A Primer on IRBC Theory

(Complete Markets, Incomplete Markets and the role of Terms of Trade in business cycles)

Enrique G. Mendoza
University of Pennsylvania & NBER

Layout of the lecture

- Study complete & incomplete markets models of response of trade balance (TB) to terms-of-trade (TOT) shocks (Harberger-Laursen-Metzler, HLM, effect) and role of TOT in business cycles

1. General introduction to HLM literature
2. Backus's complete markets IRBC framework
3. Mendoza's incomplete markets model
    1. Stylized facts on TOT, HLM and business cycles
    2. SOE multisector RBC model with Epstein-Uzawa preferences to pin down stochastic steady state
    3. RBC-style quantitative analysis

REVIEW OF THE EARLY HLM LITERATURE

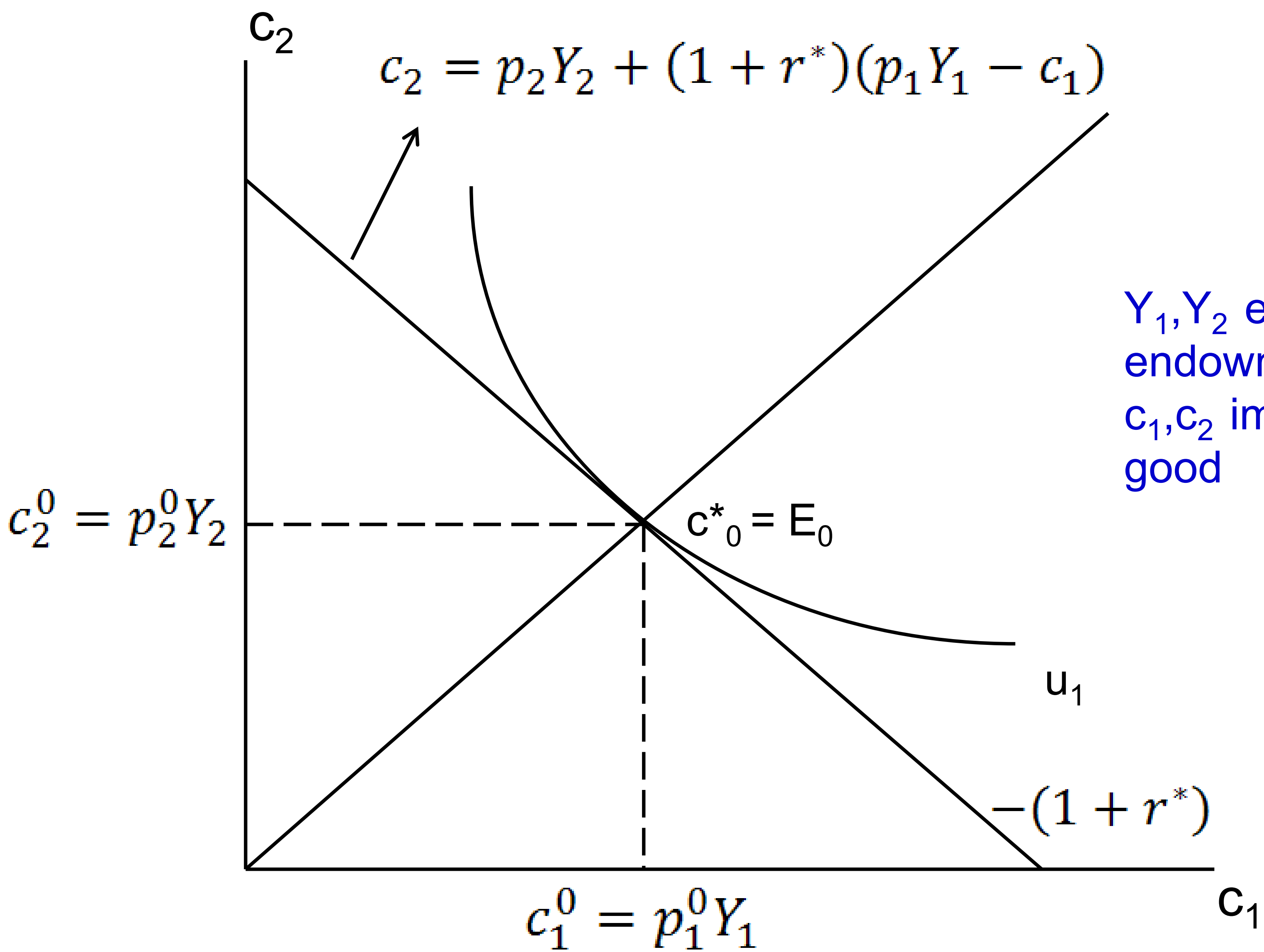
Classic HLM Argument (1950)

