Group-Shift and the Consensus Effect*

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Abstract

Individuals often tend to conform to the choices of others in group decisions, compared to choices made in isolation, giving rise to phenomena such as group polarization and the bandwagon effect. We show that this behavior, which we term the consensus effect, is equivalent to a well-known violation of expected utility, namely strict quasi-convexity of preferences. In contrast to the equilibrium outcome when individuals are expected utility maximizers, quasi-convexity of preferences imply that group decisions may fail to properly aggregate preferences and strictly Pareto-dominated equilibria may arise. Moreover, these problems become more severe as the size of the group grows.

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1 Introduction

Group decision-making is ubiquitous in social, economic, and political life. Empirical evidence suggests that individuals tend to make different choices depending on whether the outcome of interest is a result of their choice alone or also the choice of others in a group. In particular, the existing evidence largely supports the idea that these *choice shifts in groups*, which are prominent in a variety of contexts across fields, are predicted by the preference of the majority of individuals. For example, political scientists often discuss the bandwagon effect, where voters are more likely to vote for candidates who they think will win, i.e., who they believe others will vote for.[1] Another example, from the psychology and sociology literature, is the robust finding that individuals, when voting in a group, will take riskier or safer decisions vis-à-vis those taken by the individuals separately.[2] In the legal realm, jurors and judges tend to be affected by the preferences of other members of the jury or the court.[3] As an influential early article in sociology by Granovetter (1978) summarized it, "collective outcomes can seem paradoxical — that is intuitively inconsistent with the intentions of the individuals who generate them."

Models of group decisions typically analyze either private-value or common-value settings. Because, as will be explained below, with expected utility preferences in a private-value setting we should not observe choice shifts, much of the literature exploring choice shifts has focused on the common-value setting. In this context, group decisions aggregate private information regarding the relative value of possible outcomes.[4] In contrast, in this paper we maintain a private-value setting, but relax the assumption of expected utility.

To see why a violation of expected utility may generate choice shifts in groups, note that an individual choice in a group decision matters only when that individual is pivotal, that is, when his vote actually changes the outcome. However, from an

[1] Goidel and Shields (1994) found that within the United States, independents tend to vote more for a Republican candidate if that candidate is expected to win. Similarly, they found that weak Republican supporters are more likely to vote for a Democrat if that candidate is expected to win. Niemi and Bartels (1984) and Bartels (1988) discuss further evidence for this phenomenon.

[2] Stoner (1961, 1968), Nordhoy (1962) and Pruitt (1971) .

[3] Schkade, Sunstein and Kahneman (2000), Sunstein, Hastie, Payne, Schkade and Viscusi (2002), and Sunstein (2005).

[4] This literature, typified by Feddersen and Pesendorfer (1997), focuses on the ability of group decisions to aggregate private information rather than preferences. In Section 5 we contrast our findings with theirs as well as the larger literature on information aggregation in groups.

ex-ante perspective, when choosing for which option to vote, an individual does not know whether or not he will be pivotal. Thus, his choice is not a choice between receiving Option 1 or Option 2 for sure, but rather between *lotteries* defined over these two options (where if the individual turned out to be pivotal his selected option will be implemented, and otherwise the probability of each alternative to win depends on the probability that the group chooses it conditional on him not being pivotal). Violations of the independence axiom of expected utility imply that an individual may prefer Option 1 to Option 2 in isolation, yet prefer the lottery induced in the group context by choosing Option 2 over the one induced by choosing Option 1, thus accounting for the aforementioned choice shift.

In Section 2 we formally link violations of expected utility with the phenomenon of choice shifts in groups. In doing so, we provide a relationship between two types of non-standard behavior, one observed at the individual level and one at the group level. Our first result states that individuals have preferences that are strictly *quasi-convex* in probabilities if and only if they will systematically exhibit a *consensus effect* — an individual who is indifferent between two options when choosing in isolation will actually strictly prefer to vote for the option that is sufficiently likely to be chosen by the group. As discussed, the consensus effect captures the stylized fact that in group contexts individuals want to exhibit preferences that match those of the group as a whole. Quasi-convexity, on the other hand, is a well established preference pattern in decision making under risk, according to which individuals are averse toward randomization between equally good lotteries.[5] Popular models of preferences over lotteries which can exhibit quasi-convexity include rank-dependent utility (Quiggin, 1982, hereafter RDU), quadratic utility (Chew, Epstein, and Segal, 1991), and Kőszegi and Rabin's (2007) model of reference-dependence. Moreover, as observed by Machina (1984), quasi-convexity occurs if, before the lottery is resolved, the individual is allowed to take an action that determines his final utility. As long as the optimal decision is affected by a change in the probabilities, the induced maximum expected utility will be convex in the probabilities, meaning that even if the underlying

[5]Our proof shows that having quasi-convex preferences is equivalent to adopting a "threshold" rule towards the level of support that others will exhibit for any given option (i.e., the probability that any given option is chosen when a voter is not pivotal). When the level of support for an option exceeds the threshold, the individual will strictly prefer to choose it in a group situation. These thresholds have similar intuition to the reasons provided for similar consensus effects in other fields; for example, Granovetter (1978) specifically discusses the effect thresholds will have on aggregate versus individual behavior.

preferences are expected utility, induced preferences over the 'optimal' lotteries are quasi-convex.

In a seminal paper discussing choice shifts in groups, Eliaz, Ray, and Razin (2006, hereafter ERR) used the same model of group decision making but focused on group choices between particular pairs of options, safe and risky, where the former is a lottery that gives a certain outcome with probability one. They further confined their attention to RDU preferences and established an equivalence between specific types of choice shifts and Allais-type behavior, one of the most documented violation of expected utility at the individual level. Since choice shifts in groups are observed in experiments even when all lotteries involved are non-degenerate, our results suggest that the choice shifts discussed in ERR are actually manifestations of the consensus effect. In Section 3 we turn to relating our results to theirs. We extend their results for RDU preferences, but also demonstrate why the relation to Allais paradox is restricted to that specific class of preferences. In particular, the consensus effect is in general consistent not only with Allais-type behavior, but also with the opposite pattern of choice, and, similarly, Allais-type behavior does not rule out the anti-consensus effect.

In Section 4 we analyze what type of observable equilibrium behavior results from quasi-convex preferences in conjunction with strategic considerations. We describe a majority voting game as a collection of individuals, each of whom receiving one vote to cast in favor of option p or option q (no abstentions are allowed). Following the conventions of much of the recent literature (e.g. Krishna and Morgan, 2012 and Feddersen and Pesendorfer, 1999), we use the Poisson model introduced by Myerson (1998, 2000), according to which the number of voters is a random variable drawn from a Poisson distribution with mean N. Individuals' preferences (types) are drawn from a known distribution of preferences. After observing their own preferences, but no other information, individuals vote. Whichever option receives the majority of the votes is implemented. (If the vote is tied, then the winner is decided by coin flip.)

As previously mentioned, expected utility maximizers do not alter their choice in the context of group choice. In contrast, the fact that individuals with quasi-convex preferences do not like to randomize implies that voting games take on the properties of a *coordination games*. These individuals benefit from coordinating their votes with others because it reduces the amount of "randomness" in the election. They typically face a tradeoff between having the option they prefer selected and reducing the uncertainty regarding the identity of the chosen outcome.

We prove the existence of equilibria and describe their main properties. We also examine how the set of equilibria depends on the distribution of types, the voting rule, and the size of the electorate. For example, in contrast to the results under expected utility, when individuals exhibit the consensus effect, group decisions may fail to aggregate preferences properly and strictly Pareto-dominated equilibria may result. Moreover, these problems become more severe as the size of the group grows.

In Section 5 we relate our results to commonly discussed phenomena such as group polarization and the bandwagon effect, and provides foundations for the previously discussed empirical findings. For example, our model generates an *endogenous* benefit of conformity (for being in the 'winning side'), which can be behaviorally distinguished from an analogous *exogenous* benefit term that is added to the expected utility of an alternative. We also discuss how our model relates to, and can be distinguished from, alternative models in the larger literature on voting, including both common-value and private-value settings.

2 The Consensus Effect and Quasi-Convex Preferences

Our aim is to link an individual's private ranking of objects with his ranking of these same objects in a group context. We assume that any individual has preferences over monetary lotteries. Formally, let $X \subset \mathbb{R}$ be an interval of monetary prizes, and denote by Δ the set of lotteries with finite support over X. We identify an individual with his complete, transitive, and continuous preference relation $\succcurlyeq$ over Δ, which is also monotonic with respect to first-order stochastic dominance. Throughout the paper we denote by x, y, z generic elements of X and by p, q, r generic elements of Δ.[6]

In describing group decision problems, we extend the model suggested by ERR (see Section 3). Let I be a group of individuals. We identify a group decision problem as perceived by an individual $i \in I$ with a quadruple (p, q, α, β), consisting of two lotteries $p, q \in \Delta$ and two scalars $\alpha \in (0, 1)$ and $\beta \in [0, 1]$; α is the probability that individual i's decision is pivotal in choosing between p and q, and β is the probability

[6] We assume the reduction of compound lotteries axiom to only analyze single-stage distributions.

that the group chooses p conditional on i *not* being pivotal.[7,8] For now, both α and β are exogenous and fixed; accordingly, we can interpret the choice from any such quadruple as determining an individual's best-response function. In Section 4 they will be derived as part of the equilibrium analysis.

If, in the group context, the individual votes for q, the effective lottery he faces is

$$q^* = \alpha q + (1-\alpha)\left(\beta p + (1-\beta)q\right) = \left[\alpha + (1-\alpha)(1-\beta)\right] q + (1-\alpha)\beta p$$

And if the individual votes for p, the effective lottery he faces is

$$p^* = \alpha p + (1-\alpha)\left(\beta p + (1-\beta)q\right) = (1-\alpha)(1-\beta)q + [\alpha + (1-\alpha)\beta]p$$

A choice shift is thus the joint statement of $p \sim q$ but $q^* \succ p^*$ or $q^* \prec p^*$. An individual decision problem can be identified with the case $\alpha \equiv 1$.

Our definition of the *consensus effect* below suggests a specific type of choice shift, whereby an individual tends to draw towards what others would do in the absence of him being pivotal. In particular, it captures the idea that if other members of the group are likely enough to choose p when the individual is not pivotal, then the individual himself will prefer to choose p as well.[9]

Definition 1. *The individual exhibits a consensus effect at (p, q, α, β^*) if $p \sim q$ and $\beta > \beta^*$ (resp. $\beta < \beta^*$) implies that $q^* \succ p^*$ (resp., $q^* \prec p^*$). The individual exhibits the consensus effect if for all p, q, α with $p \sim q$, there exists β^* such that he exhibits the consensus effect in (p, q, α, β^*).*

Anti-consensus effect at (p, q, α, β^*) and general anti-consensus are similarly defined.

Observe that if preferences $\succcurlyeq$ satisfies the following *betweenness* property, $p \sim q$ implies $\gamma p + (1-\gamma)q \sim q$,[10] then the individual will never display any choice shift in group. This property is weaker than the standard independence axiom,[11] which

[7] We omit the index i till Section 4, where we explicitly study strategic interactions between members of the group.

[8] Note that the alternatives we consider are lotteries. For example, in a voting context, we associate a candidate with a lottery over policies.

[9] The consensus effect is defined where $p \sim q$. By continuity, the choice patterns that we study when the options are indifferent will persist even when one option is strictly preferred to the other.

