University of Pennsylvania   **Enrique G. Mendoza**

Economics 252
International Finance

This part of the course covers basic elements of open-economy macroeconomics that are needed to study financial globalization and financial crises. These theoretical principles are based on the "microfoundations" approach to macroeconomics. This approach seeks to build a theory about macroeconomic phenomena that is based on microeconomic principles. In particular, we will develop a framework for understanding how saving, investment, the current account, international capital flows, and the demand for money are determined in the equilibrium of a small open economy by modeling the decision-making process of the economic agents that take part in that economy: (a) households, which make consumption, saving, and money demand decisions, (b) firms, that choose the use of inputs for production, the level of production, and the accumulation of physical capital, and (c) the government, which sets the levels of taxes, government expenditures and transfers, and manages monetary and exchange rate policies.

The decision-making process of economic agents leads them to make "optimal" decisions, in the sense that the choices they make reflect solutions to well-defined "constrained maximization problems." These maximization problems can be summarized as follows. Households wish to choose the allocations of consumption, saving, and money demand so as to maximize utility subject to their budget constraint. Firms wish to choose their demands for capital and labor so as to maximize the present value of profits, subject to a constraint imposed by existing production technologies. The government sets the levels of its policy instruments so as to make sure that it will remain solvent over time (i.e., that the present discounted value of its total revenue equals the present discounted value of its expenditures).

The interaction of the optimal choices of all agents produces the macroeconomic equilibrium that we observe in aggregate data and is the end-result we are interested in studying. Hence, we study how international flows of financial capital, including those that triggered the waves of financial crises since the 1990s and the recent record-high current account deficits of the United States, occur as an equilibrium outcome that results from the optimal actions taken by economic agents. For instance, in this framework agents will decide to engage in speculative attacks against a currency at a particular date, under particular circumstances because it is the optimal course of action at that point in time. Note, however, that the word "equilibrium" as used in this context does not have any normative connotation. It simply refers to a situation in which economic agents are doing their best given the conditions they are facing.

We will examine the macroeconomic equilibrium of an economy that is assumed to be small and open. This implies that agents that inhabit the economy have unrestricted access to a global financial market from which they can borrow or lend at a given interest rate, and to globalized goods markets in which they can trade goods and services freely with the rest of the world. The "openness" reflects the unrestricted access to global capital and goods markets. The economy is "small" because it is one of many economies that take part in those markets, and hence decisions to borrow or lend, or to import or export goods, by this single economy cannot influence the world real interest rate or the world prices of internationally-traded goods.

1. Macroeconomic equilibrium in a small open economy without monetary distortions:

A. The Households Saving Decision (Ch. 4, S&L)

Simplifying assumptions:

A.1) Output consists of one "composite commodity" **Q** with constant price = 1

A.2) Households live **T** periods, produce output stream $\{Q_1,..,Q_T\}$ and choose consumption stream $\{C_1,..,C_T\}$

A.3) Households have access to the global capital market to borrow or lend, so **C** and **Q** need not be identical in every period

A.4) The capital market is simplified to one-period real bonds **B** which allow for borrowing and lending at an interest rate **r** in terms of units of the good.

A.5) Credit markets do not allow "Ponzi" games (i.e. debt cannot grow to infinity, borrow to service perpetually growing debt is not allowed)

=> Income in any period is given by: $Y = Q + rB_{-1}$

=> The flow of saving can therefore be defined as follows:

$$B = B_{-1} + (Y - C) = B_{-1} + (Q+rB_{-1}-C) = (1+r)B_{-1} + Q - C$$

$$B - B_{-1} = S = Y - C$$

The Household's Intertemporal Problem:

Households choose a consumption stream that (a) maximizes utility, U, and (b) is feasible (i.e., given income stream and credit market structure, the present discounted value, PV, of consumption equals that of income):

Max. $U(C_1,...,C_T)$
s.t. $PV[C_1,...,C_T] = PV[Y_1,...,Y_T] \equiv W$

where W denotes **Wealth**

We will focus mostly on a simple case of this problem known as the ***two-period model***. In this model, households live for two periods. In its most basic form, the two-period model also assumes that households do not give or receive bequests. This case implies:

T=2, B_0=0, B_2=0

Note that the two-period case makes it clear that the decision households face is not so much between saving or borrowing, but ***when*** to save and ***when*** to borrow. To see why, consider:

At t=1: $B_1 - B_0 = B_1$ (since $B_0=0$) $= S_1$

At t=2: $B_2 - B_1 = -B_1$ (since $B_2=0$) $= S_2$

which implies that: $S_1 = -S_2$.

In the two-period case, households face two ***period-by-period budget constraints***:

At t=1 $S_1 = Y_1 - C_1 = Q_1 - C_1 = B_1$

At t=2 $S_2 = Y_2 - C_2 = Q_2 + rB_1 - C_2$

These constraints are combined in the ***intertemporal (or wealth) budget constraint***. To combine the constraints, consider that $S_1 = -S_2$ and plug in the two period-by-period constraints:

$$Q_1 - C_1 = -[Q_2 + rB_1 - C_2]$$

$$Q_1 - C_1 = C_2 - Q_2 - r(Q_1 - C_1)$$

$$Q_1 + Q_2/(1+r) = C_1 + C_2/(1+r)$$

$$\mathbf{W_1 \equiv PV[Q_1, Q_2] = PV[C_1, C_2]}$$

This intertemporal constraint is the same studied in an Economics Principles class in terms of goods (x,y), except that here x is date-1 consumption, y is date-2 consumption, the price of x is 1, the price of y is $1/(1+r)$, and the price of y in terms of x is $(1+r)$.

Bequests (positive or negative) can be added easily to the intertemporal constraint:

$$W_1 \equiv (1+r)B_0 + Q_1 + Q_2/(1+r) = C_1 + C_2/(1+r)$$

In this case, wealth is the PV of output stream plus interest and principal on inherited bonds (B_0). It is also easy to consider the intertemporal constraint of infinitely-lived households:

$$W_1 \equiv (1+r)B_0 + Q_1 + Q_2/(1+r) + ... + Q_\infty/(1+r)^\infty$$

$$= C_1 + C_2/(1+r) + ... + C_\infty/(1+r)^\infty$$

Households could leave positive bequest but its present value is virtually zero, and ruling out Ponzi schemes rules out any negative bequest that is not zero in present value. This last assumption can be stated mathematically as follows: $\text{Lim}_{t\to\infty}[B_t/(1+r)^t] = 0$

The intertemporal budget constraint of the two-period model with no bequests can be illustrated in the following graph:

$$Q_1(1+r) + Q_2 = C_1(1+r) + C_2$$

Period 2

Q2+Q1(1+r)

Q

Q2

-(1+r)

Q1   W1   Period 1
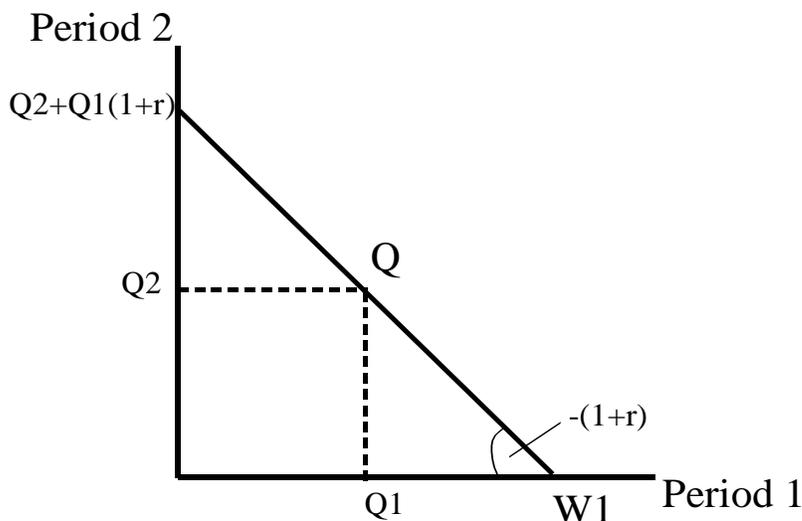
Intertemporal preferences and optimal consumption-saving decisions

We assume an ***intertemporal utility function*** of the form:

$$U(C_1, C_2)$$

with the standard properties:

A)   Positive marginal utility: $MUC = \partial U / \partial C_t > 0$ for t=1 or 2

B)   Marginal utility diminishes as consumption rises: $\partial^2 U / \partial C_t^2 < 0$ for t = 1 or 2

With these assumptions, the intertemporal indifference curves in the (C_1, C_2) space are negatively-sloped and convex. The slope of the indifference curves is called ***the intertemporal marginal rate of substitution*** ($IMRS = MUC_1 / MUC_2$), which measures the rate at which households are "happy" to substitute C_2 for C_1 so as to keep utility constant

Given that households wish to choose the affordable consumption plan that yields the highest level of utility, it follows that the ***optimal consumption-saving decision*** is determined at the tangency point between the highest attainable indifference curve and the budget line. At this point, the households are acquiring the combination of C1 and C2, among those they can afford, that makes them the "happiest." Intuitively, this also implies that at that tangency point the marginal cost of sacrificing an extra unit of current consumption equals the marginal benefit of the extra future consumption that the additional saving would yield. In other words:

$$IMRS = (1+r)$$

Graphically, the optimal consumption saving decision can be illustrated as follows:
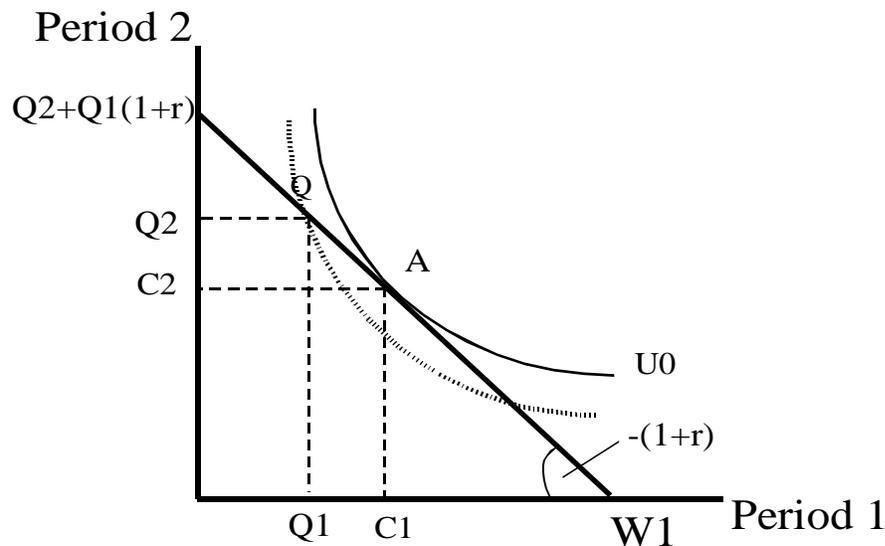
Three important characteristics of the optimal saving decision:

1) Consumption depends on wealth (i.e., the entire current and expected future income stream) and *not* just on current income. If we had uncertainty about future income, wealth would have to be determined on the basis of expectations of future Q=s.

2) Households are net borrowers or lenders at t=1 depending on the relationship between the optimal consumption stream, the income stream, and the interest rate

3) Households attain a higher level of welfare by borrowing or lending than by not accessing international credit markets (i.e., opting for the "autarky" equilibrium Q)

Calculus can also be used to solve for the optimum consumption-saving choice. The problem is a standard problem of constrained optimization:

$$\text{Max}_{\{C1,C2\}} \quad U(C1,C2)$$

$$\text{s.t.} \quad W1 = C1 + C2/(1+r)$$

This problem is easy to solve by substitution. Substitute C2 in U using the wealth constraint:

$$\text{Max}_{\{C2\}} \quad U[W1-C2/(1+r), C2]$$

Now simply maximize U with respect to C2. The first-order condition is:

$$-[\partial U/\partial C1]/(1+r) + [\partial U/\partial C2] = 0$$

$$[\partial U/\partial C1]/[\partial U/\partial C2] = (1+r)$$

which is the same condition derived intuitively earlier: IMRS = (1+r).

Consumption smoothing

The previous chart of the optimal saving choice suggests that households like to keep consumption in the two periods relatively constant, compared to what happens with their exogenous output stream. This property is known as ***consumption smoothing***, and the incentive for it to be "optimal" is the declining nature of IMRS (i.e., Y_1 and Y_2 are traded off in linear fashion at the rate $(1+r)$, whereas there is an increasing utility cost in trading C_2 for C_1).

If we restrict the utility function in a certain way, we can obtain a special case known as ***Perfectly Smooth Consumption,*** in which C is identical in every period. This requires:

1)   "Isoelastic," time-separable and homothetic utility function:

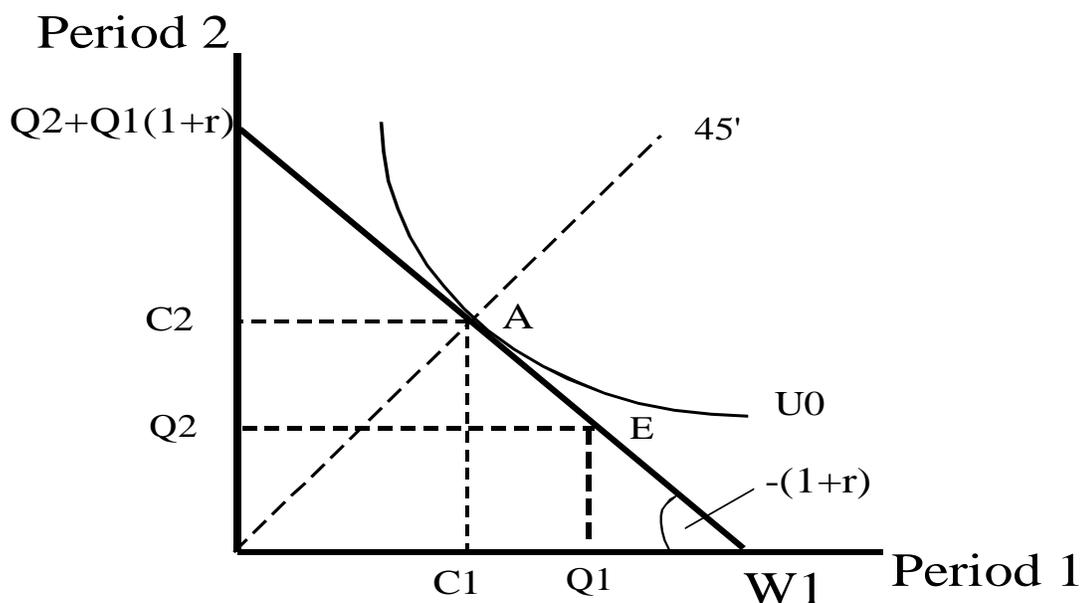$$U(C_1, C_2) \ = \ u(C_1) + (1/1+\delta)u(C_2)$$

where δ is the subjective rate of time preference

2)   A rate of time preference equal to the real interest rate $\delta = r$

If we impose these assumptions on our mathematical result we obtain:

$$[\partial u/\partial C_1]/[\partial u/\partial C_2] \ = \ (1+r)/(1+\delta)$$

$[\partial u/\partial C_1]/[\partial u/\partial C_2] = 1$ or $[\partial u/\partial C_1] = [\partial u/\partial C_2]$ which implies $C_1 = C_2 = C$

The level of consumption at which consumption is "perfectly smoothed" is solved for using the intertemporal budget constraint:

$$C + C/(1+r) = Q1 + Q2/(1+r) \equiv W$$

$$C = [(1+r)/(2+r)] W$$

Hence, households consume a constant fraction of their wealth in every period. Saving in period 1 would be given by $S1 = Q1 - C$.