- Keynesian import demand: positive but less-than-unitary marginal prop. to import (mpm)
- HLM effect: A worsening of TOT worsens the trade deficit: $\rho(TB, TOT) > 0$
- A fall in TOT causes a fall in disposable income in terms of imports, and since 0 < mpm < 1, imports fall by less than disposable income and the value of exports $\rightarrow$ trade balance falls

Intertemporal models of the 1980s

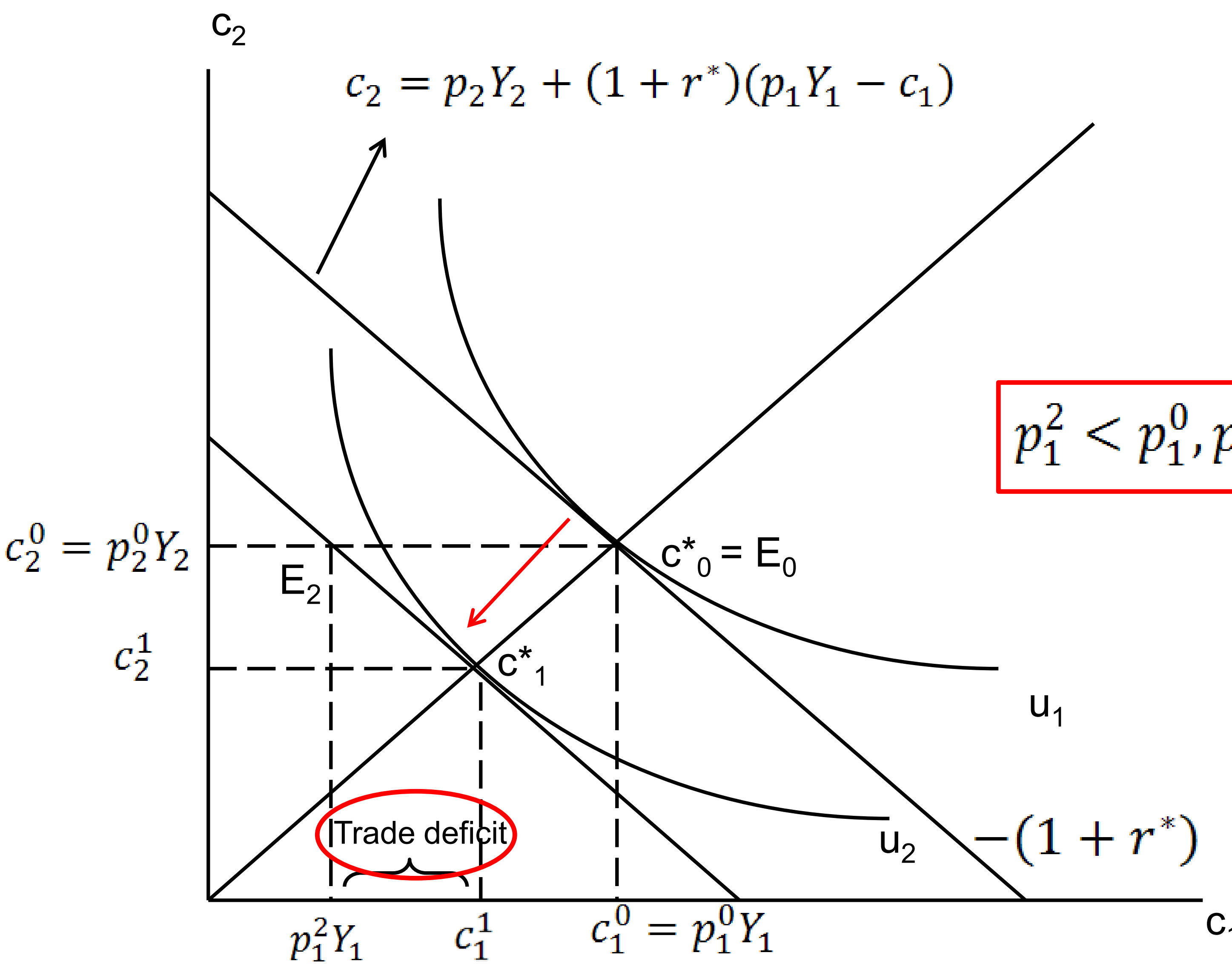
- Obstfeld (1982), Svensson and Razin(1983) and a large collection of papers that followed them
- Deterministic intertemporal equilibrium models in the class of Workhorse Model No. 1
- Main prediction: Response of TB to TOT shocks depends on duration of TOT shocks
  - Transitory shocks yield HLM effects
  - As persistence increases, co-movement weakens
  - Permanent shocks have zero or even negative co-movement (depending on discount factor)

