



# Financial innovation, the discovery of risk, and the U.S. credit crisis<sup>☆</sup>



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## ABSTRACT

Financial innovation and overconfidence about the risk of new financial products were key factors behind the 2008 U.S. credit crisis. We show that a model with a collateral constraint in which learning about the risk of a new financial environment interacts with Fisherian amplification produces a boom–bust cycle in debt, asset prices and consumption. Early realizations of a high-borrowing-ability regime turn agents optimistic about the persistence probability of this regime. Conversely, the first realization of a low-borrowing-ability regime turns agents unduly pessimistic. The model predicts large increases in household debt, land prices and excess returns during 1998–2006 followed by a collapse.

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*“Booms become busts because justifiable confidence becomes foolish optimism.”* – Robert J. Samuelson, “Causes of the Crisis,” *Washington Post*, 3/19/2012.

## 1. Introduction

The U.S. financial crisis was preceded by sharp increases in household credit, residential land prices, and leverage ratios (see Fig. 1).<sup>1</sup> Between 1997 and 2006, the year in which the crisis started as home prices began to decline nationwide, the

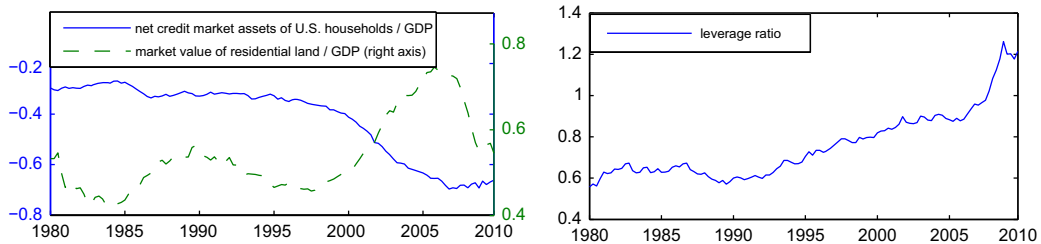
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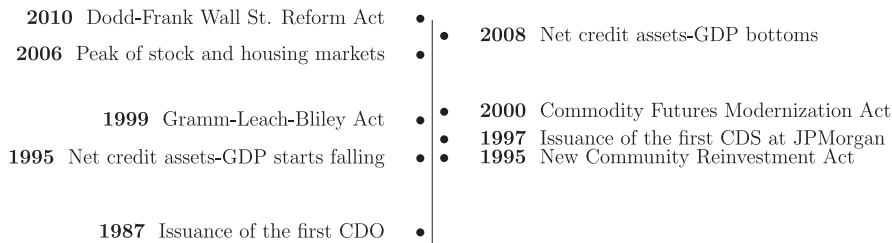
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<sup>1</sup> High leverage in financial institutions also played a critical role. Still, understanding the mechanisms that drove household credit is critical, because household mortgage origination was the anchor of the housing-related securities driving the surge in leverage of financial institutions.



**Fig. 1.** Net credit market assets, value of residential land and leverage ratio.

Sources: Net Credit Market Assets: Flow of Funds Accounts of the United States, Board of Governors of the Federal Reserve System. Value of Residential Land: Davis and Heathcote (2007).



**Fig. 2.** Timeline of events during the run-up to the U.S. credit crisis.

net credit assets of U.S. households fell from  $-35$  to  $-70\%$  of GDP and the market value of residential land surged from  $45\%$  to nearly  $75\%$  of GDP.<sup>2</sup> By contrast, these ratios were quite stable in the previous two decades. Debt grew much faster than land values, however, because the ratio of the two, a macroeconomic measure of household leverage, rose from  $0.68$  in  $1997$  to  $0.93$  in  $2006$ . The crisis then resulted in a sudden increase in leverage, as land prices fell faster than the ability to reduce debts, and leverage continued hovering around  $1.2$  after that.

As Fig. 2 shows, the U.S. credit boom started with a period of significant financial innovation characterized by new financial instruments that “securitized” the payment streams generated by a wide variety of assets, particularly home mortgages, and by far-reaching reforms that radically changed financial regulations. The gradual introduction of collateralized debt obligations (CDOs) dates back to the early 1980s, but the securitization boom that fueled the growth of household debt started in the mid 1990s with the introduction of residential mortgage backed securities (RMBSs) and collateralized mortgage obligations (CMOs). This process was greatly amplified by the introduction of credit default swaps (CDSs) on the payments of CMOs by the mid 2000s. By the end of 2007, the market of CDSs alone was worth about \$45 trillion (or 3 times U.S. GDP). The financial reforms introduced in the 1990s were the most significant since the Great Depression, and in fact aimed at removing the barriers separating bank and non-bank financial intermediaries set in the 1933 Glass–Steagall Banking Act. Three Acts were particularly important for the housing boom: The 1995 New Community Reinvestment Act, which strengthened the role of Fannie Mae and Freddie Mac in mortgage markets and facilitated mortgage securitization; the 1999 Gramm–Leach–Bliley Act, which removed the prohibition on bank holding companies from owning other financial companies; and the 2000 Commodity Futures Modernization Act, which left over-the-counter financial derivatives beyond the reach of regulators.