The effects of income shocks on optimal saving-consumption plans (perfectly-smooth case)

Consider for simplicity a case in which at the initial equilibrium the economy had constant levels of Q and C over time. We can use the framework of the perfectly-smooth case to study the effects of the following shocks:

1)   Temporary negative shock to current income (Q1) => reduce S1, borrow more

2)   Permanent negative shock to current and future income (Q1 and Q2) => S1 unaffected

3)   Anticipated negative shock to future income (Q3) => increase S1, lend more
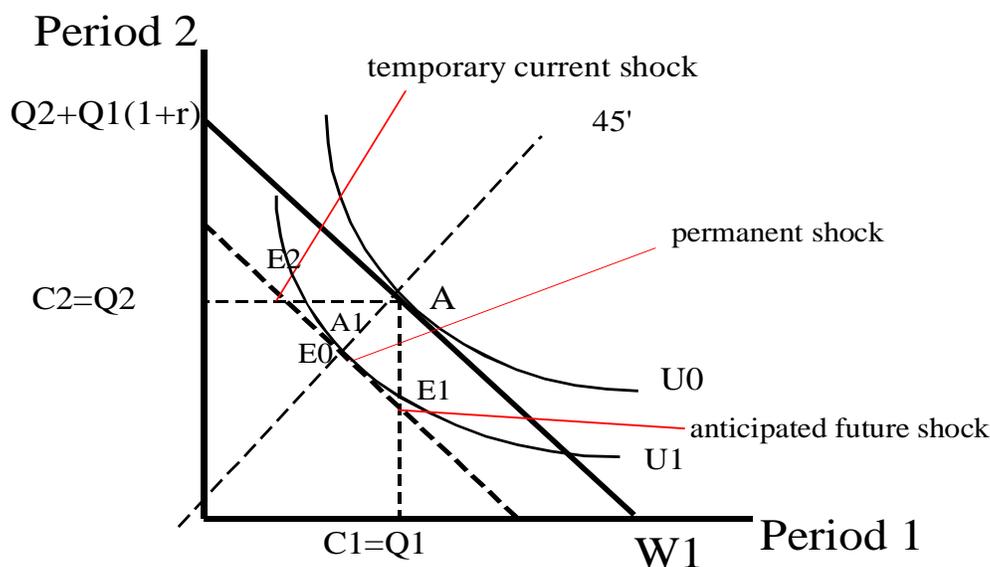
The initial equilibrium is point A. The shocks have been assumed to be of such magnitude that each of the three cases moves the economy to the same revised (dashed) budget line. E2 is for a temporary fall in Q1 leaving Q2 constant, E0 cuts both Q1 and Q2 by identical amounts, and E1 is for an anticipated fall in Q2 leaving Q1 constant. In all three cases the optimal consumption choice is the same at A1, but the saving decisions differ in each case.

Liquidity/Borrowing Constraints

If agents cannot borrow against future income, and the constraint is binding, they may be forced to consume their current income, and wealth will not be the key determinant of consumption. We observe constraints of this type in the real world because the risk of default leads lenders to design credit contracts requiring collateral, which some agents may not posses. Another reason are capital controls or other policies that restrict access to world markets.

Consumption and saving with liquidity constraints will react differently to shocks and policy changes. Current C will tend to react more than in the absence of constraints. These models are difficult to examine because of the need to determine the extent to which constraints bind and whether they remain binding in the face of policy changes and shocks.

A simple illustration of the two-period model with borrowing constraints is the following:
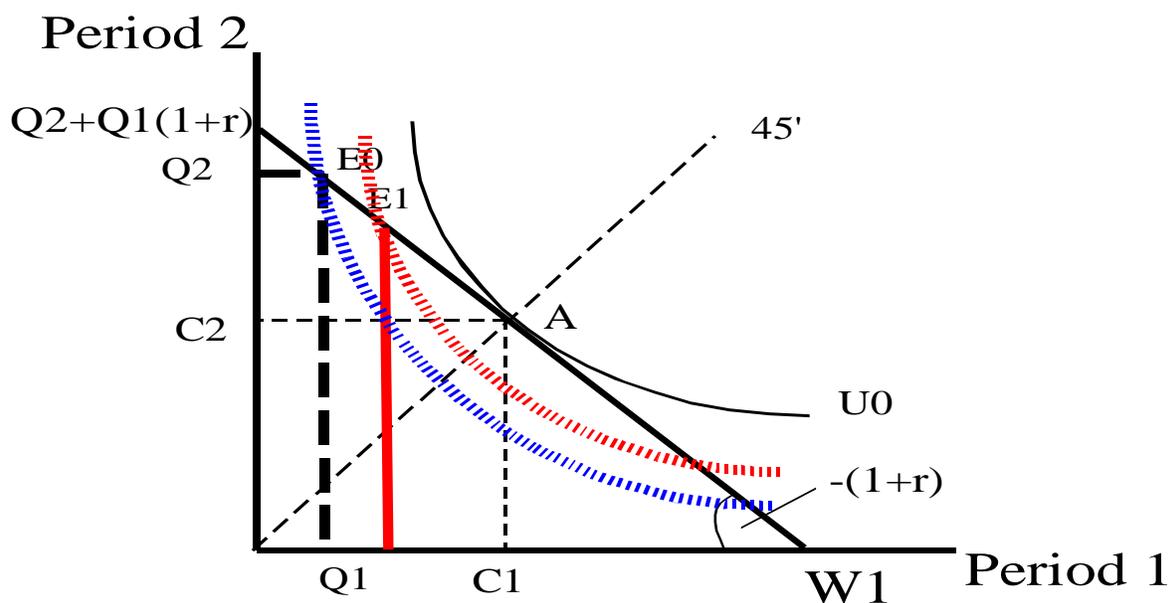
Effects of Changes in the Rate of Interest

Does saving tend to rise as interest rates increase? In theory, the effect is ambiguous because of two effects that may oppose each other:

A)   *Substitution effect*: C1 becomes more expensive than C2, so C1 should fall
       [tangency of original indifference curve with new r]

B)   *Income effect:* a rise in interest rate makes lenders richer but borrowers poorer, so
       direction of income effect depends on whether household was net lender or borrower
       [parallel inward or outward shift in budget line]

Hence, as interest rates move, households and countries can shift from net lenders to borrowers.

Net Borrower Case   *Net Lender Case*
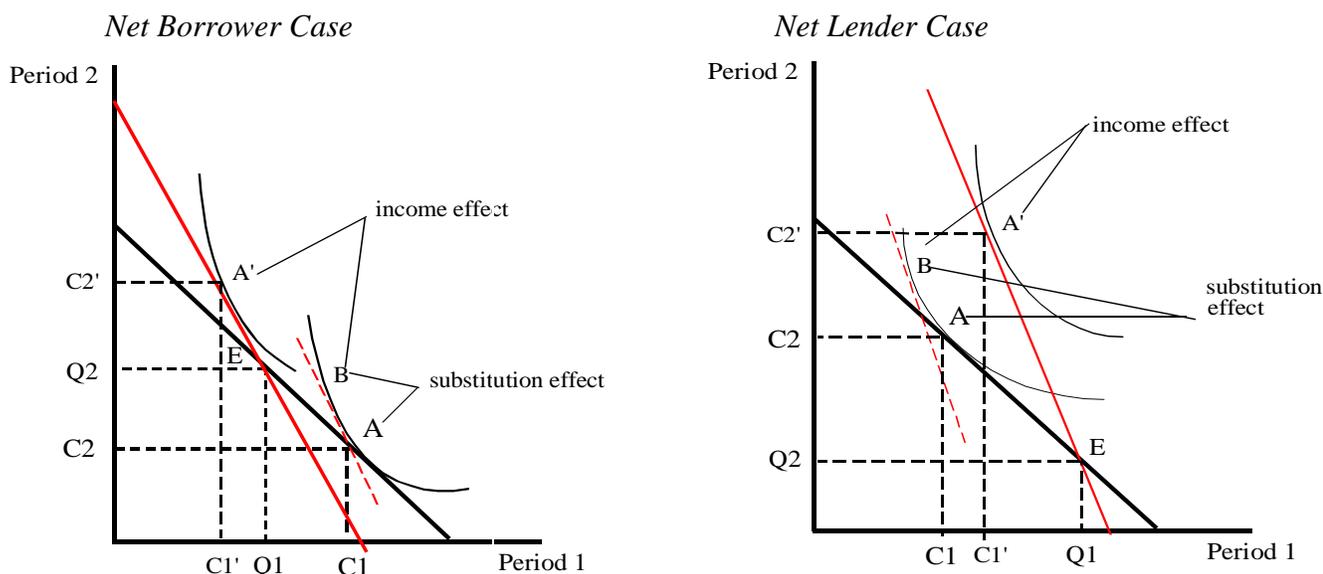
B.   The Firms Investment-Output Decisions (Ch. 5, S&L)

Investment expenditures are additions to the capital stock intended to augment future production. Investment decisions involve *fixed business investment* (plant and equipment), *inventory investment* (changes in stocks of raw materials, unfinished goods, & unsold finished goods), and *residential structures investment* (housing maintenance and new housing).

The *law of motion of the capital stock* describes the capital accumulation process:

$$K_{+1} = (1\text{-}d)K + I$$

This equation says that tomorrow=s stock of capital is the depreciated stock of capital left over from today=s production (d is the rate of depreciation) plus additions due to new investment.

The basic "microfoundations" theory of investment is also known as *the Classical theory of Investment.* The elements of this theory are the following.

Simplifying assumptions:
  B.1)   Investment decisions are made by households

  B.2)   Capital stock is homogeneous across industries

  B.3)   C and K goods are perfect substitutes, so prices of Q, C, and I are all equal to 1

  B.4)   No uncertainty (perfect foresight)

  B.5)   100 percent depreciation rate (i.e., d=1)

This setup adds to the consumption/saving setup an alternative vehicle of saving. Thus, there are now two ways to transfer purchasing power intertemporally: changes in B or in K. The manner in which B transfers purchasing power to the future is determined by the world real interest rate r. The manner in which K does it depends on the domestic production technology.

The Production Technology

The production function of the small open economy determines how much output (Q) is obtained by combinations of capital (K) and labor (L) given the state of the technology (τ):

$$Q = Q(K,L;\tau)$$

This production function has the standard properties from Microeconomics Principles:
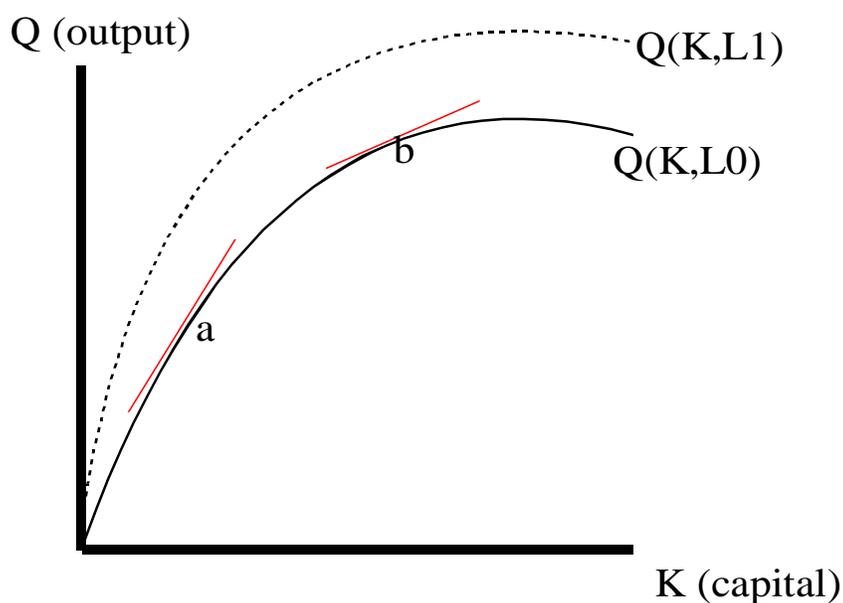
A)   Positive Marginal products:

$$MPK = \Delta Q/\Delta K > 0 \ \ or \ \ \partial Q/\partial K > 0$$

B)   Diminishing marginal products:

$$\Delta MPK/\Delta K < 0 \ \ or \ \ \partial^2 Q/\partial K^2 < 0$$

Output as a Function of Capital, for a Given Labor Input

Budget constraints with Investment Decision (two-period model):

Period-by-period constraints (two-period model with bonds and capital, L & τ constant):

at t=1 $C_1 = Q(K_1) - (B_1 + I_1)$

at t=2 $C_2 = Q(K_2) + (1+r)B_1$

Intertemporal budget constraint:

$$W_1 \equiv (Q_1 - I_1) + Q_2/(1+r) = C_1 + C_2/(1+r)$$

Two-stage optimal intertemporal decision-making:

1)   Choose I_1 and K_2 so as to **maximize total wealth** subject to resource constraint imposed by production function.

2)   Choose consumption and saving so as to **maximize intertemporal utility** given total wealth

Stage 1: Choosing investment to maximize total wealth

Given the law of motion: $I_1 = K_2 - K_1$, and taking K_1 as given ("inherited capital" or natural resource), the relevant choice variable is K_2. Following the same logic from the saving decision, the optimal investment decision is represented by the tangency point between the highest "iso-wealth" curve and the intertemporal production possibilities frontier:
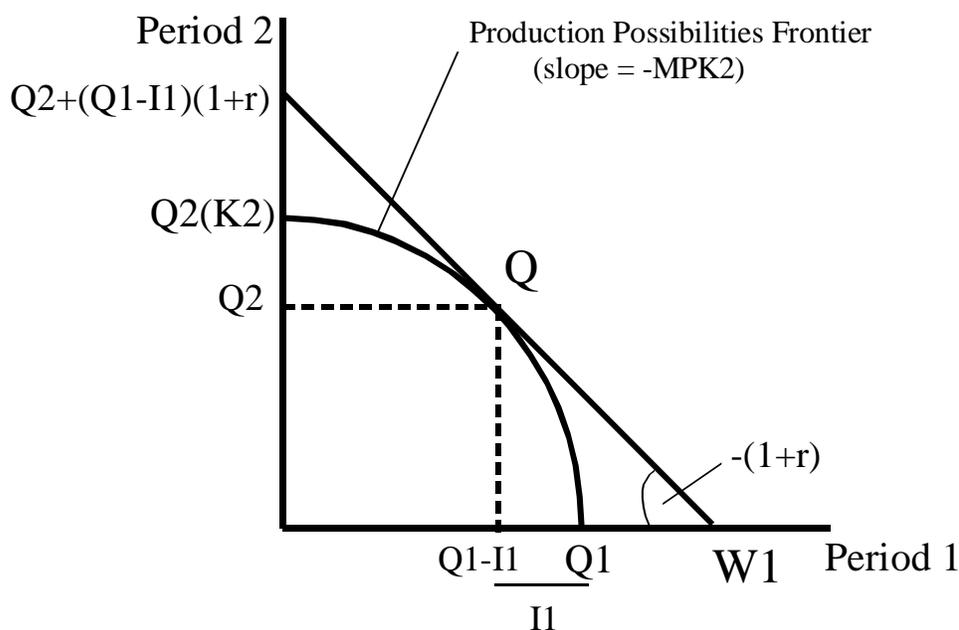
This choice of investment yields maximum wealth among those that are technologically feasible. Intuitively, the optimal investment choice equates the marginal production benefit of accumulating an extra unit of capital (i.e., the marginal product of K_2) with the marginal cost of that extra unit of capital (i.e., the world real interest rate, $1+r$). Thus, the optimal investment choice is determined by this condition:

$$MPK_2 = (1+r)$$

The optimal investment rule extends easily to a multi-period model. The basic principle that marginal costs and benefits of adding to the capital stock between two contiguous periods must be equalized continues to hold. The only difference is that now the depreciation rate appears in the optimal investment rule because the economy does not end at the end of period 2. The optimal investment rule becomes:

$$MPK_2 = (1+r) - (1-d) \quad => \quad MPK_2 = r + d$$

Calculus can be used to derive the optimal investment rule. The constrained maximization problem in the two period case with d=1 is:

$$Max_{[K2]} \quad W_1 \equiv Q(K_1) - K_2 + Q(K_2)/(1+r)$$

The first order condition is: $-1 + (\partial Q_2/\partial K_2)/(1+r) = 0$

$$\partial Q_2/\partial K_2 = (1+r) \quad => \quad MPK_2 = (1+r)$$

An the second order condition holds given the assumption of diminishing marginal products: $\partial^2 Q_2/\partial K_2^2 < 0 \quad => \quad dMPK_2/dK_2 < 0$

Investment demand is a negative function of r, *I(r,)* because of the optimality condition $MPK_2 = (1+r)$ and the diminishing marginal productivity of capital.

