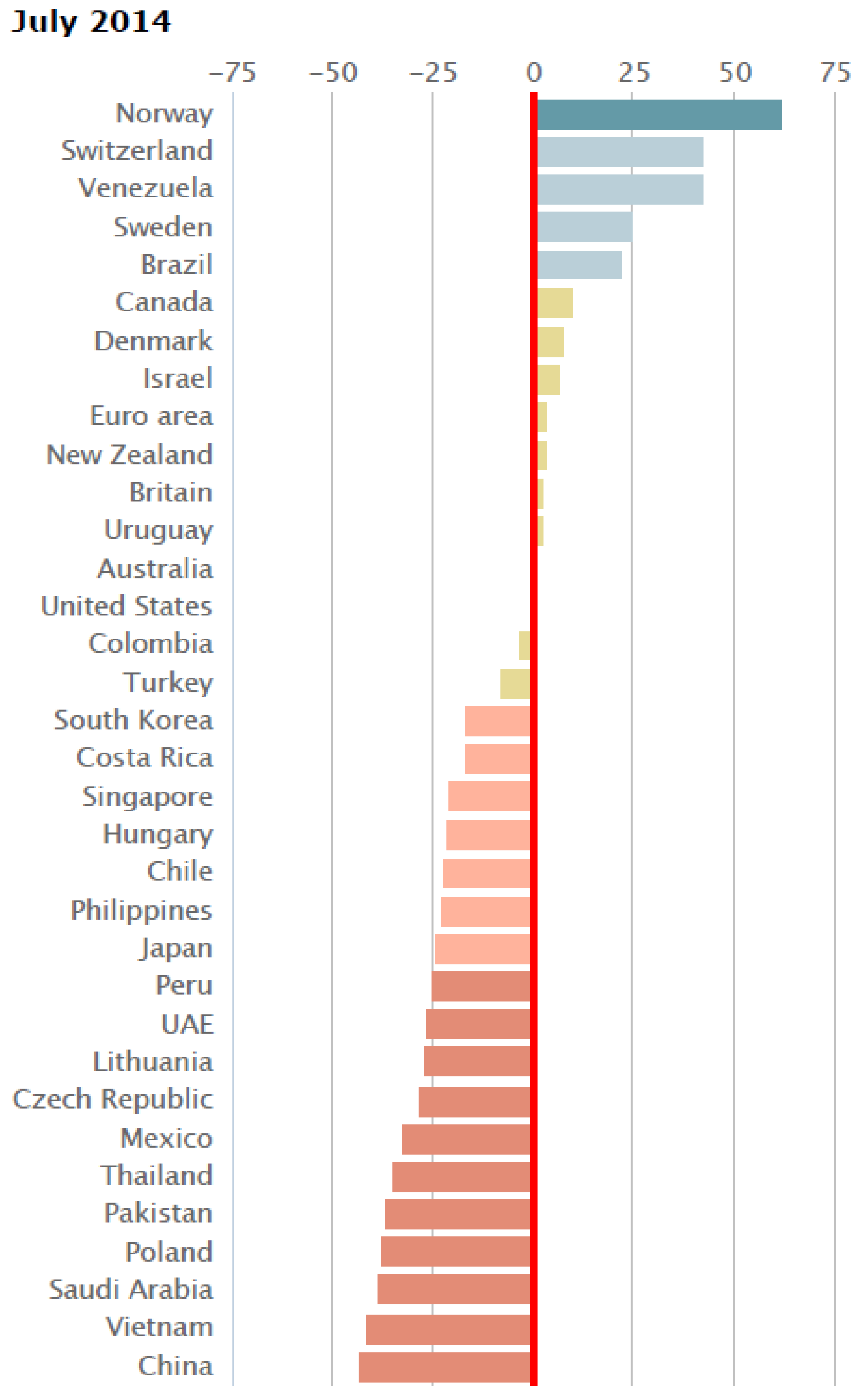
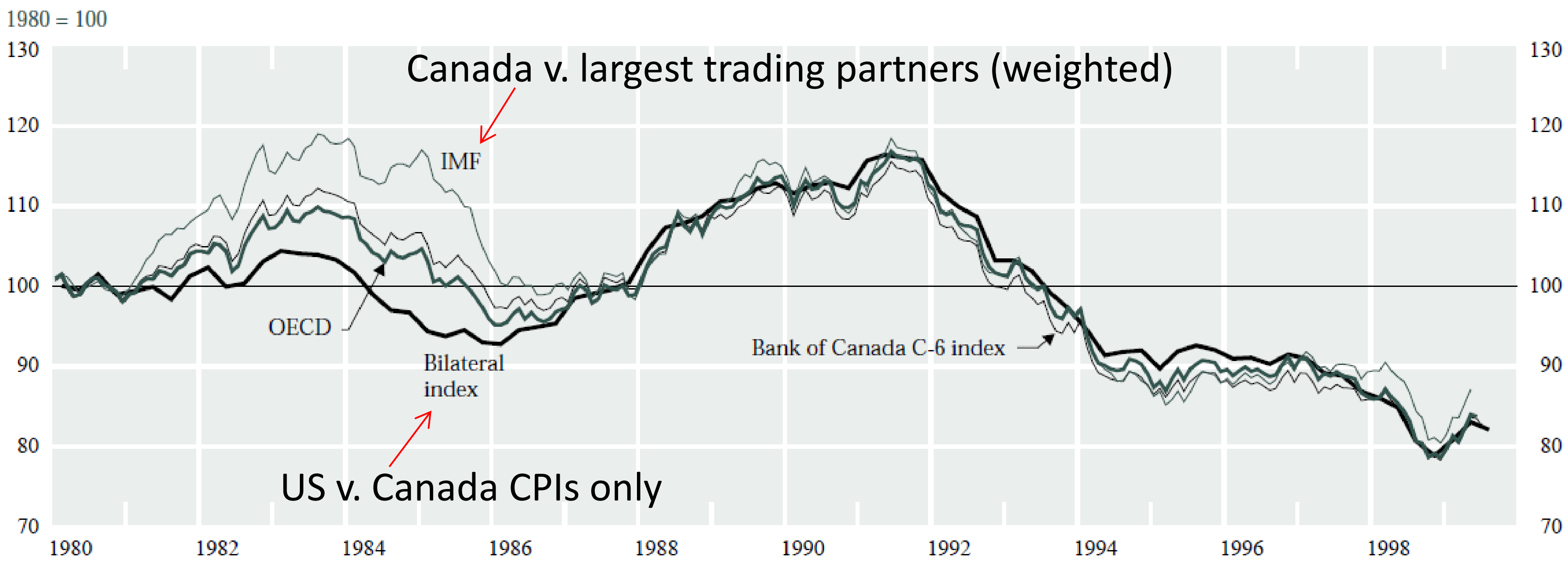
# Money, Exchange Rates and Prices under Neutrality

# The Big Mac Index

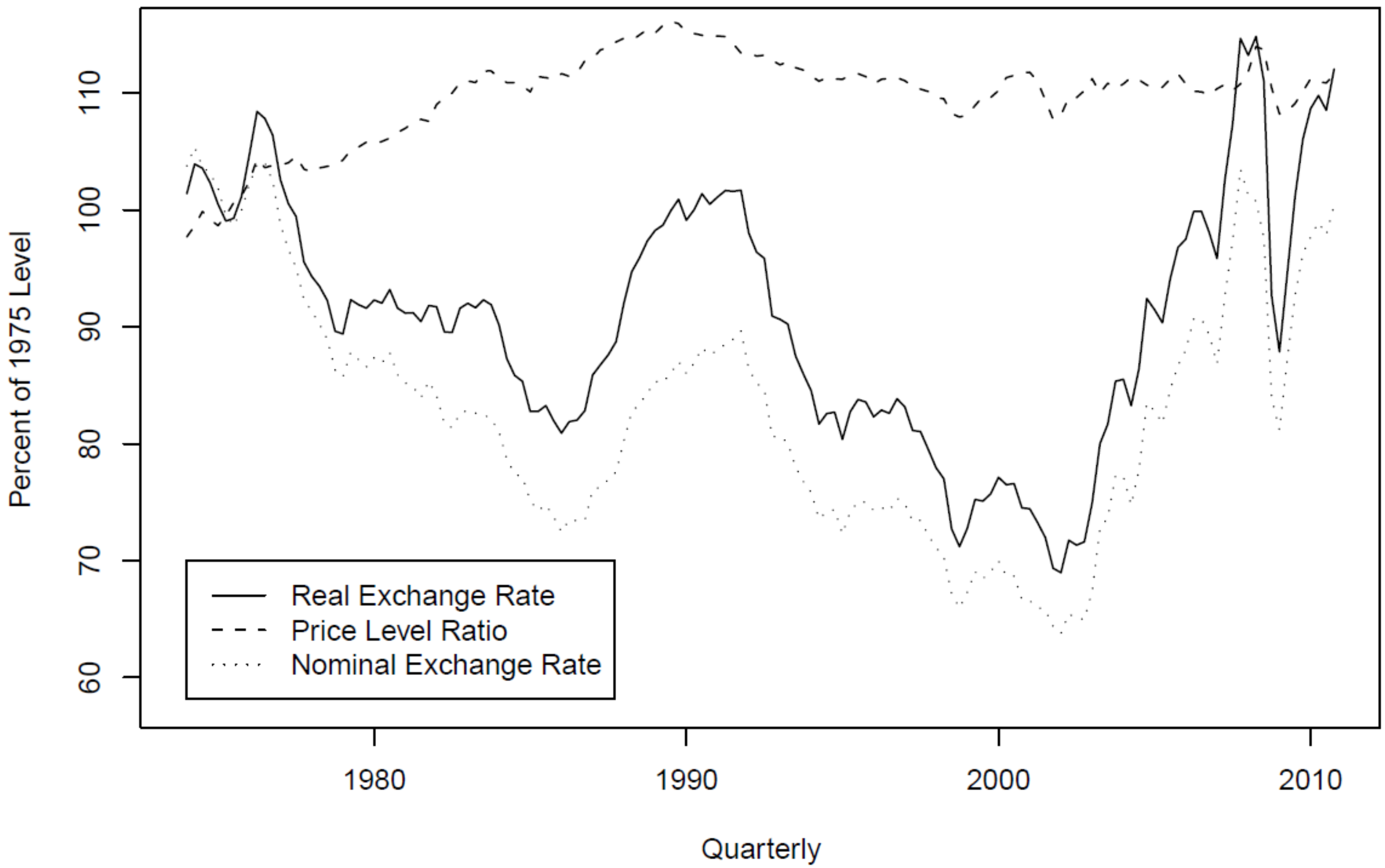


# U.S.-Canada Real Exchange Rate: 1980-1999




Increase (decrease) in the indexes measures a real appreciation (depreciation)