The pattern linking financial innovation, booms in credit and asset prices, and financial crises is not unique to the recent U.S. experience. In fact, credit booms and busts are commonly associated with large changes in the financial environment. For instance, many of the countries to which the financial crisis spread after the U.S. crash in 2008 displayed similar pre-crisis features, in terms of a large expansion of the financial sector into new instruments under new regulations, and also experienced housing booms (e.g. the United Kingdom, Spain, Iceland, Ireland). Mendoza and Terrones (2012) provide more systematic evidence of this phenomenon. They found that 35% of the credit booms observed in the 1960–2010 period across developed and emerging economies occurred after surges in capital flows, which were largely driven by reforms that liberalized capital accounts, and 25% occurred after large financial reforms. They also found that credit booms are associated with sharp cycles in economic activity and housing prices.

This paper provides an explanation for the observed relationship between financial innovation and the credit cycle. In particular, we show that financial innovation, interacting with credit constraints, leads to a “natural” underpricing of the risk associated with a new financial environment, and that this produces a surge in credit and asset prices followed by a collapse.

<sup>2</sup> Following Davis and Heathcote (2007), we focus on residential land prices instead of housing prices. They showed that land prices are significantly more important than prices of residential dwellings for explaining the evolution of U.S. housing prices between 1975 and 2006.

Undervaluing the risk is natural because of the lack of data on the performance of the new financial instruments, and on the stability of the financial system under new laws.<sup>3</sup>

We propose a model in which the true risk of the new financial environment can only be discovered with time, and this learning process interacts with a collateral constraint that limits households' debt not to exceed a fraction of the market value of their holdings of a fixed asset (i.e., residential land). Financial innovation is modeled as a structural change that introduces a regime with a higher leverage limit. Agents know that in this new environment one of the two financial regimes can materialize in any given period: one in which high ability to leverage continues, and one in which there is a lower leverage limit. They do not know the true risk of this new environment, because they lack data with which to estimate accurately the true switching probabilities across the two regimes. They are Bayesian learners, however, and so they learn over time as they observe regime realizations, and in the long-run their beliefs converge to the true regime-switching probabilities. Hence, in the long-run the model converges to the rational expectations (RE) solution, but in the short-run optimal plans and asset prices deviate from the RE equilibrium, because beliefs that differ from those of the RE solution lead to mispricing of risk.<sup>4</sup>

The model's collateral constraint introduces the classic Fisherian mechanism of financial amplification, but the analysis of its interaction with the learning dynamics is a novel feature of this paper.<sup>5</sup> In particular, the deviations of the agents' beliefs from the true RE regime-switching probabilities distort asset pricing conditions. If the constraint binds, optimistic beliefs lead agents to assign higher probabilities to states with lower excess returns, which causes a feedback loop producing higher asset prices and higher debt, and the opposite occurs when agents are pessimistic. Thus, the over- or under-pricing of assets translates into over- or under-inflated collateral values that affect the financial amplification dynamics.

Quantitative analysis shows that the process of discovery of risk in the presence of collateral constraints leads to a period of booming credit and land prices, followed by a sharp, sudden collapse. We conduct an experiment calibrated to U.S. data in which we date the start of financial innovation in the first quarter of 1998 and the beginning of the financial crisis in the first quarter of 2007. Hence, from 1998 to the end of 2006 we assume that the economy experienced the high-leverage regime, followed by a switch to the low-leverage regime in the first quarter of 2007.<sup>6</sup> Net credit assets did not rise sharply then (see Fig. 1), but the fraction of banks tightening credit standards in the Federal Reserve's *Willingness to Lend Survey* jumped from nearly zero to over 50%, and the median downpayment on conventional mortgages jumped from 5% to 13% (see February 16, *Wall Street Journal*). We also acknowledge that several factors beyond the scope of this paper played a role in the U.S. crisis (e.g. excessive leverage and exposure to counterparty risk amongst financial intermediaries, moral hazard in financial markets and rating agencies, reckless lending practices, global financial imbalances, flawed government regulation, etc.). In this paper, however, we focus exclusively on the role of financial innovation affecting households' ability to borrow in an environment with imperfect information, because we aim to show how these frictions alone cause a sharp boom–bust credit cycle.