MPK (marginal product of capital)

r1

r0

MPK=I(r)

K1   K0   K (capital)
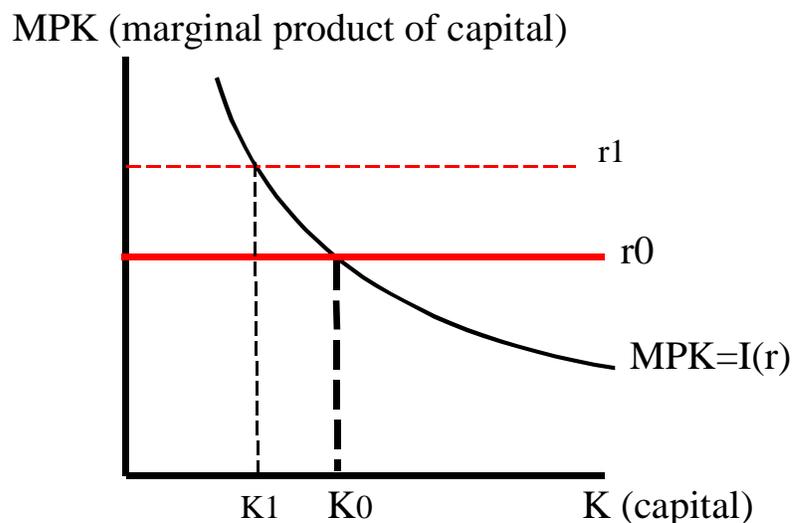
Stage 2:   Choosing consumption given maximum wealth

      Given Q_2 and W_1 from Stage 1, Stage 2 is a standard consumption/saving problem

This solution features what is known as ***Fisherian Separation:*** Investment is chosen independently of preferences.

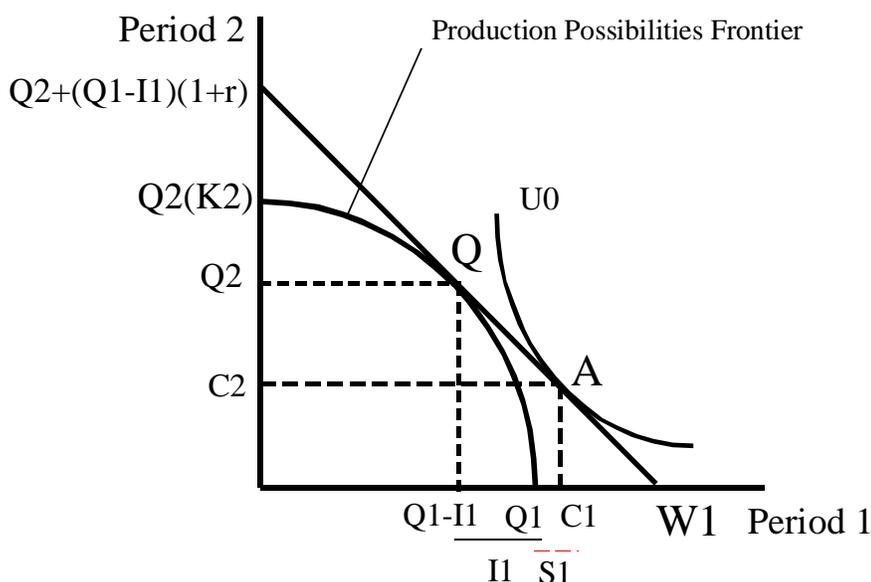*The general equilibrium of a "small open economy" without money*

      The above graph shows how C, S, and I are determined simultaneously in the small open economy given the world interest rate (1+r). The current account is also implicitly determined (as the difference between S and I, see Section C). If this were a closed economy, there could not be borrowing or lending, so r would be an endogenous price that would adjust until the PPF is tangent to an indifference curve.

Decentralizing the saving-investment equilibrium:

      We assumed before that households, or a "benevolent dictator," centralize decisions regarding S and I, but in reality households and firms make these decisions separately (i.e., in "decentralized" fashion). In an economy free of distortions the equilibria of centralized and decentralized economies is identical. In the decentralized setting the following takes place:

      A)   Firms maximize present value of profits (i.e., their market value)
      B)   Households maximize utility given wages and dividends

A key implication of this result is that firms can make optimal investment plans disregarding individual preferences of the various households that own them.

Taxes and Subsidies

The classical investment model can also be used to show how distortions resulting from taxation of capital income and investment subsidies affect capital accumulation. Define the tax rate on capital income as **t** and an investment incentive as **s** (which is a percent of the purchase price of capital).   The optimal investment condition becomes:

$$MPK(1-t) = (r + d)(1-s)$$

C. The current account (Ch. 6, S&L)

Saving and investment are identical by definition in a closed economy. In contrast, in an open economy the gap between the two represents the "current account," which is financed by foreign borrowing or lending. Moreover, since resources devoted to pay interest on foreign debt are generated by exporting goods, there is also a connection between the current account, net foreign borrowing, and the balance of trade.

Accounting Relationships between S, I and CA:

In the small open economy B (B*, in the notation of the textbook) represents net foreign borrowing, also known as "net foreign asset position." This includes all financial assets and is calculated as the sum of all claims minus all liabilities with respect of the rest of the world. If B*>0, the country is a net creditor, if B*<0 the country is a net debtor.

Following accounting principles of the Balance of Payments, and ignoring changes in foreign reserves and "error & omissions," the current account (CA) equals the negative of the capital account (KA):

$$CA = B^* - B^*_{-1} \equiv -KA$$

Note: Textbook refers to CA as net foreign borrowing, which is actually recorded in KA. This does not matter as long as changes in foreign reserves and "errors and omissions" are ignored.

Two implications of BofP accounting are worth recalling:

(1)   A *current account surplus* (*capital account deficit*) is an accumulation of foreign assets.

(2)   At any date t, the net foreign asset position equals the initial position plus the cumulative flow of all borrowing and lending up to that date: $B^*_t = B^*_0 + CA_1 + ... + CA_t$.

Consider the budget constraint for household i:

$$B^i - B^i_{-1} = Q^i + rB^i_{-1} - C^i - I^i$$

Since $Y^i = Q^i + rB^i_{-1}$ and $S^i = Y^i - C^i$ this expression simplifies to

$$B^i - B^i_{-1} = S^i - I^i$$

Add up across households in one country eliminates net domestic financing across residents so:

$$B^* - B^*_{-1} = CA = S - I$$

From the second equality we derive the key relationship between S, I and CA:

$$\mathbf{S = I + CA}$$

This equality states that domestic saving is devoted either to accumulate domestic physical capital or foreign financial assets (i.e. claims on foreign capital). If we define domestic absorption $\mathbf{A \equiv C+I}$, it also follows that:

$$\mathbf{CA = Y - A}$$

so countries run current account deficits when they absorb more than they earn

One more accounting linkage between CA and international trade follows from the accounting definition of CA: exports minus imports of goods and services plus net factor payments (NFP) abroad.

Summing up, the different accounting relations identified here imply four different ways of describing the current account:

(1)   As the change in net foreign assets: $CA = B^* - B^*_{-1}$

(2)   As the saving-investment gap: $CA = S - I$

(3)   As the income-absorption gap: $CA = Y - A$

(4)   As the trade balance plus NFP: $CA = X - IM + NFP$

Hence, once the equilibrium model determines CA, we can say that it has determined the change in holdings of foreign assets, the difference between saving and investment, the gap between income and absorption, and the sum of the trade balance plus net factor payments abroad.

Current Account and Trade Balance Implications of Budget Constraints

The intertemporal budget constraint imposes interesting restrictions on the long-run dynamics of the current account that are critical for policy discussions of financial globalization. Recall that the intertemporal budget constraint for the two-period model is:

$$C_1 + C_2/(1+r) = (Q_1 - I_1) + Q_2/(1+r)$$

Three key implications of this constraint:

(1)   If absorption exceeds income at t=1 it must be lower than income at t=2, and viceversa

(2)   If follows from (1) that if the trade balance at t=1 ($TB_1 = Q_1-C_1-I_1$) is in surplus, the trade balance at t=2 ($TB_2 = Q_2 - C_2$) must be in deficit, and viceversa. In fact, the present value of the trade balance must be zero since the intertemporal budget constraint implies:

$$TB_1 + TB_2/(1+r) = 0$$

(3)   Similarly, if a country runs a current account surplus in period 1, it must run a deficit in period 2 and viceversa. In fact, since $CA = B - B_{-1}$, it follows that in this simple two period case where $B_0=0$ and $B_2=0$:

$$CA_1 + CA_2 = 0$$

The intertemporal budget constraint also has interesting implications in the infinite-horizon case. In particular, the present discounted value of consumption must equal that of output net of investment plus initial interest and principal on foreign debt:

$$C_1 + C_2/(1+r) + ... = (1+r)B_0{}^* + (Q_1-I_1) + (Q_2-I_2)/(1+r) + ...$$

Rewriting this expression in terms of the trade balance we obtain:

$$TB_1 + TB_2/(1+r) + ... = -(1+r)B_0{}^*$$

This condition states that the present discounted value of the trade balance must be equal to the negative of the interest and principal on net foreign assets held in the initial period.

Two key implications of these results:

(1)   Debtor (creditor) countries do not need to run trade surpluses (deficits) every period, but they must combine deficits and surpluses so as to satisfy the present value constraint

(2)   No-Ponzi-scheme restriction limits growth of debt but it does not imply that debt is ever paid in full, only that interest on any perpetual debt is paid every period out of a perpetual trade surplus

The Metzler Diagram

The **Metzler diagram** is an alternative representation of the general equilibrium of the economy represented earlier with the PPF and indifference curves. The diagram uses the S and I curves as functions of "r" and illustrates the determination of CA. The difference is that the diagram focuses on the equilibrium of a single period that corresponds to the two-period model, making the determination of the current account more explicit and facilitating the analysis of the effects of exogenous shocks.

The following is an illustration of the Metzler diagram for a small open economy:
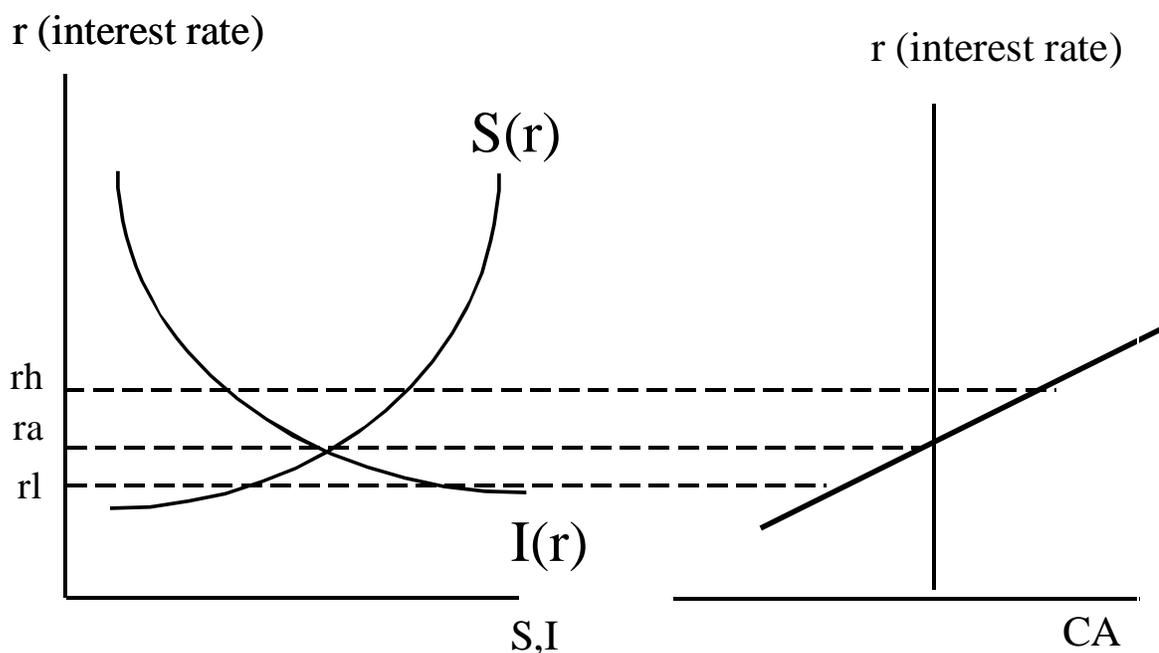
The Response of the Current Account to Exogenous Shocks

(1)   Changes in world interest rate: CA is an increasing function of r (see previous diagram)

(2)   Effects of temporary increase in Q (S to S') and anticipated increase in future Q (I to I", S to S")
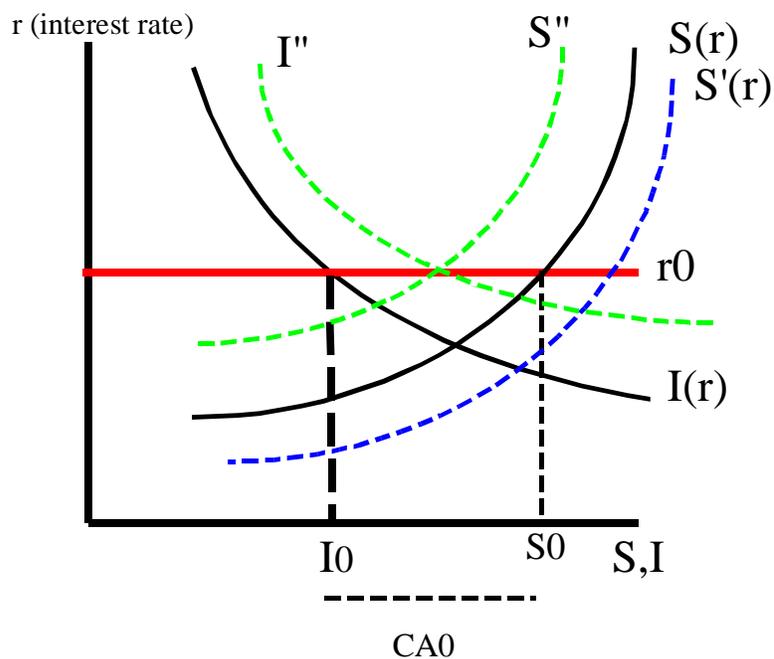
(3)   Investment shocks (oil discovery, economic reform): Positive investment shock shifts
          upward I schedule (as a rise in future productivity).

Notice the implication that current account deficits are an important ingredient of
economic reforms.

