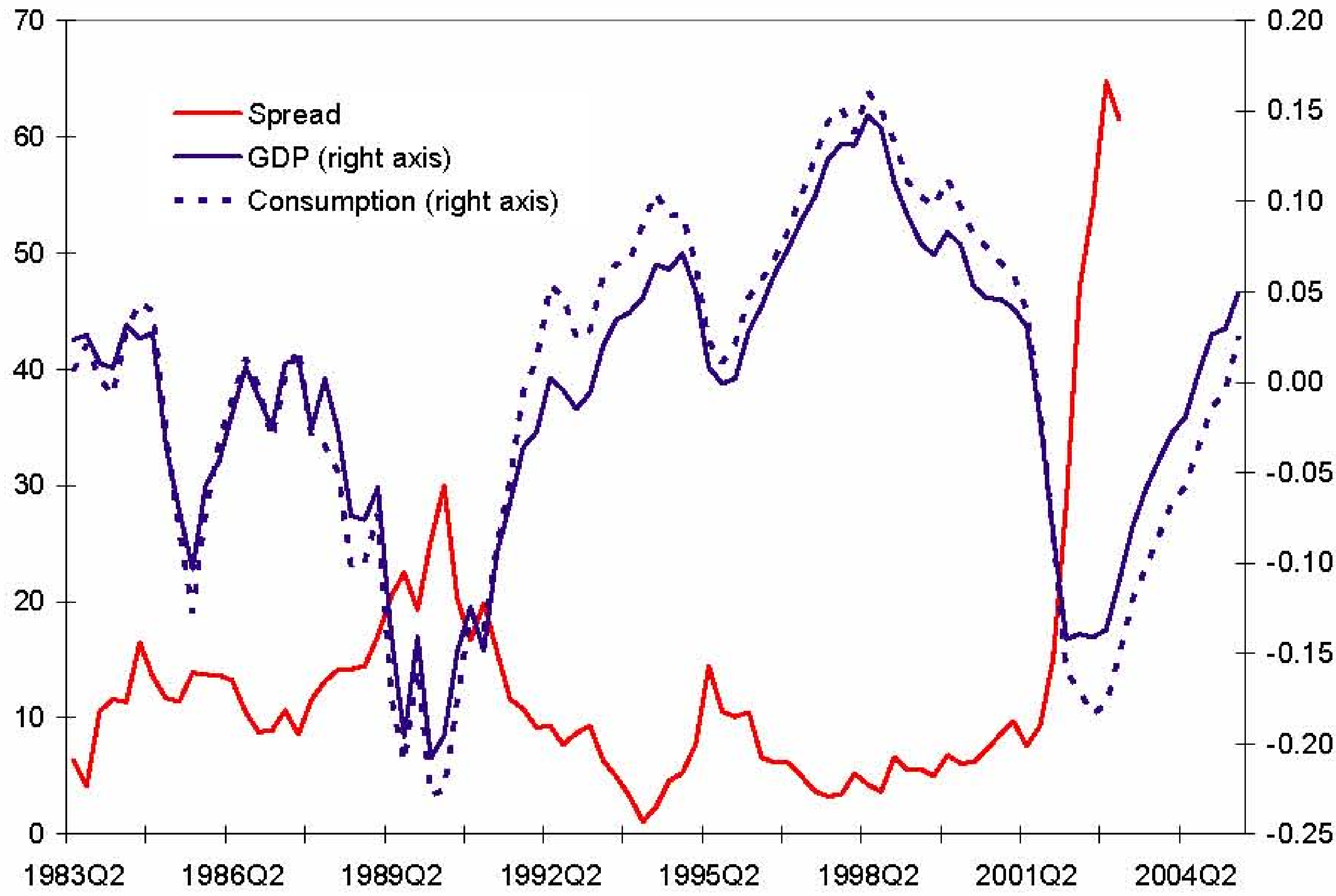
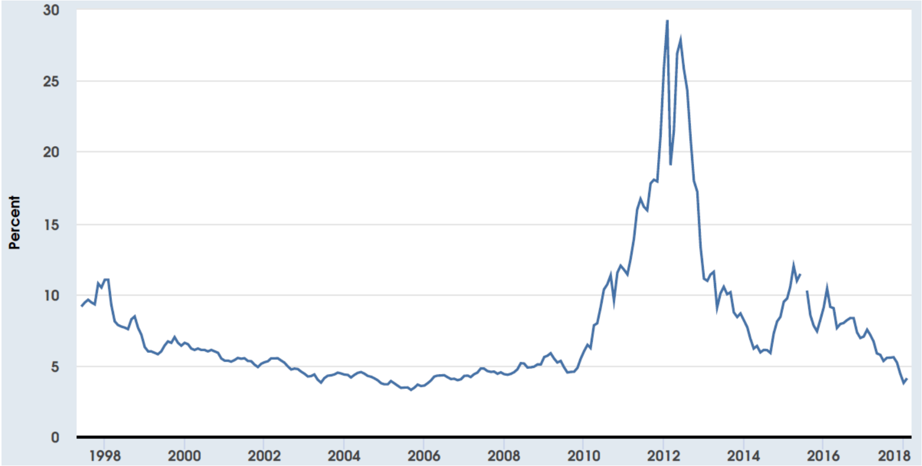
A Primer on Quantitative Models of Sovereign Default on External Debt

