An Example of Current Account Determination:
(The General Equilibrium of a Small Open Economy)

Stage 1. Maximization of Nonfinancial Wealth by Firms

1.1 Cobb-Douglas Production Technology:

$$Q = AK^{a}L^{1-a}, \quad \text{with } 0<a<1 \text{ and } A>0$$

1.2. Wealth maximization problem of the firm

$$\text{Max}_{[K2]} \quad W_1^{NF} = Q_2 - K_2 + K_1(1-d) + Q_2/(1+r)$$

given $K_1>0$ and subject to 1.1.

1.3 Maximize by susbtituting 1.1 in 1.2:

$$\text{Max}_{[K2]} \quad W_1^{NF} = A_1K_1^{a}L^{1-a} - K_2 + K_1(1-d) + (A_2K_2^{a}L^{1-a})/(1+r)$$

1.4 First-order condition:

$$-1 + (\partial Q_2/\partial K_2)/(1+r) = 0$$

$$aA_2K_2^{a-1}L^{1-a} = 1+r \quad \Rightarrow \quad MPK_2 = 1+r$$

1.5 Optimal investment and wealth:

$$K_2 = [aA_2/(1+r)]^{1/(1-a)} L$$

which implies:

$$K_2/L = [aA_2/(1+r)]^{1/(1-a)}$$

so wealth can be expressed as:

$$W_1^{NF} = A_1(K_1/L)^a L - (K_2/L) L + K_1(1-d) + [A_2(K_2/L)^a L]/(1+r)$$

1.6 Investment schedule (linear in logarithm of K_2):

$$I(r) = 1/(1-a) [Ln(aA_2) - Ln(1+r)] + Ln(L)$$

note thar for small r, $Ln(1+r) \approx r$, so we obtain:

$$I(r) = Ln(aA_2)/(1-a) - r/(1-a) + Ln(L)$$

This is the investment schedule of the S-I diagram used in the text. The effects of changes in the interest rate, the productivity parameter A, and labor are easy to interpret

Stage 2. Utility Maximization by the Household

2.1 Logarithmic, isoelastic utility function:

$$U = Ln(C_1) + \frac{Ln(C_2)}{1+\delta} \qquad 0<\delta<1$$

2.2 Wealth constraint:

$$C_1 + \frac{C_2}{1+r} = W_1 \equiv W_1^{NF} + B_0(1+r)$$

where W_1^{NF} is the solution from stage 1 obtained in 1.5.

2.3 Utility maximization:

$$MAX_{[C_1,C_2]} \quad U = Ln(C_1) + \frac{Ln(C_2)}{1+\delta}$$

subject to the constraint in 2.2

2.4 Maximization by susbtitution of C1 in 2.3 using 2.2

$$MAX_{[C_2]} \quad U = Ln\left(W_1 - \frac{C_2}{1+r} \right) + \frac{Ln(C_2)}{1+\delta}$$

2.5 First order condition:

$$\frac{\left(-\frac{1}{1+r}\right)}{\left(W_1-\frac{C_2}{1+r}\right)} + \frac{1}{C_2(1+\delta)} = 0$$

$$C_2(1+\delta) = \left(W_1-\frac{C_2}{1+r}\right)(1+r) \quad \Rightarrow \quad IMRS(C_2,C_1) = (1+r)$$

2.6 Optimal consumption decisions:

$$C_2 = \left[\frac{1+r}{2+\delta}\right]W_1$$

$$C_1 = \left[\frac{1+\delta}{2+\delta}\right]W_1$$

2.7 Saving and the saving schedule:

$$S_1(r) = Y_1 - C_1 = Q_1 + (1+r)B_0 - \frac{1+\delta}{2+\delta}W_1$$

W_1 includes present value of income stream, which is a decreasing function of r. As long as $B_0 \geq 0$, S increases with r.

3. The Current Account

Given a world interest rate r*, the current account of a small open economy is:

$$CA(r^*) = S(r^*) - I(r^*)$$

Substitute in the solutions for S and I evaluated at r=r*:

$$CA(r*) = A_1 K_1^a L^{1-a} + (1+r*)B_0 - \frac{1+\delta}{2+\delta} W_1(r*) - K_2(r*) + K_1(1-d)$$

where:

$$W_1(r*) = A_1\left(\frac{K_1}{L}\right)^a L - \frac{K_2(r*)}{L} L + K_1(1-d) + \frac{A_2\left(\frac{K_2(r*)}{L}\right)^a L}{1+r*} + (1+r*)B_0$$

and

$$K_2(r*) = \left(\frac{aA_2}{1+r*}\right)^{\frac{1}{1-a}} L$$

4. Example of Exogenous Shock: Changes in A_1

Change in A_1 is a temporary surge in period-1 output.

Consumption smoothing leads to reduce S_1 (i.e. increase in Q1 larger than increase in C1 since the output gain is allocated to C over entire life)

Investment remains constant because MPK_2 and r* are not affected by change in A_1

Since S rises and I remains constant, CA rises (improves)

Mathematical proof:

$$\frac{dCA}{dA_1} = \frac{dQ_1}{dA_1} - \left(\frac{1+\delta}{2+\delta}\right)\frac{dW_1}{dA_1}$$

$$= K_1^a L^{1-a} - \left(\frac{1+\delta}{2+\delta}\right) K_1^a L^{1-a}$$

$$= K_1^a L^{1-a}\left(\frac{1}{2+\delta}\right) > 0$$