(4)   Terms-of-trade shocks: The terms of trade are the relative price of exports in terms of
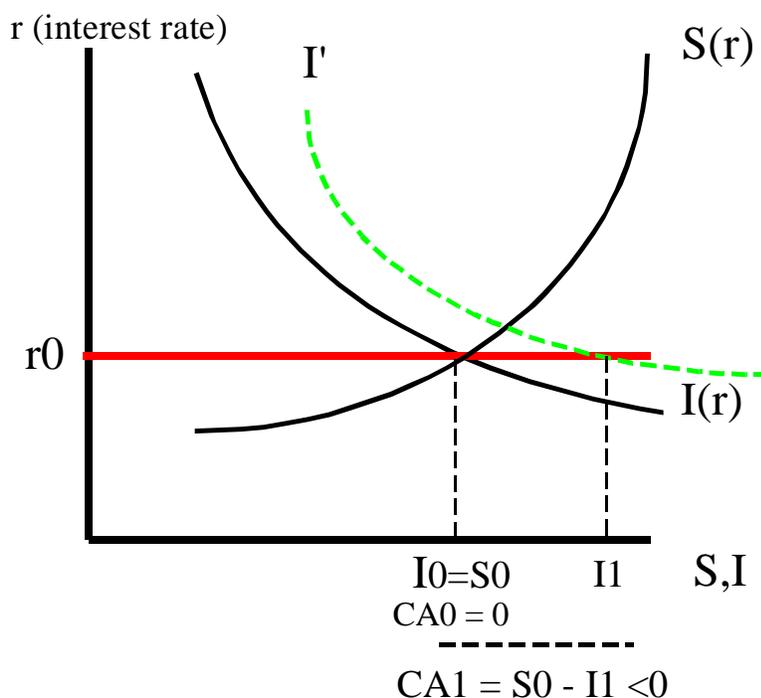   imports (TT=Px/PM).   Changes in TT are similar to output or income shocks since they
          affect the purchasing power of exports. The same logic of permanent v. transitory income
          shocks applies (this analysis is known as the Harberger-Laursen-Metzler Effect).

The "classic" policy lesson for current account adjustment: "Finance temporary shocks, adjust
permanent shocks." This lies behind one of the lending facilities of the IMF.

Limitations on Foreign Borrowing and Lending

(1)   Capital controls: government-imposed limits to access world capital markets

          If controls are complete, current account must balance each period. The interest rate is
determined as in a closed economy. Social welfare falls, even though saving may increase, as
argued earlier in Section 1.1. Shocks studied in (1)-(4) above will now affect **r** instead of **CA**,
and the economy will be isolated of the effects of changes in the world interest rate

(2)   Capital market imperfections: Uncertainty may cause risk and enforcement problems that hamper capital flows
      A.   Insolvency: A borrower may become unable to pay
      B.   Default: A borrower may willingly decide not to pay

World credit markets face the complexity of assessing solvency and complications in enforcing credit contracts. As a result, the adjustment of a small open economy to a given shock may take place in part through the current account and in part through higher domestic interest rates

D.   The Government Sector (Ch. 7, S&L)

Simplifying assumptions and government budget constraints:

      C.1)   Taxes are ***lump-sum*** payments **T** (each household pays fixed amount regardless of income or spending)

      C.2)   Government expenditures (G) do not have direct effects on utility of households or productive capacity of the economy

      C.3)   We view the choice of fiscal policy variables as an exogenous government choice, not as the outcome of explicit maximization of an objective function

      Under the above conditions, the "microfoundations" of the government sector can be studied by focusing only on the implications of government budget constraints. We assume the government can borrow from or lend to the private sector at home or foreign governments or the foreign private sector by issuing government bonds (B^g), and that the government also undertakes public investment (I^g). Hence, at any date t the government budget constraint is:

$$B^g_t - B^g_{t-1} = rB^g_{t-1} + T_t - G_t - I^g_t$$

Since governments typically have negative net asset positions (i.e. they are net debtors), we rewrite the constraint by defining **public debt $D^g \equiv -B^g$**, which yields:

$$DEF_t \equiv D^g_t - D^g_{t-1} = rD^g_{t-1} + G_t + I^g_t - T_t$$

Here, the **public deficit DEF** (i.e., the excess of total public expenditures over total public revenue) is equal, or financed by, the change in net government debt

      If we define **public saving** as $S^g \equiv T - rD^g_{-1} - G$, following the same convention as for the private sector, we obtain:

$$DEF_t \equiv D^g_t - D^g_{t-1} = I^g_t - S^g_t$$

Hence, a public deficit means that public investment exceeds saving.

The Intertemporal Government Budget Constraint:

If government faces the same credit market as households, we can combine the government budget constraints for each date into the following intertemporal constraint:

$$PV(\text{ public expenditures}) = PV(\text{ public revenues})$$

$$(G+I^g) + (G_2+I^g_2)/(1+r) + ... = T + T_2/(1+r) + ...$$

(assuming $B^g_0=0$).

The intertemporal budget constraint of the government imposes limitations on feasible choices of fiscal policy variables. Tax rates, current purchases and public investment cannot be chosen entirely at will, but must be combined to satisfy that constraint.

The Government Sector and the Current Account

Since total national saving S can be broken down into public and private saving, it is possible to establish accounting linkages relating the government sector to the current account. To do so, consider the definition of the current account:

$$CA = S - I$$

Separate both saving and investment into private and public:

$$CA = S^p + S^g - I^p - I^g = (S^p - I^p) + (S^g - I^g)$$

Apply the current-period government budget constraint:

$$CA = (S^p - I^p) - DEF$$

Hence, ***the current account equals the private saving-investment gap minus the public deficit*** Given (S^p-I^p), a rising fiscal deficit is associated with a declining current account. However, the above equation shows that the relationship between DEF and CA *depends* also on how the private saving-investment gap responds to changes in the public deficit.

The definitions of disposable income, wealth, and trade balance can also be expanded to include government:

Financial wealth of the private sector:	$B^p = D^g + B*$
Disposable income:	$Yd = Q + rB^p_{-1} - T$
Private saving:	$S^p = Yd - C = (Q + rB^p_{-1} - T) - C$
National saving:	$S = S^p + S^g$
	$= [(Q + rB^p_{-1} - T) - C] + [T - rD^g_{-1} - G]$
	$= Q + rB*_{-1} - C - G = Y - C - G$

From the definition of National saving, and the fact that I includes public and private investment, we obtain a revised definition of current account in terms of the income-absorption gap:

$$CA = S - I = Y - (C + G + I)$$

Fiscal policy in the Metzler Diagram

Fiscal policy is addedto the Metzler diagram by adding the constants S^g and I^g, which are independent of the interest rate, to the S and I curves. Once this is done we can examine the implications of basic experiments of fiscal policy. We consider in particular the effects of changes in lump-sum taxes T. With these taxes, the intertemporal budget constraint becomes:

$$\mathbf{W_1 \equiv (Q_1 - T_1) + (Q_2 - T_2)/(1+r) = C_1 + C_2/(1+r)}$$

Hence, the effects of lump-sum taxes are similar to changes in income. The effects of changes in T on C, S, and CA can be studied as temporary, permanent, and anticipated income shocks. If we adopt the **balanced budget assumption (i.e., T=G)**, the same holds for changes in G.

Case 1: Temporary Tax-Financed Increase in G

T_1 and G_1 rise by same amount, with (S^G_1, S^G_2, T_2, G_2) constant. C_1 will fall less than W_1, as household borrows from future income to smooth consumption. Private saving S^P_1 will fall and since S^G_1 did not change, S_1 also falls. Effects on I_1 & CA_1 depend on capital mobility.

Small open economy without capital controls: *I constant, CA falls*

r

$S(r)'=Sp(r)'+Sg$ $S(r)=Sp(r)+Sg$

$r0$

$I(r)$

$S1$ $I0=S0$ S,I

$CA0 = 0$

$CA1 = S1 - I0 <0$
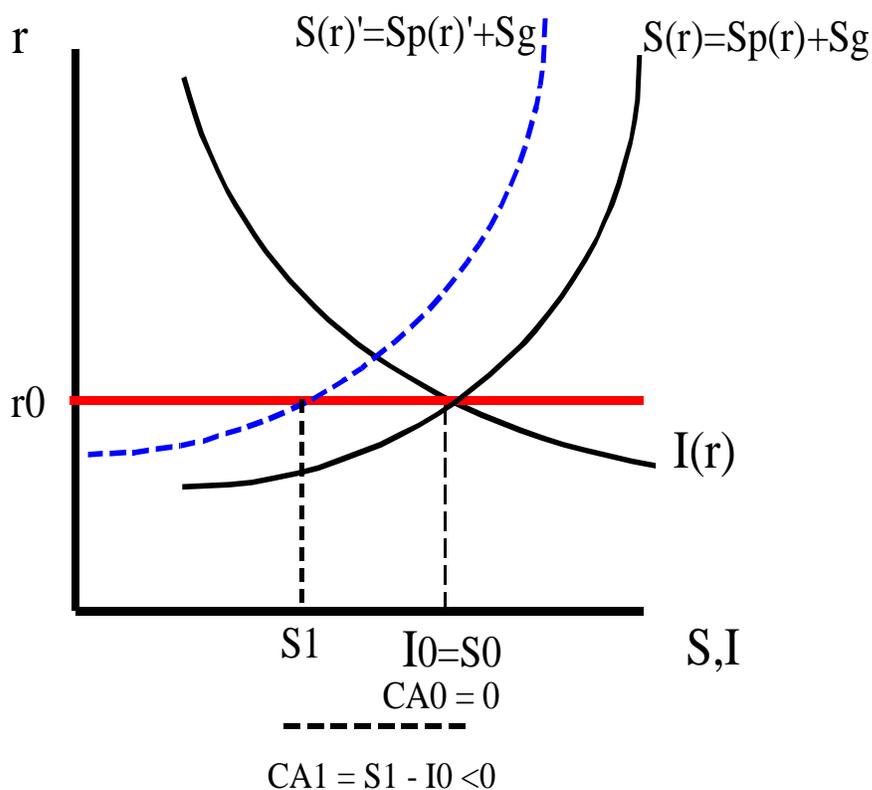
Small open economy with Capital controls: I falls, r rises, CA balanced
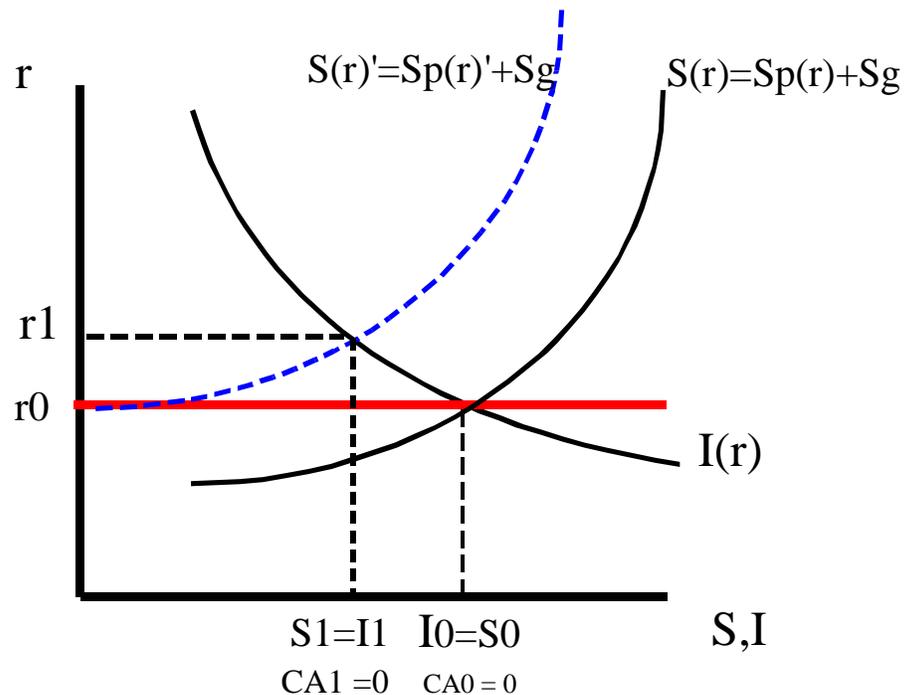
Case 2: Permanent Tax-Financed Increase in G

T and G increase by same amount every period. S_1^p is not affected because change in C_1 is identical to tax increase. To prove it, consider the perfectly smooth case where:

$$C_1 = [(1+r)/(2+r)] \, W_1$$

$$\mathbf{C_1 \; = \; [(1+r)/(2+r)] \, \{Q_1 \text{-} T_1) + (Q_2 \text{-} T_2)/(1+r)\}}$$

If T_1 and T_2 increase by X, *the effect on W_1* is measured by the present value of change in the tax liability:

$$X + X/(1+r) = X[(2+r)/(1+r)]$$

and *the effect of C_1* is equal to the effect on wealth times $\mathbf{[(1+r)/(2+r)]}$

$$\mathbf{[(1+r)/(2+r)]} \, [(2+r)/(1+r)] = 1$$

hence C_1 changes as much as W_1 and S_1^p is unaffected

Ricardian equivalence:
    If a set of strong conditions hold, the timing in which taxes are collected does not affect private consumption as long as the present value of taxes is constant. If *Ricardian Equivalence* holds, the choice between taxes or public debt at any given date is equivalent for private spending, national saving, and the current account.

A Simple proof of Ricardian Equivalence in the two-period case:

Step 1. Households' budget constraint:

$$W_1 \equiv Q_1 + Q_2/(1+r) - [T_1 + T_2/(1+r)] = C_1 + C_2/(1+r)$$

W_1 depends on PV of taxes, not on relative size of (T_1, T_2)

Step 2. Intertemporal government budget constraint (assume $D^g_0 = 0$ but let $D^g_2 \square 0$ to allow government to have a longer life horizon):

$$(G_1 + I^g_1) + (G_2 + I^g_2)/(1+r) = T_1 + T_2/(1+r) + D^g_2/(1+r)$$

Step 3. Fix PV of G and I^g and a value of D^g_2. By Step 2, the PV of tax revenue is set by the PV of total public spending. This fixes the PV of tax payments that enter into W_1 in Step 1. Since (C_1, C_2) depend on W_1, it follows then that (C_1, C_2) depend on PV of tax payments but not on the relative size of T_1 and T_2 (for a given PV of tax liability)

Ricardian Equivalence implies that *as long as the PV of the tax bill is constant*, the choice of financing G_1 using T_1 or D_1 is irrelevant. A current tax cut is anticipated to imply a future tax hike, maintaining PV of tax revenue constant, so households react to the tax cut by increasing S^p to offset exactly the fall in S^g. Note that tomorrow's taxes rise more than the current tax cut because the government must pay interest on its debt.

Ricardian Equivalence requires that the following assumptions hold:
1)   No liquidity, borrowing constraints
2)   Lump-sum taxes
3)   Equal life horizons for government and private agents
4)   Equal credit-market conditions for government and private agents

Distortionary taxation

Most taxes in the real world are not lump-sum taxes, but rather they take the form of direct and indirect taxes proportional to income, spending or wealth. These taxes distort private optimal saving-investment decisions.

Distortionary taxes drive a wedge between the various marginal rates of substitution and relative prices that characterize the equilibrium of a frictionless economy. As a result, these taxes cause misallocations of resources reflected in consumption, saving, labor, investment, and the current account, as private agents favor activities that taxes make relatively cheaper. The misallocations of resources induced by taxation are referred to as ***efficiency losses*** and the corresponding welfare loss is known as the ***deadweight loss***.

