Workhorse Models of the Small Open Economy

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Objectives and plan of the lecture

• Introduce key issues for analyzing open economy models with incomplete markets: stationarity & debt/wealth dynamics, prec. savings

• Model 1: Deterministic, 1-sector endowment SOE

• Model 2: Stochastic variant of Model 1 but with incomplete markets

• Quantitative example using a variant of Model 2

• Limitations of using local v. global methods, based on Model 2 (FiPlt method introduction)
WORKHORSE MODEL 1:
DETERMINISTIC SMALL OPEN ECONOMY MODEL
Key Assumptions

1. SOE with perfect access to world credit market
2. One-period bonds, fixed world real interest rate
3. Perfect foresight OR Complete Markets
4. Credible commitment to repay
5. Frictionless economy, no distortions
   - CA supports perfect consumption smoothing
   - Long-run NFA is simply annuity value of steady-state trade balance
Intertemporal optimization problem

- **Sequential** social planner’s problem:

\[
(I) \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[
(II) c_t = y_t - b_{t+1} + b_t R, \quad b_0 \text{ given, } \{y_t\}_{t=0}^{\infty}
\]

- Combining constraints + NPG condition yields IBC:

\[
\sum_{t=0}^{\infty} R^{-t} c_t = \sum_{t=0}^{\infty} R^{-t} y_t + b_0 R
\]

- **Recursive** planner’s problem:

\[
(III) V_t(b_t, y_t) = \max_{b_{t+1}} \{u(c_t) + \beta V_{t+1}(b_{t+1}, y_{t+1})\}
\]

subject to (II)
Equilibrium conditions

• First-order condition of the recursive problem:
  
  \[ u'(c_t) = \beta V_{1t+1}(b_{t+1}, y_{t+1}) \]
  
  – From envelope theorem (Benveniste-Sheikman eq.)
  
  \[ V_{1t+1}(b_{t+1}, y_{t+1}) = Ru'(c_{t+1}) \]
  
  – So we obtain standard Euler equation:
  
  \[ u'(c_t) = \beta R u'(c_{t+1}) \]

• Stationarity assumption: \( \beta R = 1 \Rightarrow c_t = \bar{c} \ \forall \ t \)

• Closed-form solution (using IBC):
  
  \[ \frac{\bar{c}}{(1 - \beta)} = \left[ \sum_{t=0}^{\infty} \beta^t y_t \right] + b_0 R \quad \Rightarrow \quad \bar{c} = (1 - \beta) W \]
Current account, trade balance and NFA dynamics

• The equilibrium current account is:

\[ b_{t+1} - b_t = y_t - \bar{c} + b_t r \]

• Assume output converges:

\[ y_t \to \bar{y} \text{ as } t \to \infty \]

• Stationary equilibrium of CA is zero, and steady states of NFA and NX are given by:

\[
\bar{b} = - \frac{[\bar{y} - \bar{c}]}{r} = - \frac{nx}{r} = - \frac{[\bar{y} - (1 - \beta)W]}{r} = \beta W - \frac{\bar{y}}{r}
\]
Stationarity and initial conditions

- Stationary equilibrium is unique, but since wealth depends on initial NFA, \( \bar{b} \) and \( \bar{c} \) depend on \( b_0 \) (i.e. steady state depends on initial conditions).
- Borrow when \( y_t < \bar{c} \) and save when \( y_t > \bar{c} \)
  - CA deficit with low \( y_t \)
  - CA surplus with high \( y_t \)
  - CA is procyclical!
- Not a good model of actual CA dynamics
General equilibrium extension

• Standard production function \( f(k) \) and investment w. capital adjustment costs \( (\phi^2)(k_{t+1} - k_t)^2 \) (Tobin’s Q) can be added easily

• Consumption, NFA and CA dynamics are analogous to endowment case, but evaluated at eq. sequence of net income (output minus adj. costs) implied by arbitrage cond.