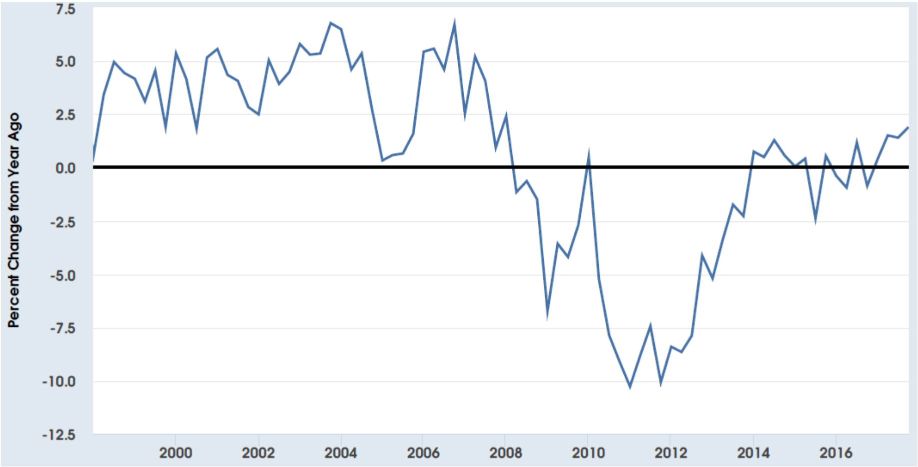
Argentina: Default risk & business cycles



Greece: Default risk & Business Cycles



a) Ten-year government bond yields



b) Quarterly real GDP growth (annual rate)

Literature on external default

- Heavily studied in the 1980s in the aftermath of defaults by many developing countries (LA, Asia, Africa)
- Most work was theoretical, non-structural empirical, or narrative.
- Main contribution was Eaton & Gerzovitz (1981): first model of strategic default by benevolent gov. unable to commit to repay debt sold to risk-neutral foreign lenders:
  - Debt is NSC, income is exogenous and stochastic
  - Gov. compares value of repayment v. autarky, defaults if latter is higher
  - Default costs: Exclusion, fixed loss of consumption
  - Prob. of default at t+1 on debt sold at t is given by prob. that, at the given debt, income at t+1 lands at a value for which default is **optimal**
  - Risk-neutral pricing implies a risk premium given by default prob.
  - Followed by several theoretical studies: Bulow & Rogoff (need of full exclusion), Calvo (self-fulfilling defaults)
- Faded away until mid 2000s, motivated by EMs defaults (Argentina, Russia, Ecuador) and role of risk premia in cycles, and more recently by European crisis. New focus on quantitative work.