[10] For any lotteries p,$q \in \Delta$, $\alpha p + (1-\alpha)q$ is the lottery that yields x with probability $\alpha p(x) + (1-\alpha)q(x)$.

[11] According to the independence axiom, $p \succcurlyeq q$ if and only if $\gamma p + (1-\gamma)r \succcurlyeq \gamma q + (1-\gamma)r$.

suggests that to accommodate such shifts, one needs to go beyond expected utility (or, more generally, beyond the betweenness class of preferences, suggested by Chew, 1983 and Dekel, 1986). To this aim, we consider the following two properties.

Definition 2. *The preference relation* $\succcurlyeq$ *is strictly quasi-convex if for all distinct* $p, q \in \Delta$ *and* $\lambda \in (0,1)$,

$$p \sim q \Rightarrow \lambda p + (1-\lambda) q \prec p$$

and is strictly quasi-concave if

$$p \sim q \Rightarrow \lambda p + (1-\lambda) q \succ p$$

Quasi-convexity implies aversion towards randomization between equally good prospects; whereas quasi-concavity implies affinity to such randomization. Betweenness preferences are those that satisfy both weak quasi-convexity and weak quasi-concavity.[12]

Our main result links violations of expected utility in the individual level with a specific pattern of choices in group situations.

Proposition 1. *Preferences are strictly quasi-convex (resp., strictly quasi-concave) if and only if they exhibit the consensus (resp., anti-consensus) effect.*

We now discuss the implications of Proposition 1 for some popular non-expected utility models. (We focus throughout the paper on the quasi-convex case, although the results naturally extend, modulo standard reversal, to quasi-concavity.)

Rank-Dependent Utility (RDU): Order the prizes $x_1 < x_2 < ... < x_n$. The functional form for RDU is:

$$V_{RDU}(p) = u(x_1) + \sum_{i=2}^{n} g\left(\sum_{j \geq i} p(x_j)\right)[u(x_i) - u(x_{i-1})] \tag{1}$$

where the weighting function $g : [0,1] \to [0,1]$ is bijective and strictly increasing. If $g(l) = l$ then RDU reduces to expected utility.

[12]The experimental evidence on quasi-convexity versus quasi-concavity is mixed. While it is a stylized empirical finding that betweenness is often violated, most of the experimental literature that documents violations of linear indifference curves (e.g., Coombs and Huang, 1976) found deviations in both directions, that is, either preference for or aversion to randomization. Camerer and Ho (1994) find support for a complicated pattern with quasi-convexity over gains and quasi-concavity over losses.

RDU preferences are quasi-convex if and only if the weighting function is convex (see Wakker, 1994). Convexity of the weighting function is typically interpreted as a type of pessimism: improving the ranking position of an outcome decreases its decision weight. This suggests the following corollary.

Corollary 1. *Suppose preferences are RDU. Then the individual is strictly pessimistic (g is strictly convex) if and only if he exhibits the consensus effect.*

The consensus effect as in Definition 1 is weak, in the sense that it does not determine how likely it has to be that the group chooses p in the absence of the individual being pivotal. However, if we put more structure on preferences we can have stronger results. This motivates introducing the class of quadratic preferences.

Quadratic Utility: A utility functional is quadratic in probabilities if it can be expressed in the form

$$V_{\mathbb{Q}}(p) = \sum_x \sum_y \phi(x,y)\ p(x)p(y)$$

where $\phi : X \times X \to \mathbb{R}$ is symmetric.[13] The quadratic functional form was introduced in Machina (1982) and further developed in Chew, Epstein, and Segal (1991, 1994).

The following result establishes that in the class of quadratic preferences, the consensus effect becomes a *majority effect* — β^* *always* equals .5, independently of the two options under consideration. So, when pivotal, the individual simply chooses the option he believes the group is most likely to choose when he is not pivotal.

Proposition 2. *Suppose preferences can be represented by a quadratic functional. Then preferences are strictly quasi-convex if and only if the individual exhibits the consensus effect at* $(p, q, \alpha, .5)$.

The following class of preferences was introduced by Kőszegi and Rabin (2007) and Delquié and Cillo (2006).

Choice Acclimating Personal Equilibrium: The value of a lottery p is

$$V_{\mathbb{CPE}_M}(p) = \underbrace{\sum_x u(x)p(x)}_{\text{consumption utility}} + \underbrace{\sum_x \sum_y \mu(u(x) - u(y))\ p(x)p(y)}_{\text{gain-loss utility}}$$

[13] There is no loss of generality in restricting ϕ to be symmetric, since an arbitrary $\phi(x,y)$ can always be replaced in the quadratic formula by $\frac{\phi(x,y)+\phi(y,x)}{2}$.

where u is an increasing utility function over final wealth and

$$\mu(z) = \begin{cases} z & \text{if } z \geq 0 \\ \kappa z & \text{if } z < 0 \end{cases}$$

is a gain-loss function with $0 \leq \kappa \leq 2$ denoting the coefficient of loss aversion. Loss aversion occurs when $\kappa \geq 1$. Masatlioglu and Raymond (2016) show that these preferences are the intersection of RDU and quadratic utility, and that they are quasi-convex if and only if $\kappa \geq 1$.

Corollary 2. *Suppose preferences have a representation $V_{\mathbb{CPE}_M}$. Then individuals are loss averse if and only if they exhibit the consensus effect at $(p, q, \alpha, .5)$.*

Corollary 2 links notions of reference dependence in individual choice with a similar notion (the consensus effect) in group choice. If the group is more likely to choose p than q when an individual is not pivotal, then this likely choice would naturally serve as a reference point when the individual is deciding how to make his own choice (which will only matter in the case where he is pivotal). This almost exactly mirrors the underlying intuition many researchers have provided for a preference for conformity — it is a type of external (i.e. based on the actions of others) reference point.

3 Risky and Cautious Shifts, and Allais Paradox

In this section we focus on group choices between particular pairs of options, s(afe) and r(isky), where s is a degenerate lottery, that is, a lottery that yields a certain prize $x \in X$ with probability 1, and r is some nondegenerate lottery. A group decision problem is then (r, s, α, β), where, as before, $\alpha \in (0, 1)$ is the probability that the individual is pivotal and $\beta \in [0, 1]$ is the probability that the group decides on the safe option s, conditional on the individual not being pivotal. In this context, we refer to *risky shift* (resp., *cautious shift*) as the joint statement $r \sim s$ and $r^* \succ s^*$ (resp., $r^* \prec s^*$), where

$$r^* = [\alpha + (1 - \alpha)(1 - \beta)]\, r + (1 - \alpha)\beta s$$

and

$$s^* = (1-\alpha)(1-\beta)r + [\alpha + (1-\alpha)\beta]s$$

These shifts are clearly a subset of the more general shifts discussed under the consensus effect. For a particular r, s, and α, there exists a β^* where an individual always exhibits a risky shift for $\beta \leq \beta^*$ and a cautious shift for $\beta \geq \beta^*$ if and only if the individual exhibits the consensus effect at (s, r, α, β^*).[14]

ERR used this setting and focused on RDU preferences. Below we review their contribution and use our results to understand whether, and how, their results generalize to other types of non-expected utility preferences. Segal (1987) showed that within RDU, a convex distortion function g in equ.(1) implies (and is implied by) behavior that accommodates a version of Allais paradox — also known as the common consequence effect — which is one of the most prominent evidence against expected utility. Formally, fix any three prizes $x_h > x_m > x_l$ and denote by (p_l, p_m, p_h) the lottery that yields prize x_i with probability p_i. The following definition formalizes this notion of the Allais paradox.[15]

Definition 3. *An individual exhibits the Allais paradox if for every pair of lotteries* $(1-\alpha, \alpha, 0)$ *and* $(1-\beta, 0, \beta)$ *with* $\alpha > \beta$, $(1-\alpha, \alpha, 0) \sim (1-\beta, 0, \beta)$ *implies* $(1-\alpha-\gamma, \alpha+\gamma, 0) \succ (1-\beta-\gamma, \gamma, \beta)$ *for all* $\gamma \in (0, 1-\alpha]$.

Theorem 1 in ERR states that within RDU, an individual exhibits the Allais paradox if and only if for any $r \sim s$ and $\alpha \in (0,1)$ there exists $\beta^* \in (0,1)$ such that he exhibits risky (resp., cautious) shift if $\beta < \beta^*$ (resp., $\beta > \beta^*$). ERR thus suggest an equivalence between a commonly known violation of expected utility and a robust phenomenon in the social psychology of groups when choosing between risky and safe options. Notice that because Allais-type behavior is equivalent to the convexity of the weighting function and therefore to quasi-convexity of preferences, it is also the case that within RDU we have additional equivalences, as the following corollary summarizes.

Corollary 3. *Consider the rank dependent utility model (equ.(1)). The following statements are equivalent:*

[14] Again, because of continuity, our results naturally extend to situations where $r \nsim s$.

[15] In Allais' original questionnaire, $x_h = 5M; x_m = 1M$, and $x_l = 0$. Subjects choose between $A = (0, 1, 0)$ and $B = (0.1, 0.89, 0.01)$, and also between $C = (0, 0.11, 0.80)$ and $D = (0.1, 0, 0.9)$. The typical pattern of choice is the pair (A, D). Definition 3 is more general than the original paradox proposed by Allais, since it puts behavioral restrictions also when no certain outcome is involved.

1. *An individual exhibits the Allais paradox*

2. *For all s, r, and α there exists β^* such that the individual exhibits the consensus effect at (s, r, α, β^*)*

3. *An individual's preferences satisfy quasi-convexity*

4. *An individual exhibits the consensus effect*

Although these equivalences are quite strong (in the sense that they link specific behavior regarding r and s to arbitrary behavior for any p and q) and have an intuitive appeal (in that they link preferences for a risky versus safe option in Allais questionnaire to similar preferences in group choice), we emphasize that these logical equivalences — as well as ERR original results — are derived in the narrow context of RDU preferences. We will now argue, by means of examples, that they are specific to this class and do not hold in general. In other words, empirical evidence that refutes RDU also challenges the aforementioned relationship between Allais-type behavior and the consensus effect.

To demonstrate this, first observe that the pattern of risky and cautious shifts discussed in ERR is implied by the consensus effect. Thus, in constructing our examples, we will show that both quasi-convexity and quasi-concavity are consistent with both Allais-type behavior and with the opposite pattern of individual choice. We will consider quadratic preferences, already discussed in the previous section. We further use the following observation: Any lottery p over fixed three outcomes $x_l < x_m < x_h$ can be represented as a point (p_l, p_h) in a two-dimensional unit simplex, where the probability of x_l (p_l) is on the x-axis and that of x_h (p_h) is on the y-axis. Showing that indifference curves become steeper, or fanning out, in the 'north-east' direction is sufficient for Allais-type behavior, while the opposite pattern, fanning in, is sufficient for anti-Allais-type behavior.

Example 1: Our first example is of preferences which are quasi-concave but exhibit Allais-type behavior. Consider the utility functional,

$$V(p) = \mathrm{E}\left[v(p)\right] \times \mathrm{E}\left[w(p)\right]$$

which is quasi-concave (since $\log V$ is concave).[16] For three outcomes, $x_l < x_m <$

[16] In this example, $\phi(x, y) = \frac{v(x)w(y)+v(y)w(x)}{2}$.

x_h, define v and w as follows: $v(x_l) = 1, v(x_m) = 2, v(x_h) = 4$; $w(x_l) = 2, w(x_m) = 3, w(x_h) = 4$. We show in the Appendix that the indifference curves of this utility functional fan out.