The initial priors of the Bayesian learning process are calibrated to match observed excess returns on Fannie Mae's RMBS at the beginning of 1998, and the high- and low-leverage limits are set equal to the actual leverage ratios before 1998 and at the end of 2006. Under these assumptions, agents become very optimistic about the probability of persistence of the high-leverage regime soon after 1998, and remain so until they observe the switch to the low-leverage regime. During this “optimistic phase,” debt, leverage and land prices rise significantly above what the RE equilibrium predicts.<sup>7</sup> In fact, the model accounts for 64% and 49% of the 1998–2006 rise in net household debt and residential land prices respectively, and it matches well the observed dynamics of RMBS excess returns. Conversely, when agents observe the first realization of the low-leverage regime, they respond with a sharp correction in their beliefs and become unduly pessimistic, causing sharp downward adjustments in credit, land prices and consumption.

The transition to the low-leverage regime is exogenous, and thus part of the credit crisis in the model is exogenous. However, the equilibrium declines in credit and prices in the model also reflect the endogenous amplification operating through the interaction of the collateral constraint and the agents' beliefs. This amplification mechanism is very strong and accounts for most of the drop in credit and prices predicted by the model. Moreover, the effects on debt and asset prices are nearly twice as large when learning and credit frictions interact than when we remove either one.

We model learning following the approach proposed by Cogley and Sargent (2008b). They offer an explanation of the equity premium puzzle based on persistent pessimism caused by the Great Depression. They assume high and low states for exogenous consumption growth, with the true transition probabilities across these states unknown. Agents learn the true

<sup>3</sup> In the U.S. securitization boom, the strategy of “layering of risk,” mixing tranches of top-rated assets with riskier assets, justified the belief that the new instruments were so well diversified that they were virtually risk free, assuming that the risk was priced correctly.

<sup>4</sup> We follow the standard practice of referring to the perfect information equilibrium as the rational expectations equilibrium, even though the Bayesian learning equilibrium is also a rational expectations equilibrium.

<sup>5</sup> Fisher (1933) gave a prominent role to changes in optimism and gloom of economic agents, but modern formulations of financial accelerators abstract from fluctuations in beliefs. He, however, assigned a limited role to changes in beliefs *except* when they interact with his credit amplification mechanism: “*I fancy that over-confidence seldom does any great harm except when, as, and if, it beguiles its victims into debt.*” (Fisher (1933)).

<sup>6</sup> We take this switch to the low-leverage as exogenous. One way to endogenously generate such a switch is to explicitly model the freeze of the interbank market as in Gertler and Kiyotaki (2010).

<sup>7</sup> The degree of optimism peaks just before agents observe the first realization of the low-leverage regime, because, when the new financial environment is first introduced, agents cannot rule out the possibility of the high-leverage regime being absorbent until they experience the first low-leverage state.

probabilities over time as they observe consumption growth. Similarly, in our setup, the true probabilities of switching across leverage regimes are unknown, and agents learn about them over time.

This paper is also related to the large Macro and Finance literatures on learning models. On the Macro side, the literature tends to focus on learning from noisy signals (e.g. Blanchard et al., forthcoming; Boz, 2009; Boz et al., 2011; Edge et al., 2007; Lorenzoni, 2009; VanNieuwerburgh and Veldkamp, 2006, and the survey by Evans and Honkapohja, 1999). The informational friction in our models is different, because agents observe realizations of the relevant variables without noise. Instead, there is imperfect information about their true transition probabilities. The U.S. credit crisis provides a natural laboratory to study learning models of this class, because the creation of the late 1990s brand-new financial regime implied that there was no useful data to estimate the true probability of its potential failure. Our work is also reminiscent of the literature on Knightian uncertainty, where agents do not know the true model with which to assess the future (see Caballero and Krishnamurthy, 2008 for an application to financial crises).

The imperfect information studies by Adam et al. (2011), Gennaioli et al. (2012) and Zeira (1999) are closer to this paper's argument. Adam et al. (2011) study housing booms and current account imbalances in G7 countries using a learning model with a collateral constraint in which Bayesian learning about housing prices amplifies the effects of interest rate cuts. Gennaioli et al. (2012) study how underestimating the probability of rare events that have large negative effects on risky asset returns causes overborrowing in an environment with endogenous financial innovation. In their model, the informational friction is in the form of "local thinking," by which agents assign zero probability to those rare events. In contrast, we model Bayesian learning about transition probabilities across financial regimes, with agents always assigning non-zero probability to all states of nature. The papers also differ in that we evaluate the ability of our model's quantitative predictions to match U.S. data while they focus mainly on theoretical analysis. Zeira argued that financial liberalization or structural changes in productivity could be followed by booms and crashes because of "informational overshooting." As in our setup, agents need to learn the true characteristics of a new asset pricing environment, but in Zeira's model this is captured by an increase in dividend growth of unknown duration and agents updating their beliefs about a future date in which high dividend growth will end. As long as they observe high dividend growth, their beliefs about future dividends increase, leading to a boom in stock prices. Then when agents finally observe the end of the dividend boom, expectations of future profits fall and prices collapse.