One example: taxes on capital income, which imply that the opportunity cost of capital and the intertemporal relative price of consumption is not $(1+r)$ but $(1+r(t-t))$. This tax is analogous to a fall in **r**, so in theory it has an ambiguous effect on S but always a negative effect on I. Evidence suggests investment tends to fall more than saving leading to a fall in CA

The Laffer Curve and Supply Side Economics

Tax policy sets effective tax rates, τ, via marginal tax rates, credits, exemptions and deductions, but cannot fully control actual tax revenue:

$$\text{Income Tax Revenue} = \tau Y(\tau)$$

Income taxes affect labor, investment and saving, and hence affect income. According to the *Laffer Curve*: tax revenue is increasing in tax rates at low tax rates, when distortionary effects are weak, and decreasing at high tax rates, when distortionary effects are strong.
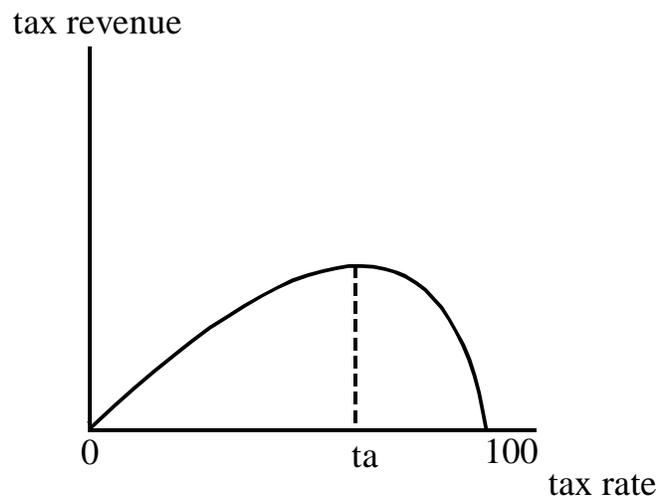
3.   Equilibrium in The Small Open Economy with Money

We now introduce money into the macroeconomic framework we have been developing. Money is added by modeling explicitly its unique role in the economy so as to study the microfoundations of money demand decisions and the process that determines money supply.

Money is the set of financial assets that serves as:

1)   *Medium of exchange* (legal tender that can be used to settle all debts)
2)   *Unit of account* (common denominator for all relative prices)
3)   *Store of value* (maintains its physical value and it is inexpensive to store)
The empirical definition of money changes over time and depends on criteria used to differentiate money. The key criterion is degree of *liquidity* (ability to exchange an asset for cash promptly and at low cost). "Monetary aggregates" in Table 8-1 are ordered from the most liquid to the less liquid.

A.   Interest Rates and Prices in a Monetary Economy (Ch. 8, S&L)

Inflation (π) measures percent change in general price level in a given period. Maintaining the assumption of one-good economy, where prices of (C, I,Q) are identical:

$$\pi = (P - P_{-1}) / P_{-1}$$

Since interest rates are set in terms of monetary units (i.e., nominal terms), the concept of inflation helps distinguish real from nominal interest rates. In particular, for a ***nominal interest rate*** i, the ***real interest rate*** r is:

$$(1+r) = P\,(1+i)/P_{+1}$$

which using the definition of inflation simplifies to

$$(1+r) = (1+i)/(1+\pi_{+1})$$

For small inflation and interest rates these reduces to

$$r = i - \pi_{+1}$$

This real interest rate is an ***ex ante real interest rate*** that embodies a measure of "expected" inflation between dates t and t+1.

B. Money and the Households' Budget Constraint (Ch. 8, S&L)

Assume that money pays zero interest. Since money is a store of value, the households' budget constraints need to be modified to take into account this new vehicle of saving. To illustrate this more easily, we re-define B to be a bond set in nominal terms. Since this is a one-good model, the same price index will apply to convert all real variables into nominal variables.

We still assume only two periods, so the previous assumptions that no assets are inherited at the beginning of life or left as inheritance at the end imply now: $M_0=I_2=B_0=B_2=0$, but we keep $M_2>0$ because, as we will see later, money is needed in both periods as a medium of exchange. Hence, the period-by-period constraints become:

Period 1: $P_1C_1 = P_1(Q_1\text{-}T_1) - P_1I_1 - (B_1+M_1)$

Period 2: $P_2C_2 = P_2(Q_2\text{-}T_2) + (1+i)B_1 + M_1\text{-}M_2$

$$P_2C_2 = P_2(Q_2\text{-}T_2) + (1+i)(B_1+M_1) - iM_1\text{-}M_2$$

The period-2 constraint shows that bonds ***dominate*** money as a store of value because the real return of saving via bonds exceeds that of money by the amount i/P_2. Combining the two constraints yields the ***intertemporal budget constraint***:

$$P_1C_1 + P_2C_2/(1+i) = P_1(Q_1\text{-}T_1\text{-}I_1) + P_2(Q_2\text{-}T_2)/(1+i) - iM_1/(1+i) - M_2/(1+i)$$

divide by P_1:

$$C_1 + P_2C_2/P_1(1+i) = (Q_1\text{-}T_1\text{-}I_1) + P_2(Q_2\text{-}T_2)/P_1(1+i) - iM_1/P_1(1+i) - M_2/P_1(1+i)$$

apply the definition $(1+r) = P_1(1+i)/P_2$ and simplify:

$$C_1 + C_2/(1+r) = (Q_1\text{-}T_1\text{-}I_1) + (Q_2\text{-}T_2)/(1+r) - i(M_1/P_2)/(1+r) - (M_2/P_2)/(1+r) \equiv W_1$$

Hence, the intertemporal budget constraint of households is the same as before except for the terms $-i(M_1/P_2)/(1+r)$ and $-(M_2/P_2)/(1+r)$. The former captures the real ***opportunity cost of holding money balances*** in period 1 in present value (each unit of M_1 has nominal opportunity cost i which is deflated using P_2 because the interest is lost in the second period). Wealth is now $W_1 \equiv W_1^{NF} - i(M_1/P_2)/(1+r) - (M_2/P_2)/(1+r)$, where W_1^{NF} stands for "nonfinancial wealth" and corresponds to the definition of wealth in the economy without money. Note that the real opportunity cost in period 1, $-i(M_1/P_2)/(1+r)$, can also be expressed as $-iM_1/P_1(1+i)$.

C. The Demand for Money: A Transactions Costs Approach

Different theories of money demand propose alternative approaches to model the benefits of holding money (money as a direct source of utility, money as a means to economize transactions costs, money required for transactions or cash-in-advance, and money as an asset). Several of these approaches yield similar results, so we focus on a transactions costs setup.

The Transactions Costs Function

The main element of the transactions costs approach is the assumption that undertaking transactions to buy goods is costly, but holding real money balances ($m \equiv M/P$) helps reduce these costs. The function $S(V)$ determines the real transactions costs per unit of C as a function of "transactions velocity" V, where $V \equiv C/m$. Hence, the more m is held for a given C, the lower the cost. Given $V \equiv C/m$, this is equivalent to assuming that S is increasing in velocity.

We assume a specific functional form for unitary transactions costs: $S=bV^{\gamma}$. Thus, the ***total transactions cost*** is $TC= C(1+bV^{\gamma})$, which using the definition of V simplifies to:

$$TC = C + bm^{-\gamma} C^{1+\gamma}$$

The ***marginal transactions cost*** of an extra unit of expenditures is the derivative of TC with respect to C:

$$MTC = \frac{\partial TC}{\partial C} = 1 + (1+\gamma)bm^{-\gamma} C^{\gamma} = 1 + (1+\gamma)b(C/m)^{\gamma}$$

The ***marginal gain of holding money*** is the marginal *cut* in TC that results from holding an extra unit of m, and is given by the *absolute value* of the derivative of TC with respect to m

$$MTm = \frac{\partial TC}{\partial m} = \gamma b\, m^{-(\gamma+1)} C^{1+\gamma} = \gamma b(C/m)^{1+\gamma}$$

The Intertemporal Budget Constraint and the Optimization Problem of the Monetary Economy

The existence of transactions costs must be considered in the cost of consumption purchases in the intertemporal budget constraint, which becomes:

$$(1+bV_1^{\gamma})C_1 + \frac{(1+bV_2^{\gamma})C_2}{1+r} = W_1 = W_1^{NF} - i_1 \frac{\left(\dfrac{M_1}{P_2}\right)}{1+r} - \frac{\left(\dfrac{M_2}{P_2}\right)}{1+r}$$

Using the definitions $(1+r)\equiv(1+i)/(1+\pi)$, $(1+\pi)\equiv P_2/P_1$, and $V\equiv C/m$ the budget constraint simplifies to:

$$C_1 + bm_1^{-\gamma}C_1^{1+\gamma} + \frac{C_2 + bm_2^{-\gamma}C_2^{1+\gamma}}{1+r} = W_1 = W_1^{NF} - \frac{i_1}{1+i_1}m_1 - \frac{m_2}{1+r}$$

We adopt for simplicity a log-utility function consistent with consumption smoothing:

$$MAX\ U = Ln(C_1) + \frac{Ln(C_2)}{1+\delta}$$

In this case, the marginal utilities of consumption are $MUC_1 = 1/C_1$ and $MUC_2 = 1/[C_2(1+\delta)]$ in date 1 and date 2 respectively.

Households will now choose C_1, C_2, m_1 and m_2 so as to maximize log utility subject to the simplified intertemporal budget constraint derived above. The maximization can be solved using calculus, but it is easier to solve by using the microfoundation principles learned earlier. In particular, households face three tradeoffs along which to optimize: (1) equating the marginal cost and benefit of saving, (2) equating the marginal cost and benefit of holding money in date 1, and (3) equating the marginal cost and benefit of holding money in date 2.

Intertemporal consumption-saving tradeoff:

The marginal cost of sacrificing a unit of C_1 is MUC_1/MTC_1, while the marginal benefit of the extra unit of saving that results from that sacrifice is $[MUC_2/MTC_2](1+r)$. Both the marginal cost and benefit include the terms that capture changes in marginal transactions costs that money helps to manage. To be at an optimum, households equate the marginal cost with the marginal benefit, that is $MUC_1/MTC_1 = [MUC_2/MTC_2](1+r)$. Given the form of the utility function and the transactions costs function, this equality can be expressed as:

$$\frac{C_2}{C_1} = \frac{(1+r)\left[1+(1+\gamma)b\left(\frac{C_1}{m_1}\right)^\gamma\right]}{(1+\delta)\left[1+(1+\gamma)b\left(\frac{C_2}{m_2}\right)^\gamma\right]} = \frac{(1+r)\left[1+(1+\gamma)bV_1^\gamma\right]}{(1+\delta)\left[1+(1+\gamma)bV_2^\gamma\right]}$$

This expression illustrates why we think of money as a distortion. In the absence of money, the intertemporal relative price of consumption that agents face is $(1+r)$, whereas in the monetary economy this price is multiplied by the terms in square brackets. Notice that these terms depend on V, which in turn will depend on the nominal interest rate. The monetary distortion thus plays a key role in explaining how fluctuations in international capital flows affect the economy.

Period-1 money demand tradeoff:

At an optimum, households equate the marginal opportunity cost of holding money in the first period with the corresponding marginal benefit. The cost is the discounted value of the interest given up by holding money, the gain is the marginal cut in transactions costs given by MTm_1:

$$\frac{i_1}{1+i_1} = \gamma b \left(\frac{C_1}{m_1}\right)^{1+\gamma}$$

This expression can now be solved for the demand for real balances in period 1:

$$m_1 = C_1 \left(\frac{i_1}{1+i_1} \frac{1}{\gamma b}\right)^{-\frac{1}{1+\gamma}}$$

This is a money demand equation with an elasticity of $1/(1+\gamma)$ with respect to $i/(1+i)$ and a unitary elasticity with respect to C. The latter implies that instead of solving for money demand, we can think of this as a solution for velocity represented as an increasing function of i:

$$\frac{C_1}{m_1} \equiv V_1 = \left(\frac{i_1}{1+i_1}\left(\frac{1}{\gamma b}\right)\right)^{\frac{1}{1+\gamma}}$$

Period-2 money demand tradeoff:

Again, the opportunity cost of holding money must equal the marginal benefit. Since this is a two-period model, money balances held at the end of period 2 are worthless and thus the marginal opportunity cost converges to 1 (i.e., consider the term $i/(1+i)$ when i goes to infinity). The marginal benefit of holding money is given by MTm_2:

$$1 = \gamma b \left(\frac{C_2}{m_2}\right)^{1+\gamma}$$

This implies that in period 2 money demand is inelastic with respect to the interest rate but still displays unitary elasticity with respect to C. In addition, velocity is now a constant given by:

$$\frac{C_2}{m_2} \equiv V_2 = \left(\frac{1}{\gamma b}\right)^{\frac{1}{1+\gamma}}$$

General properties of Velocity and Money Demand

The money demand equations we derived satisfy the following properties:

(1)   *Homogeneity* (absence of "money illusion"): if P goes up and everything else is constant, M^d increases by the same amount (i.e. agents demand *real* balances)

(2)   *Elasticity with respect to expenditures* is positive and equal to 1: if C increases by 1%, demand for real balances increases 1%.

(3)   *Interest elasticity* is negative and equal to $-1/(1+\gamma)$ in period 1: if i goes up 1 percent, demand for real balances falls by $1/(1+\gamma)$. In period 2 the interest elasticity is zero, but this is an implication of the finite life horizon of the economy.

The transactions costs approach can also be thought of as a theory explaining velocity instead of money demand. In particular, this approach predicts that:

(1)   Velocity is independent of the price level

(2)   Velocity at date 1 depends positively on nominal interest rate (i.e., $v(i)$, with $v'(i)>0$)

(3)   Velocity increases with real consumption and at the same rate

Given these results, we can adopt the following form of money demand for further applications:

$$m^d = C/v(i)$$

D.   The Money Supply Process (Ch. 9, S&L)

The process by which the supply of money is determined has two components:

1)   Monetary policy: actions of the central bank that determine the narrowest definition of money, known as ***monetary base or high-powered money***, *Mh*. *Mh* is defined as the currency in circulation plus any reserves deposited by commercial banks at the central bank. *Mh* can also be defined from the assets side of the central bank's balance sheet as the sum of foreign reserves plus net domestic credit

2)   The Money Multiplier: the process by which deposit-taking and lending operations of commercial banks enlarge the monetary base into broader money supply aggregates.

The Balance Sheet of the Central Bank

Assets	**Liabilities**
Gold and foreign exchange reserves ($\mathbf{B^*_c}$)	Currency (notes and coins)
Net domestic assets	Deposits of commercial banks
Loans to commercial banks ($\mathbf{L_c}$)	Deposits of the Government
Government securities ($\mathbf{D^g_c}$)	Other Liabilities
Other assets	Net worth

In today's *fiat money systems* currency is a liability of the central bank in an accounting sense, since there is no right to convert it into anything else. Exchange rate policy can change this (i.e., with fixed exchange rates there is a right to change domestic for foreign currency at fixed price).

The Fundamental Equation for Changes in the Money Stock

The fundamental equation for changes in base money from the asset side of the balance sheet is:

$$Mh - Mh_{-1} = (D^g_c - D^g_{c-1}) + E(B^*_c - B^*_{c-1}) + (L_c - L_{c-1})$$

where ***E is the nominal exchange rate in units of domestic currency per unit of foreign currency***. The fundamental equation states that changes in high powered money have three sources:

(1)   Changes in holdings of government securities (called ***open market operations***)

These are purchases or sales of government securities held by public by the central bank. Purchases (sales) are made by increasing (reducing) high-powered money.   By altering the supply of bonds these operations affect bond prices and interest rates (recall that interest rates fall when bond prices rise, for example a one-period bond that pays X will cost $Pb = X/(1+i)$).