Example 2: Our second example is of preferences which are quasi-convex but exhibit anti-Allais-type behavior. Consider again three fixed outcomes, $x_l < x_m < x_h$, and the utility functional be defined as

$$V(p_l, p_h) = -6p_l + p_l^2 + 7.82p_h - 3.2p_lp_h + 2.56p_h^2$$

We show in the Appendix that the indifference curves of this utility functional fan in.[17]

These two examples show that Allais-type behavior and risky/cautious shifts (and the consensus effect more generally) are not necessarily related outside RDU. In the appendix we provide another example, which demonstrates that even the equivalence between risky and cautious shifts and quasi-convexity (and so the consensus effect) that Corollary 3 describes does not extend. While quasi-convexity is a sufficient condition for ERR's risky/cautious shifts, it is not necessary.

The reason that the ERR's result does not extend beyond RDU is that the set of comparable lotteries (s and r) is not 'dense enough' in the space of all lotteries. Essentially having the consensus effect at all pairs of r and s is equivalent to quasi-convexity holding along convex combinations of pairs of r and s, which, in the absence of additional restrictions, is not enough to imply global quasi-convexity. Relying on this intuition, it is easy to see that the following pair of axioms — which are weaker than quasi-convexity — are equivalent to ERR's risky and cautious shifts.

> **Axiom 1:** Suppose $s \sim r$ and $\lambda s + (1-\lambda)r \sim \delta s + (1-\delta)r$, for $\lambda \neq \delta$ and $\lambda, \delta \in [0,1]$. If $\lambda s + (1-\lambda)r \sim \gamma s + (1-\gamma)r$ for $\gamma \in [0,1]$, then either $\gamma = \lambda$ or $\gamma = \delta$.
>
> **Axiom 2:** If $s \sim r$, then $\lambda s + (1-\lambda)r \prec s$ for all $\lambda \in (0,1)$.

Axiom 1 guarantees that along any convex combination of r and s utility is either single-peaked or single-valleyed. Axiom 2 guarantees that it is single-valleyed.

[17] V is a quadratic functional, with $\phi(x_l, x_l) = -5$, $\phi(x_m, x_l) = -3$, $\phi(x_h, x_l) = 2.51$, $\phi(x_m, x_m) = 0$, $\phi(x_h, x_m) = 3.91$, and $\phi(x_h, x_h) = 10.38$.

Proposition 3. *$\succcurlyeq$ satisfies Axiom 1 and Axiom 2 if and only if for all s, r, and α there exists β^* such that the individual exhibits the consensus effect at (s, r, α, β^*).*

4 The Consensus Effect in Equilibrium

Our analysis so far has been restricted to understanding the behavior of an individual who is facing a fixed, exogenous decision process. While, similar to ERR, our interpretation of the environment is of a group decision problem, the exact same analysis would apply also if the environment reflects a situation where the individual gets to choose with some probability, and with the remaining probability a computer chooses for him. To explicitly captures the strategic interaction, in this section we extend our analysis to a full equilibrium setting, and in doing so refer to individuals as voters.

We will show that, in contrast to settings where voters are expected utility maximizers, quasi-convex preferences can lead to phenomena such as group polarization, the bandwagon effects, preference reversals, and multiple equilibria. This is driven by the fact that quasi-convex preferences give the voting game properties of a *coordination game*.

We describe a majority voting game as a collection of individuals, each of whom receives one vote to cast in favor of either p or q (no abstentions are allowed).[18] Whichever option receives the majority of the votes is implemented (in Proposition 9 we relax the assumption of a simple majority rule). If the vote is tied, the winner is decided by a coin flip. In line with recent literature on voting (e.g. Krishna and Morgan, 2012 and Feddersen and Pesendorfer, 1999), we assume that the number of voters is a random variable which is distributed according to a Poisson distribution with mean N: the probability that there are exactly n voters is $\frac{e^{-N}N^n}{n!}$.[19]

Suppose there are three *types* of individuals. Those that prefer p to q (Type P), those that prefer q to p (Type Q), and those that are indifferent (Type $\widetilde{I}$). Each

[18] Identical results will be obtained if voting is assumed instead to be voluntary but costless.

[19] Other than the uniqueness result of Proposition 6, our results do not depend on this assumption; they carry over to situations where the number of voters is known. While there exist equilibria that involve coordination motives even when individuals are expected utility maximizers, these are not robust to uncertainty regarding the number of voters. In contrast, the equilibria we study with quasi-convex preferences are robust to this uncertainty. Simply focusing on the *voting equilibrium* of the game (i.e., a symmetric Nash equilibrium in which voters do not use weakly dominated strategies), as Feddersen and Pesendorfer (1997), is not sufficient because of the value of coordination in our setting.

individual is drawn at random from each of the three types with probabilities f_P, f_Q and $f_{\widetilde{I}}$, respectively, where $f_P + f_Q + f_{\widetilde{I}} = 1$. We denote the vector of probabilities by F. Each individual observes his own type and votes for either p or q.

As a benchmark, we first review the set of equilibria that emerge if all voters have expected utility preferences.

Proposition 4. *An equilibrium always exists. Moreover, a set of strategies is an equilibrium if and only if*

1. *Type Ps vote for p*
2. *Type Qs vote for q*
3. *Any given i of Type $\widetilde{I}$ votes for p with probability $r_i \in [0, 1]$*

Observe that in this equilibrium people vote for the option they favor in individual choice, or arbitrarily randomize between outcomes they are indifferent between.

We now turn to voters with quasi-convex preferences. Types P and Q can now come in different sub-types. We call them $P1$, $P2$ (and $Q1$, $Q2$). Types $P1$ and $Q1$ have monotone preferences between q and p. For example, $P1$ (resp., $Q1$) strictly prefers $\lambda p + (1 - \lambda)q$ to $\delta p + (1 - \delta)q$ if and only if $\lambda > \delta$ (resp., $\lambda < \delta$).[20]

In contrast, $P2$'s preferences are non-monotonic between q and p. By quasi-convexity, $P2$'s preferences are single-valleyed between p and q and there exists a threshold λ^* such that $\lambda^* p + (1 - \lambda^*)q \sim q$. Thus, for all $\lambda < \lambda^*$ it is the case that $\lambda p + (1 - \lambda)q \prec q$, which means that even though $P2$'s prefer p to q, so long as p^* and q^* are both close enough to q the individual will prefer q^* to p^*. Similarly, for $Q2$, there exists λ^* such that $\lambda^* p + (1 - \lambda^*)q \sim p$, and $\lambda p + (1 - \lambda)q \prec p$ for all $\lambda > \lambda^*$. We will refer to types $P1$ and $Q1$ as monotone types and to the others as non-monotone types. Figure 1 illustrates the utility of each type over all convex combinations of q ($\lambda = 0$) and p ($\lambda = 1$).[21]

We will assume that individuals within each type have the same preferences, so that given a group problem, λ_i^* is the same for all i of type $P2$ (similarly for $Q2$ and $\widetilde{I}$) and hence β_i^* is the same as well.[22]

[20] Expected utility preferences must be monotonic between q and p.

[21] The normalization of the utility levels from p and from q in Figure 1 and having the same convex combination give the minimum utility for both $P2$ and $Q2$ are unnecessary.

[22] We focus on the situation where all individuals in each type have the same preferences for analytic convenience, although the results naturally extend to situations where they do not.

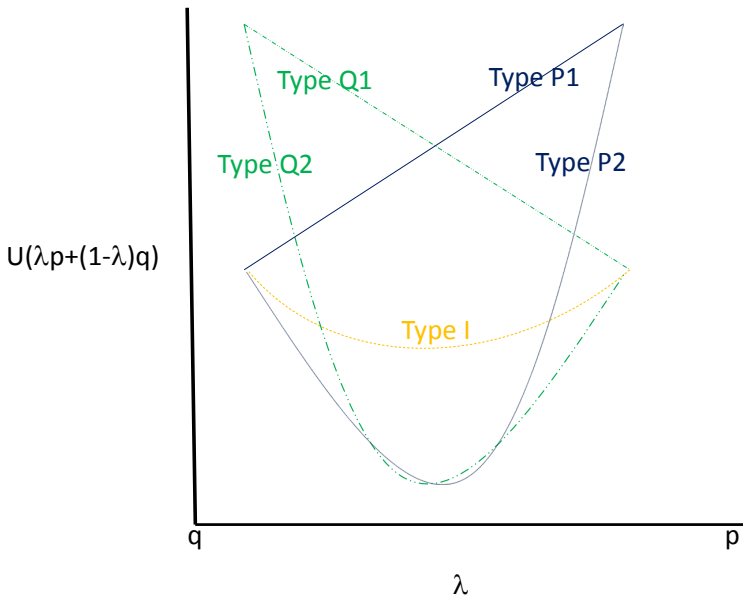

Figure 1: Five types of preferences

Before formally describing some of the properties of the equilibria, we informally discuss how the majority voting game changes with quasi-convex preferences. In particular, with quasi-convex preferences, the majority voting game takes on aspects of a coordination game — non-monotone types experience benefits from coordinating their votes with others because it reduces the amount of "randomness" in the election.[23]

We turn now to studying some of the properties of the Nash equilibria of the voting game. First, we demonstrate that an equilibrium always exists. In particular, we prove the existence of an "anonymous Nash equilibria," that is, a Nash equilibrium in which each individual's strategy depends only on his preferences (i.e. his type) and not on his identity. Although the exact set of equilibria will depend on the distribution F, we will highlight some of the salient features that differ from the expected utility case.

Proposition 5. *An anonymous Nash equilibrium always exists. Moreover, in any equilibrium (not necessarily anonymous)*

[23] A key technical aside; as Crawford (1990) points out, games in which individuals have quasi-convex preferences may oftentimes admit no Nash equilibrium. He suggest a new notion "equilibrium in beliefs" which coincides with standard Nash equilibrium under expected utility, but also exists when players have quasi-convex preferences. We simply focus on the Nash equilibrium, which, as we show in Proposition 5, always exist because of the benefits of coordination.

1. *Generically, all individuals strictly prefer to vote for one option or the other. Moreover, no individuals randomize*

2. *Type $P1s$ vote for p*

3. *Type $Q1s$ vote for q*

In contrast to Proposition 4, here no individual randomizes and, in fact, strictly dislikes randomizing. Thus, we will expect to observe choice shifts in the group — individuals who are indifferent between p and q in individual situations strictly prefer one or the other in a group setting.[24] Proposition 5, however, does not specify whether the shift would be towards q or towards p.