The credit constraint used in our model is similar to those widely examined in the macro literature on financial frictions and the international macro literature on Sudden Stops, see for example, Jermann and Quadrini (2006), Mendoza (2010) and Durdu et al. (2009). When these credit constraints are used in RE models, precautionary savings reduce significantly the long-run probability of states in which the constraints bind. In our learning model, however, agents have much weaker incentives for building precautionary savings than under rational expectations, until they attain the long-run equilibrium in which they know the true risk of the financial environment. That is, the process of discovery of risk generates sizable overborrowing (relative to the RE decentralized equilibrium), because of the optimistic expectations of agents during the optimistic phase. Since agents borrow "too much" during this phase, the economy is also more vulnerable to suffer a credit crunch when the first switch to a low-leverage regime occurs. In addition, our model differs from most financial crisis models in that it aims to explain both the boom and bust credit cycles, whereas crisis models typically focus only on the latter.

Finally, this paper is also related to some of the recent literature on the U.S. crisis that emphasizes learning frictions, financial innovation and deregulation, particularly the work of Howitt (2011), Favilukis et al. (2010) and Ferrero (2012). Howitt studies the interaction of expectations, leverage and a solvency constraint in a representative agent setup similar to ours, but he uses adaptive learning about asset returns to show how this leads to periods of "cumulative optimism" followed by "cumulative pessimism," and how this can lead to a crisis.<sup>8</sup> Favilukis et al. (2010) analyze the macroeconomic effects of housing wealth and housing finance in a heterogeneous-agents, DSGE model with credit constraints. They study transition dynamics from an environment with high financial transaction costs and tight credit limits to one with the opposite features. Ferrero (2012) has a similar flavor in that he studies the effects of relaxing LTV requirements. Similar to these two studies, our paper emphasizes the role of a relaxation of borrowing constraints, but we focus on the effects of imperfect information and learning, while they study rational expectation models.

The remainder of the paper proceeds as follows: Section 2 describes the model and the learning process. Section 3 examines the model's quantitative implications. Section 4 concludes.

## 2. A model of financial innovation with learning

Consider a representative-agent economy where risk-averse individuals formulate optimal plans facing exogenous income fluctuations, the risk of which cannot be fully diversified because asset markets are incomplete. Individuals have access to two assets: a non-state-contingent bond and an asset in fixed supply (land). The credit market is imperfect,

<sup>8</sup> Interestingly, Howitt's work and ours differ in terms of the motivation for focusing on learning to study the financial crisis. Howitt argues that learning matters because agents learn about the process of asset returns in an unchanged financial regime, while we argue that it matters because agents learn gradually the true persistence of a new financial regime with perfect information about the process driving dividends.

because individuals' ability to borrow is limited not to exceed a fraction  $\kappa$  of the market value of their land holdings. That is,  $\kappa$  imposes an upper bound on the agents' leverage ratio.

The main feature that differentiates this model from other macro models with credit frictions is that agents have imperfect information about the regime-switching probabilities that drive fluctuations in  $\kappa$ . Specifically, we model a situation in which financial innovation starts with an initial shift from a low-leverage regime ( $\kappa^l$ ) to a regime with higher ability to leverage ( $\kappa^h$ ), and agents do not know the true regime-switching probabilities between  $\kappa^l$  and  $\kappa^h$ . They are Bayesian learners, and in the long-run they learn the true probabilities, but in the short-run they form expectations with the posteriors they construct as they observe realizations of  $\kappa$ .

We assume that the risk-free interest rate is exogenous in order to keep the interaction between financial innovation and learning tractable. At the aggregate level, this assumption corresponds to an economy that is small and open with respect to world capital markets. This is in line with recent evidence suggesting that in the era of financial globalization even the U.S. risk-free rate has been significantly influenced by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998 (see [Warnock and Warnock, 2006](#); [Bernanke, 2005](#); [Durdu et al., 2009](#); [Mendoza et al., 2009](#)). Moreover, data from the *Flow of Funds of the United States* shows that, while pre-1980s the U.S. economy was in virtual financial autarky, because the fraction of net credit of U.S. nonfinancial sectors financed by the rest of the world was close to zero, about one-half of the surge in net credit since the mid-1980s was financed by the rest of the world (see [Mendoza and Quadrini, 2010](#)). Alternatively, our setup can be viewed as a partial equilibrium model that studies the effects of financial innovation on household debt and residential land prices, taking the risk-free rate as given, as in [Corbae and Quintin \(2009\)](#) and [Howitt \(2011\)](#). Still, we will examine how our main results vary if we allow the interest rate to fall as financial innovation starts, in line with what was observed in U.S. 1998–2006 data.