(2)   Changes in foreign reserves (called ***foreign exchange interventions)***

Central bank buys or sells gold or foreign currency reserves in exchange for domestic currency. A purchase (sell) of foreign currency increases (reduces) Mh. Interventions are identical in accounting terms under fixed or flexible exchange rates. However, in a truly clean floating rate the central bank should not intervene at all.

Since changes in foreign reserves are also the outcome of the balance of payments, the regime of capital mobility plays a role in the link between reserves and base money. With strict capital controls, private capital account is closed and (assuming the public capital account is constant) changes in reserves equal changes in the balance of trade.

(3)   Changes in interest rates on loans to private banks
Through the ***discount window*** the central bank lends to private banks (and in some developing countries even to private firms). In the U.S. the interest rate the Fed charges on these loans is the ***discount rate***. Reducing (increasing) the rate induces banks to borrow more (less) from the central bank thus leading to an increase (fall) in Mh

Each of the above sources of changes in Mh has an element of policy (which is stronger for open market operations), but also a component beyond the control of the government. Hence, not even the narrowest concept of money can be regarded as a truly exogenous policy choice.

The Money Multiplier and the Money Supply (see S&L for a detailed derivation).

The money multiplier is the process of "money creation" by the banking system that links Mh to measures of money supply. For example, M1 (which is the sum of currency in circulation plus demand deposits held by the private sector in commercial banks) can be expressed as:

$$M1 = \varphi\ Mh,$$

where φ is the M1 money multiplier.   Commercial banks create money because their lending operations provide households with funds that are partially returned to banks as new deposits.

Using the definitions of M1 and Mh it transpires that:

$$\varphi\ =\ M1/Mh\ =\ (CU+D)/(CU+R)\ =\ [(CU/D)+1]\ /\ [(CU/D)+(R/D)]$$

where CU is currency in circulation, D are demand deposits held at commercial banks, and R are reserve deposits by commercial banks at the central bank. The above result shows that the money multiplier is a negative function of two ratios: CU/D and R/D.

The central bank plays a key role in determining the supply of money, but it cannot control it totally. The central bank has even less control over the multiplier than over *Mh* because the multiplier depends on decisions of the private sector regarding CU/D and of commercial banks regarding R/D.

E.   Money Supply and the Consolidated Government Budget Constraint (Ch. 9, S&L)

A joint analysis of the fundamental equation of *Mh* and the government budget constraint illustrates crucial linkages between fiscal and monetary policies that are critical for understanding the effects of international capital flows. We derive the consolidated constraint as follow:

(1)   Consider the government budget constraint in nominal terms:

$$D^G - D^G_{-1} = P(G + I^G - T) + iD^G_{-1}$$

(2)   Separate public debt into holdings of the central bank and the private sector

$$D^G - D^G_{-1} = (D^G_C - D^G_{C-1}) + (D^G_P - D^G_{P-1})$$

(3)   Introduce the "fundamental equation" of changes in Mh:

$$Mh - Mh_{-1} = (D^G_C - D^G_{C-1}) + E(B^*_C - B^*_{C-1}) + (L_C - L_{C-1})$$

(4)   Combine (2) and (3), assuming constant L_C for simplicity:

$$D^G - D^G_{-1} = Mh - Mh_{-1} - E(B^*_C - B^*_{C-1}) + (D^G_P - D^G_{P-1})$$

(5)   Combine (1) and (4) to obtain:

$$Mh - Mh_{-1} - E(B^*_C - B^*_{C-1}) + (D^G_P - D^G_{P-1}) = P(G + I^G - T) + iD^G_{-1}$$

Considering that: (a) governments often do not pay interest to the central bank and (b) central bank typically transfers interest on foreign reserves to the government, we finally obtain:

$$Mh - Mh_{-1} - E(B^*_C - B^*_{C-1}) + (D^G_P - D^G_{P-1}) = P(G + I^G - T) + iD^G_{P-1} - E(i*B^*_{C-1})$$

This equation is the ***consolidated budget constraint of the public sector*** (i.e., it puts together the deficits of the treasury and the central bank). This equations yields the key result that ***there are only three ways of financing the public deficit***:

a.   Printing money
b.   Losing reserves at the central bank
c.   Selling government bonds to the private sector

In episodes of very high inflation (e.g., Russia in 1997-98), in which a country has no foreign reserves and households do not buy public debt, the only source of government financing is "printing money." This is know as ***monetization of the fiscal deficit***. Note, however, that in principle adjustments in $(G+I^G-T)$ are always an alternative to monetization.

F.   Equilibrium in the Money Market (Ch. 9, S&L)

In equilibrium, the demand for money must equal the supply of money:

$$M^D = P \ (C/v(i)) = .EQ. = \varphi Mh = M^S$$

Assuming a constant multiplier φ and that Mh is an exogenous policy choice, we can illustrate this equilibrium as follows:
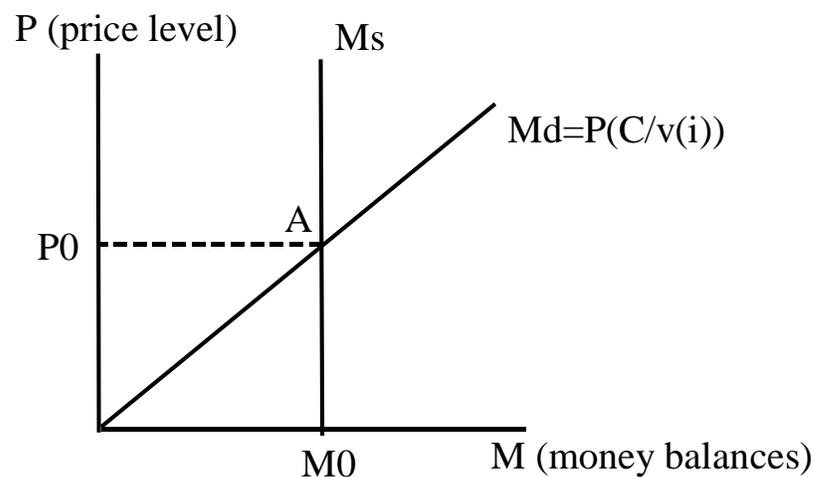
Money demand is plotted as a straight line from the origin in the (M,P) space because agents demand real balances (i.e., for constant $C/v(i)$, the demand for real money balances is constant) and hence P is a constant proportion of M.

Adjustment Mechanisms of the Money Market:

If there is an increase in the supply of money, the money market may adjust in four different ways, or a combination of them:
a.   Rise in prices
b.   Fall in interest rates
c.   Rise in income and consumption
d.   Fall in the supply of money

Which adjustment prevails depends on the assumed relationship between the money market and the rest of the economy, and on whether one allows for the presence of real or nominal frictions disturbing the adjustment of markets. Here are some possibilities:

Case 1.   Small open economy with flexible exchange rate, no capital controls and monetary neutrality   =>   adjustment via prices and the exchange rate

Case 2.   Small open economy with fixed exchange rate, capital controls and Keynesian frictions (i.e., sticky wages and prices) =>   adjustment via a rise in income and consumption and a fall in interest rates (leading to an increase in money demand)

Case 3.   Small open economy with free capital mobility and a fixed exchange rate => adjustment via an endogenous re-adjustment of money supply

G.   Money, Exchange Rates, and Prices (Ch. 10, S&L)

We now consider the interaction between the exchange rate regime and the equilibrium price level and quantity of money. The objective is to illustrate the different dynamics of adjustment that result under fixed and flexible exchange rates. This analysis also serves to introduce two key concepts: purchasing power parity (PPP) and interest rate parity (IRP).

Exchange Rate Arrangements.

a.   *Floating exchange rate regimes*: allow the laws of supply and demand in currency markets to determine E freely. A rise (fall) in E is a *depreciation (appreciation)*.

b.   *Managed exchange rate regimes*: central bank aims to maintain E at a constant level (*fixed exchange rate regime*) or inside a pre-determined range (*exchange rate band*). A rise (fall) in E is a *devaluation (revaluation)*

c.   *Currency arrangements:* a country (or a group of countries) agrees to use a foreign currency (or a common currency), and give up its own domestic currency. These include currency boards, dollarization unilateral or by agreement, and currency unions like EMU.

In practice exchange rate regimes are not as easily classified. Several countries maintain floating regimes, but central banks intervene (*dirty floats),* other countries have fixed exchange rates with *adjustable pegs*, that allow for pre-announced adjustments of the exchange rate or fluctuations within predetermined bands.

The Operation of a Fixed-Exchange-Rate Regime

In a typical fixed-exchange rate regime the central bank announces a commitment to fix the relative price of the domestic currency in terms of one foreign currency or a basket of foreign currencies. In *convertible* fixed exchange rate regimes, the central bank buys or sells domestic money for foreign money at the announced rate. China, for example, does not have a convertible currency.

=>   Central bank intervenes when the currency market rate is pressured to deviate from announced rate (or outside announced band)

=>   Intervention takes place via foreign exchange interventions that alter the bank's holdings of foreign reserves and hence affect Mh and M.   If there is pressure for a devaluation (revaluation), the central bank sells (buys) foreign reserves in the currency market.

Purchasing Power Parity

The concept of Purchasing Power Parity (PPP) is related to the ***law of one price:*** any commodity traded in a unified market free of distortions must have a single price. If international trade takes place in markets of this kind, traded goods must have the same price when expressed in a common currency:

$$P = EP*$$

Arbitrage supports the law of one price: the law of one price must hold because otherwise agents could make gains by buying or selling the commodity for which there is more than a single price.

Purchasing Power Parity is the doctrine that aims to extend the law of one price for single traded goods to the baskets of goods included in national price indexes. It requires four strong assumptions:

(1)   No natural barriers to trade (no transportation costs)
(2)   No artificial barriers to trade (no tariffs or quotas)
(3)   All goods are traded internationally
(4)   All price indexes have identical goods & weights

If assumptions fail, but the corresponding phenomena are constant over time, a weaker version of PPP maintains that the rate of change of the exchange rate must reflect inflation differentials:

$$(1+\pi)/(1+\pi^*) = (1+\varepsilon) \qquad \text{or} \qquad \pi - \pi^* \approx \varepsilon$$

The ratio ***e=EP*/P*** is the ***real exchange rate*** and it measures the relative purchasing power of currencies. ***e*** is also used as a measure of international competitiveness: a rise (fall) in ***e*** indicates that foreign goods are becoming more expensive (cheaper) than domestic goods. A rise (fall) in ***e*** is referred to as a ***real depreciation (real appreciation)***

If PPP holds, ***e*** should be constant over time. The data show, however, that in the short run there are large deviations from PPP (both in levels or in rates of change) but in the long run movements in exchange rates match closely inflation differentials.

International Interest Arbitrage (Interest Rate Parity, IRP)

If there is free international capital mobility, agents will arbitrage the returns on interest-bearing assets of different countries so that they all yield the same return in a common currency. Suppose agents can invest in a domestic nominal bond B and foreign bond denominated in foreign currency B*. B yields $(1+i)$ while B* yields $(E_{+1}/E)(1+i^*)$, hence arbitrage requires:

$$(1+i) = (E_{+1}/E)(1+i^*) \qquad \text{or} \qquad i \approx i^* + \varepsilon_{+1}$$

If this did not occur, agents could make a gain by shifting from bonds of one country to bonds of the other country. Note that under uncertainty, IRP holds substituting the actual change in the exchange rate for expectations of devaluation or depreciation plus a premium to carry this risk.

Equilibrium Determination of P, E and M

The equilibrium determination of prices, money and the exchange rate has 3 components:

(1)   Money-market equilibrium condition: $M^D = P\,(C/v(i)) = M$

(2)   Purchasing power parity: $P = EP^*$

(3)   Interest rate parity (assume $\varepsilon_{+1}=0$): $i = i^*$

From these conditions it follows that in equilibrium the following holds:

$$Mv(i^*) = EP^*\,C$$

Depending on the exchange rate regime, this equation determines equilibrium value of M *or* E:

A)   Fixed Exchange Rate: M endogenous, E exogenous

$$M = EP^*(C/v(i^*))$$

B)   Flexible exchange rate: M exogenous, E endogenous

$$E = [Mv(i^*)] / P^*C$$

If the equilibrium of the real sector of the economy (in this case real consumption *C*) were determined independently from the monetary equilibrium (i.e., if money were "neutral"), then we could take *C* as given and simply focus on the above relationships and the determinants of money demand and supply to discuss the effects of capital flows. However, since money distorts real activity, this in general can only be an approximation.

Monetary Policy and the Exchange Rate Regime

We will assume for the rest of this section that money *is* neutral and focus on the adjustment mechanism of the money market to a change in monetary policy under alternative exchange rate regimes. Consider an example in which high-powered money increases as a result of an open-market operation:

$$Mh - Mh_{-1} = D_C{}^G - D_C{}^G{}_{-1}$$

The open-market operation causes excess supply of M. Households get rid of "excessive" money by exchanging it for foreign currency. Pressure builds up for exchange rate adjustment

a.   Adjustment under Fixed Exchange Rates

Since E is not allowed to adjust, the central bank intervenes by selling foreign-exchange reserves so as to maintain E constant. As reserves fall, Mh falls and the initial monetary expansion is reabsorbed. The central bank must sell reserves until M returns to original level.
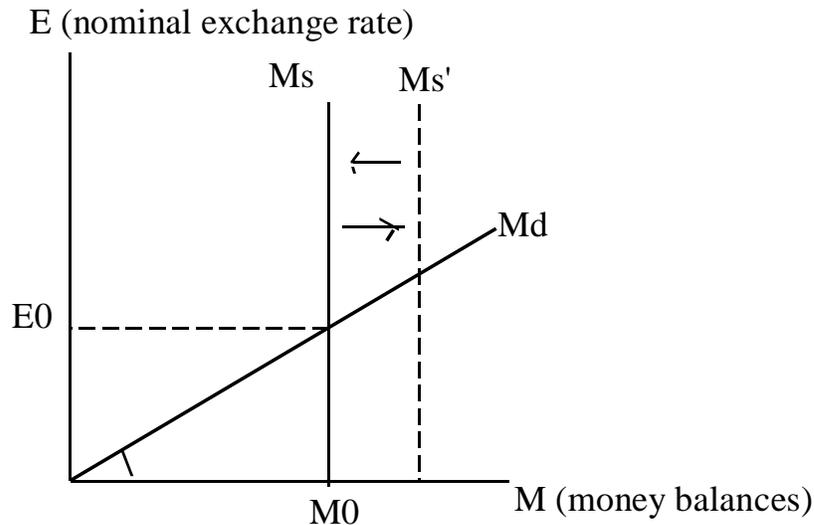
This implies:

$$E(B_C^* - B_{C-1}^*) = -(D_C^G - D_{C-1}^G)$$

so that:

$$Mh - Mh_{-1} = (D_C^G - D_{C-1}^G) + E(B_C^* - B_{C-1}^*) = 0$$

This situation can be illustrated graphically as follows:

E (nominal exchange rate)

Ms   Ms'

Md

E0

M0   M (money balances)

In a fixed-exchange-rate regime with perfect capital mobility the central bank cannot affect M. Attempts to do so only result in losses of foreign reserves. In practice this principle may fail in the very short run because of limits to capital mobility. One can measure an **offset coefficient**: $OC = - [E(B_C^* - B_{C-1}^*)] / [D_C^G - D_{C-1}^G]$, which may be less than 1 in the short run and may also differ across countries, but in the long run, and as long as E is not devalued, it should approach 1

b. Adjustment under Flexible exchange rates

In this case, the rise in the supply of money pressures the exchange rate to depreciate but the central bank does not intervene and lets depreciation take place. As E increases, P also increases (due to PPP), and the process continues until real money balances return to their original level (M and P increase by same proportion)

E (nominal exchange rate)

Ms   Ms'

Md

E1

E0

M0   M1   M (money balances)
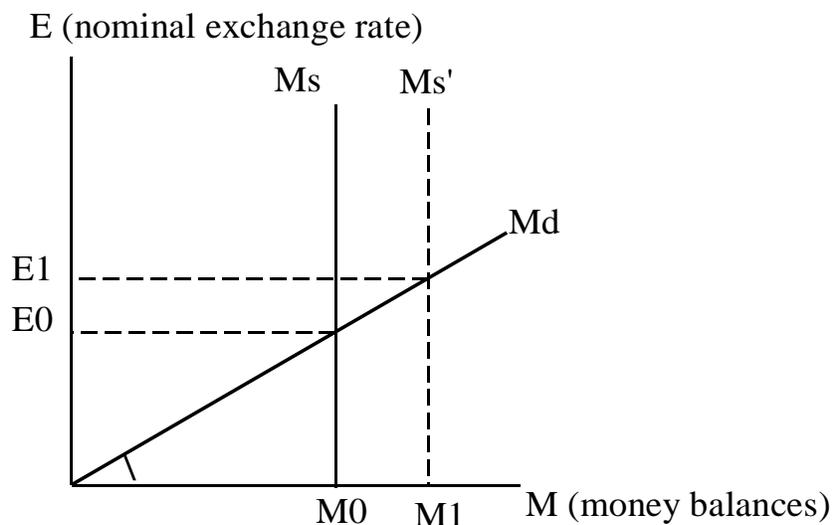
c.   Adjustment under Currency Arrangements

These exchange-rate regimes involve interactions between monetary policies of different countries. Consider the case of a unilateral currency board. Argentina pegs its peso against the U.S. dollar and in the U.S. monetary equilibrium implies:

$$P^* = M^* v^* / C^*$$

Imposing PPP implies:

$$P = E \, (M^* v^* / C^*)$$

Introducing Argentine monetary equilibrium we obtain:

$$M = EM^* \, (v/v^*) \, (C/C^*)$$

An economy aiming to maintain a unilateral peg must adjust to changes in the supply or demand for money in the country to which they are pegging. In particular, the economy takes the P as determined in the foreign economy.