• Fisherian separation: Decision rule for \( k \) is independent of \( b \) but dec. rule for \( b \) depends on \( k \)
Recursive social planner’s problem

\[ V(k, b) = \max_{\{k', b', c\}} [u(c) + \beta V(k', b')] \]

s.t.

\[ c = f(k) - (k' - k) \left[ 1 + \frac{\phi}{2} (k' - k) \right] - b' + bR \]

• With a solution characterized by decision rules:

\[ \hat{k}'(k, b), \hat{b}'(k, b) \]
Euler equations

• Bonds

\[ u'(t) = \beta Ru'(t + 1) \]

• Capital

\[
[1 + \phi(k_{t+1} - k_t)]u'(t) = \beta u'(t + 1)[f'(k_{t+1}) + 1 + \phi(k_{t+2} - k_{t+1})]
\]
Four key properties

1. $k_{ss}$ is unique and independent of initial conditions, but $c$, NFA dynamics, and $b$ still depend on $b_0$

2. Fisherian separation: Investment and production dynamics determined by this arbitrage condition:

$$\frac{d' + q'}{q} \equiv \frac{f'(K') + 1 + \phi(K'' - K')}{1 + \phi(K' - K)} = 1 + r^*$$

3. Well-defined dynamics, unique steady-state
   
   – But steady-state Euler eq. does not yield a solution for $b$. Instead, we solve jointly with model’s dynamics

4. Local methods around det. steady states are not useful for solving these models
   
   – Even temporary shocks have permanent effects
   
   – But shooting methods do work
Time-series dynamics
(and a gains from trade argument)

\[ nx_0 = b_1 - b_0 R \]

\[ nxss = -r \times Ass \]
Effects of Shocks

1. Additive (e.g. government expenditures)
   - Permanent: No effect on debt or capital dynamics, equal effects on income profile and consumption.
   - Transitory: No effect on investment dynamics but affects debt dynamics through the effect on permanent income and steady state of $b$.

2. Multiplicative (e.g. productivity, terms of trade)
   - Permanent or transitory: Affect both investment and debt dynamics and steady state of $b$, but only permanent shocks affect $k_{ss}$.

- CA can turn countercyclical (e.g. persistent TFP shocks induce borrowing for investment)
WORKHORSE MODEL 2: STOCHASTIC MODEL WITH INCOMPLETE MARKETS
Uncertainty and Incomplete Markets

• NFA are non-state-contingent, one-period “real” bonds chosen from a finite state space defined by a discrete grid:

\[ B = [b_1 < b_2 < \ldots < b_z] \]

• Income and world interest rate are exogenous

• Income follows exogenous Markov process with “m” states and known transition prob. matrix:

\[ \bar{Y} = [y_1 < y_2 < \ldots < y_m] \quad P(y_i, y_j) \]

• Asset markets are incomplete: \( B \) cannot provide full insurance against income fluctuations
Sequential planner’s Problem

- Choose \( \{b_{t+1}\}_{t=0}^\infty \) so as to

\[
\max E_0 \left[ \sum_{t=0}^\infty \beta^t u(c_t) \right]
\]

s.t.

\[
c_t = y_t - b_{t+1} + b_t R
\]

\[
b_{t+1} \in B \quad P(y_t, y_{t+1}) \text{ known}
\]

\[(b_0, y_0) \text{ given,}\]

...looks very similar to Model 1, but it is very different!
Aiyagari’s natural debt limit

• $u(.)$ is twice differentiable, concave and satisfies the Inada condition:

$$\lim_{c \downarrow 0} u'(c) = \infty$$

• Implies that consumption must be positive at all times, and hence budget constraint yields NDL:

$$b' \geq -\left[\frac{y_{min}}{R - 1}\right]$$

  – Otherwise the agent is exposed to the risk of zero consumption with positive probability
  – Highlights “global” nature of decision-making under incomplete markets (all potential future histories matter)
  – Could also have ad-hoc debt limit $b' \geq -\phi \geq NDL$
Recursive planner’s problem

\[
V(b_n, y_i) = \max_{b' \in B} \left\{ u(y_i - b' + b_nR) + \beta \sum_{j=1}^{m} P(y_i, y_j) V(b', y_j) \right\}
\]

s.t. \( b' \geq -\phi \geq NDL \)

for each of the \( m \times z \) pairs \( (b_n, y_i) \), with \( b_1 = -\phi \).