The OSR argument

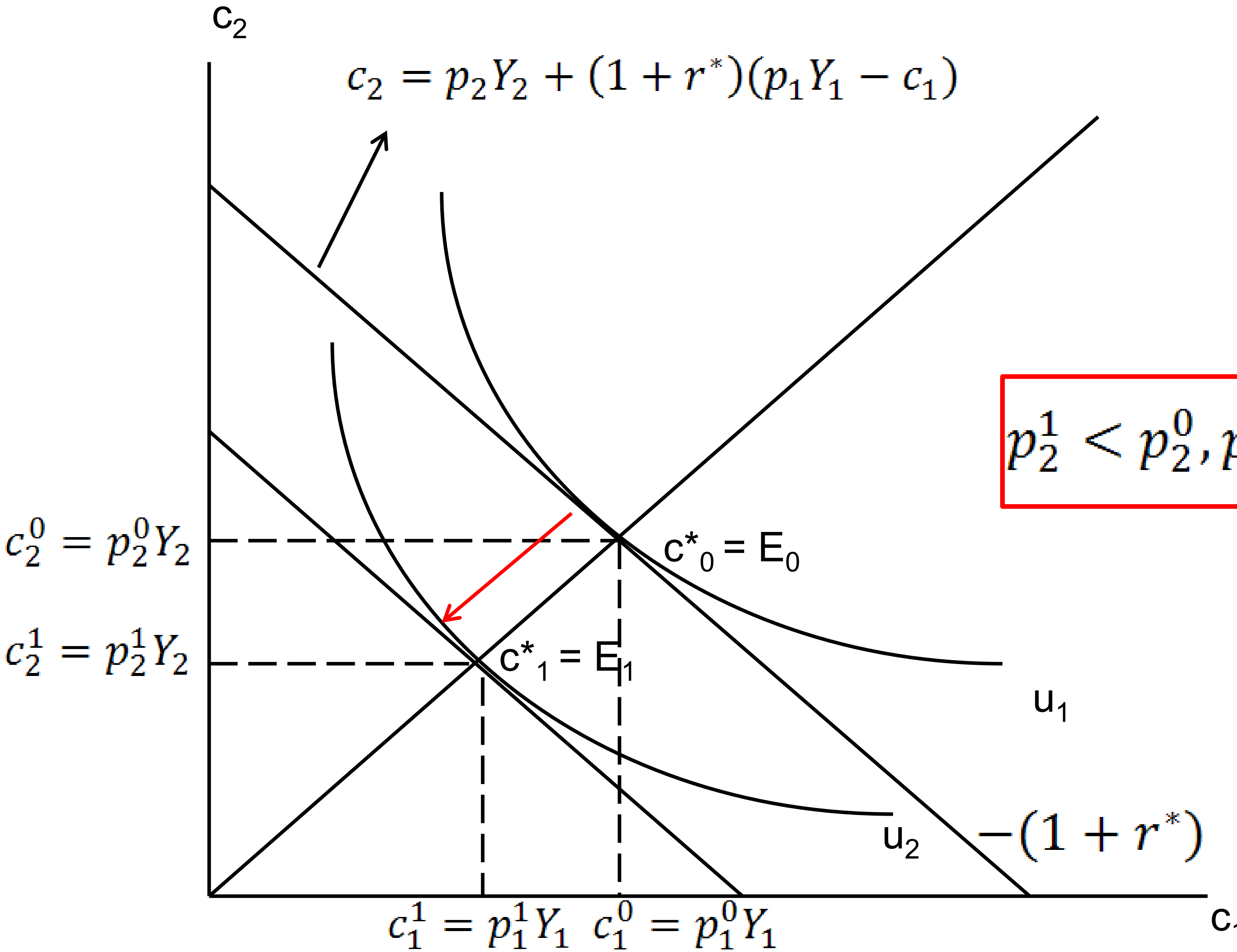


Y_1, Y_2 exportable endowment
c_1, c_2 importable good

The OSR argument: transitory shock



The OSR argument: permanent shock



$$p_2^1 < p_2^0, p_1^1 < p_1^0$$

Formal derivation of the OSR results

- Rewrite Workhorse model No. 1 with two goods:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t^m)$$

$$s.t. \quad c_t^m = p_t^x \bar{x} - b_{t+1} + b_t(1+r^*) \qquad \lim_{t\to\infty} \frac{b_t}{(1+r^*)^t} = 0$$

$$\beta(1+r^*) = 1 \qquad p_t^x = \overline{p^x}(1 \pm \epsilon_t)$$

$$u'(c_t^m) = \beta(1+r^*)u'(c_t^m) \Rightarrow \overline{c^m} \ \forall\, t$$

$$\overline{c^m} = (1-\beta)\overline{p^x}\bar{x}\left(\sum_{t=0}^{\infty} \frac{(1+\epsilon_t)}{(1+r^*)^t}\right) + b_0 r^*$$

The OSR results

- Permanent TOT fall: Largest cut in PV of income, makes imports fall as much as value of exports.

$$\overline{c^m} = \frac{(1-\beta)\overline{p^x}\bar{x}(1+\epsilon)}{(1-\beta)} \Rightarrow \downarrow \overline{c^m} = \downarrow p_t^x \bar{x}$$

- Transitory TOT fall: Fall in PV of income is smaller than with permanent shock, hence imports fall less, and thus must fall less than value of exports at date t (consumption smoothing), so $p_t^x \bar{x} - c_t^m$ falls
- Transitory fall in TOT reduces TB (HLM effect), permanent fall has no effect

THE RESPONSE OF TB TO TOT SHOCKS IN MODELS WITH UNCERTAINTY AND COMPLETE MARKETS

Backus's Two Country Model

- The relationship between TOT and TB depends on source of TOT variations and structure of preferences
- It DOES NOT depend on persistence of TOT!
- Argues that his results differ from OSR because of *uncertainty*, but it is really because of *uncertainty AND complete markets*!