Monetary Effects of an Unexpected Devaluation

Begin from an equilibrium $M = EP^* [C/v(i^*)]$, and assume that an unexpected and permanent increase in E takes place (i.e., a fixed exchange rate prevails after the devaluation).

=>   PPP implies that P rises immediately by the same proportion

=>   Higher E also implies agents need more M (given P*, C and i*)

=>   Agents try to acquire extra M by selling foreign assets to acquire foreign currency and exchange it for domestic currency, pressure for an appreciation surges

=>   Central bank intervenes to prevent the appreciation by selling domestic currency to buy the extra foreign currency agents wish to trade. The central bank gains foreign reserves and agents alleviate the shortfall of money with the bank's sale of domestic currency

=>   The process ends when M has increased by as much as E and P

Central bank is made wealthier and the private sector poorer. Central bank gained real reserves while keeping constant its real liabilities (i.e., M/P), and households sold B* only to maintain M/P unchanged.

Adjustment under Capital Controls

Capital controls break IRP condition and as a result the effects of a monetary expansion (as the one triggered by the open-market operation) are altered as follows:

(1)   Fixed Exchange Rates: Excess supply of M induces excess demand for bonds, driving up bond prices and down interest rates. The fall in *i* to *i'* restores money market equilibrium according to the following condition:

$$\Delta M + M = (EP^*C)/V(i')$$

The lower *i*, with unchanged P, lowers r and leads to a fall in saving and increase in investment, which lowers the current account (smaller surplus or larger deficit). This induces a loss of foreign exchange reserves (since the capital account is closed) and leads to the re-absorption of the increase in money supply.

$$Mh - Mh_{-1} = E(B_c^* - B_c^*{}_{-1}) = S(r) - I(r) = CA(r)$$

As M falls, nominal and real interest rates begin to increase until they return to their original situation. The current account deficits that persist while interest rates remain low are adjustment mechanism. This differs from the sudden adjustment of the capital account in the absence of capital controls. Full adjustment, however, cannot be avoided

(2)   Flexible exchange rates: Adjustment mechanism is still a depreciation of E that results in an immediate increase in P reducing M/P back to the initial equilibrium.

4.   Inflation, Unsustainable Policies, and Balance-of-Payments Crises

We now use the framework developed in the last three sections to study the effects of reversals in international capital flows and currency crises, including some the effects on the real sector of the economy. We first examine the connection between fiscal deficits, monetary policy and inflation under fixed and flexible exchange rates.

A.   Government Deficits and Inflation (Ch. 12, S&L)

The analysis of the determination of money supply showed that, once the public sector is integrated with the central bank, the following condition holds:

$$Mh - Mh_{-1} - E(B_C^* - B_{C-1}^*) + (D^G{}_P - D^G{}_{P-1}) = P(G + I^G - T) + iD^G{}_{P-1} - E(i^*B^*{}_{C-1})$$

Thus, a public deficit is financed by (i) printing money, (ii) losing reserves, or (iii) borrowing from the private sector. We now use this result to study the interaction between fiscal policy, monetary policy and inflation stabilization under alternative exchange rate regimes.

Simplifying Assumptions
a.   Government cannot increase its credit from private agents (i.e., $(D^G{}_P - D^G{}_{P-1}) = 0$)
b.   Mh=M (i.e. the money multiplier equals 1)

With these assumptions, and defining the real public deficit as **DEF**, the consolidated government budget constraint reduces to:

$$M - M_{-1} - E(B_C^* - B_{C-1}^*) = P(DEF)$$

Budget Deficits under Fixed Exchange Rates

We showed earlier that with fixed E the quantity of money is demand-determined. In particular, the equilibrium M is given by:

$$M = EP^* \, [C / v(i^*)]$$

Considering fixed E, PPP, and starting the analysis at a stationary equilibrium with $\pi=0$, we know that: $E=E_{-1}$, $P^*=P_{-1}^*$, $C=C_{-1}$, $i^*=i^*_{-1}$. As a result, $M=M_{-1}$, and hence we know from the consolidated public sector constraint that:

$$- E(B^*_C - B^*_{C-1}) = \ P(DEF)$$

If $M=M^d$ is constant, and if government can only borrow from abroad or from the central bank, then effectively government can only borrow from abroad (central bank financing leads to fall in reserves). Moreover, if private sector financing is not available to the government and the government maintains a deficit, a fixed exchange rate will not be sustainable in the long run because the central bank will exhaust its foreign reserves

=> A government financing its deficit with the central bank avoids inflation as long as the central bank has foreign reserves (PPP implies $\pi=\pi^* = 0$ as long as $\varepsilon=0$)

=> When the central bank runs out of reserves it will have to either devalue the currency and start again with a fixed rate, or move to a floating exchange rate. The collapse of the fixed exchange rate is referred to as a ***Balance-of- payments Crisis***

Budget Deficits under Floating Exchange Rates

If after reserves run out the government switches to a floating rate, money is no longer demand-determined and foreign reserve losses are no longer a source of fiscal deficit financing. Hence $M-M_{-1}= P(DEF)$, or

$$(M - M_{-1})/P \ = \ DEF$$

The real value of the public deficit equals the real value of the change in the quantity of money. In other words, in this case the government is following a policy of ***monetization of the deficit.***

Monetization of the deficit causes inflation through its effect on the monetary equilibrium. To see how rewrite the consolidated public sector constraint as:

$$DEF \ = \ [(M-M_{-1})/M] \ (M/P)$$

Since monetary equilibrium implies $M=PC/v$, and assuming the same "stationarity" conditions as before regarding DEF, C and v, it follows that C/v is constant over time. Hence, M/P is also constant, which implies that the growth rate of M equals inflation. The above condition becomes:

$$DEF = [\pi/(1+\pi)] \ (M/P)$$

Thus, under a floating exchange rate there is a direct relationship between inflation and fiscal deficits: a higher deficit leads to a higher rate of inflation (or of depreciation of the currency since with $\pi^* = 0$, PPP implies $\pi = \varepsilon$)

The budget deficit is being financed here with an ***inflation tax*** on real money balances at the rate $\pi/(1+\pi)$. The inflation tax is a special tax because it is the only tax that does not require an explicit law to authorize it, and it is collected without a collection agency and at a trivial cost.

$\Rightarrow$ Chain of causation of inflation tax: $\uparrow DEF \Rightarrow \uparrow M \Rightarrow \uparrow M^s > M^D \Rightarrow \uparrow B^*_p \Rightarrow \uparrow E \Rightarrow \uparrow P$

B.   Balance-of-Payments Crises: Collapse of Fixed Exchange Rates (Ch. 12, S&L)

If a government adopts a fixed exchange rate policy but fails to eliminate its fiscal deficit, we have established above that it will eventually run out of reserves and be forced to make a policy change. We now consider the process by which this takes place in more detail. The process of collapse of a fixed exchange rate, or currency peg, has three parts:

1.   While ***the exchange rate is fixed***, central bank reserves fall at a smooth, constant rate:

$$- E(B^*_C - B^*_{C-1}) = P(DEF)$$

The central bank will not be able to keep the exchange rate fixed indefinitely, as reserves will be eventually depleted.

2.   Once ***the peg is abandoned***, the exchange rate will have to depreciate at a rate that reflects an inflation rate such that the inflation tax revenue finances the deficit:
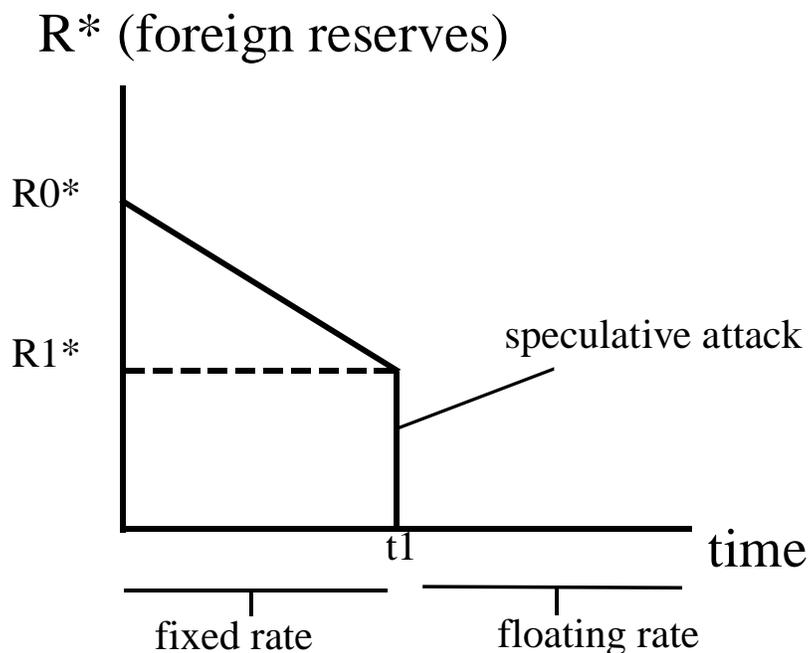
$$DEF = [\pi/(1+\pi)] (M/P) = [\pi/(1+\pi)](C/v(i^*+\varepsilon))$$

π and ε will increase from $\pi = \pi^* = 0$ when $\varepsilon = 0$ to a value such that the inflation tax finances DEF (note that under a floating regime, $\pi = \varepsilon$).

3.   The higher ε after the collapse will result in an increase in the opportunity cost of holding money determined by i, because IRP implies $i = i^* + \varepsilon_{+1}$. Thus, the demand for money will collapse exactly on the date in which the peg is abandoned (i.e., *t1*). The sudden drop in money demand is a sudden ***speculative attack*** on the central bank's foreign reserves, as agents seek to exchange domestic money balances, which are about to be devalued, for foreign assets isolated from devaluation risk. The size of this attack can be measured as:

$$\Delta m = [C/v(i^*)] - [C/v(i^*+\varepsilon_{+1})]$$

Agents anticipate the shift to the floating rate and increase in π. They know that if they hold the excess real balances (corresponding to fixed E) at the end of date *t1* they will suffer capital losses. Before *t1* the lower interest rate makes it optimal to maintain higher real balances. As a result, there is a sudden collapse in money demand and a speculative attack at *t1*. Both the date of the attack and its amount are endogenous in this framework. To see why, consider the following graph illustrating the dynamics of a balance-of-payments crisis:

R* (foreign reserves)

R0*

R1*   speculative attack

                                                                t1   time

                        fixed rate   floating rate

Can Domestic Borrowing be Used to Avoid Inflation or Currency Crises?

YES IF   public debt is sold to private agents and later higher taxes or reduced G are enacted so as to ensure that the present-value government budget constraint holds.

NO IF   fiscal policy is not tightened (or perceived to be tightened) in the future. If fiscal policy does not generate future surpluses, public debt only serves to postpone the day in which the inflation tax needs to be used. In fact, since public debt has to be serviced, borrowing today postpones inflation at the risk of even higher future inflation.

C.   The Inflation Tax and Seigniorage (Ch. 12, S&L)

Inflation tax: loss of purchasing power of real balances $IT \equiv [\pi/(1+\pi)]\,(M/P)$

Seigniorage: purchasing power of newly issued money $SE \equiv (M - M_{-1})/P = [(M-M_{-1})/M]\,(M/P)$

In a stationary situation in which inflation equals the growth rate of money, IT =SE, but this is not true in general. For example, consider a world interest rate shock to a small open economy with fixed exchange rate to a foreign country with zero inflation. PPP implies $\pi=0$, hence IT=0, but a fall in i* would increase money demand and hence SE would have a one-shot increase.

Inflation Tax, the Household's Budget Constraint and Inflation=s Laffer Curve

        By manipulating period-by-period household budget constraints of an economy with money one can show that in an equilibrium with $M/P=M_{-1}/P_{-1}$ (S & L pp. 340-341):

$$C = [Q + r(B_{-1}/P_{-1}) - T] - [(B/P) - (B_{-1}/P_{-1})] - IT$$

Thus, to maintain stock of M/P constant in an environment with inflation, agents yield some of disposable income that otherwise could be used for consumption. Inflation erodes real balances, and hence households need to save merely to keep money demand at the desired level.

As a distortionary tax, inflation features a Laffer curve. A persistent increase in inflation implies an increase in the nominal interest rate, and hence implies that demand for real balances falls. Thus, increases in π reduce the base of the inflation tax. At low π the increase in inflation offsets the fall in money demand, increasing inflation tax revenue, but at high π the fall in money demand is larger than the increase in the tax rate and revenue falls.

Seigniorage under fixed exchange rates

There are certain situations under which an economy with a fixed exchange rate can collect seigniorage. Two examples are:

1)   Global inflation: domestic economy inflates to keep PPP, and required increase in
        domestic M generates seigniorage
2)   Domestic boom: if domestic economy experiences faster growth in Q, demand for money
        rises and government collects seigniorage due to increased equilibrium M (even though in
        this case inflation and IT are zero).

If instead of the fixed exchange rate the economy opted for dollarization or a currency union, it would be effectively giving up this source of government revenue.

D.   Real and Monetary Effects of Balance-of-Payments Crises

Until now we have examined currency crises assuming money is neutral (i.e., assuming that changes in inflation, interest rates, or the exchange rate do not affect the real sector) . However, we developed before an equilibrium framework in which the effects of monetary nonneutrality can be explored. We will consider now a simple example that explores this issue.