• The solution is characterized by:
  1. Decision rule \( b' = g(b, y) \)
  2. Value function \( V(b_n, y_i) \)
  3. Unconditional stationary distribution of \((b, y)\)

\[
\lambda(b, y) = \text{Prob}(b_t = b, y_t = y)
\]

• Fast and easy to solve w. FiPIT method
Law of motion of conditional probabilities

- $P(y_t, y_{t+1})$ and $b' = g(b, y)$ induce a law of motion for conditional transition probabilities from date-$t$ states $(b,y)$ to date-$t+1$ states $(b',y')$:

$$
\lambda_{t+1}(b',y') = \text{Prob}(b_{t+1} = b', y_{t+1} = y')
$$

$$
= \sum_{b_t \in B} \sum_{y_t \in \bar{y}} \text{Prob}(b_{t+1} = b'|b_t = b, y_t = y) \times \text{Prob}(y_{t+1} = y'|y_t = y) \times \text{Prob}(b_t = b, y_t = y)
$$
Equilibrium Transition Probabilities

• But since \( b' = g(b, y) \) is a unique recursive function of \((b, y)\), the law of motion becomes:

\[
\lambda_{t+1}(b', y') = \sum_{b} \sum_{y} \lambda_t(b, y) \text{Prob}(y_{t+1} = y' | y_t = y) \gamma(b', b, y)
\]

\[
\gamma(b', b, y) = \begin{cases} 
1 & \iff b' = g(b, y) \\
0 & \text{otherwise}
\end{cases}
\]

– Which can be rewritten as:

\[
\lambda_{t+1}(b', y') = \sum_{y} \sum_{\{b : b' = g(b, y)\}} \lambda_t(b, y) P(y, y')
\]
Stochastic Stationary State

- The stochastic steady state is a joint stationary distribution of NFA and income, $\lambda(b, y)$, which is the fixed point of the law of motion

$$
\lambda_{t+1}(b', y') = \sum_y \sum_{\{b': b' = g(b, y)\}} \lambda_t(b, y) P(y, y')
$$

- Methods to solve for $\lambda(b, y)$:
  - Iterating to convergence in the law of motion
  - Computing Eigen values of $(mxz)^2$ trans. prob. matrix
  - Powering to convergence transition prob. Matrix

- Use it to compute moments and IRFs:

$$
E[b] = \sum_{(b, y) \in B \times Y} \lambda(b, y) b \\
E[c] = \sum_{(b, y) \in B \times Y} \lambda(b, y)(y - b'(b, y) + Rb) \\
E_t[b] = \sum_{(b, y) \in B \times Y} \lambda_t(b, y) b \\
E_t[c] = \sum_{(b, y) \in B \times Y} \lambda_t(b, y)(y - b'(b, y) + Rb)
$$
Precautionary savings  
(failure of the standard stationarity condition)

• Standard stationarity assumption $\beta R = 1$ fails  
  – Euler eq. implies “constant consumption,” but income is always stochastic and NFA is non-state-contingent.  
  – Formally: marginal benefit of saving $\beta^t R^t u'(t)$ follows a Supermartingale process, and since Supermartingales converge, it follows that $b' \to \infty$

• Agents self insure, build precautionary savings  
  – If $\beta R < 1$, force pushing to borrow and force pushing for prec. savings support stationary distribution  
  – Natural Debt Limit imposes lower bound on NFA  
  – But the deterministic st. state is always the debt limit!
Equilibrium & stationary NFA demand curve

\[ \rho = \frac{1}{\beta} - 1 \]

autarky equilibrium

small open economy eq.
Remarks about solving models with incomplete markets

- Solving these models generally requires global methods that can track dynamics of wealth distributions.
- Certainty equivalence fails (e.g., higher variance or persistence of shocks increases average NFA).
- Local methods feature a unit root unless a “stationarity inducing” assumption is added (Schmitt G & Uribe (03)).
- But local solutions differ significantly from global solution (de Groot, Durdu & Mendoza (20)).
- Prec. savings also affects portfolio structure (wealthier agents/countries tolerate more risk, hold larger shares of risky assets at lower premia).
Example from Durdu, Mendoza & Terrones (2008)

• SOE with exogenous Markov endowment:

\[ V(b, \varepsilon) = \max_{b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \exp(-v(c)) E[V(b', \varepsilon')] \right\} \]

s.t. \[ c = \varepsilon y - b' + bR + A \]

\[ b_{t+1} \geq \phi \geq -\min(\varepsilon ty + A) / r \]