Uncertainty setup

- Follows Lucas (1982,1984) in setting recursive (Markov) structure of choice under uncertainty
- Two country, stochastic endowment economy
- Uncertainty is characterized by events z_t drawn from finite set Z_t, so that at each date $t = 0,1,\ldots T$ the state of nature is described by
  - z^t: history of events $z^t \equiv (z_1, \ldots, z_t)$ with $z^t \in Z^t$
  - z_0, the initial event.

Markov endowments

- Two countries specialized by commodity Markov processes of endowments of diff. goods
  - $\{x(z^t)\}$, country 1. $\{y(z^t)\}$, country 2.
- z_t follows Markov process characterized by a time-invariant Markov chain:
  1. $z_t \in Z_t \in \mathcal{R}^n$ for *n* events (realization vector)
  2. *P* *n x n* transition matrix
  3. π_0, *n x 1* initial prob. Vector for each $z \in Z$
- Markov property: Probability of state z^t conditional on z_0 is given by:

$$\pi(z^t) = \pi(z_t|z_{t-1})\pi(z_{t-1}|z_{t-2}) \ldots \pi(z_1|z_0)$$

Preferences

- Households in each country consume both goods and have CES expected utility:

$$(I) \quad u_i = \sum_{t=0}^{T} \beta^t \sum_{z^t \in Z^t} \pi(z^t) \left[\frac{a_i(z^t)^{1-\alpha} + b_i(z^t)^{1-\alpha}}{1-\alpha} \right]$$

$$\alpha > 0 \quad i = 1,2$$

${}^{1}/_{\alpha} \equiv$ Atemporal & intertemporal elasticity of substitution

Complete Markets of Contingent Claims

- Assume complete markets in state-contingent (Arrow-Debreu) claims, which are secs. traded at date 0 promising to deliver goods at particular future dates and states
- A-D markets pin down **date-0 prices** of home and foreign goods defined as $q(z^t)$ and $p(z^t)$
- Agents maximize utility subject to date 0 budget constraints that incorporate all A-D trades