Research on country risk & business cycles

- **RBC models w. working capital and exogenous country risk**:
  - Uribe & Yue (06), Perri & Neumeyer (06), Oviedo (06)
  - Observed country risk as exogenous int. rate shock
  - Labor paid in advance, int. rate shocks affect labor costs
  - Effects of country risk are large w. large shocks + *all* labor paid in advance
  - Caveats: country risk is endogenous, and working capital financing is likely to be much smaller than in these models
- **EG models of strategic default with exogenous output shocks**:
  - Arellano (08), Aguiar & Gopinath (06), Yue (06), Bi (08a,b), Chatterjee & Eyigungor (12), D'Erasmo (08), Cuadra & Sapriza (08), Hatchondo et al (08), Wright (08), Benjamin & Wright (09), Pitchford & Wright (10), etc…
  - Quantitative studies of variants of EG model
  - Calibrated to output process of defaulting economies (Argentina)
  - Can't explain observed default probs. and debt ratios unless particular form of output costs are imposed exogenously
- **EG models with endogenous output** and private sector role (Mendoza & Yue (12), Sosa-P. (18), Arellano et al. (16))
- Surveys: Aguiar & Amador (13), Aguiar et al. (18)

Arellano 2008: Default Risk & Fluctuations in Emerging Economies

- Benevolent government maximizes private utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

- World credit market: NSC discount bonds B' with pricing function $q(B',y)$. Debt implies $B'<0$. Gov. gets $-q(B',y)B'$ today (transfers it to households), pays B' tomorrow *if it does not default.*

- Budget constraint under repayment:

$$c = y + B - q(B', y)B'.$$

- Budget constraint under default:

$$c = y^{def} \qquad y^{def} = h(y) \leq y \qquad \text{with } h'(y) > 0$$

- Risk-neutral creditors max. profits given prices and default prob. δ :

Profits: $\phi = qB' - \frac{(1-\delta)}{1+r} B'$ Optimality cond: $q = \frac{(1-\delta)}{1+r}$

Recursive formulation

- Government and lenders act sequentially.
  - Gov. starts with B, observes y, and then chooses to default or not
  - If Gov repays, it observes a pricing function $q(B',y)$ and chooses B' optimally, and creditors choose B' taking q and δ as given

- At equilibrium the pricing function satisfies:

$$q(B', y) = \frac{(1 - \delta(B', y))}{1 + r}$$

  - For $B' \geq 0$, default prob. is zero, bonds pay world interest rate $1+r$
  - q lies in the interval $[0,1/(1+r)]$ and $1/q$ is country interest rate

- Value function for gov. with default option:

$$v^o(B, y) = \max_{\{c,d\}} \left\{ v^c(B, y), v^d(y) \right\}$$

  - "c" means continuation (staying in the credit contract)
  - "d" means default

Recursive formulation contn 'd

- **Value of default**

$$v^d(y) = u(\boxed{y^{def}}) + \beta \int_{y'} \left[\theta \boxed{v^o(0, y')} + (1-\theta)\boxed{v^d(y')}\right] f(y', y)dy'$$

  - θ = exogenous re-entry prob. (otherwise default is absorbent)

- **Value of continuation** (i.e. conditional on not defaulting)

$$v^c(B, y) = \max_{(B')} \left\{ u(\boxed{y - q(B', y)B' + B}) + \beta \int_{y'} \boxed{v^o(B', y')} f(y', y)dy' \right\}$$

  – subject to lower bound on debt $B' > Z$

- Repayment and default income sets (for given *B*)

$$A(B) = \{y \in Y : v^c(B, y) \geq v^d(y)\} \qquad D(B) = \{y \in Y : v^c(B, y) < v^d(y)\}$$

  – A default income threshold $y^*(B)$ divides $\{y, B\}$ space into repayment & default regions

Recursive Equilibrium

- **Recursive equilibrium** is defined by (i) a consumption plan $c(s)$, for $s=\{B,y\}$, (ii) a policy function for sovereign debt $B'(s)$, (iii) repayment and default sets $A(B),\ D(B)$, and (iv) a bond pricing function $q(B',y)$ such that:
  a) Given (ii), $c(s)$ satisfies the budget constraint
  b) Given (iv), $B'(s),\ A(B)$ and $D(B)$ solve sovereign's problem
  c) The pricing function $q(B',y)$ reflects default probabilities and satisfies the lenders' no-arbitrage condition

- Resources generated by debt $-q(B',y)B'$ follow Laffer curve.
  – Default prob. rises as *B'* falls because value of continuation is increasing in *B'* and value of default is independent of *B* (Arellano proves using default & repayment sets, without differentiability)

Default probability and default sets

- At equilibrium, prob. of default at t+1 on *B'* cond. on date t information is:

$$\delta(B',y) = \int_{D(B')} f(y',y)dy'$$

- If default set is empty, default prob=0. If default set includes all of Y, default prob=1. But in general:

Proposition 1. (Default sets are shrinking in assets.) *For all $B^1 \leq B^2$, if default is optimal for B^2, in some states y, then default will be optimal for B^1 for the same states y, that is, $D(B^2) \subseteq D(B^1)$.*

  - Akin to E+G proposition showing that default prob is increasing in debt.