• Allows for 2 formulations of rate of time pref.:
  1. Uzawa-Epstein endogenous rate of time preference
  2. Bewley-Aiyagari-Hugget setup with \( \beta R < 1 \)

\[ v(c) = \rho^{UE} \ln(1 + c) \text{ or } \ln(1 + \rho^{BAH}) \]
Calibration

• Discrete state space:

\[(b, b') \in B = \{b_1 < b_2 < \ldots < b_n\}\]

\[\varepsilon \in E = \{\varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_j\}\]

\[\pi(\varepsilon_{t+1} | \varepsilon_t)\]

\[n=1000\]

• Income process (set to Mexico’s detrended GDP)

\[y_t = \rho_y y_{t-1} + \epsilon_t\]

\[\sigma_y = 3.301\%\]

\[\rho_y = 0.597\]

\[\sigma_{\epsilon} = \sqrt{\sigma_y^2 (1 - \rho_y^2)} = 2.648\text{ percent}\]

– Discretized using Tauchen-Hussey quadrature method with j=5 (yields process with 3.28% s.d. and AR=0.55)

– Can also use canonical Markov chains (e.g. “simple persistence” rule) to discretize time-series processes
• $E[y] = 1$ for simplicity (variables are GDP ratios)

• $E[b] = -0.44$ Mexico’s average NFA/GDP 1985-2004 in Lane & Milesi Ferretti (06)

• $E[c] = 69.2$ Mexico’s average C/GDP 1965-2005

• $R = 1.059$ Mexico’s country real interest rate from Uribe and Yue (06)

• It follows that $A = y + b(R - 1) - c = 0.282$.

• Discount factors and rates of time preference:
  - **UE:** $\rho^{UE} = \ln(R) / \ln(1 + c) = 0.109 \quad (1 + c)^{-0.109} = 0.944$
  - **BAH:** $\rho^{BAH} = 0.064$ set by searching for values of ad-hoc debt limit & discount factor that match $E[b]=-0.44$ and $sd(c)=3.28\%$ ( $\phi =-0.51 \quad \beta=0.94$)
Calibrated state space

- Vector of income realizations

```
1    -0.075642
2    -0.035892
3       0.0
4    0.035892
5    0.075642
```