In order to provide intuition about the actual pattern of voting that can be observed in equilibrium, we will analyze the best response function of a voter. We index the number of possible voting combinations by m. Consider voting pattern $\mathbb{V}^m$ and suppose individual i is a member of type Γ. Given this, observe that F and $\mathbb{V}^m$ generate a probability $\alpha(\mathbb{V}^m, F)$ of an individual being pivotal, and so a threshold probability $\beta^*(\mathbb{V}^m, \Gamma, F)$. Denote the set of types that vote for p (resp., q) given $\mathbb{V}^m$ as $\mathbb{P}(\mathbb{V}^m)$ (resp., $\mathbb{Q}(\mathbb{V}^m)$).[25]

The probability that p is chosen when i is not pivotal is:

$$\beta_{i,\mathbb{V}^m,F} = \sum_{n=1}^{\infty} \frac{e^{-N}N^n}{n!} \left(\frac{\sum_{k=\lceil \frac{n}{2}+2 \rceil}^{n} \binom{n}{k} (\sum_{\tau \in \mathbb{P}(\mathbb{V}^m)} f_\tau)^k (\sum_{\tau \in \mathbb{Q}(\mathbb{V}^m)} f_\tau)^{n-k}}{1 - \sum_{k=\lceil \frac{n}{2}-1 \rceil}^{\lfloor \frac{n}{2}+1 \rfloor} \binom{n}{k} (\sum_{\tau \in \mathbb{P}(\mathbb{V}^m)} f_\tau)^k (\sum_{\tau \in \mathbb{Q}(\mathbb{V}^m)} f_\tau)^{n-k}} \right)$$

Individual i's best response is to choose p if $\beta_{i,\mathbb{V}^m,F} > \beta^*(\mathbb{V}^m, \Gamma, F)$ and q if the inequality is reversed. (And these two values are generically not equal.) Thus, a voting pattern is an equilibrium if it is the case that $\mathbb{V}^m$ generates $\beta_{i,\mathbb{V}^m,F}$ that are consistent with it.[26]

[24] However, is not necessarily the case that if we sum up the total number of choices for p less the total number of choices for q in the individual choice problem, and compare it to the vote totals in the majority voting game, that the latter is farther from 0.5 than the former. See Section 5 for a discussion of this.

[25] In defining $\beta_{i,\mathbb{V}^m,F}$, we assume that all individuals of the same type behave the same; a similar construction — albeit more complicated — can be performed without assuming anonymity.

[26] Quasi-convexity of preferences alone does not restrict the order of the thresholds $\beta^*(\mathbb{V}^m, \Gamma, F)$ across the different non-monotone types. However, additional restrictions, such as that all preferences are in the quadratic class, do ensure that the thresholds are ordered in the "intuitive" fashion.

The question of whether there is a unique equilibrium depends on the exact preferences and parameters of the problem. Because of the coordination nature of the majority voting game, there will often be multiplicity. However, the next proposition provides a sufficient condition for a unique pattern of voting. It states that whenever there are enough voters that strongly favor one of the options (i.e. in a monotone fashion), it is the case that all non-monotone types vote for that option as well.

Proposition 6. *There exists an N^* such that for all $N > N^*$ there is a f_{P1} (resp., f_{Q1}) sufficiently close to 1, such that the unique equilibrium is for all non-monotone types to choose p (resp., q).*

Proposition 6 thus predicts group polarization to such an extent that it actually causes *preference reversals* — individuals who in an individual problem would choose q over p will now actually choose p in the group problem (e.g., type $Q2$). The result generates an intuitive type of preference reversal — individuals coordinate on voting for an outcome strongly favored by many others.

However, individuals can also coordinate on equilibria that are not necessarily strongly favored, as shown by the next proposition. This proposition highlights how benefits from coordination generate multiple equilibria.

Proposition 7. *For any given N, if there exists a small enough proportion of non-monotone types, then generically there is a unique equilibrium.*[27] *In contrast, for large enough N, if the proportion of non-monotone types is sufficiently close to 1, then there are always at least two equilibria.*

That is, if non-monotone types form a large enough proportion of the population, they can all vote the same to ensure an outcome gets elected with very high probability. Voting against the group leads to additional uncertainty, which reduces ex-ante utility. In other words, when there is a sufficient number of any non-monotone type, the benefits of coordination become so large that multiple equilibria must exist. This can have counter-intuitive effects on voting outcomes. For example, imagine that all individuals are of type $P2$ and hence, when choosing individually, will choose p. However, when choosing as a group they could not only coordinate on an equilibrium where everyone votes for p but also on one where everyone votes for q. The latter is

[27] Moreover, if preferences are quadratic and if f_{P1} and f_{Q1} are sufficiently close to one another, then $P2$ (resp., $Q2$) types all vote for p (resp., q).

clearly Pareto sub-optimal, but exists because of the benefits of coordination. Thus, we can observe preference reversal not just because an individual knows many other voters have "extreme" preferences, but also because an individual knows that many other voters have preferences where they would like to coordinate.

Proposition 7 also demonstrates that when there are enough monotone types, uniqueness of an equilibrium is guaranteed. Importantly, the proposition does not state that in this equilibrium non-monotone types will coordinate on their actions, but only that it will be unique. The intuition is that with very few non-monotone types, although individuals may not know with certainty which option will be chosen, they know with near certainty, regardless of the voting behavior of non-monotone types, what the probability that p is chosen, that is, they know β with near certainty, which implies uniqueness.

Because voters care about what happens when they are not pivotal, and the uncertainty about what will happen hinges on the number of voters, the size of the group has important implications for behavior. If there are too few voters, then any given individual can have a large impact on the election. In the extreme case, when there are one or two voters, an individual is always pivotal. In contrast, as N grows large the probability of being pivotal goes to 0, but also the chosen outcome when a voter is not pivotal becomes known with (almost) certainty. Intuitively, as N grows large, both the proportions of each type of voters and (since voters generically do not randomize) the proportions of votes for each option are known with almost certainty. This implies that p^* and q^* are arbitrarily close to either p or q, and so all individuals will prefer to vote for either one or the other. The following proposition formalizes these assertions.

Proposition 8. *For sufficiently small N, types $P2$ and $Q2$ always vote for p and q respectively. For a sufficiently large N, generically in all equilibria, all non-monotone types take the same action.*

This proposition says that in large elections we should always expect to see preference reversals. Moreover, large elections will almost surely fail to aggregate preferences.

We can also consider what happens as the voting rules shift. Denote one option, without loss of generality p, as the status quo, and assume the threshold T needed to

replace p with q increases from fifty percent in favor of q.[28] Intuitively, as the threshold increases, the probability of q being chosen falls, and so non-monotone types become less likely to vote for it. Eventually, the unique equilibrium is for non-monotone types to votes for p. (Of course a similar result holds if q is made the default option.)

Proposition 9. *There exists an N^* such that for all $N > N^*$ there exists a T sufficiently close to 1, such that if f_{P1} is bounded away from 0, the unique equilibrium is for all non-monotone types to vote for p.*

5 Discussion

Our discussion of quasi-convex preferences has focused on preferences that are explicitly non-expected utility. However, as we mentioned in the introduction and was initially pointed out by Machina (1984), if an expected utility maximizer is allowed to take a payoff-relevant action before the lottery he faces is resolved, then his induced preferences over the ex-ante 'optimal' lotteries will be quasi-convex. To see this, suppose there are two individuals, facing two lotteries, p and q, between which they are both indifferent. There are three outcomes, and p is a binary lottery over the best and middle outcomes while q is a binary lottery over the best and worst outcomes. Both individuals are indifferent between p and q. The individuals vote as in our voting game. After voting, but before the chosen alternative is revealed, each individual can take one (and only one) of two 'insurance' action; a_1 or a_2. Action a_1 fully insures against the realization of the middle outcome, but not the low outcome, while a_2 insures against the realization of the low outcome, but not the middle outcome. Thus, even if the two individuals have expected utility preferences over lotteries, they have a strict incentive to coordinate their votes, because they would like to know which insurance action to take.

Because many applications focus on groups choosing between two options, we have also restricted our analysis to binary choices. Our results, however, are readily extended. For example, individuals will still exhibit a consensus effect. Imagine that the group must choose over Ω possible lotteries, denoted $p_1, ...p_\Omega$, and that an individual is indifferent between all of them. Then, so long as β_j is sufficiently large, the individual will vote for option j.

[28] ERR consider an extreme form of voting rule, where a unanimous agreement must be made to shift away from the status quo.

One way of interpreting our results is in line with notions of reference dependence. Kőszegi and Rabin (2007) discuss how an individual may prefer p to q if expecting p, and q to p if expecting q. In line with this, our non-monotone individuals choose p if they think it is sufficiently likely that they will receive p regardless of their choice, and similarly for q. An interpretation of this behavior is that individuals are disappointed when they receive outcomes in q while expecting better outcomes in p, and vice versa. This complements the intuition given in Section 2.

The link between quasi-convex preferences and the consensus effect has implications beyond standard voting situations. For example, suppose two candidates are running against one another. The election is just over and Candidate A was declared the winner by a small margin. However, there is the possibility of a recount. Our results imply that individuals with quasi-convex preferences who voted for Candidate B may actually prefer, after learning that it is likely A will be elected, to avoid a recount (which add additional randomness to the outcome).

Relation to Stylized Facts

The bandwagon effect, as described in the introduction, is discussed in Simon (1954), Fleitas (1971), Zech (1975), and Gartner (1976), among others. It captures the idea that if individuals believe others will vote for a certain option, they themselves are more likely to vote for that option as well. Thus, it reflects the change in the individual's behavior as a function of the group composition. Abusing notation slightly, we will denote the fraction of individuals voting for p given voting pattern V^m as $\mathbb{P}(\mathbb{V}^m)$ and for q as $\mathbb{Q}(\mathbb{V}^m)$. Let $Z = \mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)$, and observe that $Z \in (-1+f_{P1}, 1-f_{Q1})$. The bandwagon effect describes the fact that if Z is close enough to 1 (resp.,-1), then any non-monotone type individual will strictly prefer to vote for p (resp., q). For example, the results of Proposition 6 demonstrate the bandwagon effect: Z approaches 1 (resp., -1) as f_{P1} (resp., f_{Q1}) approaches 1. In Proposition 7, as $f_{P1} + f_{P2}$ approaches 0, Z can take on any number in $(-1, 1)$, thus the bandwagon effect guarantees two equilibria. The second part of Proposition 9 can also be interpreted in a similar manner — for a large enough N, Z is always sufficiently close to -1 or 1.

Much of the discussion regarding group shifts focuses on group polarization, where the group ends up having a more extreme decision than the aggregate of individuals' decisions in isolation. This has been documented in a variety of settings — for ex-

ample, Isenberg, (1986), Myers and Lamm (1976), McGarty et al. (1992), Van Swol (2009), and Moscovici and Zavalloni (1969) — and has been of particular interest to researchers examining the effects of decisions by juries (such as Main and Walker, 1973, Bray and Noble, 1978, and Sunstein, 2002). In our simple stylized setting, we can analyze when, and why, group decisions may be more polarized than individual decisions. For simplicity, suppose there are no type $\widetilde{I}$ individuals. We can then measure the degree of polarization by the difference between the proportion of people who choose p relative to those who choose q. Thus, in individual choice settings $|(f_{P1} + f_{P2}) - (f_{Q1} + f_{Q2})|$ is the relative strength of the support for p over q, while the corresponding measure of polarization in the group setting is $|\mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)|$. Group polarization occurs if $|f_{P1} + f_{P2} - (f_{Q1} + f_{Q2})| < |\mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)|$. Although it seems intuitive that the consensus effect generates group polarization, this is not necessarily true. While Proposition 6 guarantees that with large enough N the equilibrium exhibits group polarization, Propositions 7-9 do not necessarily guarantee this phenomenon, as whether or not polarization occurs depends on the shape of the preferences of the non-monotone types. In particular, fixing the distribution of types we can always find preferences for $P2$ and $Q2$ types so that we get group *de-polarization*, that is, $|f_{P1} + f_{P2} - (f_{Q1} + f_{Q2})| > |\mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)|$.

One explanation for group shifts is an explicit benefit of conformity or for being on the winning side (for example, Callander, 2007 , Hung and Plott, 2011 and Goeree and Yariv, 2007). Our model generates an endogenous benefit of conformity; individuals are willing to vote against what they would choose in isolation in order to reduce the uncertainty of the outcome, or in other words to conform to what they expect to already happen. Their interest in doing so is not explicit, but rather depends on the distribution of types and expected number of voters.

