m_V | s_V) dm_A dm_V \]
inferring the positions

- model selection

\[ p(\hat{s}_A | m_A, m_V) = p(\hat{s}_A | \hat{C}, m_A, m_V) p(\hat{C} | m_A, m_V) = 1 \]
reporting positions

model averaging

kording et al. 2007
selection bias

model averaging

weight of visual cue

Experimental paradigm
Stimulus

left
right

Buttons pressed

Visual

Auditory

perceived
visual stimulus

perceived
auditory stimulus

mean

One cause

Two causes

Position [deg]

Gain > 0

Gain < 0

-5

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