### 2.1. Agents' optimization problem and equilibrium conditions

Agents act atomistically in competitive markets and choose consumption ( $c_t$ ), land holdings ( $l_{t+1}$ ) and holdings of one-period discount bonds ( $b_{t+1}$ ), taking as given the price of land ( $q_t$ ) and the gross real interest rate ( $R$ ) so as to maximize a standard intertemporal utility function:

$$E_0^s \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (1)$$

It is critical to note that  $E_t^s$  represents expectations conditional on the representative agent's beliefs formulated with the information available up to and including date  $t$ . These beliefs will differ in general from the rational expectations formulated with perfect information, which are denoted  $E_t^a$ .

The agents' budget constraint is

$$c_t = z_t g(l_t) + q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R} + b_t \quad (2)$$

Agents operate a Neoclassical production function  $g(l_t)$  subject to a TFP shock  $z_t$ .<sup>9</sup> A linear production technology can also be used, but we will use the curvature of  $g(l_t)$  to calibrate the model so that the condition that arbitrages returns across bonds and land is consistent with U.S. data on the risk-free interest rate and the value of residential land (see [Section 3](#) for details).

TFP shocks follow an exogenous discrete Markov process, about which agents have perfect information. That is, they know the Markov transition matrix  $\pi(z_{t+1}|z_t)$  and the corresponding set  $Z$  of  $M$  possible realizations of  $z$  at any point in time (i.e.,  $z_t \in Z = \{z_1 < z_2 < \dots < z_M\}$ ). Alternatively, we could assume that TFP shocks are also affected by imperfect information.

Frictions in credit contracting that we do not model explicitly force agents to comply with a collateral constraint that limits the value of debt (given by  $b_{t+1}/R$  since  $1/R$  is the price of discount bonds) to a time-varying fraction  $\kappa_t$  of the market value of their land holdings

$$\frac{b_{t+1}}{R} \geq -\kappa_t q_t l_{t+1} \quad (3)$$

In this constraint,  $\kappa_t$  follows a "true" Markov process characterized by a standard two-point, regime-switching process with regimes  $\kappa^h$  and  $\kappa^l$ , with  $\kappa^h > \kappa^l$ , and transition probabilities given by  $F^a = p^a(\kappa_{t+1}|\kappa_t)$ .<sup>10</sup> The continuation probabilities are denoted  $F_{hh}^a \equiv p^a(\kappa_{t+1} = \kappa^h|\kappa_t = \kappa^h)$  and  $F_{ll}^a \equiv p^a(\kappa_{t+1} = \kappa^l|\kappa_t = \kappa^l)$ , and the switching probabilities are  $F_{hl}^a = 1 - F_{hh}^a$  and  $F_{lh}^a = 1 - F_{ll}^a$ . The long-run probabilities are  $\Pi^h = F_{hh}^a / (F_{hh}^a + F_{hl}^a)$  and  $\Pi^l = F_{ll}^a / (F_{ll}^a + F_{lh}^a)$  respectively, and the corresponding

<sup>9</sup> Instead of modeling utility of housing services and production of residential dwellings (see [Iacoviello, 2005](#); [Kiyotaki et al., 2011](#)), we focus on the role of residential land as an asset with dividends set by a production technology and its price as the value of collateral. This is in line with the findings of [Davis and Heathcote \(2007\)](#) about residential land prices as the main driver of home prices, and it also allows us to maintain tractability in the global, non-linear learning algorithm needed to solve the model. In principle, introducing our framework of financial innovation with learning into models in the line of Iacoviello or Kiyotaki et al. should still produce a rise (fall) in housing prices during the optimistic (pessimistic) phase to the extent that land as an input in housing production is in inelastic supply and buyers and sellers have common beliefs.

<sup>10</sup> One could also specify a continuous AR(1) process for  $\kappa$ ,  $\kappa_t = m_t + \kappa_{t-1} + \varepsilon_t$ . The different regimes could be captured with a shift in the mean:  $m \in \{m^h, m^l\}$ , which the agents could learn about. We conjecture that this setup would yield similar results as agents could turn optimistic about the persistence of the  $m^h$  regime.



mean durations are  $1/F_{hl}^a$  and  $1/F_{lh}^a$ . The long-run unconditional mean, variance, and first-order autocorrelation of  $\kappa$  follow the standard regime-switching formulae:

$$E^a[\kappa] = (F_{lh}^a \kappa^h + F_{hl}^a \kappa^l) / (F_{lh}^a + F_{hl}^a) \quad (4)$$

$$\sigma^2(\kappa) = \Pi^h (\kappa^h)^2 + \Pi^l (\kappa^l)^2 - (E[\kappa])^2 \quad (5)$$

$$\rho(\kappa) = F_{ll}^a - F_{hl}^a = F_{hh}^a - F_{lh}^a. \quad (6)$$

Using  $\mu$  to denote the Lagrange multiplier on the credit constraint, the Euler equations for bonds and land in the agents' problem are

$$u'(c_t) = \beta RE_t^s [u'(c_{t+1})] + \mu_t \quad (7)$$

$$q_t (u'(c_t) - \mu_t \kappa_t) = \beta E_t^s [u'(c_{t+1}) (z_{t+1} g'(l_{t+1}) + q_{t+1})]. \quad (8)$$