- Transition prob. matrix of income shocks

```
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<th>ROW</th>
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<th>COL 3</th>
<th>COL 4</th>
<th>COL 5</th>
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```

- Grid of bonds: spacing=0.001514, nodes=1000, lower bound=-0.5123
## Calibrated parameter values

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
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<tr>
<td>$\rho_{BAH}$</td>
<td>Rate of time preference in the BAH setup</td>
<td>0.064</td>
</tr>
<tr>
<td>$\rho_{UE}$</td>
<td>Rate of time preference elasticity in the UE setup</td>
<td>0.109</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of relative risk aversion</td>
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<td>$\phi$</td>
<td>Ad-hoc debt limit</td>
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<td>$R$</td>
<td>Gross world interest rate</td>
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<tr>
<td>$\bar{y}$</td>
<td>Mean output</td>
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<td>$c$</td>
<td>Consumption-output ratio</td>
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<td>$b$</td>
<td>Net foreign assets-output ratio</td>
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<td>$\sigma_{e}$</td>
<td>Standard deviation of output innovations</td>
<td>0.026</td>
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<td>$\rho$</td>
<td>Autocorrelation of output</td>
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<tr>
<td>$A$</td>
<td>Lump-sum absorption</td>
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</table>
Transitional and stationary distributions

Note: Initial conditions are lowest (b,y) with positive long-run probability
Transitional and stationary distributions

B. Uzawa-Epstein Preferences
(Starting from lowest 1% probability NFA position)

Note: Initial conditions are lowest (b,y) with positive long-run probability
Transitional dynamics of NFA

Note: Dynamics show forecasting function starting from lowest positive prob. B and neutral income shock and plotted as differences relative to long-run averages.
Effects of income variability on precautionary NFA demand

A. Variability

- Foreign assets in Percent of GDP
- Standard deviation of output (%)
## Unconditional moments

<table>
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<tr>
<th></th>
<th>Baseline</th>
<th>Auto Corr 0.7</th>
<th>Std Dev. 5%</th>
<th>Std Dev. 2.5%</th>
<th>Risk Aver. 5.0</th>
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<td>BAH</td>
<td>UE</td>
<td>BAH</td>
<td>UE</td>
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<td>0.10</td>
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<td>Means</td>
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<td>1.00</td>
<td>1.00</td>
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<td>-0.42</td>
<td>-0.41</td>
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<td>Trade balance(^2/)</td>
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<td>0.02</td>
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<td>Coefficients of variation (in percent)</td>
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<td>3.63</td>
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<td>3.92</td>
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<td>2.02</td>
<td>2.77</td>
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<td>Baseline</td>
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<td>Auto Corr 0.7</td>
<td>Std Dev. 5%</td>
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<td>variation (relative to output)</td>
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<td>Output correlations</td>
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<td>-0.48</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Output</td>
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<td>0.59</td>
<td>0.69</td>
<td>0.69</td>
<td>0.59</td>
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<tr>
<td>Consumption</td>
<td>0.97</td>
<td>0.84</td>
<td>0.97</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>Foreign assets</td>
<td>0.99</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Current account</td>
<td>0.57</td>
<td>0.51</td>
<td>0.67</td>
<td>0.62</td>
<td>0.57</td>
</tr>
<tr>
<td>Trade balance</td>
<td>0.67</td>
<td>0.55</td>
<td>0.76</td>
<td>0.67</td>
<td>0.67</td>