# U.S.-Canada Real Exchange Rate and Components (1975=100)




Increase (decrease) in the indexes measures a real appreciation (depreciation)

# Equilibrium of P, E and M

1. Money market equilibrium condition

$$M^D = P\ (C/v(i)) = M$$

2. Purchasing power parity condition

$$P = EP^*$$

3. Interest rate parity condition (assume $\varepsilon_{+1}=0$)

$$i = i^*$$

4. Equilibrium condition

$$Mv(i^*) = EP^*\ C$$

# Equilibrium and the Ex. Rate Regime

1. Under fixed exchange rate, solve for M (equilibrium determines M with E exogenous)

$$M = EP^*(C/v(i^*))$$

2. Under floating exchange rate, solve for E (equilibrium determines E, with M exogenous)

$$E = [Mv(i^*)] / P^*C$$

- This assumes C is also exogenous (determined separately from money market=>money is neutral)

# Monetary policy and the Exchange Rate Regime

- How does the exchange rate regime alter the effects of monetary policy?
- Consider a monetary expansion via OMO under perfect capital mobility (and assume P*=1)
- From the "fundamental equation" of the CB balance sheet:

$$Mh - Mh_{-1} = D_C^G - D_C^G{}_{-1}$$

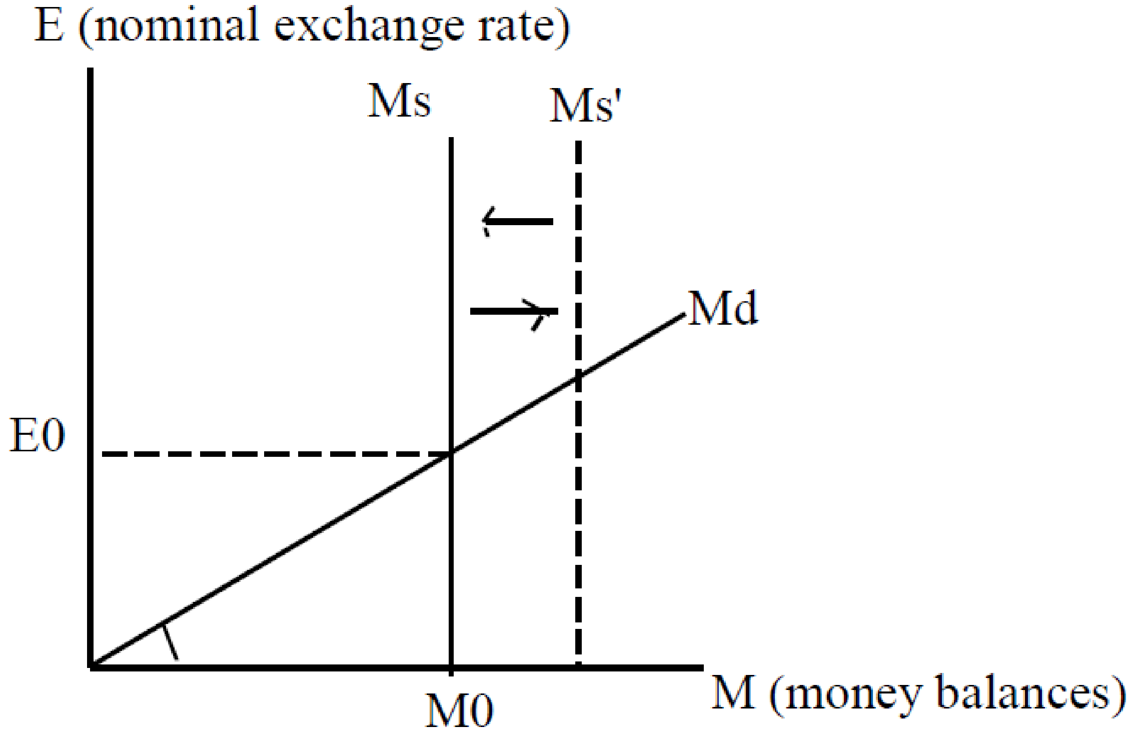
- Starting from equilibrium, this causes "excess supply" of money.
- Private agents get rid of excess money by buying foreign currency=> pressure for adjustment in E

# MP under Fixed Exchange Rates

- CB must sell foreign reserves to keep E fixed, which reduces Mh until M returns to original level

$$Mh - Mh_{-1} = (D_C{}^G - D_C{}^G{}_{-1}) + E(B_C{}^* - B_C{}^*{}_{-1}) = 0$$

$$E(B_C{}^* - B_C{}^*{}_{-1}) = -(D_C{}^G - D_C{}^G{}_{-1})$$



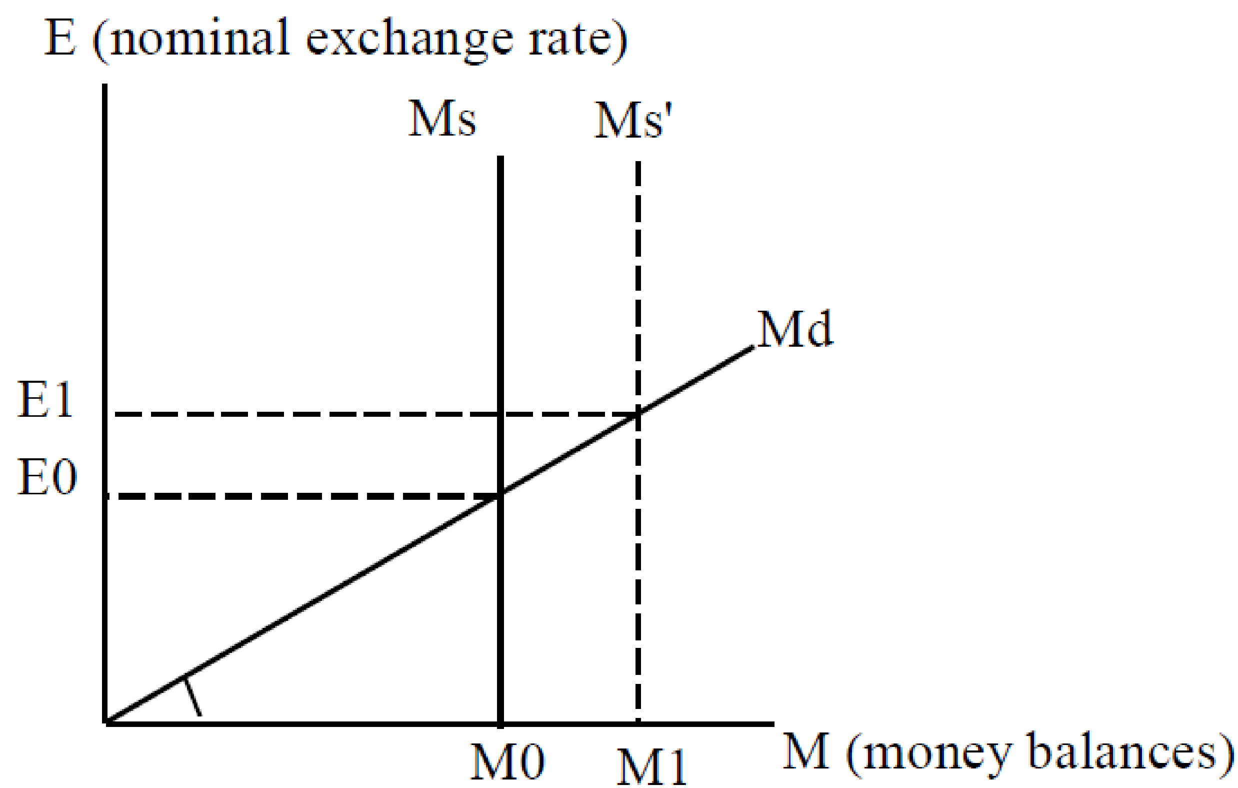
# MP under fixed Exchange Rates

- *A central bank under a fixed exchange rate regime and perfect capital mobility cannot conduct monetary policy. Any attempt to change M is fully offset by changes in reserves*
- In very short run the offset may be less than 1 but eventually, if E remains fixed, it is 1
- Offset (exchange market pressure) coefficient:

$$OC = - [E(B_C{}^* - B_C{}^*{}_{-1})] / [D_C{}^G - D_C{}^G{}_{-1}]$$

# MP under floating Exchange Rates

- Excess M supply causes depreciation: E and P rise, due to PPP, until M/P returns to original level (i.e. monetary policy is also ineffective)



# MP under Currency Arrangements

- Unilateral currency board (Argentina 91-02)

1. U.S. monetary equilibrium (floating):

$$P^* = M^* v^* / C^*$$

2. PPP implies:

$$P = E\,(M^* v^* / C^*)$$

3. Monetary equilibrium in Argentina implies:

$$M = EM^*\,(v/v^*)\,(C/C^*)$$

=>Country with unilateral board or peg must adjust to changes in supply or demand for money in the country of the currency it follows

# "Remonetization:" Monetary Effects of an unexpected devaluation

- Managed Ex. Rates are often introduced after initial surprise devaluation (e.g. Mexico 1988)

1. Start at equilibrium and introduce a unexpected, permanent rise in E
2. PPP=> immediate increase in P
3. Given P*, C and i*=> M must increase $M=EP^*[C/v(i^*)]$
4. Agents sell foreign assets to get domestic currency => pressure for appreciation
5. CB sells reserves because it is now under a peg
6. Process continues until M/P returns to original level
7. At the end, CB is richer (more foreign assets) and private sector poorer (less foreign assets, same M/P)

# Monetary expansions under Capital Controls

- IRP condition no longer applies.