Defining the return on land as  $R_{t+1}^q \equiv (z_{t+1} g'(l_{t+1}) + q_{t+1}) / q_t$  and the period marginal utility of consumption as  $\lambda_{t+1} \equiv \beta u'(c_{t+1})$ , the Euler equations can be used to derive the following expression for the excess return on land:

$$E_t^s [R_{t+1}^q - R] = \frac{(1 - \kappa_t) \mu_t - \text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)}{E_t^s[\lambda_{t+1}]} \quad (9)$$

Thus, as [Mendoza \(2010\)](#) explained, the borrowing constraint enlarges the standard premium on land holdings, driven by the covariance between marginal utility and asset returns, by introducing direct and indirect effects. The direct effect is represented by the term  $(1 - \kappa_t) \mu_t$ . The indirect effects are represented by the fact that the credit constraint hampers the agents' ability to smooth consumption, which reduces  $\text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)$ , and tilts consumption towards the future, which lowers  $E_t^s[\lambda_{t+1}]$ . Moreover, since expected land returns satisfy  $q_t E_t^s[R_{t+1}^q] \equiv E_t^s[z_{t+1} g'(l_{t+1}) + q_{t+1}]$ , we can obtain the following forward solution for the agents' land valuation as<sup>11</sup>:

$$q_t = E_t^s \left[ \sum_{j=0}^{\infty} \left( \prod_{i=0}^j \left( \frac{1}{E_t^s[R_{t+1+i}^q]} \right) \right) z_{t+1+j} g'(l_{t+1+j}) \right]. \quad (10)$$

The expressions in (9) and (10) imply that the collateral constraint lowers land prices because it increases the rate of return at which future land dividends are discounted. Note also that land valuations are reduced at  $t$  not just when the constraint binds at  $t$ , which raises  $E_t^s[R_{t+1}^q]$ , but also if the constraint is expected to bind in the future, which raises  $E_t^s[R_{t+1+i}^q]$  for some  $i > 0$ .

While the above effects are at work even when the true Markov process of  $\kappa$  is known (replacing  $E_t^s$  with  $E_t^a$ ), condition (10) also suggests that the learning process and the collateral constraint interact in an important way. For instance, suppose the credit constraint with  $\kappa^h$  binds at  $t$ . It follows that expected excess returns for  $t+1$  must be lower in states with  $\kappa^h$  than with  $\kappa^l$ , since the constraint must bind more in the latter. Hence, if beliefs are optimistic, agents assign more probability to states with lower expected returns than under true rational expectations, and this, via condition (10), translates into higher land prices, which in turn via the collateral constraint yields higher debt. The opposite is also true: if beliefs are pessimistic, agents assign higher probability to states with higher land returns, which depress current land prices more than under rational expectations, and via the collateral constraint this results in even lower debt.

Given the fixed unit supply of land, the model's equilibrium conditions are

$$u'(c_t) = \beta RE_t^s [u'(c_{t+1})] + \mu_t \quad (11)$$

$$q_t (u'(c_t) - \mu_t \kappa_t) = \beta E_t^s [u'(c_{t+1}) (z_{t+1} g'(1) + q_{t+1})] \quad (12)$$

$$c_t = z_t g(1) - \frac{b_{t+1}}{R} + b_t \quad (13)$$

$$\frac{b_{t+1}}{R} \geq -\kappa_t q_t 1 \quad (14)$$

We adapt the Bayesian learning setup proposed by [Cogley and Sargent \(2008b\)](#) (see Appendix A for details). This setup has the property that, if agents observe a long enough sample with sufficient regime switches, their beliefs converge to  $F^a$ .<sup>12</sup> Moreover, since  $\kappa$  is exogenous and observable, we are modeling passive learning from *and* about exogenous variables without noise. This facilitates the numerical solution, because it allows us to separate the evolution of beliefs from the agents' dynamic optimization problem, since agents cannot benefit from experimenting with the latter in order to improve the former. Accordingly, we follow a two-stage solution strategy: First, we use Bayesian learning to generate the sequence of

<sup>11</sup> Following [Mendoza \(2010\)](#), it is also possible to use Eq. (8) to express the forward solution for  $q_t$  in terms of the future stream of dividends discounted at the stochastic discount factors adjusted for the shadow value of the credit constraint.

<sup>12</sup> Time alone does not determine how fast agents learn. The order in which  $\kappa$  realizations occur and the number of switches across regimes also matters.

posterior density functions  $\{f(F^s|\kappa^t)\}_{t=1}^T$  over  $T$  periods in which a history of realizations of  $\kappa$  is observed. Each of these density functions is a probability distribution over possible transition matrices  $F^s$ . Since agents do not know  $F^s$ , the density function changes with the history of realizations observed up to date  $t$  (i.e.,  $\{\kappa^t, \kappa^{t-1}, \dots, \kappa^1\}$  where  $\kappa^t = (\kappa_t, \kappa_{t-1}, \dots, \kappa_1)$ ) and with the date-0 priors, as we explain below.