Budget constraints with A-D claims

$$(IIa)\ Home\ \sum_{t,z^t}[p(z^t)a_1(z^t) + q(z^t)b_1(z^t) - p(z^t)x(z^t)] \leq 0$$

$$(IIb) Foreign\ \sum_{t,z^t}[p(z^t)a_2(z^t) + q(z^t)b_2(z^t) - q(z^t)y(z^t)] \leq 0$$

- Terms of trade → $TOT(z^t) \equiv \dfrac{p(z^t)}{q(z^t)}$

Competitive Equilibrium with A-D Claims

- $\{p(z^t), q(z^t)\}_{t,z^t}$ and $\{a_1(z^t), a_2(z^t), b_1(z^t), b_2(z^t)\}_{t,z^t}$ such that
  - Households max *(I)* s.t. *(IIa), (IIb)*
  - Commodity markets clear in all states:

$$a_1(z^t) + a_2(z^t) = x(z^t) \quad b_1(z^t) + b_2(z^t) = y(z^t)$$

$$\forall \; z^t \in Z^t$$

- In the absence of distortions, Mantel & Negishi showed that A-D equilibrium can be characterized as solution to a planner's problem that chains "seemingly static" problems

Planner's Intertemporal Problem

$$\max \sum_i \lambda_i u_i = \sum_i \lambda_i \sum_{t=0}^{T} \beta^t \sum_{z^t \in Z^t} \pi(z^t) u(a_i(z^t), b_i(z^t))$$

s.t.

$$a_1(z^t) + a_2(z^t) \leq x(z^t)$$

$$b_1(z^t) + b_2(z^t) \leq y(z^t) \quad \forall\, z^t \in Z^t$$

for some choice of λ_i's

$A(z^t)$ multiplier on $x(z^t)$ constraint ($A(z^t) = p(z^t)$)

$B(z^t)$ multiplier on $y(z^t)$ constraint ($B(z^t) = q(z^t)$)

Planner's First-Order Conditions

$$a_1(z^t): \;\lambda_1 \beta^t \pi(z^t) u_1\big(a_1(z^t), b_1(z^t)\big) = A(z^t)$$

$$b_1(z^t): \;\lambda_1 \beta^t \pi(z^t) u_2\big(a_1(z^t), b_1(z^t)\big) = B(z^t)$$

$$a_2(z^t): \;\lambda_2 \beta^t \pi(z^t) u_1\big(a_2(z^t), b_2(z^t)\big) = A(z^t)$$

$$b_2(z^t): \;\lambda_2 \beta^t \pi(z^t) u_2\big(a_2(z^t), b_2(z^t)\big) = B(z^t)$$

- Define **date-t, state-t prices** (Arrow secs prices)

$$\frac{A(z^t)}{\beta^t \pi(z^t)} = P(z^t) \qquad \frac{B(z^t)}{\beta^t \pi(z^t)} = Q(z^t)$$

Sequential Planner's Problem

- Identical prices and allocations follow from solving independent problems for each state z^t. (eq. allocations and prices are not history dependent!)

- $P(z^t)$ and $Q(z^t)$ are Lagrange multipliers in these sequential Lagrangians for each z^t:

$$\mathcal{L} = \lambda_1 u\big(a_1(z^t), b_1(z^t)\big) + \lambda_2 u\big(a_2(z^t), b_2(z^t)\big)$$

$$+P(z^t)[x(z^t) - a_1(z^t) - a_2(z^t)]$$

$$+Q(z^t)[y(z^t) - b_1(z^t) - b_2(z^t)]$$

Sequential first-order conditions

$$a_1(z^t):\ \lambda_1 u_1(.) = P(z^t)$$

$$b_1(z^t):\ \lambda_1 u_2(.) = Q(z^t)$$

$$a_2(z^t):\ \lambda_2 u_1(.) = P(z^t)$$

$$b_2(z^t):\ \lambda_2 u_2(.) = Q(z^t)$$

$$P(z^t):\ a_1(z^t) + a_2(z^t) \leq x(z^t)$$

$$Q(z^t):\ b_1(z^t) + b_2(z^t) \leq y(z^t)$$

Equivalence is evident considering the mapping between date-t, state-t prices and date-0 prices