1. Fixed E: Adjustment is slower and via different mechanism, but increase in M is fully offset and MP is still ineffective
   a) Excess M supply leads agents to buy bonds=> bond prices rise, “i” falls: $\Delta M + M = (EP^{*}C)/V(i')$
   b) Given P, lower “i” means lower “r”=> S falls, I rises=>CA deficit=>fall in reserves via BoP

$$Mh - Mh_{-1} = E(B_c^* - B_c^*{}_{-1}) = S(r) - I(r) = CA(r)$$

2. Flexible E: Adjustment is still via depreciation (immediate rise in P via PPP)

# Inflation, Unsustainable Policies and Balance-of-Payments Crises

# Fiscal deficits and inflation

- Recall the consolidated budget constraint of the central bank and the government:

$$Mh - Mh_{-1} - E(B^*_C - B^*_{C-1}) + (D^G_P - D^G_{P-1}) = P(G+I^G-T) + iD^G_{P-1} - E(i^*B^*_{C-1})$$

  – A primary fiscal deficit can only by financed by printing money, losing reserves or borrowing from private sector

- How does this affect interaction between fiscal and monetary policies under different ex. rate regimes?
- Simplified consolidated constraint:

$$M - M_{-1} - E(B^*_C - B^*_{C-1}) = P(DEF)$$

where: $P(DEF) = P(G+I^G-T) + iD^G_{P-1} - E(i^*B^*_{C-1})$

– …also made two simplifying assumptions

1. Private sector unwilling to buy more gov. debt
2. The money multiplier equals 1 (M=Mh)

# Fiscal deficits under Fixed Exchange Rates

- From last class, with E fixed the monetary eq. determines M (demand-determined quantity of money)

$$M = EP^* \, [C / v(i^*)]$$

- Given PPP and zero inflation (start at stationary eq.):

$$E=E_{-1},\ P^*=P_{-1}^*,\ C=C_{-1},\ i^*=i^*_{-1} \quad \Rightarrow \quad M=M_{-1}$$

- Hence, the consolidated constraint implies:

$$-\,E(B^*_{C} - B^*_{C-1}) = P(DEF)$$

1. Fixed E is not sustainable in the long run if DEF>0!!
2. Financing DEF via sale of reserves prevents inflation only as long as reserves last (PPP implies $\pi=\pi^* = 0$ as long as $\varepsilon=0$)
3. When reserves are exhausted, CB is forced to devalue (BoP crisis), and then try to fix again or switch to float

# Fiscal deficits under Floating Ex. Rates

- Under floating, since M is no longer demand-determined, the consolidated constraint yields:

$$M - M_{-1} = P(DEF)$$

$$(M - M_{-1})/P = DEF$$

  - Real value of deficit equals the real value of money creation (monetization of the deficit)

- This causes inflation as we saw last class, and we can calculate how much

1. Start by rewriting the above result:

$$DEF = [(M - M_{-1})/M]\,(M/P)$$

2. Remember at stationary equilibrium C, v, DEF are constant => C/v constant=> M/P constant => growth of M must equal growth of P:

$$[(M-M_{-1})/M] = [\pi/(1+\pi)]$$

3. From 1. and 2. we get that:

$$DEF = [\pi/(1+\pi)]\ (M/P) = [\pi/(1+\pi)](C/v(i^{*}+\pi))$$

- This condition determines the eq. $\pi = \varepsilon$ at which M/P can pay for DEF
- Primary deficit is paid for by the inflation tax on real balances at the rate $\pi/(1+\pi)$
- This is a special tax because it is free of the legislative process that governs all other taxes
- Chain of causation for a higher deficit:

$$\uparrow DEF \Rightarrow \uparrow M \Rightarrow \uparrow M^{s} > M^{D} \Rightarrow \uparrow B^{*}_{p} \Rightarrow \uparrow E \Rightarrow \uparrow P$$

# BoP Crises: The Collapse of Fixed Exchange Rate Regimes

- The Salant-Henderson-Krugman model
  - A country that fixes E but keeps running a fiscal deficit will lose reserves and be forced to abandon the fixed rate
- The process has three steps:
  1. Fixed E period: CB reserves fall at constant rate

$$- E(B^{*}_{C} - B^{*}_{C-1}) = P(DEF)$$

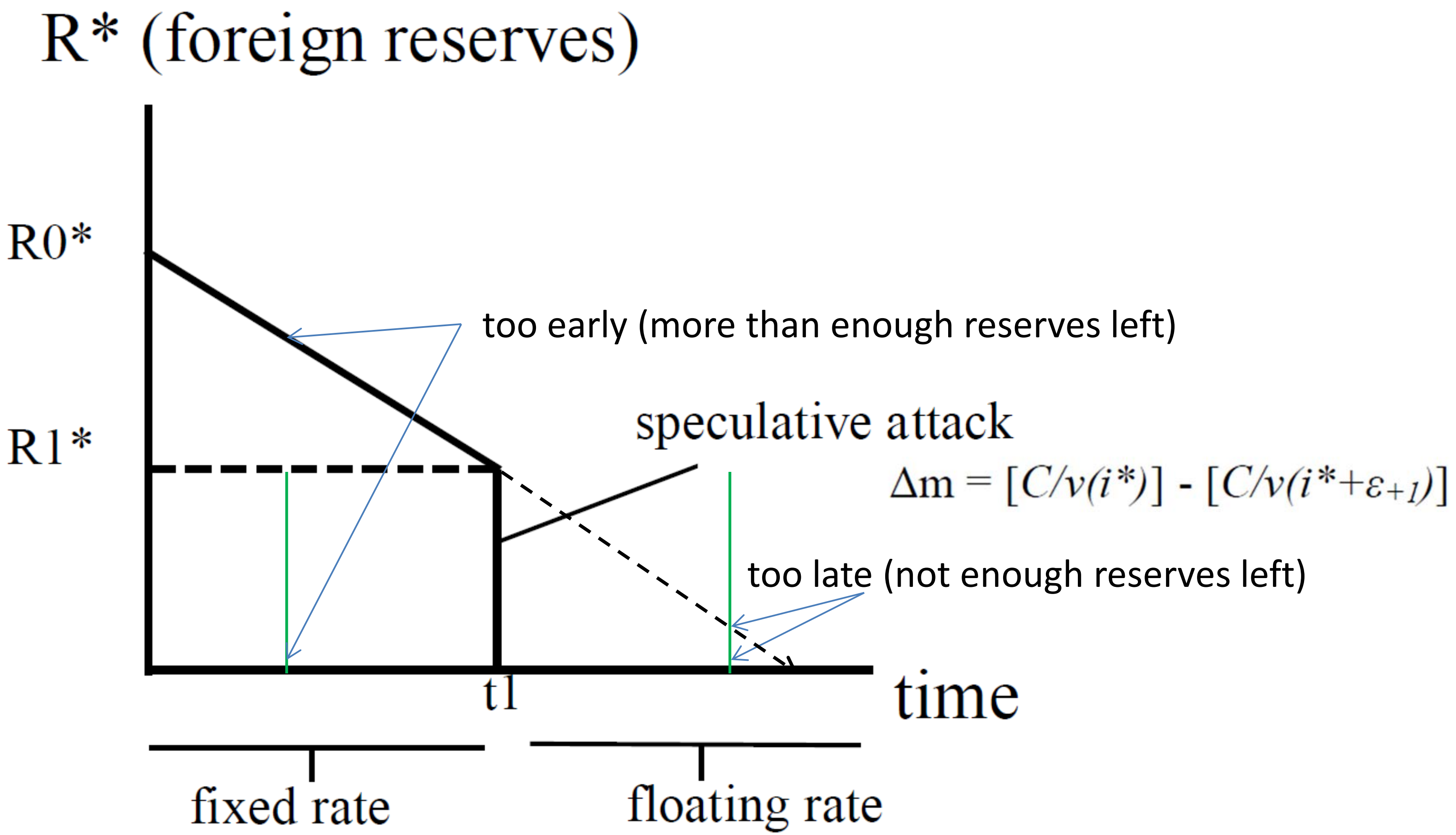
  2. Floating E period (after crisis): inflation tax phase

$$DEF = [\pi/(1+\pi)]\ (M/P) = [\pi/(1+\pi)](C/v(i^{*}+\varepsilon))$$

  3. BoP Crisis period (speculative attack): discrete drop in money demand due to sudden increase in interest rate

$$\Delta m = [C/v(i^{*})] - [C/v(i^{*}+\varepsilon_{+1})]$$

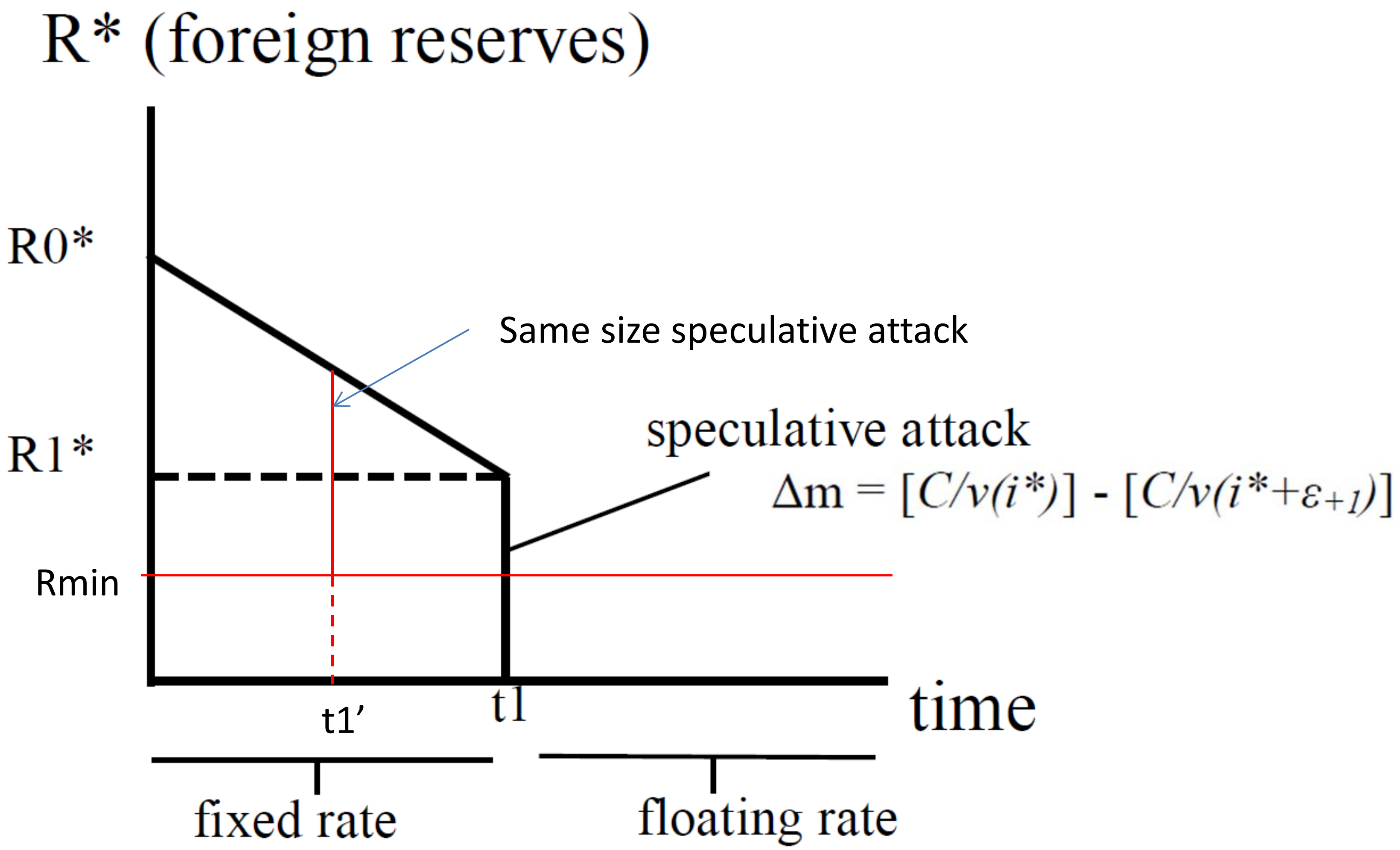
- Speculative attack: higher rate of depreciation implies higher nominal interest rate and drop in Md at an endogenous date t1
- Agents seek to get rid off domestic money about to be devalued. At higher interest rate it is suboptimal to hold higher real balances from the fixed E period
- Occurs optimally and despite perfect foresight!