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<tr>
<td>Discount factor</td>
<td>0.98</td>
<td>0.00</td>
<td>0.98</td>
<td>0.00</td>
<td>0.98</td>
</tr>
</tbody>
</table>
GLOBAL V. LOCAL METHODS FOR OPEN ECONOMY MODELS WITH INCOMPLETE MARKETS
Be careful how we “close” op. ec. models

- Schmitt-Grohe & Uribe (03) proposed three ad-hoc ways to induce stationarity so that local methods can be used:
  1. Debt-elastic interest rate (DEIR) function $r(b - \bar{b})$
  2. Resource cost of holding assets $h(b - \bar{b})$
  3. Rate of time pref. depends on “aggregate” $C(b - \bar{b})$

- They showed these are about equivalent in an RBC moment-matching exercise

- DEIR is widely used in research & policy and regarded as innocuous (assumed to yield accurate approximation)

- Using this approach, Garcia-Cicco, Pancrazi & Uribe (10) concluded that RBC-SOE model cannot explain AR behavior of net exports
Autocorrelation functions of TB/Y
Autocorr. of Net Exports: Data v. Models

- Garcia-Cicco et al.: NX is AR(1) in the data, but in RBC-SOE model solved with DEIR function it is a near-unit-root process.

- de Groot et al. (2020): this is not a property of the “exact” solution, but a limitation of using stationarity-inducing assumptions (DEIR function)

- Heuristic argument:
  1. Definition of net exports: \( t b_t = b_{t+1} - b_t R^* \)
  2. Assume AR(1) process for NFA: \( b_{t+1} = \rho b_t + \varepsilon_{t+1} \) and notice DEIR implicitly sets \( \rho \) when specifying \( r(b - \bar{b}) \). Garcia-Cicco et al. set it so that \( \rho \approx 1 \), so that DEIR is “inessential”
Autocorrelations of net exports and NFA

• Combine 1 & 2, solve for AR(1) of net exports:

\[ \rho(nx) = \frac{q^2 \rho + \rho - q - q \rho^2}{1 + q^2 - 2q \rho} \]

where \( q = 1/R^* \)

• \( \rho(nx) \) is a nonlinear function of \( \rho \), so we need “exact” solution for \( \rho \) in order to derive correct results about \( \rho(nx) \)
  – Changing \( \rho \) from 0.95 to 0.999 changes \( \rho(nx) \) from near zero to 0.999!!
  – Knowing true solution of NFA dynamics is critical
Autocorrelations of NFA and NX

First-order autocorrelation of NFA vs. First-order autocorrelation of TB

4% interest rate
8.5% interest rate
Autocorrelations of NFA and NX

Empirically relevant range of AR(1) of NX
Limitations of ad-hoc approach to induce stationarity of NFA positions

• Generally: ad-hoc approach imposes long-run and AR(1) of NFA instead of solving for it

• “Exact” global, non-linear solution is not critical for some business cycle moments, but it is critical for those directly related to NFA and for other key issues:
  1. Global imbalances (accumulation of reserves)
  2. Financial crises & macro-prudential regulation
  3. Sovereign risk
  4. Financial development
...but still ad-hoc approach is widely used

- Allows using local methods that solve quickly and can be applied to large models
- DEIR is by far more common than cost of holding bonds or endogenous discounting
- Majority sets the debt elasticity of DEIR function to “inessential value” of 0.001 following SGU (2003), others calibrate it or estimate it (0.00014-2.8 range)
- Most applications use 1OA), some have used 2OA, 3OA or risky steady state (RSS)
- Recent methods for occ. binding constraints: OccBin (Iacoviello-Guerrieri), DynareOBC (Holden)
Global v. local methods for op. ec. models
(de Groot, Durdu,& Mendoza (2020))

• Compared global solution (FiPIt) v. 1OA, 2OA, RSS, OccBin & DynareOBC for endowment economy, RBC, and Sudden Stops (RBC w. occ. binding collateral constraint)

• Local methods approximate poorly prec. savings

• Business cycle moments, IRFs, SDFs, and financial crises dynamics & frequency differ

• Best performance requires targeting NFA moments from global sol. (e.g. autocorr. of NFA)

• Various local methods differ mainly on 1st moments
Model 2 again

• Optimization problem:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad u(c_t) = \frac{c_t^{1-\sigma}}{1 - \sigma}. \]

\[ c_t = e^{z_t \bar{y}} + b_t - q b_{t+1} \]

\[ b_{t+1} \geq -\varphi. \]

• Optimality conditions in recursive form:

\[ c(b, z)^{-\sigma} \geq \beta R \sum_{z'} \pi(z', z) \left[ (c(b'(b, z), z'))^{-\sigma} \right] \]

\[ c(b, z) = e^{z \bar{y}} + b - q b'(b, z) \]
FiPIT, a Simple & Fast Global Method
Mendoza-Villalvazo (2020)

1. Start iteration $j$ with a conjectured decision rule $\hat{b}_j'(b, z)$