Closed-form solution of seq. problem

- Functional form: $u = \dfrac{a^{1-\alpha} + b^{1-\alpha}}{1-\alpha}$
- Eq. prices given welfare weights:

$$P(z^t) = \left[\frac{\sum_i \lambda_i^{\frac{1}{\alpha}}}{x(z^t)}\right]^{\alpha} \qquad Q(z^t) = \left[\frac{\sum_i \lambda_i^{\frac{1}{\alpha}}}{y(z^t)}\right]^{\alpha}$$

- Eq. allocations given welfare weights:

$$a_1(z^t) = \lambda_1^* x(z^t) \qquad a_2(z^t) = \lambda_2^* x(z^t),$$

$$b_1(z^t) = \lambda_1^* y(z^t), \qquad b_2(z^t) = \lambda_2^* y(z^t)$$

$$\lambda_i^* \equiv \frac{\lambda_i^{\frac{1}{\alpha}}}{\sum_i \lambda_i^{\frac{1}{\alpha}}} \qquad \lambda_1^* + \lambda_2^* = 1$$

Closed form solution contn'd

- Eq. TOT:

$$TOT(z^t) = \frac{p(z^t)}{q(z^t)} = \frac{P(z^t)}{Q(z^t)} = \left[\frac{y(z^t)}{x(z^t)}\right]^{\alpha}$$

which are independent of welfare weights.

- To finish characterizing equil., we need to find particular weights $(\lambda_1^*, \lambda_2^*)$ that correspond to the CE for the endowment sequence $\{x(z^t), y(z^t)\}$
- This is done by identifying weights such that optimal quantities and prices satisfy each country's budget constraint
- Weights are time and state invariant!

Equilibrium Welfare Weights

$$\sum_{t,z^t} p(z^t)\big(x(z^t) - a_1(z^t)\big) = \sum_{t,z^t} q(z^t) b_1(z^t)$$

$$\sum_{t,z^t} \beta^t \pi(z^t) \left[\frac{\sum_i \lambda_i^{\frac{1}{\alpha}}}{x(z^t)}\right]^\alpha \big(x(z^t) - \lambda_1^* x(z^t)\big) = \sum_{t,z^t} \beta^t \pi(z^t) \left[\frac{\sum_i \lambda_i^{\frac{1}{\alpha}}}{y(z^t)}\right]^\alpha \lambda_1^* y(z^t)$$

$$\sum_{t,z^t} \beta^t \pi(z^t) x(z^t)^{1-\alpha} = \lambda_1^* \left[\sum_{t,z^t} \beta^t \pi(z^t)\big(x(z^t)^{1-\alpha} + y(z^t)^{1-\alpha}\big)\right]$$

$$\lambda_1^* = \frac{\sum_{t,z^t} \beta^t \pi(z^t) x(z^t)^{1-\alpha}}{\sum_{t,z^t} \beta^t \pi(z^t)\big(x(z^t)^{1-\alpha} + y(z^t)^{1-\alpha}\big)}$$

Balance of trade

- Define trade balance in units of imported goods

$$TB(z^t) = TOT(z^t)\left[x(z^t) - a_1(z^t)\right] - b_1(z^t)$$

- Equilibrium trade balance:

$$TB(z^t) = \left[\frac{y(z^t)}{x(z^t)}\right]^{\alpha} \left[x(z^t) - \lambda_1^* x(z^t)\right] - \lambda_1^* y(z^t)$$

$$TB(z^t) = y(z^t)^{\alpha} x(z^t)^{1-\alpha}(1 - \lambda_1^*) - \lambda_1^* y(z^t)$$

- Equilibrium terms of trade:

$$TOT(z^t) = \left[\frac{y(z^t)}{x(z^t)}\right]^{\alpha}$$

The Relationship Between TB and TOT

- TOT gain caused by $(\downarrow x, \bar{y})$ or $(\uparrow y, \bar{x})$
- First case: $(\downarrow x, \bar{y})$, compare TB before and after

$$x^{1-\alpha}\bar{y}^{\alpha}(1-\lambda_1^*) - \lambda_1^*\bar{y} \lesseqgtr \bar{x}^{1-\alpha}\bar{y}^{\alpha}(1-\lambda_1^*) - \lambda_1^*\bar{y}$$

