Fisherian Deflation, Credit Collapse and Sudden Stops

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1. Basic elements of financial crises modeling
2. Principles of Fisherian models
3. Extending the small open economy model to incorporate Fisherian deflation
4. Quantitative experiments
5. Policy implications
Basics of Nonlinear Financial Crises Models
“...debt happens as a result of actions occurring over time. Therefore, any debt involves a plot line: how you got into debt, what you did, said and thought while you were there, and then—depending on whether the ending is to be happy or sad—how you got out of debt, or else how you go further and further into it until you became overwhelmed by it, and sank from view.”

(Margaret Atwood, “Debtor’s Prism,” WSJ, 09/20/2008)
Crises, “Black swans” & nonlinearities

• “Things are not conceptually out of control, this is not some mystery black swan we don’t understand and we need to rewrite all the paradigms because all the modeling is wrong. If people are acting using a linear model, what looks like a ten-sigma event can actually be a two-sigma event…”

• “Most of the models in credit, in trading desks, in macro models do quite well locally, the problem is when you stop being locally nonlinearities are really quite large,...If you want to see what happened in AIG...they wrote a whole lot of credit default swaps...the assets underlying them went down not one shock, not two shocks, not three shocks, but over and over. Each time the same size shock is going to create something even larger…”

Pricing liabilities with financial distress

Theoretical pricing function

Yield

Risk-free rate

liability position

Rationing ceiling
Amplification, nonlinearities and “macroprudential” policy

Theoretical pricing function

financial distress with MPP

regular cycle

local approximation

yield vs. liability position
Principles of Fisherian models of financial crises
Fisherian models

• Fisher (1933): narrative of fin. amplification driven by debt-deflation mechanism & interaction of innovation and beliefs
  – Modern examples: Minsky, Kiyotaki-Moore, Bernanke-Gertler, Aiyagari-Gertler, Mendoza, Brunnermeier-Sannikov...

• Collateral constraints: debt cannot exceed a fraction of market value of collateral (a market price affects borrowing capacity).

1. **DTI models** (flow constraints): debt in units of tradables limited to a fraction of market value of total income:

\[ b_{t+1} \geq -\kappa (y_t^T + p_t^N y_t^N) \]

2. **LTV models** (stock constraints): debt cannot exceed a fraction of the market value of assets:

\[ b_{t+1} \geq -\kappa q_t k_{t+1} \]
Two key features of Fisherian constraints

1. **Debt-deflation mechanism**: When constraints bind, agents fire sale assets/goods, prices fall, constraint tightens further forcing more fire sales
   - Credit crunch causes collapse in demand, and also in supply via effects on factor demands (e.g. deflation of relative prices)
   - Crises are endogenous outcomes of standard shocks not large unexpected, or financial, shocks
   - Different from Keynesian disequilibrium: price flexibility, rather than rigidity and exogenously insufficient aggregate demand

2. **Pecuniary externality**: agents do not internalize effect of individual borrowing on collateral prices
   - Dynamic externality: effect of today’s borrowing on tomorrow’s prices if there is a financial crisis
   - Market failure that justifies macroprudential regulation
Extending the SOE Model to Incorporate Fisherian DTI Constraints
Why is this important?

• In economies that are highly leveraged, Fisherian deflation amplifies real effects of credit contractions.

• Via Fisherian deflation, credit frictions induce amplification and asymmetry (i.e., “Great Depressions” or “Sudden Stops”) in response to “standard” shocks.

• The transmission mechanism features a “pure” balance sheet effect (i.e. without feedback) and Fisher’s debt-deflation process.

• Quantitatively, the contribution of the Fisherian deflation is substantial and Fisherian models provide a framework for macroprudential policy analysis.
Fisher’s debt-deflation mechanism

• Example: DTI setup with debt in units of tradables, leveraged on nontradables income

\[ b_{t+1} \geq -\kappa (y_t^T + p_t^N y_t^N) \]

• Assume a period of expansion and real appreciation driven by rising relative prices (nontradables relative to tradables)

• Debt rises and at some point DTI constraint becomes binding

• Pure balance sheet effect: borrowing constraint lowers tradables consumption and rel. price of nontradables

• Fisherian deflation: spiral of tightening debt constraints and falling nontradables prices (feedback effect)
Adding DTI constraint to SOE model

- Households consume tradables and nontradables, combining them into a composite good \( c(c_t^T, c_t^N) \). For example, if the composite good is Cobb-Douglas:
  \[
  c(c_t^T, c_t^N) = (c_t^T)^\alpha (c_t^N)^{1-\alpha}
  \]