The date of the attack is determined when there are just enough reserves left to accommodate the decline in real balances

# Caveats and extensions

1. Usually CBs don’t let reserves going to zero, but they have a target minimum=>attack of same size, occurs earlier
2. Domestic borrowing can avoid the crisis if followed by correction of the deficit, if not it only postpones crisis (and even larger crisis since debt adds to the initial deficit)
3. Incipient/implicit deficits can trigger crisis in countries without explicit primary deficits (e.g. Mexico 1994, anticipation of bank bailout)




The date of the attack is determined when there are just enough reserves left to accommodate the decline in real balances

# Inflation tax & Seigniorage

- Inflation tax: loss of purchasing power of real balances

$$IT \equiv [\pi/(1+\pi)]\ (M/P) = [\pi/(1+\pi)](C/v(i^{*}+\pi))$$

  - IT is reflected in the $m_1$ term in the household's intertemporal budget constraint PV(c) =…-(i/(1+i))$m_1$
  - IT has a Laffer curve because M/P=C/v(i), and higher inflation causes higher interest rates, lower money demand
- Seigniorage: purchasing power of new money

$$SE \equiv (M - M_{-1})/P \quad = [(M-M_{-1})/M]\ (M/P)$$

- Identical in a stationary eq., but not otherwise. Example: shock to i* with fixed E and constant P*=> PPP implies $\pi=0$, so IT=0, but lower i* reduces i as well, rises real money demand and yield higher SE

- Seigniorage with fixed exchange rates:

1. *Global inflation* allows the country to inflate while keeping PPP. Higher P requires agents to increase Md and higher M yields seigniorage for the central bank
2. *Domestic boom*: As C or Q grow faster, Md rises, and again higher M yields seigniorage for the central bank (even with zero inflation and zero inflation tax)

# Solving the BOP Crises Model

- Three equation system to determine date and size of the speculative attack, and inflation rate after the currency collapse

1. From the fixed exchange rate phase:

$$R_t{*} = R_{t-1}{*} - def\,, \quad with \;\; R_0{*} > 0,\; R_t{*} \geq R^{\min}$$

2. From the speculative attack date:

$$\Delta m = \frac{C}{v(i*)} - \frac{C}{v(i*+\pi)}$$

3. From the floating exchange rate phase:

$$def = \frac{\pi}{1+\pi}\frac{C}{v(i*+\pi)}$$

a) From 3. solve for inflation as function of *def*:

$$def = \frac{\pi}{1+\pi} \frac{C}{v(i^* + \pi)} \Rightarrow \pi(def)$$

– If we had the v(.) function given we could solve explicitly but here I leave it in implicit form

b) Use now a) in 2. to solve for size of the attack:

$$\Delta m = \frac{C}{v(i^*)} - \frac{C}{v(i^* + \pi)} = \frac{C}{v(i^*)} - \frac{1+\pi}{\pi} def$$

$$\Delta m(def) = \frac{C}{v(i^*)} - \frac{1+\pi(def)}{\pi(def)} def$$

– Also a solution expressed as a function of *def*. If we had the v(.) function it would be an explicit solution

c) From 1. we can tell that that reserves evolve as:

$$R_t^* = R_0^* - t \cdot def\,, \quad for \;\; t = 0, 1, 2, \ldots$$

d) The BOP crises model predicts that the crisis hits when reserves in excess of the minimum required equal the size of the attack. Hence, we must have:

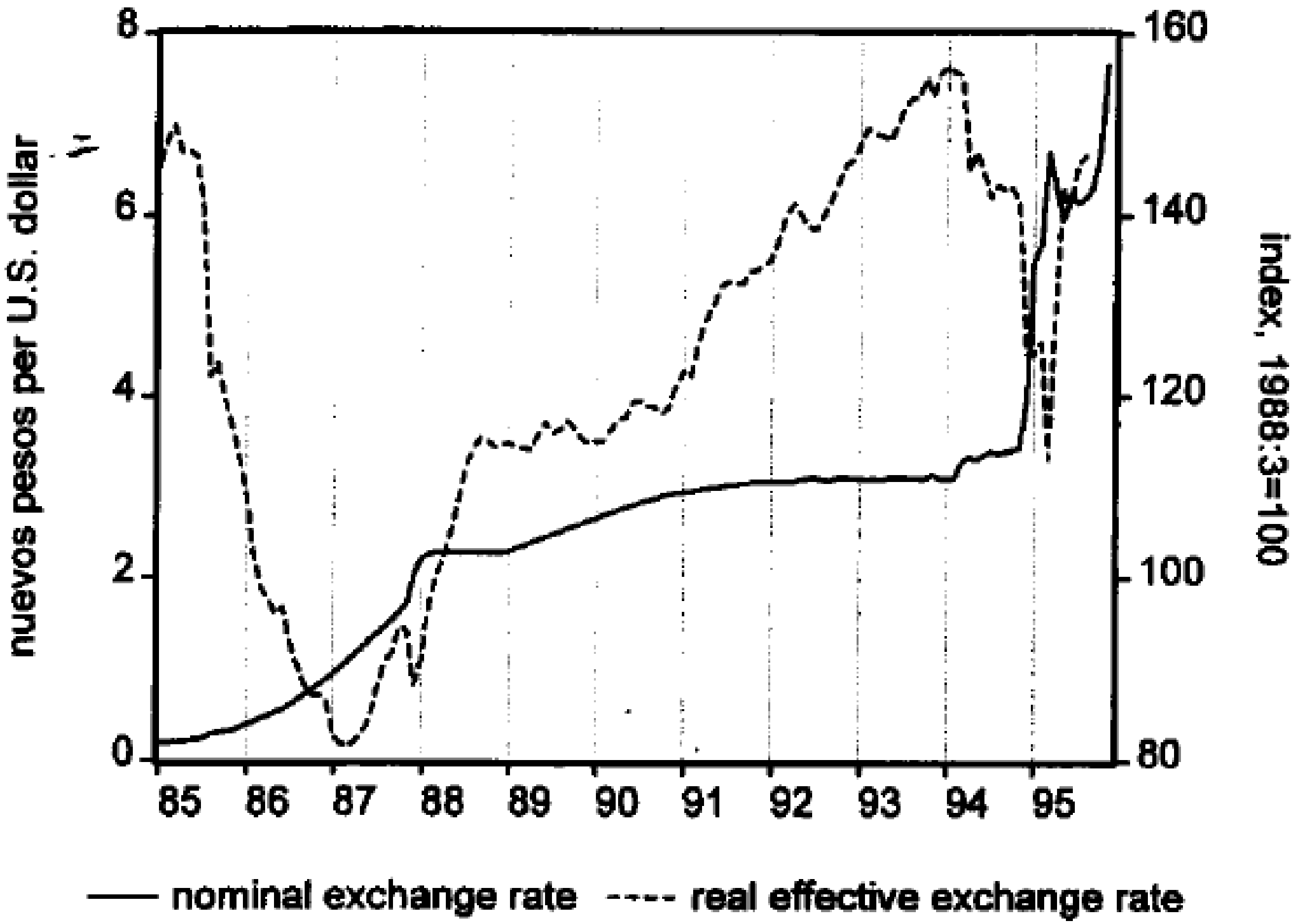
$$R_t^* - R^{\min} = \boxed{R_0^* - t^{crisis} \cdot def - R^{\min} = \Delta m(def)}$$

$$t^{crisis}(def) = \frac{R_0^* - R^{\min} - \Delta m(def)}{def}$$

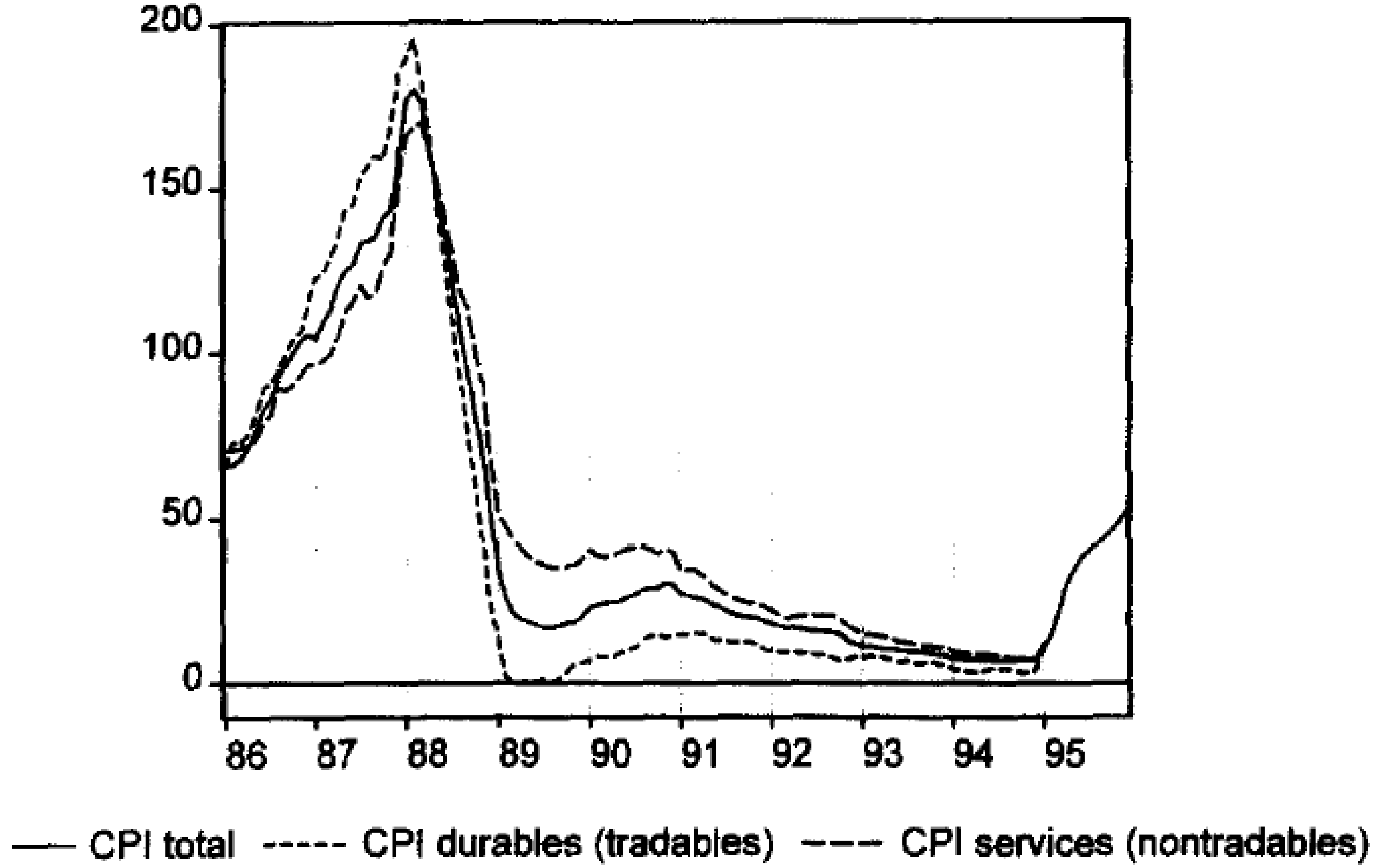
- Round down to nearest integer!! (time is discrete and the answer here doesn't have to be an integer)
- Higher minimum reserves do not change size of attack but the attack occurs sooner
- Larger *def* also makes the attack occur sooner, both because the denominator rises and because the size of the attack is larger (higher inflation needed to finance larger *def*)