One way of understanding the distinct predictions of our model relative to both the standard model and to models that incorporate exogenous benefits of conformity is to look at the appropriate utility differences between options.

To build intuition, consider the case of an expected utility maximizer, with utility function V_{EU}, who is not concerned about conformity. For this individual, $V_{EU}(p^*) = (1-\alpha)(1-\beta)V_{EU}(q)+[\alpha+(1-\alpha)\beta]V_{EU}(p)$ and $V_{EU}(q^*) = [\alpha + (1-\alpha)(1-\beta)]\, V_{EU}(q)+ (1-\alpha)\beta V_{EU}(p)$. Thus, under expected utility, $V_{EU}(p^*) - V_{EU}(q^*) = \alpha(V_{EU}(p) - V_{EU}(q))$, which implies both (i) $V_{EU}(p^*)-V_{EU}(q^*) > 0$ if and only if $V_{EU}(p)-V_{EU}(q) > 0$; and (ii) $V_{EU}(p^*)-V_{EU}(q^*)$ is falling in the individual's probability of being pivotal.

The simplest model of *exogenous* conformity benefits maintains the assumption of expected utility, but allows for an individual to directly care about the choices of others. We can capture exogenous benefit of conformity in two steps. First, we suppose that the utility gap between choosing p and q in isolation is, as before, simply $V(p) - V(q)$. However, when choosing as part of a group, an individual's payoff for voting for an option is directly effected by the probability that option is chosen when the individual is not pivotal (i.e. β or $1 - \beta$), as well as by his probability of being pivotal.

We can represent this distortion using a function $\varsigma_i(\beta, \alpha)$ for option $i \in \{p^*, q^*\}$ that is *added* to $V_{EU}(i^*)$, that is, $\widetilde{V}(i^*) = V_{EU}(i^*) + \varsigma_i(\beta, \alpha)$. We suppose $\varsigma_{p^*}(\beta, \alpha)$ (the distortion applied to p^*) is increasing in β and $\varsigma_{q^*}(1 - \beta, \alpha)$ (the distortion applied to q^*) is increasing in $1 - \beta$. Moreover, we suppose that $\varsigma_{p^*}(\beta, \alpha) - \varsigma_{q^*}(1 - \beta, \alpha)$ is decreasing in α, and $\varsigma_{p^*}(\beta, 1) - \varsigma_{q^*}(1 - \beta, 1) = 0$. This captures the intuition that as a voter becomes more likely to be pivotal (and in the limit becomes the only voter) then others' behavior do not influence his relative ranking of options. Given this distortion, the utility difference between casting a vote for either option now becomes $\widetilde{V}(p^*) - \widetilde{V}(q^*) = \alpha(V_{EU}(p) - V_{EU}(q)) + (\varsigma_{p^*}(\beta, \alpha) - \varsigma_{q^*}(1 - \beta, \alpha))$. Since the utility gap now depends also on the size of the conformity distortion, we can capture the size and direction of conformity benefits by the term $\widetilde{V}(p^*) - \widetilde{V}(q^*) - \alpha(\widetilde{V}(p) - \widetilde{V}(q))$.[29] Unlike under expected utility, this gap need not be zero (and will be either positive or negative depending on β). Moreover, our assumptions imply that this difference is increasing in β and falling in α.

We now turn to calculating the utility gap in our model of quasi-convex preferences, which captures a type of *endogenous* conformity benefits, and is given by $V(p^*) - V(q^*) - \alpha(V(p) - V(q))$. Similar to the exogenous case above, the gap need not be zero and will be rising in β. However, and unlike the exogenous model of conformity, $V(p^*) - V(q^*) - \alpha(V(p) - V(q))$ may be non-monotone (and so non-decreasing) in the pivot probability, α. It should be clear from our results that we may have $V(p) - V(q) > 0$ while $V(p^*) - V(q^*) < 0$. In this case, $V(p^*) - V(q^*)$ will indeed be non-monotone in α; the reason is that as the probability of being pivotal goes to zero (as N goes to infinity) it must be the case that $V(p^*) - V(q^*)$ converges to zero.[30] It can also be the case that although $N > 1$, $V(p^*) - V(q^*) \geq V(p) - V(q)$; al-

[29] Note that $\widetilde{V}(p) - \widetilde{V}(q) = V_{EU}(p) - V_{EU}(q)$.

[30] The non-monotonicity pattern does not rely on observing a preference reversal, that is, on

though as N goes to infinity $V(p^*) - V(q^*)$ converges to zero, it may actually (locally) increase in N in a certain region.[31]

More generally, the motivation for conformity in our model, i.e., reducing randomness, is distinct from potential other motivations, which often rely on a desire to feel socially integrated and so may depend on factors such as the observability of one's vote, or the extent to which the choice is being made by other voters (versus an objective randomization device).

Related Literature

Political scientists and economists have long recognized that in election between two options, assuming either voluntary-costless or compulsory voting implies that an equilibrium exists where all individuals vote for one of the options. These equilibria, which involve coordination with expected utility preferences, are knife-edge cases, in the sense that some individuals are exactly indifferent between voting for either of the two options. Thus, the equilibrium is not robust to small costs or to uncertainty about the number of voters (as in the Poisson model). In contrast, voters in our model may strictly prefer to exhibit preference reversals in group situations, and so our results are robust to small perturbations, in line with the fact that we obtain them while explicitly incorporating uncertainty about the number of voters. And as our results show, we do observe such coordination equilibrium in the game with quasi-convex preferences.

Our results are related to the large literature on understanding voting and the aggregation of preferences or information in elections.[32] The literature has made two assumptions regarding how individuals value outcomes.

The first, as we made in this paper, is the assumption of private values. In this case, with expected utility preferences and either compulsory or costless voting, all

$V(p) - V(q)$ and $V(p^*) - V(q^*)$ having opposite signs.

[31] A similar construction can be shown to be true for the willingness to pay to vote: even though N has increased, an individual's willingness to pay to actually vote may increase.

[32] An important distinction is that while we assume that alternatives in the voting game are lotteries, most papers suppose they are final outcomes. Of course, this complicates thinking about our results in relation to the pre-existing literature; for example, in a common-values setting, private signals would then need to be about a particular outcome in the support of p or q. We nevertheless believe our assumption is natural in many instances; for example, if voters value candidates by what policies they will implement and there is a degree of uncertainty about what campaign promises candidates will actually follow through with.

voters vote sincerely (i.e., individuals vote as part of the group in the same way they would choose in isolation) and all individuals vote. Thus preferences are aggregated, in the sense that not only is the correct outcome chosen, but the true proportion of supporters of each side is also revealed (although not the strength of preference), modulo indifference. These results stand in contrast to what we obtain, where we find equilibria (sometimes unique) in which individuals do not vote sincerely and, moreover, we cannot rule out Pareto-dominated equilibria where preferences are not aggregated properly. Thus, with violations of expected utility, even in situations most amenable to sincere voting and preference-aggregation, these two properties may fail.

If voting is costly (but individuals still have private values), then as Ledyard (1981, 1984), and Palfrey and Rosenthal (1983, 1985) point out, individuals need to trade off the cost versus the benefit of voting, namely the chance of being pivotal. This implies that the proportion of votes cast for each side will now depend not just on the fraction of supporters for each option, but also on the cost and benefit distributions of both types of supporters (Taylor and Yildrim, 2010). However, as in models with compulsory voting, conditional on voting, voters will still truthfully reveal their preferences over the options, in contrast to the preference reversals we predict.

The other assumption in the literature is that outcomes have a common-value component, and voters receive signals about it.[33] With compulsory voting, as Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) first noted, sincere voting is in fact not an equilibrium. Surprisingly, despite this, information is still aggregated in large elections, other than in knife-edge situations. The prediction is slightly different with voluntary, instead of compulsory, participation. Krishna and Morgan (2012) demonstrate that if participation is voluntary (either free or costly) then although some individuals may not vote, individuals who do vote will do so sincerely, and information is aggregated in large elections.

A key difference between the predictions of our model and the common-value literature is that in our model individuals vote insincerely to avoid randomness, whereas with a common-value component, individuals vote insincerely to help ensure the selected option is optimal given the (unknown) state. These two motivations can imply

[33] Although we consider the two assumptions about values separately, Ghosal and Lockwood (2009), Feddersen and Pesendorfer (1999), Feddersen and Pesendorfer (1997) and Krishna and Morgan (2011) consider elections in the presence of both common-value and private-value components.

very different behaviors in some circumstances. For example, adding partisan individuals who will always vote for p will push uninformed quasi-convex voters who want to match the state towards choosing p. However, as the example below illustrates, in a common-value setting individuals will instead want to more often vote against p; as Feddersen and Pesendorfer (1996) note "[uninformed independent agents] vote to compensate for the partisans". More generally, in a model with both features (quasi-convexity preferences and a common-value component) individuals may vote insincerely not only for strategic reasons but also for reasons related to their desire to reduce the randomness of the election.[34] These mixed motivations for insincere voting will impede information aggregation.

A simple example can highlight these issues. Suppose there are five voters in a majority rule election. Voters 1 and 2 are partisans and will always vote for option p (as specified below). Voters 3, 4, and 5 care about both what state will be realized and what alternative was chosen; in particular, they want to match the chosen alternative to the state. There are two equally likely states, s_p and s_q. Suppose there are three final outcomes $\bar{x} > x > \underline{x}$. The alternatives are two lotteries p and q. p (resp., q) gives x with probability ρ regardless of the state, $\bar{x}$ with probability $1-\rho$ if the state is s_p (resp., s_q), and $\underline{x}$ with probability $1-\rho$ if the state is s_q (resp., s_p). Finally, voters 3 and 4 receive perfectly revealing private signals about the state prior to voting, while voter 5 receives no signal at all.

If all voters have expected utility preferences, then consider a situation where voters 3 and 4 always vote in accordance with their perfectly revealing signals. Voter 5 now wants to condition his vote on being pivotal. Voter 5 knows that the only time he is pivotal is when the state is s_q (otherwise all other four voters are voting for p). Thus, he should always cast her vote for q. It is easy to show that such behavior on the parts of voters 3, 4, and 5 constitute an equilibrium which aggregates information.

Now, to make the minimal deviation from the standard model, suppose only voter 5 has quasi-convex preferences (everyone else still has expected utility preferences), that are non-monotone between p and q. Since states are equally likely, $p^* = p$ and $q^* = \frac{1}{2}p + \frac{1}{2}q$.[35] One can easily construct preferences such that p^* is preferred to q^*. In this case there will be no equilibrium that aggregates information.

[34] Thus, as mentioned previously, our results would apply even if, whenever an individual was not pivotal, the choice of p or q was generated randomly by a computer and so would convey no information, shutting down the common values channel.

[35] Note that in this example $\alpha = \frac{1}{2}$ and $\beta = 1$.

This intuition readily extends even when the number of voters becomes large, as in the results in the literature on information aggregation, if any given voter has an equal chance of having preferences equivalent to that of voter 1, 2, 3, 4 and 5 in our simple example. In this case, information aggregation will still fail.

Ellis (2016) also relaxes the assumption of expected utility in a voting setting. He considers a common-value voting game with subjective uncertainty, where voters have max-min utility as in Gilboa and Schmeidler (1989). Because in a subjective environment max-min utility implies a preference for hedging, he shows that voters have a desire to randomize; i.e. they exhibit an anti-consensus effect.