- Given Prop. 1 and bounded support for y, it follows that when bonds are sold with default risk they satisfy these boundaries:

$$\underline{B} \leq \overline{B} \leq 0 \qquad \underline{B} = \sup\{B: \ D(B) = Y\} \qquad \overline{B} = \inf\{B: \ D(B) = \emptyset\}$$

  - Debt is inside $\underline{B} \leq \overline{B} \leq 0$, because there is always a large (small) enough debt such that default (repayment) is optimal for all y

- This justifies that $q(.)$ depends on both y and B, but it depends on y only if the shock is not i.i.d. (E+G studied only iid)

Analytics of the i.i.d case

- Do govs. default in good times or bad times?
  - With complete markets, incentives for default are higher in good times (when is time to repay or save)
  - With incomplete markets it is possible to default in bad times
- In i.i.d. case, default incentives are stronger in bad times for sure:

Proposition 3. *Default incentives are stronger the lower the endowment. For all $y_1 \leq y_2$, if $y_2 \in D(B)$, then $y_1 \in D(B)$.*

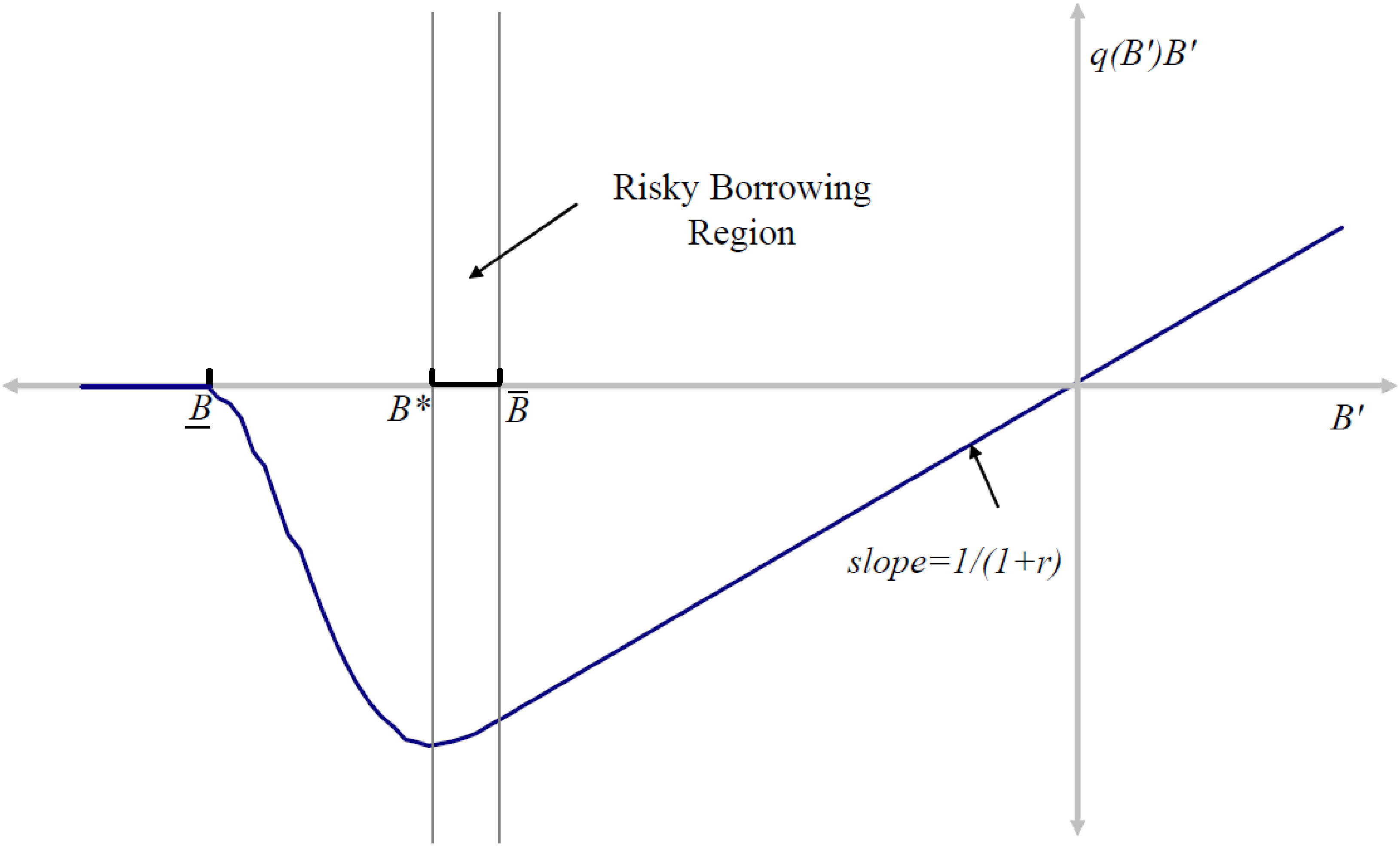
  - In bad times, contracts available are not useful insurance for a highly indebted borrower because they cannot increase c relative to y, so default may be preferable in recessions.
- Equilibrium debt Laffer Curve:

$$q(B')B' = \frac{1}{1+r}[1 - F(y^*(B'))]B'$$

  - The default income threshold $y^*(B)$ is decreasing in B due to Prop. 1.
  - At the threshold:

$$v^d(y^*(B)) = v^c(B, y^*(B)) \text{ for } B \in (\underline{B}, \overline{B})$$

Debt Laffer curve



For B'<B*, the same consumption resources can be raised with higher B' (smaller debt). Hence, risky debt exists only if risky borrowing region is non-empty

Numerical application to Argentina

Table 1. Business Cycle Statistics for Argentina

	Default episode			
	x: Q1–2002	$std(x)$	$corr(x,y)$	$corr(x,r^c)$
Interest rates spread	28.60	5.58	-0.88	
Trade balance	9.90	1.75	-0.64	0.70
Consumption	-16.01	8.59	0.98	-0.89
Output	-14.21	7.78		-0.88

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

σ is set to 2

r is set to 1.7%

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t^y,$$

$\rho = 0.945$ and $\eta = 0.025$.

"This specification for default costs gives the model flexibility such that higher default probabilities can be calibrated. Mechanically the asymmetric costs increase the range of risky borrowing because the value of autarky is a less sensitive function of the shock."

$$h(y) = \begin{Bmatrix} \hat{y} & if\ y > \hat{y} \\ y & if\ y \leq \hat{y} \end{Bmatrix}.$$

Calibration

	Values	Target Statistics
Discount factor	$\beta = 0.953$	3% default probability
Probability of re-entry	$\theta = 0.282$	Trade balance volatility 1.75
Output costs	$\widehat{y} = 0.969E(y)$	5.53% debt service to GDP

Business Cycle Statistics in the Benchmark Model

	Default Episodes	$std(x)$	$corr(x,y)$	$corr(x,r^c)$
Interest rates spread	24.32	6.36	-0.29	
Trade balance	-0.01	1.50	-0.25	0.43
Consumption	-9.47	6.38	0.97	-0.36
Output	-9.60	5.81		-0.29
Other Statistics				
Mean Debt (% output)	5.95	Mean Spread		3.58
Default Probability	3.00			

Business Cycle Statistics in the Model with Risk Averse Kernel

	Default Episodes	$std(x)$	$corr(x, y)$	$corr(x, r^c)$
Interest rates spread	53.69	10.65	-0.22	
Trade balance	-0.69	2.89	-0.15	0.17
Consumption	-8.11	7.17	0.91	-0.24
Output	-8.37	5.90		-0.22
Other Statistics				
Mean Debt (% output)	7.33		Mean Spread	10.4
Default Probability	3.1			

But Argentina defaulted on $100 million worth of external public debt, which was 37% of 2001 GDP, or 51% of GDP at end 2000 (almost 500% of exports!)