# Case Study: Mexico 1994

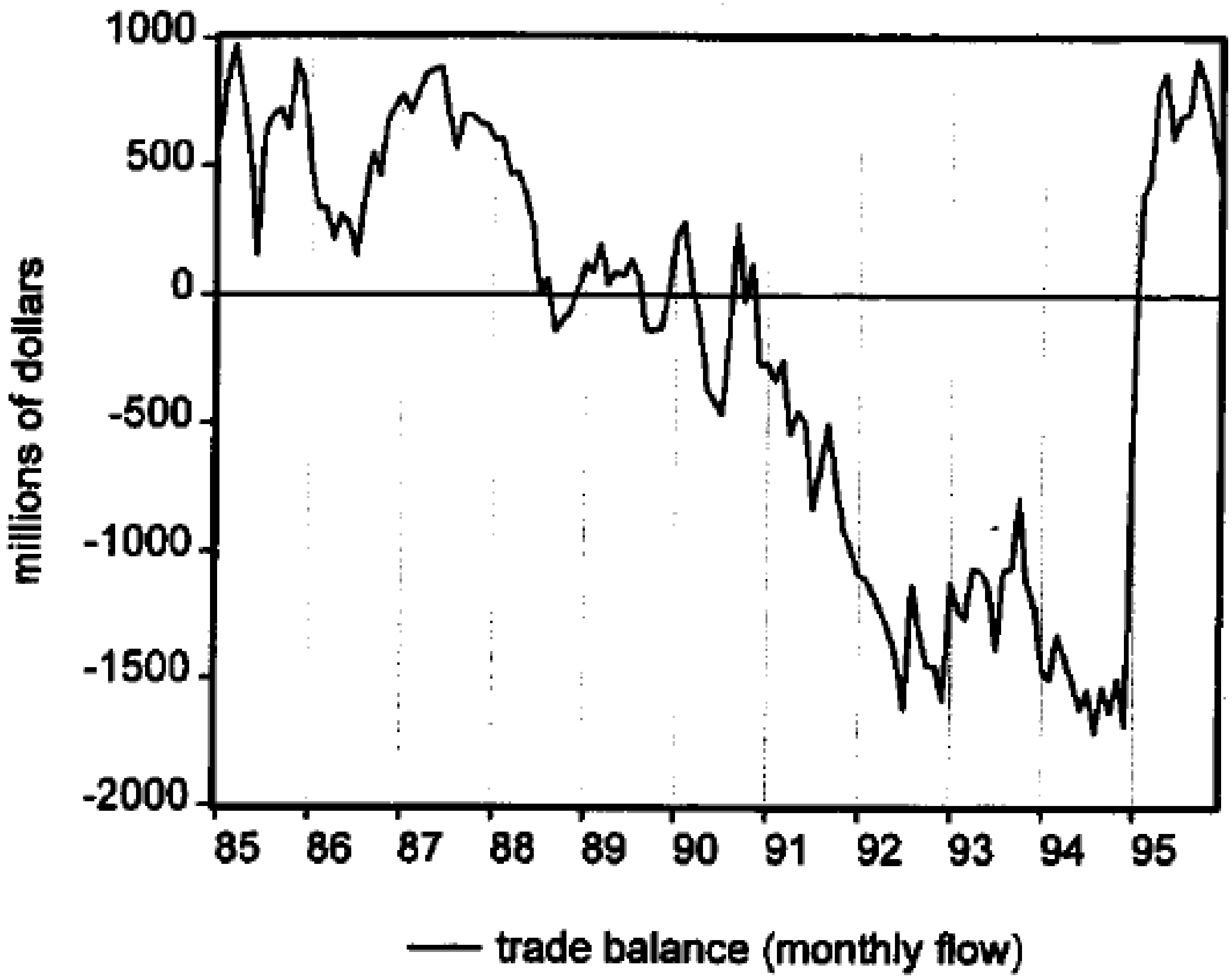
- Real and nominal exchange rates:



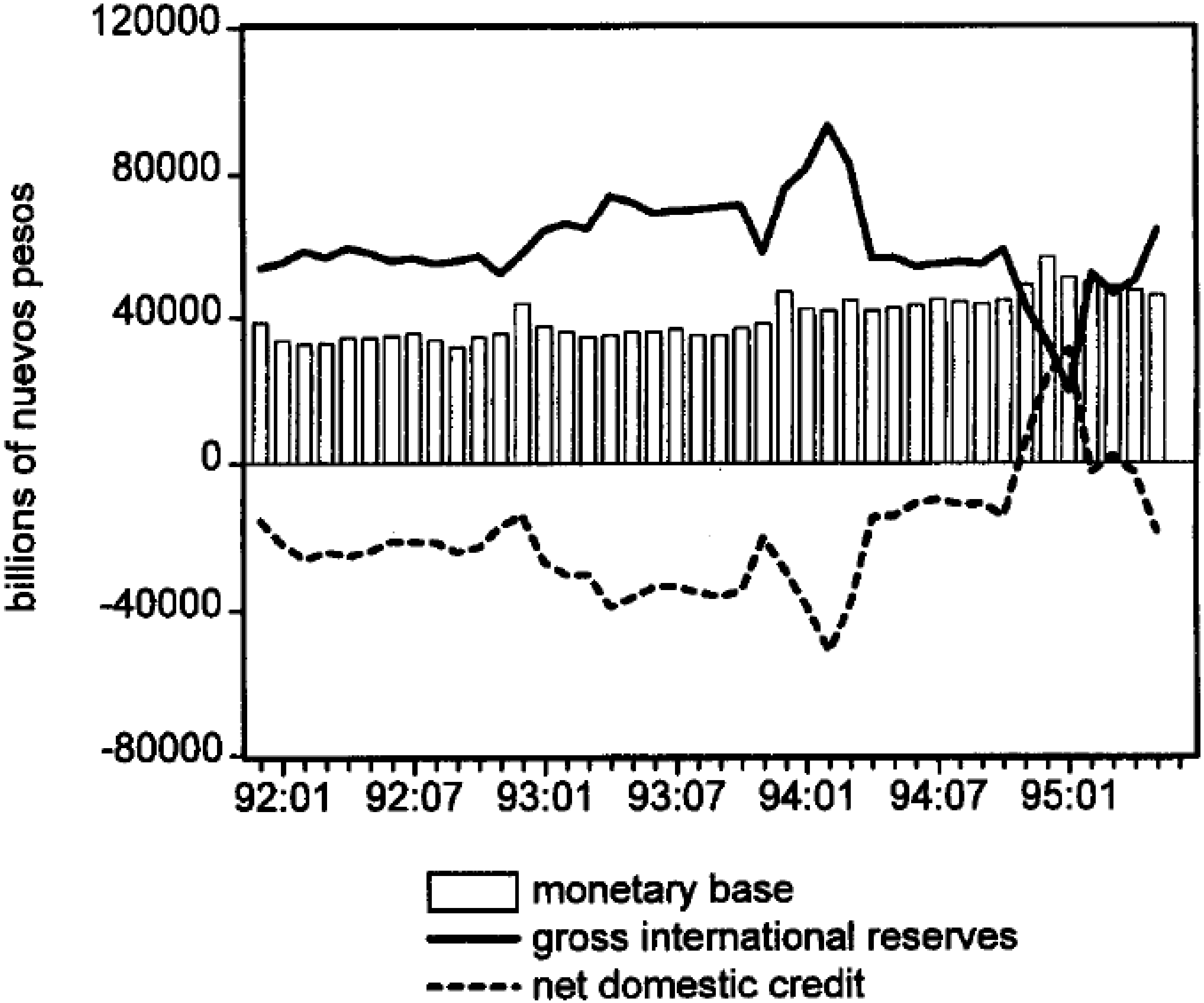
- Consumer prices (exchange-rate based stabilization)