The second stage of the solution, again following Cogley and Sargent, solves the agent's optimal plans and the model's recursive equilibrium by adopting Kreps's Anticipated Utility (AU) approach to model dynamic optimization with Bayesian learning. The AU approach focuses on combining the sequences of posterior densities obtained in the first part,  $\{f(F^s|\kappa^t)\}_{t=1}^T$ , with chained solutions from a set of "conditional" AU optimization problems. These problems are conditional on the posterior density function of  $F^s$  that agents form with the history of realizations up to each date  $t$ . The rest of this section discusses in more detail the two stages of the solution algorithm.

## 2.2. Bayesian learning setup

The Cogley–Sargent learning setup fits nicely our aim to study financial innovation as the arrival of a new state  $\kappa^h$  with imperfect information about its transition probabilities, since there was no data history with which to evaluate the creditworthiness of the new financial instruments and the stability of the new financial regime. As we show below, if agents start by observing  $\kappa^h$  for a few periods, they go through an "optimistic phase" in which they assign a probability to continuing in  $\kappa^h$  higher than the true value. This optimism by itself is a source of vulnerability, because it is quickly reversed into a "pessimistic phase" with the opposite characteristics as the first realization of  $\kappa^l$  hits the economy. In addition, during the optimistic phase, the incentives to build precautionary savings against the risk of a shift in  $\kappa$  are weaker than in the long-run RE equilibrium, which increases the agents' risk of being caught with "too much" debt when the shift to  $\kappa^l$  occurs.

The learning framework takes as given  $\kappa^T$  and a prior of  $F^s$  for date  $t=0$ , and it yields the posteriors sequence  $\{f(F^s|\kappa^t)\}_{t=1}^T$ . At every date  $t$ , the information set includes  $\kappa^t$ . Agents then form posteriors using a Bayesian beta-binomial probability model (see Appendix A). Since information is imperfect only with regard to the Markov transition matrix across  $\kappa^s$ , and because  $\kappa$  can only take two values, this boils down to imperfect information about  $F_{hh}^a$  and  $F_{ll}^a$ . The other two elements of the transition matrix of  $\kappa$  are recovered as the corresponding complements. The posteriors have distributions  $F_{hh}^s \propto \text{Beta}(n_t^{hh}, n_t^{hl})$  and  $F_{ll}^s \propto \text{Beta}(n_t^{lh}, n_t^{ll})$ , where  $n_t^{ij}$   $i, j = h, l$  are counters of the number of regime switches observed in a particular history  $\kappa^t$ .

As in Cogley and Sargent (2008b), we assume that the date-0 priors for  $F_{hh}^s$  and  $F_{ll}^s$  are independent and given by  $p(F_{ii}^s) \propto (F_{ii}^s)^{n_0^{ii}-1} (1-F_{ii}^s)^{n_0^{ii}-1}$  for  $i = h, l$ . The counter  $n_0^{ij}$  denotes the number of transitions from state  $i$  to state  $j$  assumed to have been observed before  $t=1$ . The regime counters are then updated as follows:

$$n_{t+1}^{ij} = \begin{cases} n_t^{ij} + 1 & \text{if } \kappa_{t+1} = \kappa^j \text{ and } \kappa_t = \kappa^i, \\ n_t^{ij} & \text{otherwise.} \end{cases}$$

The posterior means, which are key for the second stage of the solution, are given by

$$E_t[F_{hh}^s] = n_t^{hh} / (n_t^{hh} + n_t^{hl}), \quad E_t[F_{ll}^s] = n_t^{ll} / (n_t^{ll} + n_t^{lh}). \tag{15}$$

Thus, the posterior means of the continuation probability of a particular regime change only when that same regime is observed at date  $t$ . Since in a two-point, regime-switching setup continuation probabilities also determine mean durations, it follows that agents learn about both the persistence and mean duration of  $\kappa^l$  and  $\kappa^h$  only as they actually experience each regime.<sup>13</sup>

The potential for financial innovation to produce significant underestimation of risk can be inferred from the evolution of the posterior means. With financial innovation defined as the arrival of a brand new regime  $\kappa^h$  with unknown probabilities of shifts between  $\kappa^h$  and  $\kappa^l$ , the learning process starts from  $n_0^{ij} \approx 0$ .<sup>14</sup> It follows then from Eq. (15) that the first sequence of realizations of  $\kappa^h$  generates substantial optimism (i.e., a sharp increase in  $E_t[F_{hh}^s]$  relative to  $F_{hh}^a$ ). Moreover, it also follows that the optimism produced by any subsequent sequence of realizations of  $\kappa^h$  will be smaller. Intuitively, this is because it is only after observing the first switch to  $\kappa^l$  that agents rule out the possibility of  $\kappa^h$  being an absorbent state. Similarly, the first realizations of  $\kappa^l$  generate a pessimistic phase, in which  $E_t[F_{ll}^s]$  is significantly higher than  $F_{ll}^a$ , and pessimism in this phase is larger than in any subsequent pessimistic phase.