2. Generate the consumption dec. rule implied by that conjecture using the resource constraint

$$c_j(b, z) = e^z \bar{y} + b - q \hat{b}_j'(b, z)$$

3. Solve for a new consumption dec. rule “directly” using the Euler eq. (assuming $\varphi$ is not binding)

$$c_{j+1}(b, z) = \left\{ \beta R \sum_{z'} \pi(z', z) \left[ \left( c_j(\hat{b}_j'(b, z), z') \right)^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}}$$

– In RHS, we evaluate the j-th iteration cons. dec. rule using the values of the state variables at $t+1$
– Requires interpolation, because consumption dec. rule is only known at grid nodes
– No need for a non-linear solver as with endogenous grids method
FiPIT Method Contn’d

4. Generate new bond’s decision rule $b'_{j+1}(b, z)$ using the resource constraint. If $b'_{j+1}(b, z) \leq -\varphi$, the debt limit binds and we set $b'_{j+1}(b, z) = -\varphi$.

5. Update the initial conjecture for iteration $j+1$:

$$\hat{b}'_{j+1}(b, z) = (1 - \rho)\hat{b}'_j(b, z) + \rho b'_{j+1}(b, z)$$

– Use $0 < \rho < 1$ for unstable iterations, or $\rho > 1$ for slow convergence.

6. Iterate to convergence (until $b'_{j+1}(b, z) = \hat{b}'_j(b, z)$ up to a convergence criterion)

• Analogous to Parameterized Expectations (fixed-point iteration using simulation & regression in Step 3)
• Finite state space better than colocation (occ. bind. constraints)
• Extends easily to 2 endogenous states w. bilinear interpolation
Local methods

• **1OA, 2OA**: standard approximations of NFA dec. rule applied to approximations of same order to opt. conditions around $b^{dss}$ (DEIR with $\beta(1+r) = 1$)

• **RSS**: $b^{rss}$ obtained from 2OA of cond. expectation of steady-state Euler eq., solved *jointly* with 1OA of decision rule around $b^{rss}$ (assumes $\beta(1+r) < 1$)

• Use DEIR to support $b^{dss}$ (necessary in 1OA & 2OA)

$$\frac{1}{q_t} \equiv 1 + r_t = 1 + r + \psi \left[ e^{b^{dss}} - b_{t+1} - 1 \right]$$

– For small perturbations, debt elasticity is $\eta^r \equiv -\psi b^{dss}$

– $\psi$ can be SGU *baseline* inessential value (0.001) or *targeted* to a particular moment (e.g. autocorr. of nfa)
Local methods cont’d

• 2OA to NFA decision rule in dev. form:

$$\tilde{b}_{t+1} = h_b \tilde{b}_t + h_y \tilde{y}_t + \frac{1}{2} \left( h_{bb} \tilde{b}_t^2 + h_{yy} \tilde{y}_t^2 + h_{\sigma_z \sigma_z} \sigma_z^2 \right) + h_{by} \tilde{b}_t \tilde{y}_t$$

  a) 1OA and RSS have only the first two terms in RHS
  b) RSS uses risky ss. instead of det. ss to define devs.
  c) $h_b$ has same value regardless of approx. order
  d) $h_{\sigma_z \sigma_z} \sigma_z^2$ captures effect of income variability on NFA (prec. savings). In RSS it also matters for risky ss.
  e) Quantitatively, all other 2nd order terms are negligible

• Assuming log utility and i.i.d. income process:

$$h_b(\psi, b^*) = \frac{R + e^{b^* \psi} (1 - b^* \psi + \psi) - \sqrt{R^2 + 2e^{b^* \psi} (b^* \psi + \psi - 1)R + e^{2b^* \psi} (1 - b^* \psi + \psi)^2}}{2e^{b^* \psi}}$$

  – Hence, autocorr. of NFA is

$$\rho_b(\psi, b^*) \approx h_b(\psi, b^*)$$
# Calibration

## 1. Common parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Coefficient of relative risk aversion</td>
<td>2.0</td>
</tr>
<tr>
<td>$y$</td>
<td>Mean endowment income</td>
<td>1.00</td>
</tr>
<tr>
<td>$A$</td>
<td>Total absorption</td>
<td>0.28</td>
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<tr>
<td>$R$</td>
<td>Gross world interest rate</td>
<td>1.059</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of income (percent)</td>
<td>3.27</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autocorrelation of income</td>
<td>0.597</td>
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## 2. Global solution parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.940</td>
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<tr>
<td>$\phi$</td>
<td>Ad-hoc debt limit</td>
<td>$-0.51$</td>
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## 3. Local solution parameters

### Common parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.944</td>
</tr>
<tr>
<td>$b$</td>
<td>Deterministic steady state value of NFA</td>
<td>$-0.51$</td>
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### Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\psi$</td>
<td>Inessential DEIR coefficient</td>
<td>0.001</td>
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### Targeted calibration

<table>