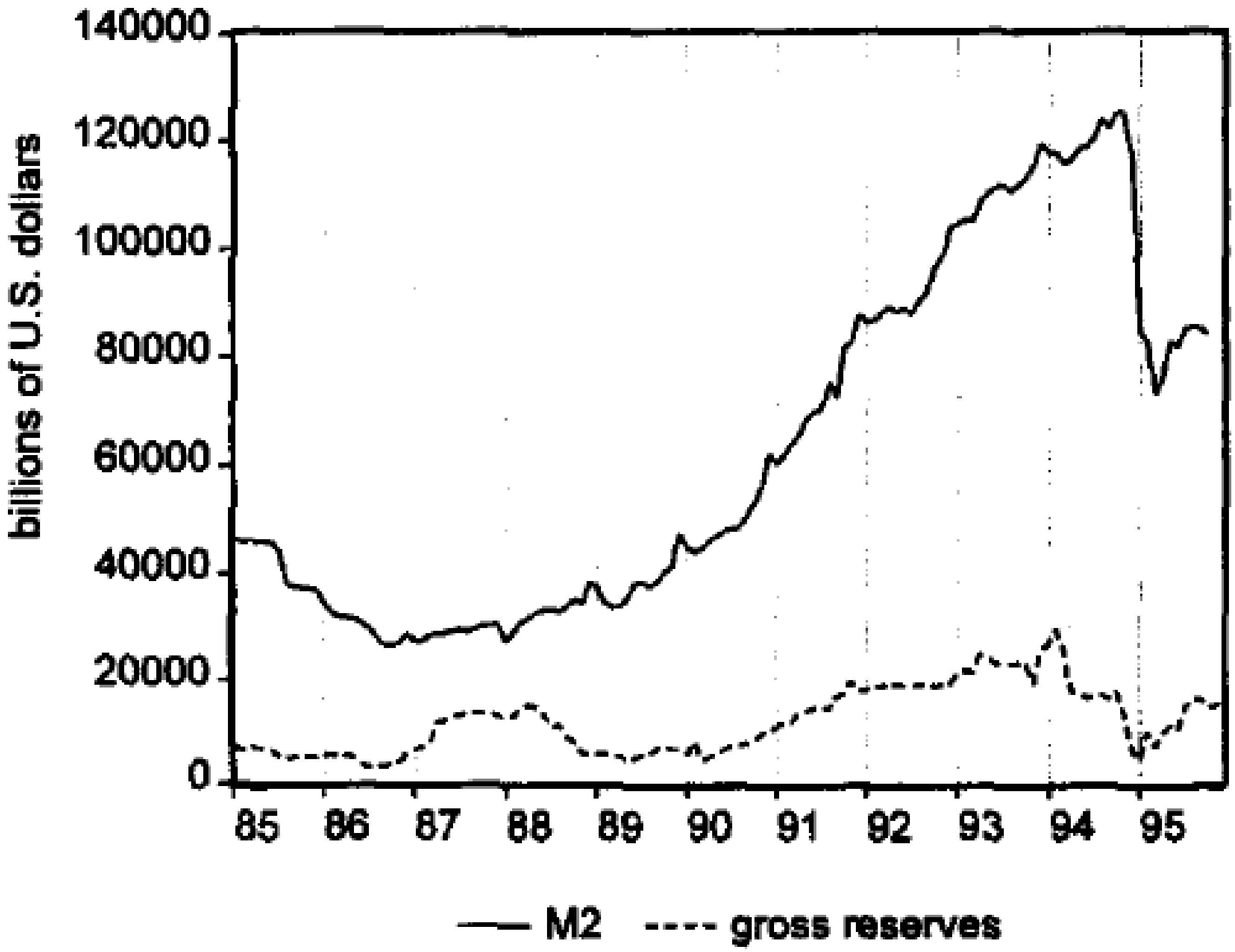
- Trade balance



• Central bank balance sheet

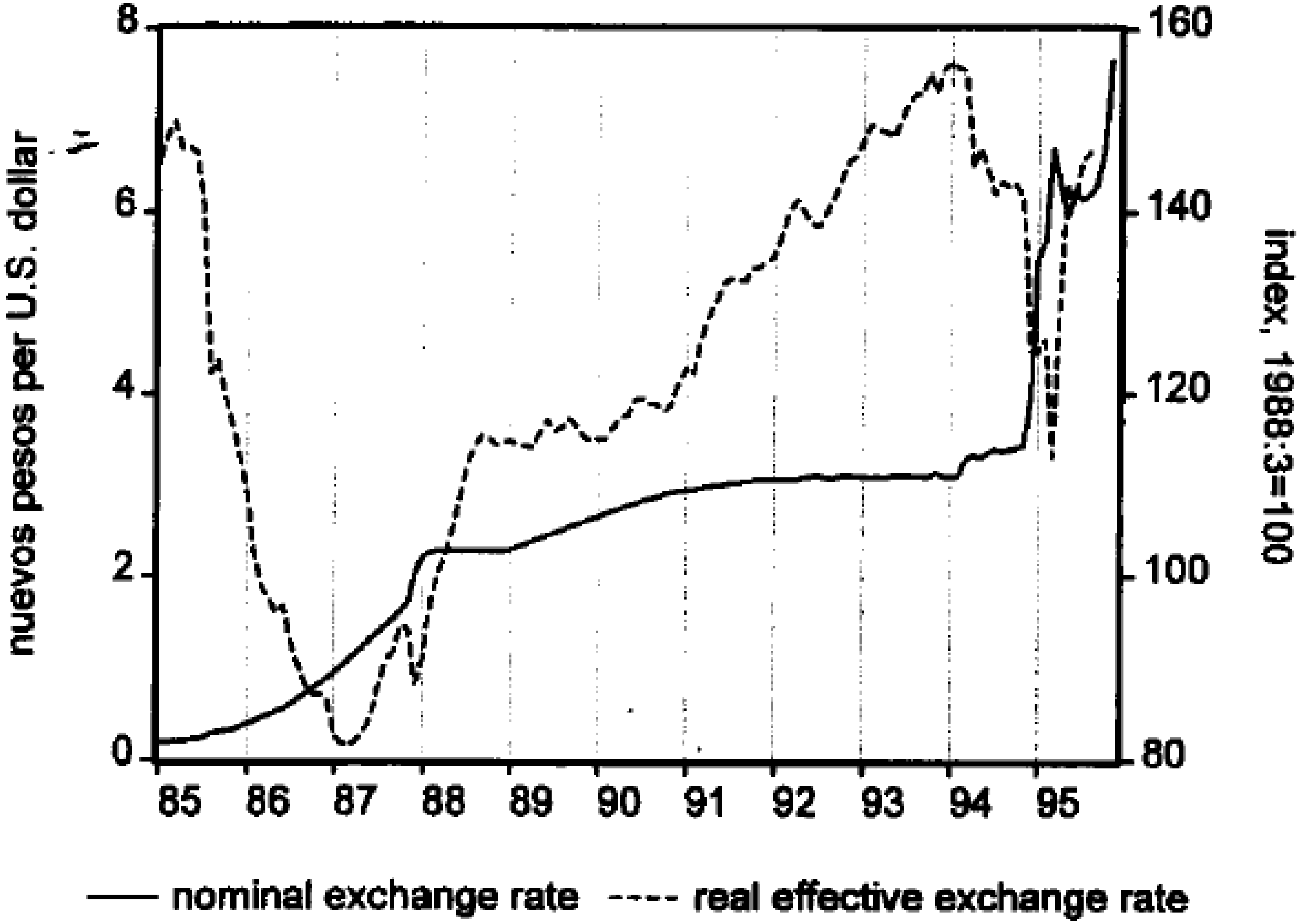


• M2 and gross reserves

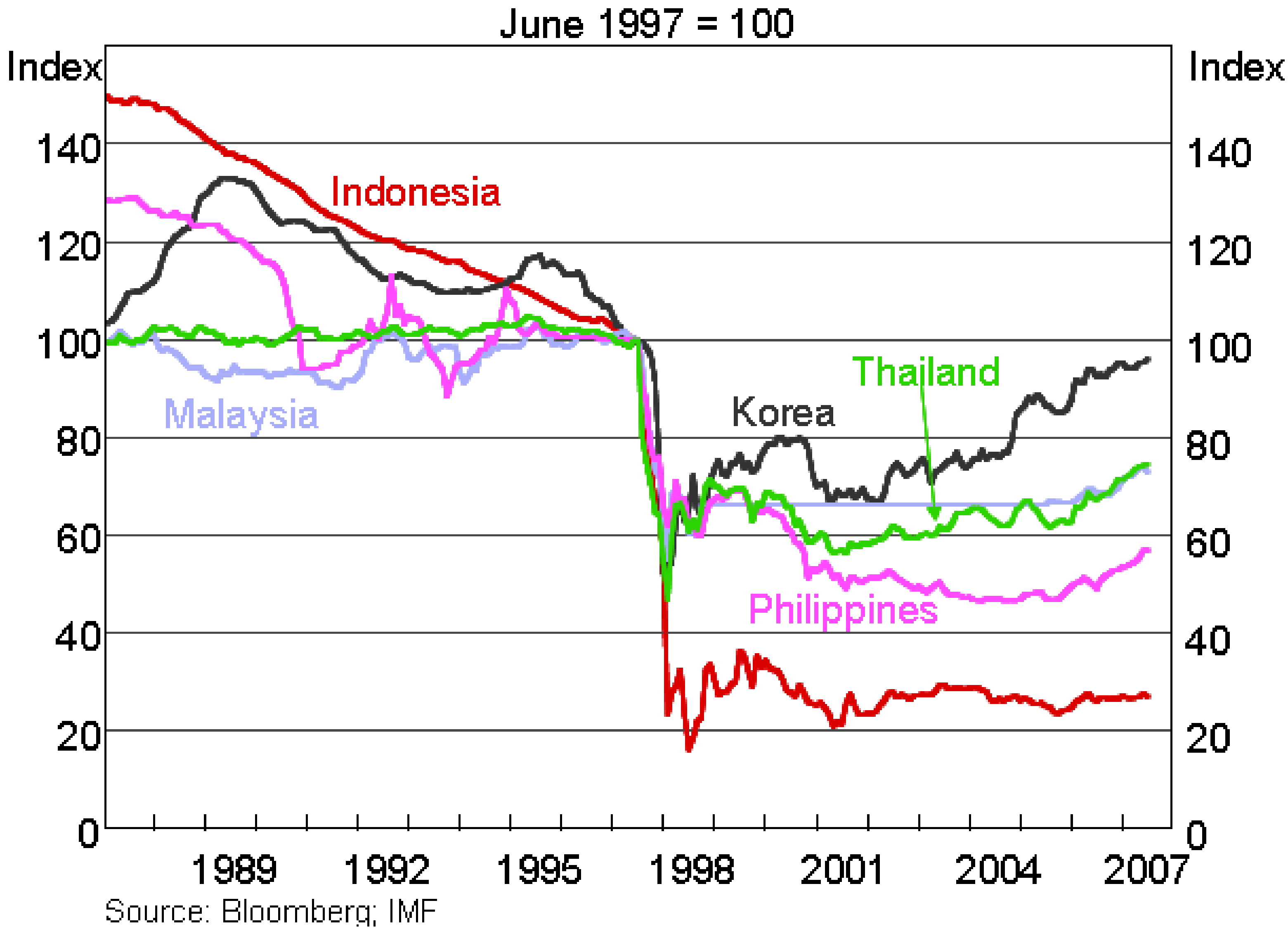


# Currency & Financial Crises: "Greatest Hits"

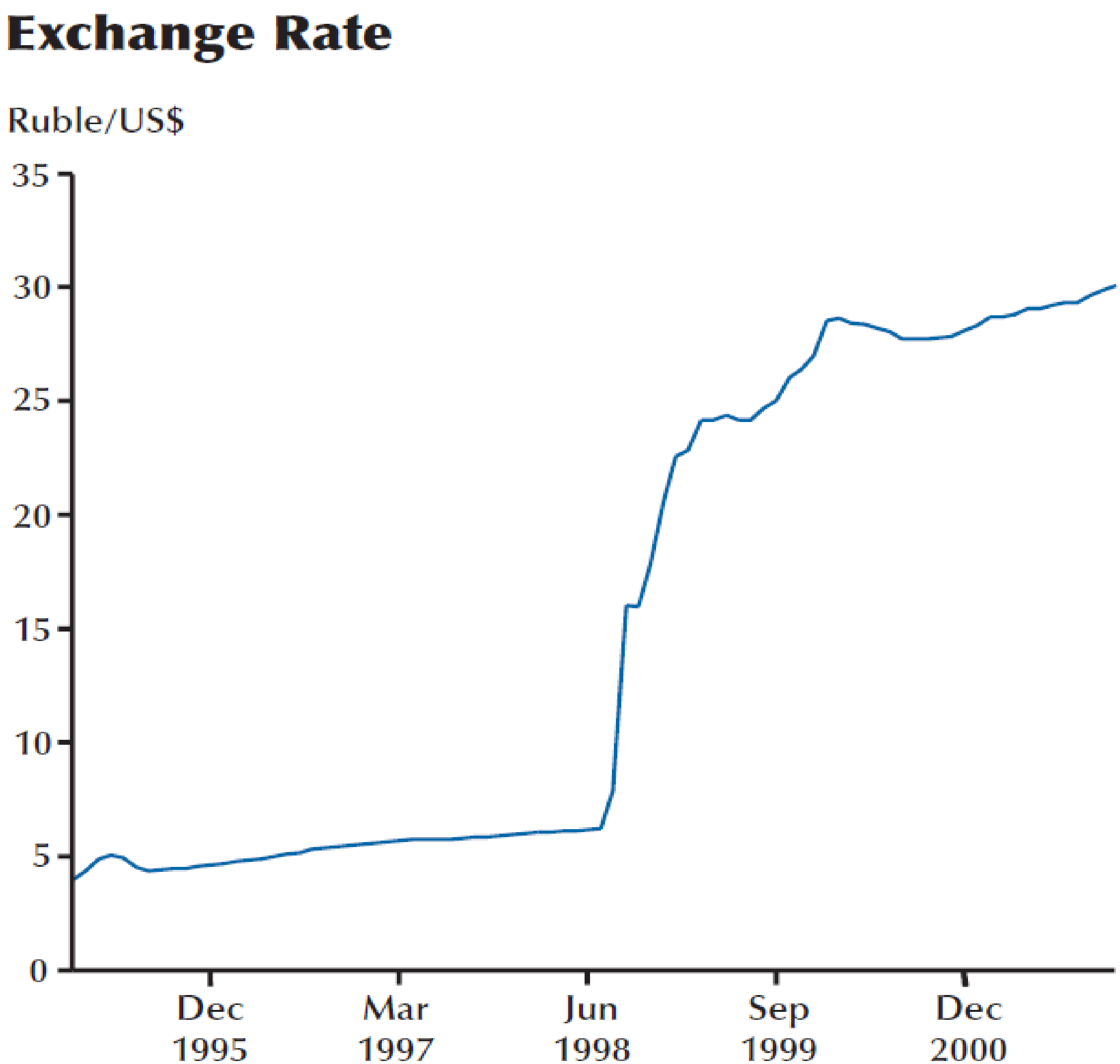
# Mexico 1994-1995



# South-East Asia 1997

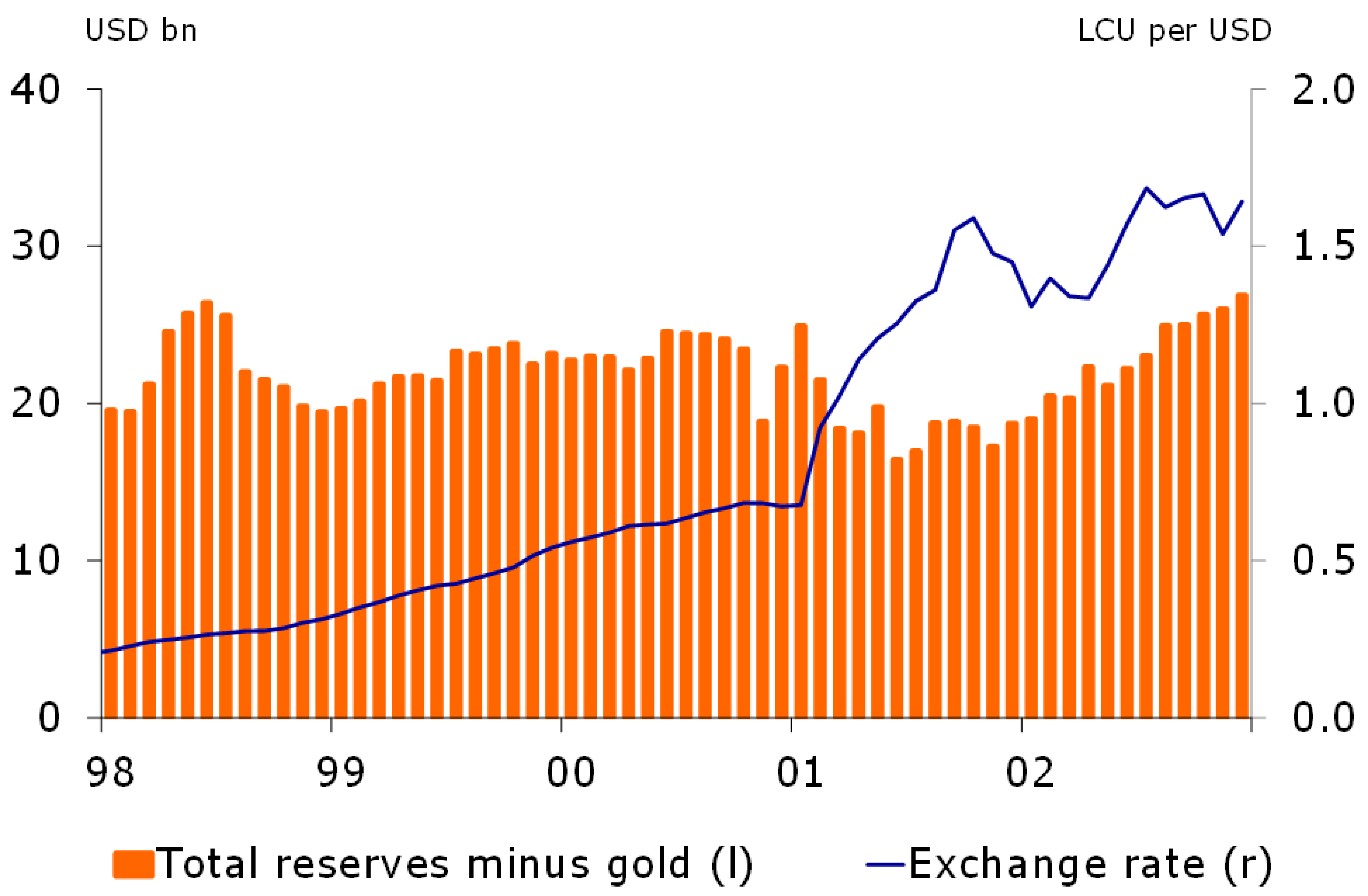