6 Appendix: Proofs

Before we discuss the proofs, whenever we consider two arbitrary options p and q, we adopt the following normalization: Recall that for all values of $\alpha, \beta \in [0,1]$, q^* and p^* are on the line segment connecting q and p in some multidimensional simplex. In order to simplify notation, we will rotate the probability simplex so that for any given p and q under consideration, this line segment runs from the origin through $e_1 = (1,0,0,0...)$ and associate q with the origin. Moreover, we can now focus on the 1 dimensional case, and think of the line segment connecting 0 and 1 where we associate q with 0 and p with 1. We will thus associate a lottery $zp + (1-z)q$ for $z \in [0,1]$ with the point z. Note that since $p^* - q^* = \alpha(p-q) = \alpha$, we have that $p^* \geq q^*$ given our normalization.

Moreover, we fix representation of the preference relation $\succcurlyeq$ for each given type V_Γ, which can depend on the type Γ (we will frequently omit the dependence on Γ to simplify notation). For $z', z'' \in [0,1]$, let $\gamma(z', z'') = V(z') - V(z'')$ measure the utility gap between z' and z''. Observe that γ depends on the exact representation V. However, we will be concerned with ordinal rather than cardinal properties of γ and V.

Lemma 1 $\succcurlyeq$ satisfies strict quasi-convexity if and only if for all p and q such that $p \sim q$ there exists a $z^* \in (0,1)$ such that V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$.

Proof of Lemma 1: First we show the *if* part. Observe that the assumption implies that $V(z) < V(p) = V(q)$ for all $z \in (0,1)$. This implies quasi-convexity since it holds for arbitrary p and q such that $p \sim q$.

We now show the *only if* part. Suppose not. Then for some pair p and q such that $p \sim q$ there is no z^* with the properties as in the premise. This implies that there exists at least one interior local *maximum*, denoted $Z \in (0,1)$. Then, by continuity, there exists a neighborhood $[\underline{z}, \bar{z}] \ni Z$ such that $V(\underline{z}) = V(\bar{z}) \leq V(Z)$, violating strict quasi-convexity. $\square$

Lemma 2 For all p and q such that $p \sim q$ there exists a $z^* \in (0,1)$ such that V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$, if and only if for all p and q such that $p \sim q$ and $\alpha \in (0,1)$, there exists a pair $z', z'' \in [0,1]$ with the following three properties:

1. $z' - z'' = \alpha$ and $\gamma(z', z'') = 0$.

2. For all $\widetilde{z}' > z'$, $\widetilde{z}'' > z''$, and $\widetilde{z}' > \widetilde{z}''$, $\gamma(\widetilde{z}', \widetilde{z}'') > 0$.

3. For all $\widetilde{z}' < z'$, $\widetilde{z}'' < z''$, and $\widetilde{z}'' < \widetilde{z}'$, $\gamma(\widetilde{z}', \widetilde{z}'') < 0$.

Proof of Lemma 2: We prove the only if part first. To see that 1 is implied, first consider all pairs z', z'' such that $z' - z'' = \alpha$. Observe that both $\gamma(1, 1-\alpha) > 0$ and $\gamma(\alpha, 0) < 0$ hold by definition. By continuity there must be a point $z \in [\alpha, 1]$ such that $\gamma(z, z-\alpha) = 0$.

To see that 2 is implied, observe that since $\gamma(z', z'') = 0$, $z^* \in [z', z'']$ (if not, then the line $[0, 1]$ would have at least two local minima, a contradiction). There are two cases. If $\widetilde{z}'' > z^*$, then by Lemma 1 we have $\gamma(\widetilde{z}', \widetilde{z}'') > 0$. In contrast, if $\widetilde{z}'' < z^*$ then $V(\widetilde{z}'') < V(z'')$, and since $V(\widetilde{z}') > V(z')$, we have $V(\widetilde{z}') > V(\widetilde{z}'')$, or $\gamma(\widetilde{z}', \widetilde{z}'') > 0$. The proof that 3 is implied is exactly analogous.

For the if part, suppose it is not the case so that there is an interior local maxima in the interval, denoted $Z \in (0, 1)$. Then, by continuity, there exists a neighborhood $[\underline{z}, \bar{z}] \ni Z$ such that $V(\underline{z}) = V(\bar{z})$. Thus there exists an α' such that $\underline{z} - \bar{z} = \alpha'$. Observe that the pair $\underline{z}, \bar{z}$ satisfies condition 1, but not conditions 2 or 3. $\square$

Proof of Proposition 1: By construction $p^* - q^* = \alpha$. Given that, Condition 1 implies that at β^* we have $p^* = z'$ and $q^* = z''$. By Conditions 2 and 3 of Lemma 2, $\beta > \beta^*$ (resp., $\beta < \beta^*$) implies that $\gamma(p^*, q^*) > 0$ (resp.,< 0). Conversely, the pair p^*, q^* at β^* satisfies the properties of $z', z'' \in [0, 1]$ in Lemma 2. $\square$

Proof of Corollary 1: Wakker (1994) shows that convexity of g is equivalent to quasi-convexity of preferences. The result follows from Proposition 1. $\square$

Proof of Proposition 2: Recall that quadratic preferences imply mixture symmetry (Chew, Epstein, and Segal, 1991). The preference relation $\succcurlyeq$ satisfies mixture symmetry if for all $p, q \in \Delta$ and $\lambda \in [0, 1]$,

$$p \sim q \Rightarrow \lambda p + (1-\lambda)\, q \sim \lambda q + (1-\lambda)\, p$$

Suppose $q \sim p$. By mixture symmetry, we have

$$q^* = [a + (1-a)(1-b)]\, q + (1-a)bp \sim (1-a)bq + [a + (1-a)(1-b)]\, p \equiv \widehat{q}$$

If $\beta < 0.5$, $k = \frac{(1-a)(1-2b)}{a+(1-a)(1-2b)} \in (0, 1)$ and we have $p^* = kq^* + (1-k)\widehat{q}$. By strict quasi-convexity $q^* \succ p^*$.

Moreover, by mixture symmetry we have

$$p^* = (1-a)(1-b)q + [a + (1-a)b]p \sim [a + (1-a)b]q + (1-a)(1-b)p \equiv \widehat{p}$$

If $\beta > 0.5$, $l = \frac{(1-a)(2b-1)}{a+(1-a)(2b-1)} \in (0,1)$ and we have $q^* = lp^* + (1-l)\widehat{p}$. By strict quasi-convexity $p^* \succ q^*$.

And if $\beta = 0.5$ and $q \sim p$ then, by mixture symmetry,

$$q^* \sim \widehat{q} = (1-a)bq + [a + (1-a)(1-b)]\, p = (1-a)(1-b)q + [a + (1-a)b]\, p = p^*$$

and hence $q \sim p \Rightarrow q^* \sim p^*$

To show the other direction, suppose preferences do not satisfy strict quasi-convexity everywhere. If preferences satisfy betweenness someplace, then in that region the decision-maker is indifferent to convexification. If preferences satisfy strict quasi-concavity somewhere, then we observe an anti-consensus effect in that region. □

Proof of Corollary 2: Masatlioglu and Raymond (2016) show that under $\mathbb{CPE}_M$, individuals are loss averse if and only preferences are strictly quasi-convex. Moreover, they show that if preferences can be represented with $V_{\mathbb{CPE}_M}$ then they also have a quadratic representation. The result follows. □

Proof of Corollary 3: The equivalence of 1, 2, and 3 is shown by ERR. The equivalence of 3 and 4 is Proposition 1. □

Proof of Example 1: This utility functional does not exhibit Allais-type behavior. To see this, denote the probability of x_h by p_h and the probability of x_l by p_l. The utility of a lottery $(x_h, p_h; x_m, 1 - p_l - p_h, x_l, p_l)$ is then

$$\begin{aligned}
&p_l^2[\phi(x_m, x_m) - 2\phi(x_m, x_l) + \phi(x_l, x_l)] \\
&+ p_l p_h[-2\phi(x_h, x_m) + 2\phi(x_h, x_l) + 2\phi(x_m, x_m) - 2\phi(x_m, x_l)] \\
&+ p_h^2[\phi(x_h, x_h) - 2\phi(x_h, x_m) + \phi(x_m, x_m)] \\
&+ p_l[-2\phi(x_m, x_m) + 2\phi(x_m, x_l)] \\
&+ p_h[2\phi(x_h, x_m) - 2\phi(x_m, x_m)] \\
&+ \phi(x_m, x_m)
\end{aligned}$$

First, we will normalize the utility values. Chew, Epstein, and Segal (1991) show

that ϕ is unique up to affine transformation. So we will set $\phi(x_m, x_m) = 0$ and $\phi(x_m, x_l) = \phi(x_l, x_m) = -1$ (recall that $\phi(x_m, x_m) \geq \phi(l, m)$ by monotonicity). The other relevant values of ϕ will be stated below.

Second, recall that Allais-type behavior is equivalent to indifferent curves fanning out in the probability simplex, where the value of p_l is on the horizontal axis and that of q_h on the vertical axis. Fanning out is equivalent to the slopes of the indifference curves becoming less steep moving horizontally in the simplex. The slope of the indifference curves, denoted by $\mu(p_l, p_h)$ below, is equal to

$$-\frac{2p_l[2+\phi(x_l,x_l)] + p_h[-2\phi(x_h,x_m) + 2\phi(x_h,x_l) + 2] - 2}{p_l[-2\phi(x_h,x_m) + 2\phi(x_h,x_l) + 0 + 2] + 2p_h[\phi(x_h,x_h) - 2\phi(x_m,x_m)] + [2\phi(x_h,x_m)]}$$

Taking the derivative $\frac{\partial\mu(p_l,p_h)}{\partial p_l}$ and observing that its denominator is always positive, we know that to determine its sign (which tells us whether we get fanning out or fanning in) we only need to consider its numerator.

First, we focus on fanning out along the p_l-axis, and so will set $p_h = 0$ after calculating $\frac{\partial\mu(p_l,p_h)}{\partial p_l}$. Note that the derivative of the numerator of $\mu(p_l, p_h)$ with respect to p_l is $2[2 + \phi(x_l, x_l)]$, while the derivative of the denominator of $\mu(p_l, p_h)$ with respect to p_l is $[-2\phi(x_h, x_m) + 2\phi(x_h, x_l) + 2]$. We also have that at $p_h = 0$, the numerator of $\mu(p_l, p_h)$ equals $2p_l[2 + \phi(x_l, x_l)] - 2$ and the denominator of $\mu(p_l, p_h)$ equals $p_l[-2\phi(x_h, x_m) + 2\phi(x_h, x_l) + 2] + [2\phi(x_h, x_m)]$. Therefore, the numerator of $\frac{\partial\mu(p_l,p_h)}{\partial p_l}$ equals $-4\phi(x_h, x_m) - 4\phi(x_l, x_l)\phi(x_h, x_m) - 4\phi(x_h, x_l) - 4$, meaning that we get fanning out horizontally along $q = 0$ if and only if

$$-\phi(x_h, x_m) - \phi(x_h, x_l) - 1 - \phi(x_l, x_l)\phi(x_h, x_m) < 0$$

Given our specified v and w functions, we can represent ϕ using a matrix

$$\begin{pmatrix} \phi(x_l,x_l) & \phi(x_l,x_m) & \phi(x_l,x_h) \\ \phi(x_l,x_m) & \phi(x_m,x_m) & \phi(x_m,x_h) \\ \phi(x_l,x_h) & \phi(x_m,x_h) & \phi(x_h,x_h) \end{pmatrix}$$

Substituting in our actual values (only for the lower triangle, because of the sym-

metry of ϕ) gives

$$\begin{pmatrix} 2 & \phi(x_l, x_m) & \phi(x_l, x_h) \\ 3.5 & 6 & \phi(x_m, x_h) \\ 6 & 10 & 16 \end{pmatrix}$$

To normalize $\phi(x_m, x_m) = 0$ and $\phi(x_m, x_l) = -1$, we subtract 6 from all payoffs and then divide by 2.5. This yields the ϕ matrix

$$\begin{pmatrix} -8/5 & \phi(x_l, x_m) & \phi(x_l, x_h) \\ -1 & 0 & \phi(x_m, x_h) \\ 0 & 8/5 & 4 \end{pmatrix}$$

We then have $-\phi(x_h, x_m) - \phi(x_h, x_l) - 1 - \phi(x_l, x_l)\phi(x_h, x_m) = -1/25 < 0$, so indifference curves are fanning out. This proves fanning out along the line $p_h = 0$.