$$x^{1-\alpha} \lesseqgtr \bar{x}^{1-\alpha}$$

$$\left(\frac{x}{\bar{x}}\right)^{1-\alpha} \lesseqgtr 1 \Rightarrow \begin{cases} \alpha < 1 \Rightarrow \left(\frac{x}{\bar{x}}\right)^{1-\alpha} < 1 \\ \alpha = 1 \Rightarrow \left(\frac{x}{\bar{x}}\right)^{1-\alpha} = 1 \\ \alpha > 1 \Rightarrow \left(\frac{x}{\bar{x}}\right)^{1-\alpha} > 1 \end{cases} \quad \rho(TOT, TB) \lesseqgtr 0 \Leftrightarrow \alpha \lesseqgtr 1$$

The Relationship Between TB and TOT

- Second case: $(\uparrow y, \bar{x})$

$$TB(y) \lesseqgtr TB(\overline{y})$$

$$(1-\lambda_1^*)\bar{x}^{1-\alpha}(y^\alpha - \bar{y}^\alpha) \lesseqgtr \lambda_1^*(y-\bar{y})$$

$$\alpha \to 0 \Rightarrow \uparrow y \Rightarrow (\uparrow TOT, \downarrow TB)$$

$$\alpha \ \ \text{large} \Rightarrow \uparrow y \Rightarrow (\uparrow TOT, \uparrow TB)$$

Main result

- The relationship between TOT and TB in a complete-markets setup depends on
  - Nature of the shock driving the two variables (*x* or *y*)
  - Preference parameters
  - <u>Not on duration of shocks</u>
- HLM effect requires “relatively” low atemporal & intertemporal elasticities of substitution

Why is this different from OSR?

- TOT shock is a shock incorporated into Z^t given z^0. In this sense, it is somewhat "anticipated" (realization of a random process with known support)
- We can show that for an "unanticipated or parametric" change in sequence of endowments not included in Z^t" (i.e., an *MIT shock*) the results are similar to OSR (permanent shock has no effect on TB).

An “unexpected permanent shock”

- Consider a permanent endowment shock in perfect foresight.
- Initial equilibrium: $x_t = y_t = 1 \quad \forall\ t$

$$\lambda_1^* = \lambda_2^* = 1/2$$

$$p_t = q_t = \beta^t \qquad \text{TOT=1}$$

$$a_{1t} = a_{2t} = b_{1t} = b_{2t} = 1/2 \quad \forall\ t$$

$$TB_t = [x_t - a_{1t}] - b_{1t} = 0$$

Effects of “unexpected permanent shock”

- At $t = \tau$ $\quad y_1 < 1 \quad \forall\ t \geq \tau$

$$TOT_t = {y_1}^{\alpha} \quad \forall\ t \geq \tau$$

$$TB_t = {y_1}^{\alpha}(1 - \lambda_1^*) - \lambda_1^* y_1$$

$$= {y_1}^{\alpha} - \frac{{y_1}^{\alpha}}{1 + {y_1}^{1-\alpha}} - \frac{y_1}{1 + {y_1}^{1-\alpha}}$$

$$= \frac{{y_1}^{\alpha}(1 + {y_1}^{1-\alpha}) - ({y_1}^{\alpha} + y_1)}{1 + {y_1}^{1-\alpha}} = 0$$

Why are predictions different under complete markets?

- Adding uncertainty without AD claims to OSR setup implies assuming incomplete markets
- *b* is the only financial asset, and does not allow state-contingent trading
  - Adds potentially powerful wealth effects of nondiversified shocks and prec. savings
- In last example, λ_1^* changes from ½ to $\dfrac{1}{1+{y_1}^{1-\alpha}}$
  - Reflects wealth effect.
- Incomplete markets limit agent's ability to adjust behavior to "possibility of change"

What if we try solving the incomplete markets case?

- Backus shows that in the bond-economy case:

$$TB(z^t) = TOT(z^t)[x(z^t) - \lambda_1^*(z^t)x(z^t)] - \lambda_1^*(z^t)y(z^t)$$

where $\lambda_1^*(z^t)$ is the *state-contingent* welfare weight that captures wealth effect (without a closed-form solution for $\lambda_1^*(z^t)$)

- HLM effect of the OSR setup in a stochastic model requires incomplete markets (not MIT shocks, but non-insurable shocks)

- This is necessary, but is it sufficient?