# Russia 1998




SOURCE: IMF (end of period data).

# Turkey 2001

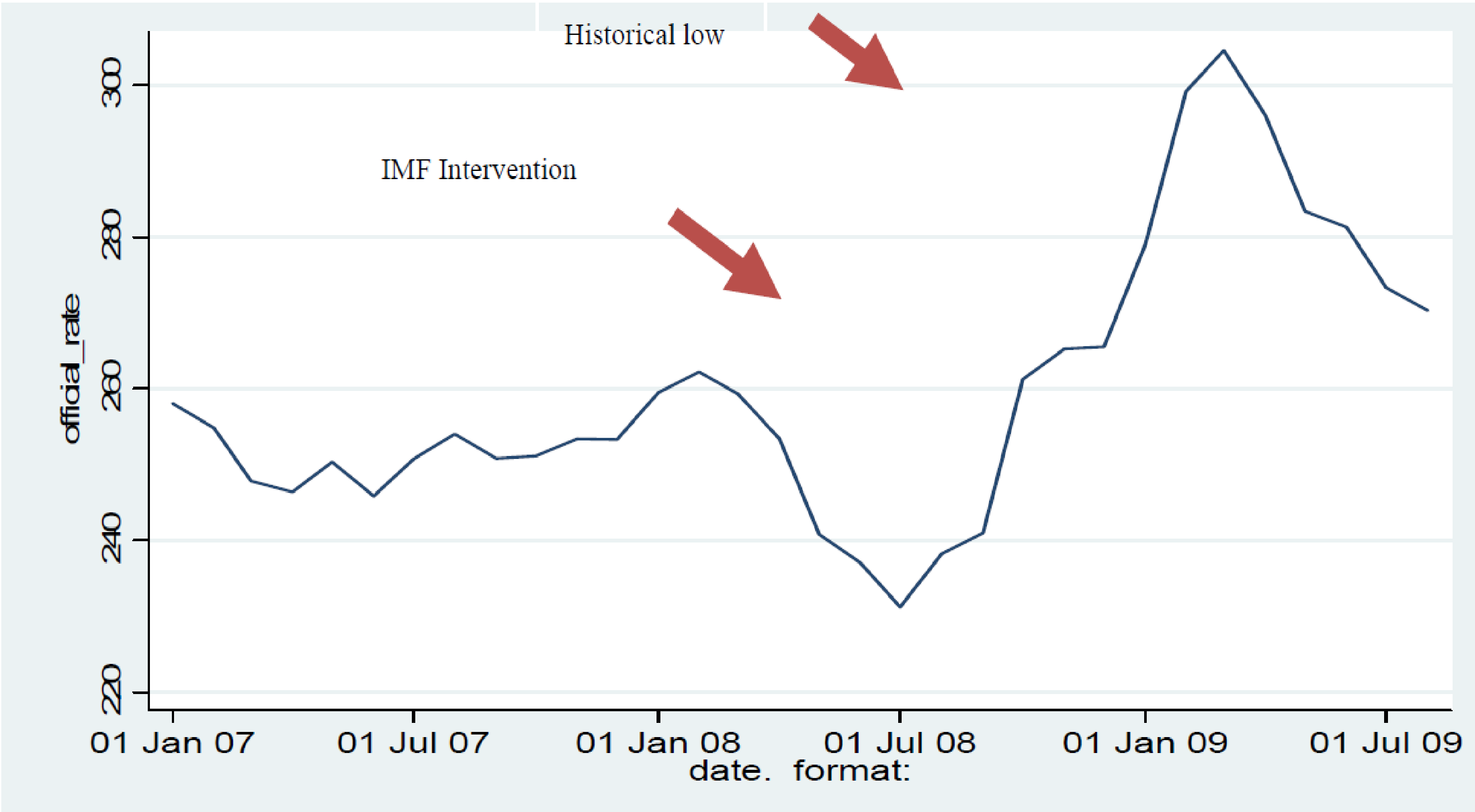


USD bn
LCU per USD
40
30
20
10
0
2.0
1.5
1.0
0.5
0.0
98
99
00
01
02
Total reserves minus gold (l)
Exchange rate (r)

# Argentina 2001-2002

# Iceland 2008

# Hungary 2009



etcetera, etcetera, etcetera….