In order to extend fanning out throughout the unit simplex, we use the notion of expansion paths, defined by Chew, Epstein, and Segal (1991). We will use their definition, tailored to our example, which is as follows.

Given three outcomes $x_l < x_m < x_h$, consider the probability simplex (i.e. triangle) over those three outcomes, as described in the text (where p_h denotes the probability of h and p_l the probability of l). Suppose that indifference curves in this space are always differentiable inside the simplex, where, as above, $\mu(p_l, p_h)$ denotes the slope of the indifference curve passing through any given point (p_l, p_h). An expansion path collects the set of all points, the indifference curve through which have the same slope (that is, (p_l, p_h) and (p'_l, p'_h) are on the same expansion path if $\mu(p_l, p_h) = \mu(p'_l, p'_h)$.)

Chew, Epstein, and Segal (1991) show that for quadratic preferences which are not expected utility, expansion paths are linear (in the case of expected utility all points in the simplex are in the same expansion path). Moreover, they show that either,[36]

- no two expansion paths intersect (in other words expansion paths are parallel); or,
- all expansion paths intersect at a single point (i.e., if two expansion paths intersect at (p'_l, p'_h) then all expansion paths must intersect there), which may

[36] See Lemmas A2.2-5 in their paper.

or may not be inside the unit simplex (i.e., it is possible that the point where they intersect has p_l and p_h values greater than 1 or less than 0)

We now turn to applying expansion paths to our example. In Example 1, the "reduced form" utility function over lotteries defined over the three outcomes (taking into account our normalized values) is:

$$V(p_l, p_h) = -2p_l + \frac{2p_l^2}{5} + \frac{16p_h}{5} - \frac{6p_l p_h}{5} + \frac{4p_h^2}{5}$$

Observe that $\left(\frac{-6}{5}\right)^2 - 4 \times \frac{2}{5} \times \frac{4}{5} = \frac{36}{25} - \frac{32}{25} = \frac{4}{5} > 0$, and so we know the indifference curves take the shape of *hyperbolas*, and thus all expansion paths intersect at a single point.[37] To find this point of intersection, we simply need to find the critical point of the utility function.[38] The first order conditions demonstrate that this is at $p_l = 4, p_h = 1$. Thus, all expansion paths must intersect there, which in turns implies that, within the unit simplex, all expansion paths are positively sloped (and do not intersect within the simplex).

Consider moving from some point (p_l, p_h) to (p_l', p_h) in the probability simplex, with $p_l < p_l'$. Denote the expansion path (p_l, p_h) is on as E_1 and the expansion path (p_l', p_h) is on as E_2. Then we can find points $(\hat{p}_l, 0)$ and $(\hat{p}_l', 0)$ such that the former is on expansion path E_1 and the latter is on expansion path E_2. Since the expansion paths cannot cross anywhere other than $(4, 1)$, $\hat{p}_l < \hat{p}_l'$. But we know from our previous reasoning that, regardless of the initial value of p_l, when increasing p_l and moving along the line $p_h = 0$, the slopes of the indifference curves decrease. So the slope of the indifference curve is lower at $(\hat{p}_l', 0)$ than $(\hat{p}_l, 0)$, meaning that the slope of the indifference curve must be lower at (p_l', p_h) than (p_l, p_h). Therefore, we get fanning out as p_l increases, regardless of p_h, inside the probability simplex. □

Proof of Example 2: We consider the functional over (p_l, p_h) given by

$$V = -6p_l + p_l^2 + 7.82p_h - 3.2p_l p_h + 2.56p_h^2$$

Since $3.2^2 - 4 \times 2.56 = 10.24 - 10.24 = 0$, the indifference curves of V take the shape of *parabolas*, which have the same axis of symmetry. Thus all indifference

[37] For details, see Chew, Epstein, and Segal (1991). Intuitively, the expansion paths all must intersect at center of the hyperbolas, or, in other words, at the point of intersection of the asymptotes.

[38] This follows from the fact that the asymptotes of the hyperbola must be on the same level set.

curves either have lower contour sets that are (strictly) convex or upper contour sets that are (strictly) convex. In our case, because the axis of symmetry has a positive slope and lies below the unit simplex, preferences have convex lower contour sets and hence satisfy quasi-convexity.

Moreover, $\frac{\partial V}{\partial p_l} = -6 + 2p_l - 3.2p_h$ and $\frac{\partial V}{\partial p_h} = 7.82 - 3.2p_l + 5.12p_h$. Thus, the slope of the indifference curves is $\mu(p_l, p_h) = -\frac{-6+2p_l-3.2p_h}{7.82-3.2p_l+5.12p_h}$.

Along the set of lotteries where $p_h = 0$, $\mu(p_l, p_h)$ reduces to $-\frac{-6.+2p_l}{7.82-3.2p_l}$. Taking the derivative of this with respect to p_l gives $\frac{0.347656}{(2.44375-p_l)^2} > 0$, so indifference curves are fanning in. This proves fanning in along the line $p_h = 0$.

In order to extend fanning in throughout the probability simplex, we use expansion paths in a similar way to Example 1. Since the indifference curves are parabolas, it is the case that the expansion paths are parallel.[39] Moreover, because the axis of symmetry of the indifference curves is an expansion path, the expansion paths have positive slopes.

Consider moving from some point (p_l, p_h) to (p_l', p_h), where $p_l < p_l'$. Denote the expansion path (p_l, p_h) is on as E_1 and the expansion path (p_l', p_h) is on as E_2. Then we can find points $(\hat{p}_l, 0)$ and $(\hat{p}_l', 0)$ such that the former is on expansion path E_1 and the latter is on expansion path E_2. Since the expansion paths cannot cross $\hat{p}_l < \hat{p}_l'$. But we know from our previous reasoning that, regardless of the starting value of p_l, when increasing p_l and moving along the line $p_h = 0$ the slope of the indifference curves increase. So the slope of the indifference curves is higher at $(\hat{p}_l', 0)$ than at $(\hat{p}_l, 0)$, which, in turns, implies that the slope of the indifference curves must be higher at (p_l', p_h) than (p_l, p_h). So we get fanning in as p_l increases, regardless of p_h. $\square$

Example 3: quasi-convex is not implied by only risky and cautious shifts Observe that while risky or cautious shifts imply that along each line connecting a degenerate lottery to any other lottery (in the multi-dimensional probability simplex) quasi-convexity must hold, it need not hold along lines which do not pass though a degenerate lottery. In particular, consider the two-dimensional probability simplex, so that r has support with no more than three outcomes, one of which is s. Now consider two lotteries p and q which have support over the same three outcomes as r and the one associated with s. Moreover, suppose that $p \sim q \nsim r$. Thus any convex combination of p and q must also have support over the same three outcomes.

[39] Again, see Chew, Epstein, and Segal (1991).

Similarly, any two lotteries p' and q', convex combinations of which can generate both p and q, must also have support over the same three outcomes. Thus, there is no safe outcome which is an affine combination of p and q. It may be that the indifference curve passing through p and q is not quasi-convex (e.g. linear), yet all indifference curves that lie along convex combinations of all s and r are quasi-convex. Figure 2 provides a graphical example of such a situation in the Marschak-Machina triangle. Consider the region which contains all lines connecting the middle outcome to any point on indifference curve I (the set of points indifferent to the middle outcome). Within this region, indifference curves are strictly quasi-convex. Thus, we obtain ERR's risky and cautious shifts. However, we do not have universal quasi-convexity (notably, in the region nearest the best outcome).

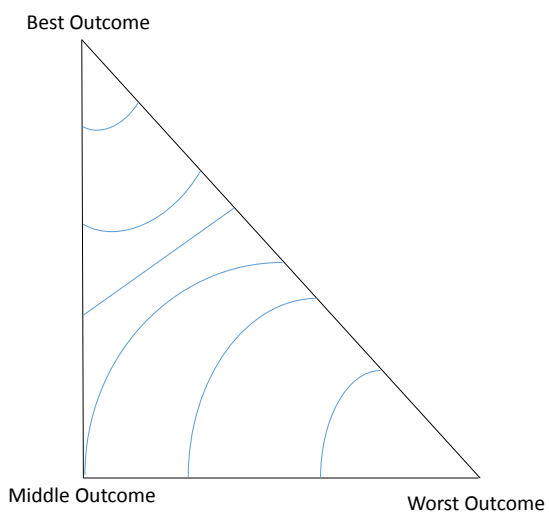

Figure 2: A Demonstration of Example 3

□

Proof of Proposition 3: First, we will let $s = p$ and $r = q$ and use our normalization described at the beginning of the Appendix. We will show that Axioms 1 and 2 hold if and only if for all s and r such that $s \sim r$, there exists a $z^* \in (0, 1)$ such that V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$.

Suppose that for all s and r such that $s \sim r$ there exists a $z^* \in (0, 1)$ such that V is strictly decreasing on $[0, z^*]$, strictly increasing on $[z^*, 1]$, and $\lambda s + (1 - \lambda)r \sim$

$\delta s + (1-\delta)r$, for $\lambda \neq \delta$ and $\lambda, \delta \in [0,1]$. If $\lambda s + (1-\lambda)r \sim \gamma s + (1-\gamma)r$ for $\gamma \in [0,1]$, then $V(\lambda) = V(\gamma) = V(\delta)$, which can only be true if γ equals either λ or δ. This proves Axiom 1.

Second, observe that since $V(0) = V(1)$ and V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$ then $V(z) < V(0)$ for all $z \in (0,1)$, which implies Axiom 2.

To show the other direction, observe that if the implication is false then there exists a local maximum $Z \in (0,1)$, that is, there exists $[\underline{z}, \bar{z}] \ni Z$ such that $z \in [\underline{z}, \bar{z}]$ implies $V(Z) \geq V(z)$. If $V(Z) > V(0)$ then Axiom 2 is violated; and if $V(Z) \leq V(0)$ then there are at least four points in (0,1) which have the same V value, contradicting Axiom 1.

Given this equivalence we have shown, we now simply use a modified Lemma 2 with $s = p$ and $r = q$, which proves the result on (r, s) pairs. □

Proof of Proposition 4: For any distribution F over types, consider the strategies as specified in the Proposition. Type $\widetilde{I}$ voters are indifferent between all possible outcomes and hence will be indifferent between any randomization over p and q. Since the number of voters is a random variable, there is always a non-zero probability any given individual is pivotal. Thus type A voters will always strictly prefer to vote for p and type B voters will always strictly prefer to vote for q. □

Before proceeding to the rest of the proofs, we denote the induced lotteries faced by individual i of type Γ given voting pattern m and distribution F by $p_i^*((\mathbb{V}^m), \Gamma, F)$ and $q_i^*((\mathbb{V}^m), \Gamma, F)$. We sometime refer to non-monotone types, that is, types $P2$, $Q2$, or $\widetilde{I}$, by NM.