- Utility function can be log as before, with infinite life horizon:
  \[
  U(c^T, c^N) = \ln \left( c(c_0^T, c_0^N) \right) + \frac{\ln \left( c(c_1^T, c_1^N) \right)}{1 + \delta} + \frac{\ln \left( c(c_2^T, c_2^N) \right)}{(1 + \delta)^2} + \ldots
  \]

- Budget constraint (with \( R \equiv 1 + r \) & tax on nontradables purchases):
  \[
  c_t^T + (1 + \tau_t) p_t^N c_t^N = y_t^T + p_t^N y_t^N - b_{t+1} + b_t R + T_t
  \]

- DTI Fisherian credit constraint:
  \[
  b_{t+1} \geq -\kappa (y_t^T + p_t^N y_t^N)
  \]
• Government budget constraint:

\[ \tau_t p^N_t c^N_t = \bar{g}^T + p^N_t \bar{g}^N + T_t \]

• Nontradables market clearing:

\[ c^N_t + \bar{g}^N = y^N_t \]

• Tradables resource constraint (obtained by combining budget constraints of households and government):

\[ c^T_t + \bar{g}^T = y^T_t - b_{t+1} + R b_t \]

• Focus on the case that gave perfectly smooth consumption before (i.e. assume \( 1 + \delta = R \)), and for simplicity assume also \( y^N_t = \bar{y}^N \) so that market-clearing implies \( \bar{c}^N = \bar{y}^N - \bar{g}^N \)
Frictionless perfectly smooth equilibrium (FPSE)

- Recall optimal savings plan equates marginal cost and benefit of savings (IMRS=R). Since bonds are in units of tradables we get:

\[
\frac{\partial u(c(c_t^T, \tilde{c}^N))}{\partial c_t^T} = \frac{(1 + r) \cdot u(c(c_{t+1}^T, \tilde{c}^N))}{(1 + \delta) \cdot \frac{\partial u(c(c_{t+1}^T, \tilde{c}^N))}{\partial c_{t+1}^T}}
\]

- Hence assuming \( \delta = r \) still yields perfectly smooth consumption:

\[
\frac{\partial u(c(c_t^T, \tilde{c}^N))}{\partial c_t^T} = \frac{\partial u(c(c_{t+1}^T, \tilde{c}^N))}{\partial c_{t+1}^T} \iff c_t^T = c_{t+1}^T = \tilde{c}^T
\]

- Tradables intertemporal resource constraint implies same result as before: \( c_t^T \) is a constant fraction of tradables wealth

\[
\tilde{c}^T = (1 - \beta)[W_0 + Rb_0] - \tilde{g}^T \quad \text{with} \quad W_0 \equiv \sum_{t=0}^{\infty} R^{-t} (y_t^T)
\]
New item we need to determine is the equilibrium $p_t^N$

Optimal choices of $c_t^T$ and $c_t^N$ equate marginal cost and benefit of reallocating consumption:

$$
\frac{\partial u(c(c_t^T,c_t^N))}{\partial c_t^T} = \frac{\partial u(c(c_t^T,c_t^N))}{\partial c_t^N} (1 + \tau_t) p_t^N
$$

$$
\Phi(c_t^T/c_t^N) \equiv \frac{\partial u(c(c_t^T,c_t^N))}{\partial c_t^T} \bigg/ \frac{\partial u(c(c_t^T,c_t^N))}{\partial c_t^N} = (1 + \tau_t) p_t^N
$$

$\Phi(c_t^T/c_t^N)$ is the MRS between the two goods

Using solutions for $c_t^T$ & $c_t^N$ we get:

$$
p_t^N = \bar{p}^N = \Phi(\bar{c}^T/\bar{c}^N)(1 + \tau_t)^{-1}
$$

with \( \Phi(\bar{c}^T/\bar{c}^N) = \frac{1 - \alpha \bar{c}^T}{\alpha \bar{c}^N} \) for Cobb–Douglas $c(.)$
• Frictionless perfectly-smooth equilibrium (if credit constraint never binds):

\[
\bar{c}^N = \bar{y}^N - \bar{g}^N
\]

\[
\bar{c}^T = (1 - \beta)[W_0 + Rb_0] - \bar{g}^T, \quad W_0 \equiv \sum_{t=0}^{\infty} R^{-t}y_t^T
\]

\[
\bar{p}^N = \frac{\Phi(\bar{c}^T / \bar{c}^N)}{(1 + \tau)}
\]

\[
b_{t+1} = y_t^T + Rb_t - (1 - \beta)[W_0 + Rb_0]
\]

– This a “social optimum:” yields the highest welfare the economy can attain given its resources
Wealth-neutral shocks (WNS) to $y_0^T$