# Financial crises without devaluation

1. Argentina 1994
2. Hong Kong 1997
3. United States 2008
4. Spain 2009-
5. Greece 2009-
6. Ireland 2008-

…..etcetera, etcetera, etcetera

- But like with currency crises, countries have large reversals of capital flows, real depreciation, collapse in asset prices deep recession and very often banking crises

# Solving the General Equilibrium of an Economy with a *Sustainable* Fixed Exchange Rate
## (two period model)

Later we will use the same setup to analyze effects of an unsustainable peg

# Simplifying assumptions

1. 100% depreciation of K (d=1)
2. Zero initial financial assets ($B_0$ = $M_0$ =0)
3. Two-period life horizon ($K_3$=$B_3$=$M_3$=0)
4. Inelastic labor supply at the amount L
5. Zero foreign inflation and $P^*=1$, so $i^*=r$
6. Exchange rate is fixed at E=1

   by PPP: $\pi=0$ and by IRP: $i=i^*=r$

7. Gov. starts with reserves $R_0$ and transactions costs are gov. revenue
8. Gov. expenditures are exogenous, taxes endogenous to balance budget constraint

# Max. of nonfinancial wealth by firms

1.1 Cobb-Douglas Production Technology:

$$Q_t = AK_t^{a}L_t^{1-a}, \quad \text{with } 0<a<1 \text{ and } A>0$$

1.2. Wealth maximization problem of the firm

$$\text{Max}_{[K2]} \quad W_1^{NF} = Q_1 - K_2 + Q_2/(1+r)$$

given $K_1>0$ and subject to 1.1.

1.3 Maximize by substituting 1.1 in 1.2:

$$\text{Max}_{[K2]} \quad W_1^{NF} = A_1K_1^{a}L^{1-a} - K_2 + (A_2K_2^{a}L^{1-a})/(1+r)$$

1.4 First-order condition:

$$-1 + (\partial Q_2/\partial K_2)/(1+r) = 0$$

$$aA_2K_2^{a-1}L^{1-a} = 1+r \quad \rightarrow \quad MPK_2 = 1+r$$

1.5 Solve for optimal investment and wealth:

$$K_2 = [aA_2/(1+r)]^{1/(1-a)} L$$

$$W_1^{NF} = A_1(K_1/L)^a L - (K_2/L) L + [A_2(K_2/L)^a L]/(1+r)$$

# Utility Maximization by Households

2.1 Logarithmic, isoelastic utility function:

$$U = \mathrm{Ln}(C_1) + \frac{\mathrm{Ln}(C_2)}{1+\delta} \qquad 0 < \delta < 1$$

2.2 Money as a means to economize transactions costs:

$$TC = C + bm^{-\gamma} C^{1+\gamma}$$

2.3 Wealth constraint:

$$(1 + bV_1^{\gamma})C_1 + \frac{(1 + bV_2^{\gamma})C_2}{1+r} = W_1 = W_1^{NF} - i_1 \frac{m_1}{1+r} - T_1 - \frac{T_2}{1+r} - \frac{m_2}{1+r}$$

where $W_1^{NF}$ is from 1.5 and T are lump-sum taxes levied by government.

2.4 Using the definitions $(1+r)=(1+i)/(1+\pi)$ and $(1+\pi)=P_2/P_1$, and substituting the definition of $V_t$ the budget constraint simplifies to:

$$C_1 + bm_1^{-\gamma} C_1^{1+\gamma} + \frac{C_2 + bm_2^{-\gamma} C_2^{1+\gamma}}{1+r} = W_1 = W_1^{NF} - \frac{i_1}{1+i_1} m_1 - T_1 - \frac{T_2}{1+r} - \frac{m_2}{1+r}$$

2.5 Utility maximization: choose $C_1$,$C_2$, $m_1$ and $m_2$ so as to maximize 2.1 subject to the simplified budget constraint in 2.4

$$\underset{C_1, C_2, m_1 \text{ and } m_2}{\text{MAX}}\ U = \text{Ln}(C_1) + \frac{\text{Ln}(C_2)}{1+\delta}$$

$$C_1 + bm_1^{-\gamma} C_1^{1+\gamma} + \frac{C_2 + bm_2^{-\gamma} C_2^{1+\gamma}}{1+r} = W_1^{NF} - \frac{i_1}{1+i_1} m_1 - \frac{m_2}{1+r}$$

Recall we solved this problem using the Lagrange method

## 2.6 Optimality conditions of the Households Problem:

***A) Intertemporal consumption-saving tradeoff:***

$$\frac{C_2}{C_1} = \frac{(1+r)\left[1+(1+\gamma)b\left(\frac{C_1}{m_1}\right)^{\gamma}\right]}{(1+\delta)\left[1+(1+\gamma)b\left(\frac{C_2}{m_2}\right)^{\gamma}\right]} = \frac{(1+r)\left[1+(1+\gamma)bV_1^{\gamma}\right]}{(1+\delta)\left[1+(1+\gamma)bV_2^{\gamma}\right]} \quad \textbf{(I)}$$

Monetary distortion $H(i_1)$, increasing in $i_1$

***B) Period-1 money demand tradeoff:***

V$_1$(i$_1$ ) increasing in i$_1$

$$\frac{i_1}{1+i_1} = \gamma b\left(\frac{C_1}{m_1}\right)^{1+\gamma} \Rightarrow \frac{C_1}{m_1} \equiv V_1 = \left(\frac{i_1}{1+i_1}\left(\frac{1}{\gamma b}\right)\right)^{\frac{1}{1+\gamma}} \quad \textbf{(II)}$$

***C) Period-2 money demand tradeoff:***

$$1 = \gamma b\left(\frac{C_2}{m_2}\right)^{1+\gamma} \Rightarrow \frac{C_2}{m_2} \equiv V_2 = \left(\frac{1}{\gamma b}\right)^{\frac{1}{1+\gamma}} \quad \textbf{(III)}$$

# Government budget constraints and aggregate resource constraints

3.1 Consolidated budget constraints of gov. and central bank:

Period budget constraints

$$G_1 = bm_1^{-\gamma} C_1^{1+\gamma} + m_1 + T_1 - R_1 + (1+r) R_0$$

$$G_2 = bm_2^{-\gamma} C_2^{1+\gamma} + T_2 + (1+r) R_1 + m_2 - \frac{m_1(1+r)}{1+i_1}$$

Intertemporal budget constraint

$$G_1 + \frac{G_2}{1+r} = bm_1^{-\gamma} C_1^{1+\gamma} + \frac{bm_2^{-\gamma} C_2^{1+\gamma}}{1+r} + \frac{i_1}{1+i_1} m_1 + T_1 + \frac{T_2}{1+r} + (1+r) R_0 + \frac{m_2}{1+r}$$

4.1 Aggregate resource constraint (combine gov. and households constraints):

$$C_1 + \frac{C_2}{1+r} = W_1^{NF} - \left( G_1 + \frac{G_2}{1+r} \right) + (1+r) R_0$$

# Equilibrium consumption allocations

1. Fixed E and IRP imply $i=i^*=r$, which we use in consumption-saving tradeoff to solve for $\frac{C_2}{C_1}$:

$$\frac{C_2}{C_1}=\frac{(1+r)}{(1+\delta)}\left[\frac{1+(1+\gamma)bV_2^{\gamma}\left(\frac{r}{1+r}\right)^{\frac{\gamma}{1+\gamma}}}{1+(1+\gamma)bV_2^{\gamma}}\right]$$

This is a fraction <1

$$H(r)=\left[\frac{1+(1+\gamma)bV_2^{\gamma}\left(\frac{r}{1+r}\right)^{\frac{\gamma}{1+\gamma}}}{1+(1+\gamma)bV_2^{\gamma}}\right]<1 \quad \Rightarrow \quad \frac{C_2}{C_1}=\frac{1+r}{1+\delta}H(r)$$

<u>But notice that with an infinite horizon we would have H(r)=1!!!!</u>

2. The result from 1. and the aggregate resource constraint in 4.1 form a two-equation system in two unknowns. Solve:

$$C_2 = \left[ \frac{1+r}{1+\delta+H(r)} \right] H(r) \left( W_1^{NF} - \left( G_1 + \frac{G_2}{1+r} \right) + (1+r) R_0 \right)$$

$$C_1 = \left[ \frac{1+\delta}{1+\delta+H(r)} \right] \left( W_1^{NF} - \left( G_1 + \frac{G_2}{1+r} \right) + (1+r) R_0 \right)$$

These are the equilibrium solutions for consumption in periods 1 and 2.
All the terms in the right-hand-sides of these expressions are either exogenous parameters or solutions obtained in the previous steps.

# Monetary equilibrium allocations

6.1 Given consumption solutions, substitute back into money demand equations to get:

$$m_1 \equiv \frac{M_1}{P_1} = C_1 \left( \frac{r}{1+r} \left( \frac{1}{\gamma b} \right) \right)^{-\frac{1}{1+\gamma}} \qquad m_2 \equiv \frac{M_2}{P_2} = C_2 \left( \frac{1}{\gamma b} \right)^{-\frac{1}{1+\gamma}}$$

6.2 Since E is fixed, quantity of money is demand-determined, so money supplies are:

$$M_1^S = (EP^*)\, C_1 \left( \frac{r}{1+r} \left( \frac{1}{\gamma b} \right) \right)^{-\frac{1}{1+\gamma}} \qquad M_2^S = (EP^*)\, C_2 \left( \frac{1}{\gamma b} \right)^{-\frac{1}{1+\gamma}}$$

6.3 How does this change if we consider flexible E?

Endogenous $i_1 = r + \varepsilon$ replaces "r" and supply of money is exogenous, and consumption allocations and $\varepsilon$ are determined together, given an initial price level or initial E.

6.4 *Exchange rate regime equivalence*: there is always a monetary policy under a floating regime that can produce the same consumption allocations and value of E of a fixed exchange rate (i.e. the monetary policy that yields endogenously $\varepsilon$ =0)

# Current account and bond holdings

- Since CA=S-I, and we have already solved for both S (from households problem) and I (from firms' problem), we already have the solution
- Bond holdings follow from period budget constraints of households and government
- Effects of shocks

(1) Productivity shocks have similar effects as in the model without money (this is because we did not assume that money balances are used for investment expenditures)

(2) Shocks to $r$ have income and substitution effects on S and I as in the nonmonetary model, but in addition they affect the monetary equilibrium in period 1, and also alter the intertemporal relative price of consumption (adding an extra effect on $C_1$,$C_2$,$S_1$ and $CA_1$).