Proof of Proposition 5: First, by same arguments as in the proof of Proposition 4, it is clear that in any equilibrium, $P1$ and $Q1$ types will behave like expected utility maximizers, which implies points 2 and 3.

To show the existence of an anonymous equilibrium, notice that actions can't depend on an individual's identity, just their type. Thus $\alpha_i(\mathbb{V}^m, F) = \alpha(\mathbb{V}^m, F)$ and so $\beta_i^*(\mathbb{V}^m, \Gamma, F) = \beta^*(\mathbb{V}^m, \Gamma, F)$ for all i. We prove existence by contradiction, that is we will suppose no such equilibrium exists and show a contradiction occurs. We do this in several steps.

- Initially we suppose all NM types vote for p. Call this voting pattern $(1:1)$.[40]

[40] In the proof we induct on the number of types (the number on the left), and within each type,

We will order the three NM types by increasing order of the threshold required to vote for q (given this voting pattern): $\mathbb{I}$, $\mathbb{III}$ and $\mathbb{IIII}$. Thus, if type $\mathbb{IIII}$ wants to switch their vote to q then all other NM types would as well. Since, by assumption, we are supposing this is not an equilibrium, then at least one of the three NM types wants to deviate to voting for q. Clearly individuals of type $\mathbb{I}$ must want to switch (because of our ordering assumption).

We now order all possible individuals $1, 2, \ldots$. Observe that only a subset of these individuals will be realized in the Poisson distribution. We will consider each individual's strategy, conditional on him being of type $\mathbb{I}$ and induct on the order of the individuals. Begin with individual 1. By construction, in the proposed voting pattern, $\beta^*(\mathbb{V}^{(1:1)}, \mathbb{I}, F) > \beta(\mathbb{V}^{(1:1)}, \mathbb{I}, F)$ or, equivalently, $q^* \succ p^*$. So individual 1 in type $\mathbb{I}$ would prefer to switch to voting for q. Denote this voting pattern $(1:2)$.

Observe that under voting pattern $(1:2)$, we have that for all other individuals both $p^*(\mathbb{V}^{(1:2)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(1:2)}, \Gamma, F)$ are closer to q than $p^*(\mathbb{V}^{(1:1)}, \Gamma, F)$ and $p^*(\mathbb{V}^{(1:1)}, \Gamma, F)$, respectively. Therefore, because all individuals in type $\mathbb{I}$ preferred to deviate from voting for p to voting for q under voting pattern $(1:1)$, it is now the case that $q^*(\mathbb{V}^{(1:2)}, \mathbb{I}, F)$ is strictly preferred to $p^*(\mathbb{V}^{(1:2)}, \mathbb{I}, F)$. Thus individual 2, if realized as type $\mathbb{I}$, will also have a strict incentive to switch his vote from p to q.

We continue by simply inducting on the number of individuals. After all individuals with index smaller than k have switched, we have voting pattern $(1:k)$. It is clear using the reasoning described above that all individuals in type $\mathbb{I}$ with index greater than k strictly prefer $q^*(\mathbb{V}^{(1:k)}, \mathbb{I}, F)$ to $p^*(\mathbb{V}^{(1:k)}, \mathbb{I}, F)$ and the same for those with index less than k, which guarantees that they will not switch back to vote for p. Thus, we conclude this step by having a potential anonymous equilibrium where of the NM types, types $\mathbb{I}$ vote for q and the other NM types vote for p.

- Suppose again, continuing our contradiction, that this voting pattern (where of the NM types, types $\mathbb{I}$ vote for q and the other NM types vote for p) isn't an equilibrium. Denote this voting pattern by $(2:1)$. Now, we re-order the two remaining NM types that are voting for p under voting pattern $(2:1)$, calling

on the number of individuals within it (the number on the right).

them types $\mathbb{III}$ and $\mathbb{IIII}$.[41] Under our assumption that voting pattern $(2:1)$ is not an equilibrium, it must be the case that $\mathbb{III}$ types want to switch from voting for p to q.

We now repeat the inductive process from the previous step but for individuals in type $\mathbb{III}$; order all individuals, and conditional on them drawing that type, switching them one by one from voting for p to voting for q. Observe that after individual k in type $\mathbb{III}$ switches from voting for p to q, that for all other individuals both $p^*(\mathbb{V}^{(2:k+1)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(2:k+1)}, \Gamma, F)$ are both closer to q than $p^*(\mathbb{V}^{(2:k)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(2:k)}, \Gamma, F)$ respectively. This means that (i) conditional on drawing type $\mathbb{III}$ no individual has an incentive to switch their votes, and (ii) conditional on drawing type $\mathbb{I}$ no individual would want to switch their vote back to p after any step in the inductive process. We conclude this step by having a potential equilibrium where of the NM types, types $\mathbb{I}$ and $\mathbb{III}$ vote for q and the type $\mathbb{IIII}$ vote for p.

- Lastly, we repeat the same exercise above, applying to type $\mathbb{IIII}$ voters. We will then conclude that we have an equilibrium in which all NM types vote for q, and have a strict preference to do so. This equilibrium is obviously anonymous, contradicting the assumption that no such equilibrium exists.

We now turn to proving the properties of the equilibrium. We have already proved parts 2 and 3. Suppose that an equilibrium exists with voting pattern $\mathbb{V}^m$ which induces, for each individual i, a pivot probability $\alpha(\mathbb{V}^m, F)$ and a threshold $\beta^*(\mathbb{V}^m, \Gamma, F)$. To see that 1 is true, observe that in the space of distributions F, generically $\beta^*(\mathbb{V}^m, \Gamma, F) \neq \beta_{i,\mathbb{V}^m,F}$. If in fact $\beta_i^*(\mathbb{V}^m, \Gamma, F) = \beta_{i,\mathbb{V}^m,F}$ then because of quasi-convexity the decision-maker still prefers not to randomize between the two. $\square$

Before proceeding we prove another useful Lemma.

Lemma 3 For all $\epsilon > 0$ there exists an N^*, such that $N \geq N^*$ implies $\alpha = p^* - q^* \leq \epsilon$.
Proof of Lemma 3: As N goes to infinity the probability of being pivotal goes to 0. Thus α goes to 0. $\square$

Proof of Proposition 6: Recall that regardless of the voting patterns of others, each voter faces a non-zero chance of being pivotal, and so Types $P1$ and $Q1$ will

[41] Types $\mathbb{III}$ and $\mathbb{IIII}$ need not correspond to the same groups as under voting pattern $(1:1)$; the ranking of the threshold to switch from p to q may be lower in one group under $(1:1)$ but higher under $(2,1)$.

always vote for p and q respectively. Let $\bar{z}^*$ indicate the highest value of z^*_Γ across all NM types, which means $\bar{z}^* \in (0,1)$.[42] For a large enough f_{P1} and large enough N, in any voting pattern it is very likely, for each individual i, that p is chosen whenever i is not pivotal. Thus, for all individuals $p^*(\mathbb{V}^m, \Gamma, F)$ and $q^*(\mathbb{V}^m, \Gamma, F)$ are both in $(\bar{z}^*, 1)$, meaning that all NM individuals will choose p^*. We can conduct a similar exercise for f_{Q1}. □

Proof of Proposition 7: First part: Each voting pattern generates a $\beta_{i,\mathbb{V}^m,F}$. Observe that since types $P1$ and $Q1$ always vote for p and q, respectively, as the proportion of NM types goes to 0 it is the case that $\beta_{i,\mathbb{V}^m,F}$ approaches some constant $\hat{\beta}$ regardless of the voting pattern of the NM types. Similarly, $\beta^*(\mathbb{V}^m, \Gamma, F)$ approaches $\beta^*(\Gamma)$. Generically, in the space of preferences, these two are not equal and thus each NM type will have a unique best response regardless of the strategy of any other NM type.

Second part: Observe that if $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to 1 then all individuals will vote p. Similarly if it is arbitrarily close to 0, all individuals will vote q. If the proportion of NM types approaches 1 and all NM types vote for p, then $\beta_{i,\mathbb{V}^m,F}$ approaches 1 and so we have an equilibrium. Similar logic applies if all NM types vote for q. □

Proof of Proposition 8: For the first part: Clearly when N is small enough, conditional on being realized as an actual voter, an individual puts arbitrarily high probability on being the only person, and so their vote is pivotal. Thus $P2$ and $Q2$ will almost surely determine the outcome and so always vote for p and q respectively.

We prove the second part of Proposition 9 in two steps. First, we show that it holds for all anonymous equilibria. Recall that in all anonymous equilibria, all individuals of the same type take the same action. For large enough N, the proportion of each type in the total number of voters is known with near certainty (equals f_Γ). Moreover, fixing an equilibrium it is known exactly what action each type takes. This means that with near certainty we know what proportion of the total number of voters choose p and what proportion choose q, and hence the outcome of the voting game is known with near certainty; in other words, for all individuals $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to either 1 or to 0. Without loss of generality suppose $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to 1. Then $p^*(\mathbb{V}^m, \Gamma, F)$ is arbitrarily close to p and since N is large, Lemma 3

[42] Recall that for any type Γ, z^*_Γ is such that V_Γ is strictly decreasing on $[0, z^*_\Gamma]$ and strictly increasing on $[z^*_\Gamma, 1]$.

implies that $q^*(\mathbb{V}^m, \Gamma, F)$ is also arbitrarily close to $p^*(\mathbb{V}^m, \Gamma, F)$ (and so to p). This immediately implies that for any Γ, $q^*(\mathbb{V}^m, \Gamma, F)$ and $p^*(\mathbb{V}^m, \Gamma, F)$ are both greater than z^*_Γ, and so all NM types prefer to choose p. This proves the second part in the case of anonymous equilibria.

The next step is to prove that with large N, generically all equilibria are anonymous. Consider two different individuals, i and j, who are considering their strategies, conditional on drawing the same type Γ. For large enough N, even if they choose different strategies, $\alpha_i(\mathbb{V}^m, F)$ is arbitrarily close to $\alpha_j(\mathbb{V}^m, F)$ (and both are arbitrarily close to 0). Moreover, $\beta^*_i(\mathbb{V}^m, \Gamma, F)$ is arbitrarily close to $\beta^*_j(\mathbb{V}^m, \Gamma, F)$. Thus, for large enough N if $p^*_i(\mathbb{V}^m, \Gamma, F) \succ q^*_i(\mathbb{V}^m, \Gamma, F)$ then $p^*_j(\mathbb{V}^m, \Gamma, F) \succ q^*_j(\mathbb{V}^m, \Gamma, F)$. We can iterate this argument over all individuals of a given type, and we obtain an anonymous equilibrium.

Thus, the only situation where we may have non-anonymous equilibria is where we have an (infinite) sequence of N along which $p^*_i(\mathbb{V}^m, \Gamma, F) \sim q^*_i(\mathbb{V}^m, \Gamma, F)$ holds. $\square$

Proof of Proposition 9: Suppose $f_{P1} > \epsilon > 0$. Recall that we need a proportion of at least T people to vote for q in order for it to be chosen. But even if all NM types vote for q, as T goes to 1 the probability that the proportion of votes for q is greater than T goes to 0. Thus $p^*(\mathbb{V}^m, \Gamma, F)$ and $q^*(\mathbb{V}^m, \Gamma, F)$ both go to p, so p^* is preferred over q^* by all NM types. Thus in equilibrium all NM must vote for p. $\square$

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