- $(y_0^T, y_1^T)$ such that $\downarrow y_0^T \& \uparrow y_1^T$ relative to $\bar{y}^T$ keeping $W_0$ constant:

$$\frac{(y_1^T - \bar{y}^T)}{1 + r} = (\bar{y}^T - y_0^T) \text{ with } \bar{y}^T \equiv \frac{rW_0}{1 + r} \text{ is permanent income}$$

- In the FPSE, WNS shocks are irrelevant for consumption, prices and welfare. They only alter pattern of borrowing and lending:

$$\hat{b}_1 - b_0 = y_0^T - \bar{y}^T < 0$$

$$\hat{b}_2 - \hat{b}_1 = -(\hat{b}_1 - b_0) > 0,$$

$$\hat{b}_t = b_0 \text{ for } t \geq 2$$

- If for a given WNS the Fisherian credit constraint does not bind we preserve FPSE: Agents borrow at 0, repay at 1. But if it binds, FPSE cannot be maintained (credit is constrained)
Fisherian deflation equilibrium (FDE)

- Unanticipated wealth neutral shock $\downarrow y_0^T$
- As $y_0^T$ falls agents borrow more, and if $y_0^T$ falls below critical level $\hat{y}^T$, credit constraint becomes binding

$$\hat{y}^T = \frac{\bar{y}^T - b_0 - \kappa \bar{p}_0 N \bar{y}^N}{1 + \kappa}$$

- A devaluation of the real ex. rate can be proxied as a tax hike ($\uparrow \tau \rightarrow \downarrow \bar{p}_0^N$). This increases $\hat{y}^T$, so “real devaluations” can trigger credit constraints for smaller shocks to $y_0^T$
- $\kappa$ has an upper bound, above which the credit constraint never binds for positive income, and a lower bound below which $c_0^T \leq 0$. 
Solving Fisherian deflation equilibrium (FDE)

- FDE yields an equilibrium with “Sudden Stop.” The solution follows from two equations:

1. Date-0 tradables resource constraint implies:
   \[ c_0^T = y_0^T - \bar{g}^T + \kappa [y_0^T + p_0^N \bar{y}^N] + Rb_0 < \bar{c}^T \]

2. Date-0 price of nontradables implies:
   \[ p_0^N = \Phi \left( \frac{c_0^T}{\bar{y}^N - \bar{g}^N} \right) (1 + \tau_0)^{-1} < \bar{p}_0^N \]

- The above two form a possibly non-linear eq. in \( c_0^T \):
  \[ c_0^T = y_0^T - \bar{g}^T + \kappa \left[ y_0^T + \left( \Phi \left( \frac{c_0^T}{\bar{y}^N - \bar{g}^N} \right) (1 + \tau_0)^{-1} \right) \bar{y}^N \right] + Rb_0 \]

- The date-0 current account implies:
  \[ b_1 - b_0 = -\kappa [y_0^T + p_0^N \bar{y}^N] - b_0 > \bar{b}_1 - b_0 \]
\[ \ddot{c}^T = (1 - \beta)[W_0 + Rb_0] - \ddot{g}^T \]
\[ c_0^T = \hat{y}^T - \ddot{g}^T + \kappa[\hat{y}^T + p_0^N\ddot{y}^N] + Rb_0 \]
\[ p_0^N = \Phi \left( \frac{c_0^T/\ddot{c}^N}{1 + \tau} \right) \]
Graph of Equilibrium with Fisherian deflation

\[ c_0^T = y_0^T - \bar{g}^T + \kappa [y_0^T + p_0^N \dot{y}^N] + Rb_0 \]

\[ y_0^T < \dot{y}^T \]

\[ p_0^N = \frac{\Phi(c_0^T/\bar{c}^N)}{(1 + \tau)} \]
Main features of FDE

- Asymmetric response to positive v. negative shocks
- FDE amplifies response to negative shocks beyond “pure” balance sheet effect
- “Devaluation” or price shocks neutral for FPSE, not FDE
- Nonexistence and multiplicity:
  - If shock is “too large”, economy cannot borrow (no FDE exists) and moves to autarky solution
  - Unique equilibrium guaranteed if SS is steeper than PP around PSE (point A)
- Endogenous volatility of $pn$ drives volatility of $rer$ and causes Sudden Stops
Quantitative Experiments
Functional forms & calibration

- **Functional forms:**

\[ u(c) = \frac{(c^{1-\sigma})}{(1 - \sigma)} \quad c = [a(c^T)^{-\mu} + (1 - a)(c^N)^{-\mu}]^{-1/\mu} \]