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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\psi$</td>
<td>DEIR coefficient for 2OA</td>
<td>0.0469</td>
</tr>
<tr>
<td>$\psi$</td>
<td>DEIR coefficient for RSS</td>
<td>0.0469</td>
</tr>
</tbody>
</table>
NFA autocorr. & the three local methods

- $\rho_b(\psi, b^*)$ describes eq. mapping between debt elasticity parameter and NFA autocorr. in local solutions
  - If $\psi = 0$, it has two roots given by $(1+r,1)$, so NFA is non-stationary.

- Given $(R, b^*)$, $\rho_b(\psi, b^*)$ is a U-shaped function of $\psi$, but in quantitatively relevant range is downward sloping, convex.

- Plot $\rho_b(\psi, b^*)$ as $\psi$ varies for $b^*$=0, -0.51 (det. ss.) and -0.41 (risky ss.)

- For $0 \leq \psi \leq 0.1$, $\rho_b(\psi, b^*)$ nearly identical for 1OA, 2OA & RSS!

- Since 2nd order terms (except $h_{\sigma_z\sigma_z} \sigma_z^2$) are negligible, all three methods have very similar 2nd & higher-order moments and IRFs, and pruning is irrelevant!
Elasticity of DEIR function & NFA dec. rule
## Comparison of long-run moments

<table>
<thead>
<tr>
<th></th>
<th>GLB</th>
<th>Baseline Calibration</th>
<th>Targeted Calibration</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>2OA</td>
<td>RSS DEIR βR &lt; 1 DEIR</td>
<td>2OA RSS DEIR DEIR</td>
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<tr>
<td>(\psi = )</td>
<td>na</td>
<td>0.001</td>
<td>na 0.001 0.0469 0.0469</td>
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<tr>
<td><strong>Averages</strong></td>
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<tr>
<td>(\mu(c))</td>
<td>0.694</td>
<td>0.701 0.093 0.692</td>
<td>0.689 0.689</td>
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<tr>
<td>(\mu(nx/y))</td>
<td>0.022</td>
<td>0.015 0.625 0.025</td>
<td>0.028 0.028</td>
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<tr>
<td>(\mu(b/y))</td>
<td>-0.413</td>
<td>-0.282 -11.210 -0.451</td>
<td>-0.502 -0.506</td>
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<tr>
<td><strong>Standard deviations relative to standard deviation of income</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma(c)/\sigma(y))</td>
<td>0.992</td>
<td>1.594 1.161 1.617</td>
<td>1.001 0.997</td>
</tr>
<tr>
<td>(\sigma(nx)/\sigma(y))</td>
<td>0.660</td>
<td>1.327 1.202 1.346</td>
<td>0.730 0.730</td>
</tr>
<tr>
<td>(\sigma(nx/y)/\sigma(y))</td>
<td>0.643</td>
<td>1.311 1.161 1.331</td>
<td>0.709 0.709</td>
</tr>
<tr>
<td>(\sigma(b)/\sigma(y))</td>
<td>7.461</td>
<td>62.327 1.706 40.078</td>
<td>6.647 6.576</td>
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<tr>
<td>(\sigma(b/y)/\sigma(y))</td>
<td>7.735</td>
<td>61.989 1.892 40.213</td>
<td>7.174 7.118</td>
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<tr>
<td><strong>Income correlations</strong></td>
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<tr>
<td>(\rho(y,c))</td>
<td>0.755</td>
<td>0.202 0.188 0.197</td>
<td>0.684 0.684</td>
</tr>
<tr>
<td>(\rho(y,nx))</td>
<td>0.729</td>
<td>0.572 0.312 0.567</td>
<td>0.705 0.708</td>
</tr>
<tr>
<td>(\rho(y,nx/y))</td>
<td>0.704</td>
<td>0.572 0.006 0.567</td>
<td>0.705 0.708</td>
</tr>
<tr>
<td>(\rho(y,b))</td>
<td>0.449</td>
<td>0.128 0.070 0.124</td>
<td>0.489 0.488</td>
</tr>
<tr>
<td>(\rho(y,b/y))</td>
<td>0.549</td>
<td>0.156 0.445 0.149</td>
<td>0.593 0.592</td>
</tr>
<tr>
<td><strong>First-order autocorrelations</strong></td>
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<tr>
<td>(\rho_c)</td>
<td>0.840</td>
<td>0.995 0.996 0.995</td>
<td>0.929 0.929</td>
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<tr>
<td>(\rho_{nx})</td>
<td>0.543</td>
<td>0.819 0.934 0.823</td>
<td>0.583 0.582</td>
</tr>
<tr>
<td>(\rho_{nx/y})</td>
<td>0.551</td>
<td>0.826 0.995 0.830</td>
<td>0.591 0.590</td>
</tr>
<tr>
<td>(\rho_b)</td>
<td>0.977</td>
<td>0.999 0.999 0.999</td>
<td>0.977 0.977</td>
</tr>
<tr>
<td>(\rho_{b/y})</td>
<td>0.961</td>
<td>0.998 0.953 0.998</td>
<td>0.958 0.959</td>
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</tbody>
</table>
Effect of higher income variability on mean NFA

As SGU (2003) showed, DEIR and bond adjustment cost are analogous up to 1OA. Hence higher $\psi$ is akin to higher adj. cost, which keeps NFA close to its mean.
Impulse response functions

a. NFA/Output Baseline

b. NFA/Output Targeted

c. Consumption Baseline

d. Consumption Targeted

e. NX/Output Baseline

f. NX/Output Targeted