Simplifying assumptions:

(A) $d = 1$ (full depreciation of capital)

(B)   Zero initial assets ($B_0 = M_0 = 0$)

(C)   Two period life horizon ($K_3 = M_3 = 0$)

(D)   Labor supplied inelastically by the amount L

(E)   Foreign inflation is zero and the foreign price level is1.
        ($\pi^* = 0$, $P_1^* = 1$, and $i^* = r$ where r is the world interest rate)

(F)   Exchange rate is fixed at a rate of 1 to 1 ($\varepsilon = 0$, $E = 1$)
        =>   by PPP: $\pi = 0$ and by IRP: $i = i^* = r$

1.   *Maximization of Nonfinancial Wealth by Firms*

1.1   Cobb-Douglas Production Technology:

$$Q_t = AK_t{}^a L_t{}^{1-a}, \quad \text{with } 0<a<1 \text{ and } A>0$$

1.2.   Wealth maximization problem of the firm

$$\text{Max}_{[K2]} \quad W_1{}^{NF} = Q_1 - K_2 + Q_2/(1+r)$$

given $K_1>0$ and subject to 1.1.

1.3   Maximize by substituting 1.1 in 1.2:

$$\text{Max}_{[K2]} \quad W_1{}^{NF} = A_1 K_1{}^a L^{1-a} - K_2 + (A_2 K_2{}^a L^{1-a})/(1+r)$$

1.4   First-order condition:

$$-1 + (\partial Q_2/\partial K_2)/(1+r) = 0$$

$$aA_2 K_2{}^{a-1} L^{1-a} = 1+r \quad \rightarrow \quad MPK_2 = 1+r$$

1.5   Solve for optimal investment and wealth:

$$K_2 = [aA_2/(1+r)]^{1/(1-a)} L$$

$$W_1{}^{NF} = A_1(K_1/L)^a L - (K_2/L) L + [A_2(K_2/L)^a L]/(1+r)$$

2.   *Utility Maximization by the Household*

2.1   Logarithmic, isoelastic utility function:

$$U = \text{Ln}(C_1) + \frac{\text{Ln}(C_2)}{1+\delta} \qquad 0 < \delta < 1$$

where the marginal utilities of consumption (MUC) are:

$$MUC_1 = 1/C_1$$

$$MUC_2 = 1/[(C_2(1+\delta)]$$

2.2   Money as a means to economize transactions costs:

Assume a unitary transactions costs function $S(V) = bV^\gamma$, where $V=C/m$ and $m=M/P$. Thus, total transactions costs are $TC = C(1+bV^\gamma)$, which using V simplifies to:

$$TC = C + bm^{-\gamma} C^{1+\gamma}$$

The marginal transactions costs is:

$$MTC = \frac{\partial TC}{\partial C} = 1 + (1+\gamma)bm^{-\gamma}C^{\gamma} = 1 + (1+\gamma)b(C/m)^{\gamma}$$

The marginal *cut* in transactions cost of an extra unit of real money demand is:

$$MTm = \frac{\partial TC}{\partial m} = \gamma b\, m^{-(\gamma+1)}C^{1+\gamma} = \gamma b(C/m)^{1+\gamma}$$

2.3 Wealth constraint:

$$(1+bV_1^{\gamma})C_1 + \frac{(1+bV_2^{\gamma})C_2}{1+r} = W_1 = W_1^{NF} - i_1\frac{m_1}{1+r} - T_1 - \frac{T_2}{1+r} - \frac{m_2}{1+r}$$

where W_1^{NF} is from 1.5 and T are lump-sum taxes levied by government.

2.4 Using the definitions (1+r)/(1+i)/(1+π) and (1+π)/P$_2$/P$_1$, and substituting the definition of V_t the budget constraint simplifies to:

$$C_1 + bm_1^{-\gamma}C_1^{1+\gamma} + \frac{C_2 + bm_2^{-\gamma}C_2^{1+\gamma}}{1+r} = W_1 = W_1^{NF} - \frac{i_1}{1+i_1}m_1 - T_1 - \frac{T_2}{1+r} - \frac{m_2}{1+r}$$

2.5 Utility maximization: choose C_1, C_2, m_1 and m_2 so as to maximize 2.1 subject to the simplified budget constraint in 2.4

2.6 Optimality conditions of the Households Problem:

A) *Intertemporal consumption-saving tradeoff:*

$$\frac{C_2}{C_1} = \frac{(1+r)\left[1+(1+\gamma)b\left(\frac{C_1}{m_1}\right)^{\gamma}\right]}{(1+\delta)\left[1+(1+\gamma)b\left(\frac{C_2}{m_2}\right)^{\gamma}\right]} = \frac{(1+r)\left[1+(1+\gamma)bV_1^{\gamma}\right]}{(1+\delta)\left[1+(1+\gamma)bV_2^{\gamma}\right]} \qquad \textbf{(I)}$$

B) *Period-1 money demand tradeoff:*

$$\frac{i_1}{1+i_1} = \gamma b\left(\frac{C_1}{m_1}\right)^{1+\gamma} \qquad \textbf{(II)}$$

Which implies a money demand equation with an elasticity of $1/(1+\gamma)$ with respect to $i/(1+i)$ and a unitary elasticity with respect to C. The latter implies that velocity can be represented as:

$$\frac{C_1}{m_1} \equiv V_1 = \left(\frac{i_1}{1+i_1} \left(\frac{1}{\gamma b} \right) \right)^{\frac{1}{1+\gamma}}$$

C) **Period-2 money demand tradeoff:**

$$1 = \gamma b \left(\frac{C_2}{m_2} \right)^{1+\gamma} \tag{III}$$

Money demand is inelastic with respect to the interest rate but still displays unitary elasticity with respect to C. Velocity will be a constant given by:

$$\frac{C_2}{m_2} \equiv V_2 = \left(\frac{1}{\gamma b} \right)^{\frac{1}{1+\gamma}}$$

3. Government Budget Constraints

3.1 The central bank starts with foreign exchange reserves $R_0 > 0$. Seigniorage and transactions costs are assumed to be government revenue. The government sets government expenditures G_1 and G_2, and uses lump-sum taxes T_1 and T_2 to make sure that the present value of government expenditures equals that of government revenue plus $(1+r)R_0$. Hence, the period-by-period government budget constraints are:

$$G_1 = bm_1^{-\gamma} C_1^{1+\gamma} + m_1 + T_1 - R_1 + (1+r)R_0$$

$$G_2 = bm_2^{-\gamma} C_2^{1+\gamma} + T_2 + (1+r)R_1 + m_2 - \frac{m_1(1+r)}{1+i_1}$$

These imply that the intertemporal government budget constraint is:

$$G_1 + \frac{G_2}{1+r} = bm_1^{-\gamma} C_1^{1+\gamma} + \frac{bm_2^{-\gamma} C_2^{1+\gamma}}{1+r} + \frac{i_1}{1+i_1} m_1 + T_1 + \frac{T_2}{1+r} + (1+r)R_0 + \frac{m_2}{1+r}$$

The term in the left-hand-side of this equation is an exogenous policy choice of the government, and the last term in the right-hand-side, $(1+r)R_0$, is an exogenous initial condition. In contrast, the terms that represent the present value of seigniorage and transactions costs are endogenous. Hence the need to assume that T_1 and T_2 are set by the government so that the constraint holds (i.e., lump-sum taxes are not an arbitrary choice).
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4.   *Aggregate Resource Constraint of the Economy*

4.1   Combining the wealth constraint of the household in 2.4 with the intertemporal
government budget constraint in 3.1 we obtain the intertemporal resource constraint:

$$C_1 + \frac{C_2}{1+r} = W_1^{NF} - \left(G_1 + \frac{G_2}{1+r} \right) + (1+r)R_0$$

5.   *Equilibrium Consumption Allocations*

5.1   The fixed exchange rate combined with IRP implies that i=i*=r. Using this result, the
consumption-saving tradeoff (equation (I)) simplifies to:

$$\frac{C_2}{C_1} = \frac{(1+r)}{(1+\delta)} \left[\frac{1 + (1+\gamma)bV_2^{\gamma}\left(\frac{r}{1+r}\right)^{\frac{\gamma}{1+\gamma}}}{1 + (1+\gamma)bV_2^{\gamma}} \right]$$

The term in [.] reflects the monetary distortion on the intertemporal allocation of
consumption. The lower opportunity cost of holding money in period 1 implies a higher
m_1 and lower V_1 compared to date 2.   C is therefore cheaper in period 1 and hence the
intertemporal relative price of consumption favors C_1 vis-a-vis C_2.

Define the monetary distortion as H, where H is:

$$H(r) = \left[\frac{1 + (1+\gamma)bV_2^{\gamma}\left(\frac{r}{1+r}\right)^{\frac{\gamma}{1+\gamma}}}{1 + (1+\gamma)bV_2^{\gamma}} \right] < 1$$

Hence, the consumption-saving tradeoff becomes:

$$\frac{C_2}{C_1} = \frac{1+r}{1+\delta} H(r)$$

5.2   The above equation is solved together with the resource constraint 4.1 to find C_1 and C_2:

$$C_2 = \left[\frac{1+r}{1+\delta+H(r)} \right] H(r) \left(W_1^{NF} - \left(G_1 + \frac{G_2}{1+r} \right) + (1+r)R_0 \right)$$

$$C_1 = \left[\frac{1+\delta}{1+\delta+H(r)} \right] \left(W_1^{NF} - \left(G_1 + \frac{G_2}{1+r} \right) + (1+r)R_0 \right)$$

6.   Monetary Equilibrium

6.1   Given C_1 and C_2 from 5.2, the IRP condition ($i_1=r$), and the money demand tradeoffs

$$m_1 \equiv \frac{M_1}{P_1} = C_1 \left(\frac{r}{1+r} \left(\frac{1}{\gamma b} \right) \right)^{-\frac{1}{1+\gamma}}$$

(equations (II) and (III)), the demand for money in periods 1 and 2 is given by:

$$m_2 \equiv \frac{M_2}{P_2} = C_2 \left(\frac{1}{\gamma b} \right)^{-\frac{1}{1+\gamma}}$$

6.2   With P* constant and E fixed, PPP implies that domestic prices are given by P=EP*. This, combined with money demand solutions from 6.1, implies that in order to keep E fixed the central bank must supply M in the amounts:

$$M_1^S = (EP^*) C_1 \left(\frac{r}{1+r} \left(\frac{1}{\gamma b} \right) \right)^{-\frac{1}{1+\gamma}}$$

$$M_2^S = (EP^*) C_2 \left(\frac{1}{\gamma b} \right)^{-\frac{1}{1+\gamma}}$$

6.3   If this were a flexible exchange rate system, the solutions would change. First, note that IRP now implies $i_1=r+\varepsilon$ and by PPP $\varepsilon = \pi=(P2-P1)/P1$. This implies that the nominal interest rate which was exogenous under a fixed exchange rate (facilitating the solutions for V and C) now is endogenous. Second, the monetary equilibrium now takes M_1^S and M_2^S as exogenous, and determines (jointly with the equilibrium conditions for consumption) the domestic price levels P_1 and P_2. Once the two price levels are solved for, PPP determines the nominal exchange rate as E=P/P* in each period.

6.4.   *Exchange Rate Regime Equivalence*: An important corollary of 6.3 and 6.2 is that in the model we explored there is always a monetary policy under a floating exchange rate regime that yields identical results as a particular managed exchange rate regime. For instance, if the managed regime is a fixed exchange rate (i.e., $\varepsilon=0$), one could generate the same equilibrium outcome with a flexible exchange rate by setting monetary policy so that $P_1=P_2$, and hence $\pi=0$. The lesson from this is that what is relevant is not so much the exchange rate regime but the combination of policies that are behind it.

7.   *The Current Account*

7.1   The exercise already includes all the results necessary to show the determination of the equilibrium level of the current account (i.e., the S-I gap):

(1)   Productivity shocks have similar effects as in the model without money (this is because we did not assume that money balances are used for investment expenditures)

(2)   Shocks to *r* have income and substitution effects on S and I as in the nonmonetary model, but in addition they affect the monetary equilibrium in period 1, and also alter the intertemporal relative price of consumption (adding an extra effect on C_1, C_2, S_1 and CA_1).

8.   *Balance-of-Payments Crises*

If the central bank tries to supply more money than indicated by the solutions found in 6.2, it will induce a fall in foreign reserves, and if reserves are depleted it will trigger the collapse of the fixed exchange rate. The process can be described as originating in a fiscal deficit in a manner analogous to the one explored earlier. Note, however, that in this two period model the speculative attacks can only occur in date 1. Hence, instead of exploring how long a fixed exchange rate can last given a fiscal deficit, we determine how large a fiscal deficit needs to be to trigger a speculative attack in date 1.

This is how a balance-of-payments crisis would work in the two-period model. Consider a situation in which the government widens the date-1 public deficit by lowering T_1, hoping to increase T_2 so as to satisfy its intertemporal government budget constraint. Since this switch would in principle leave optimal money demand and consumption choices unaltered, it follows from the date-1 government budget constraint that foreign reserves fall (T_1 and R_1 fall by the same amount). If R_1 needed to fall to zero (or to some critical minimum level the central bank decides it must keep), then the fixed exchange rate regime would not be sustainable. The exchange rate would be devalued as needed for inflation and the higher nominal interest rate that it implies to close the budget gap.

While with the two period model we lost the ability to determine the date of the crisis as we did before, we can now discuss the effects of the crisis on the real sector of the economy. We established that, in equilibrium, C falls as the interest rate increases (see the solution for C_1). Hence, the collapse of money demand that occurs on the date of a speculative attack against the fixed exchange rate will be larger that when we assumed monetary neutrality, because the volume of consumption transactions will decline as the interest rate increases. We can infer from the graphical analysis of the balance of payments crisis that in an infinite-horizon case this would also make the crisis occur earlier than in the case of monetary neutrality. The current account improves on the date of the speculative attack, since consumption will fall while output and investment remain constant (i.e. the gap S-I widens).

If the model were generalized to include investment expenditures in the transactions costs function and an endogenous labor supply decision by the households, the increase in the nominal interest rate would also act like a distortionary tax on capital accumulation and the real wage, leading to a fall in investment and labor supply.

9.   *Uncertainty, Credibility, and the Syndrome of Exchange Rate Based Stabilizations*

One key feature of managed exchange rate regimes is that economic agents, living in an uncertain world, form expectations of how long such regimes will be maintained. When agents attach some probability to the collapse of a managed exchange rate regime, we say that the regime lacks *"credibility."* The expectations of devaluation will be reflected in the nominal interest rate because of the ex-ante nature of the IRP condition. Hence, an economy with a fixed

exchange rate but less than fully credible will not feature a nominal interest rate equal to the world interest rate, as we assumed before, but will pay a "credibility" premium. The higher interest rate alters money demand and the equilibrium allocations of the real sector of the economy (for example, the monetary distortion H identified in 5.1 would be triggered simply by the expectation of devaluation, even if the prevailing regime is that of a fixed exchange rate).

A more general version of the model we examined here can be used to explore whether the theory can account for the currency crises and economic fluctuations that we observe in countries with fixed exchange rate regimes that lack credibility. This is an important issue because several developing economies have attempted disinflation programs anchored on fixed-exchange rate regimes that have been observed to trigger a striking pattern of boom-recession cycles. Moreover, several of these disinflation plans have ended up collapsing with a devaluation of the currency. Two articles by Mendoza and Uribe ("Devaluation Risk and the Syndrome of Exchange Rate Based Stabilizations," NBER WP 7014, 1999 and "The Business Cycle of Balance of Payments Crises," NBER WP 7028, 1999) show that lack of credibility can be an important factor in accounting for the empirical regularities of Mexico's failed exchange rate based stabilization plan of 1987-1994.