- **Parameter values:**
  - \( \beta = 0.96, \quad \sigma = 2 \)
  - \( a = 0.342 \)  Mendoza’s (02) estimate for Mexico
  - \( \mu = 0.204, \quad 1/(1+ \mu) = 0.83 \)  at upper bound of range of existing estimates for Latin America
  - \( yT/(pNyN) = 1.543, \quad cT/yT = 0.66, \quad cN/yN = 0.71 \)  from Mendoza’s (02) estimates for Mexico
  - Initial debt set at 1/3 of GDP
  - “Permanent” output normalized to 1 (results as shares)
Wealth Neutral Shocks to $y_0^T$ with $\kappa=0.34$
Wealth Neutral Shocks to $y_0^T$ with $\kappa=0.34$
...and it gets worse (output & employment effects)

- Firms that produce NT goods experience a collapse of the relative price of their output and sales
- Value of marginal product (i.e. demand) for inputs they use (capital, labor, int. goods) falls, and so does output
- Falling supply of NT goods has two effects:
  - Weakens deflation by reducing supply
  - Worsens the credit crunch as now both prices and output fall
- “Vulnerable” agents that were “o.k.” before are hit by the crisis as they become unemployed and/or hit credit constraint because of falling incomes and prices
How bad can it get in the U.S.? The Great Depression

Diagram showing the GDP and Credit Index from 1929 to 1936.
Deflation during the U.S. Great Depression

The graph illustrates the GDP index and CPI inflation during the U.S. Great Depression. The GDP index shows a sharp decline from 1929 to 1933, reaching its lowest point in 1933, and then a recovery towards 1936. Similarly, the CPI inflation also undergoes a significant drop during the same period, indicating deflation.
Policy Implications
Macroprudential pecuniary externality

• Optimality condition equating costs and benefits of borrowing/saving without regulation:

\[
\frac{\partial u(c(c_t^T, \tilde{c}^N))}{\partial c_t^T} = \frac{(1 + r)}{(1 + \delta)} E_t \left[ \frac{\partial u(c(c_{t+1}^T, \tilde{c}^N))}{\partial c_{t+1}^T} \right] + \mu_t
\]

where \(\mu_t\) is Lagrange multiplier of credit constraint. It is >0 if it binds. In normal times, \(\mu_t=0\) => standard condition.

• Macroprudential pecuniary externality: Regulator considers how \(p_{t+1}^N\) responds to debt chosen at \(t\) if constraint binds at \(t+1\). Its optimality condition in normal times is:

\[
\frac{\partial u(c(c_t^T, \tilde{c}^N))}{\partial c_t^T} = \frac{(1 + r)}{(1 + \delta)} E_t \left[ \frac{\partial u(c(c_{t+1}^T, \tilde{c}^N))}{\partial c_{t+1}^T} \right] + \mu_{t+1} \kappa \tilde{y}^N \frac{\partial p_{t+1}^N}{\partial c_{t+1}^T}
\]

• Social marginal cost of borrowing exceeds private one, causing overborrowing in the absence of regulation!
Optimal Macroprudential policy

• “Macroprudential” debt tax: Assume borrowing carries a tax to make debt costlier so as to prevent credit booms.

• The optimal tax would make the private marginal cost of borrowing match social MC of borrowing:

$$\tau_t = E_t \left[ \mu_{t+1} \kappa \bar{y}^N \frac{\partial p_t^N}{\partial c_T} \right] / E_t \left[ \frac{\partial u(c(c_T^{t+1}, \bar{c}^N))}{\partial c_T^{t+1}} \right]$$

- $\tau_t > 0$ only if the constraint is expected to bind with some probability at $t+1$ (this is why it is “prudential”).

• Equivalent instruments: capital requirements, new CCyB of the BIS, regulatory LTV or DTI ratios.
Optimism and “Fisherian inflation”

- Technological and/or financial *innovation* can lead to optimistic valuation of collateral
- Rising collateral values trigger credit boom (the constraint binds also in the upswing)
- Shocks that trigger the constraint have even larger effects because of higher leverage
- Recessions can be even deeper if they cause pessimistic valuation of collateral
- But modeling optimism/pessimism requires additional structure (e.g. Bayesian learning)
- Boz and Mendoza (2014) proposed a model of the U.S. crisis with these features
1. In highly leveraged economies with credit frictions, endogenous deflation of relative prices causes Sudden Stops (deep recessions, CA reversals)

2. Credit frictions induce amplification and asymmetry in response to “standard” shocks

3. The contribution of the Fisherian deflation is substantial, and it has large adverse welfare effects

4. Pecuniary externality via collateral values justifies macroprudential regulation (but optimal policy is complex and simple rules are much less effective)