Fig. 3 illustrates the model's learning dynamics using a simple example. Here, we set  $F_{hh}^a = 0.95$  and  $F_{ll}^a = 0.5$ , initial priors  $n_0^{ij} = 0.1$  for all  $i, j = h, l$ , and use a sample of 300  $\kappa$  realizations produced by a stochastic simulation of the true Markov-switching process. The chart shows the  $\kappa$  realizations, the time paths of  $E_t[F_{hh}^s]$  and  $E_t[F_{ll}^s]$ , and the true regime-switching probabilities.

The striking result from this example is that financial innovation leads to significant underestimation of risk, because the initial sequence of realizations of  $\kappa^h$  (the first "optimistic phase" up to  $t=30$ ) generates a high level of optimism that builds quickly. In fact, agents update their beliefs about the persistence of  $\kappa^h$  from 0.5 to 0.916 just after observing  $\kappa_1 = \kappa^h$ , and then

<sup>13</sup> If priors, as well as  $F_{hh}^a$  and  $F_{ll}^a$ , are correlated, learning would be faster, because agents would update both  $F_{hh}^s$  and  $F_{ll}^s$  every period. But this is akin to removing some of the informational friction by assumption. In an extreme case, with perfectly correlated priors and  $F_{hh}^a = F_{ll}^a$ , agents know from the start that the transition matrix is symmetric, which violates the initial premise stating that they do not know any of its properties.

<sup>14</sup> A truly "new" financial regime has  $n_0^{ij} = 0$ , but in this case the binomial distribution is not defined.

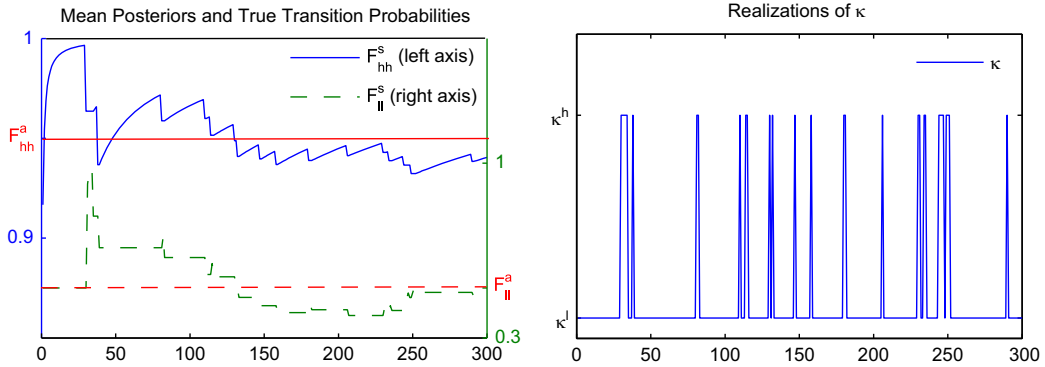


Fig. 3. Evolution of beliefs.

in the subsequent 29 periods  $E_t[F_{hh}^s]$  continues rising to peak at around 0.999. As explained above, optimism never grows as large during the optimistic phases that occur later on (e.g. between dates 40 and 80). The first realizations of  $\kappa^l$  generates a strong “pessimistic phase,” in which  $E_t[F_{ll}^s]$  is significantly higher than  $F_{ll}^a$  and raises quickly towards 1, so the period of highly optimistic expectations is followed by a period of highly pessimistic expectations.

Fig. 3 also reflects the result indicating that  $E_t[F_{hh}^s]$  and  $E_t[F_{ll}^s]$  are updated only when the economy is in the high- or low-leverage state. This also explains why in this example  $E_t[F_{hh}^s]$  converges to its RE counterpart faster than  $E_t[F_{ll}^s]$ . Given that  $\kappa^l$  is visited much less frequently, since  $F_{hh}^a > F_{ll}^a$ , it takes longer to learn its true persistence.

### 2.3. Recursive anticipated utility competitive equilibrium

We define the AU competitive equilibrium in recursive form. Since in the quantitative analysis we solve the model by policy function iteration on the equilibrium conditions (11) and (14), we formulate the recursive equilibrium using these conditions (see Appendix B for details). The state variables are defined by the triple  $(b, z, \kappa)$ . Because the law of iterated expectations still holds (see Appendix B in Cogley and Sargent, 2008b), we divide the problem into a set of AU optimization problems (AUOP) for  $t = 1, \dots, T$ , each conditional on  $E_t[F_{hh}^s]$  and  $E_t[F_{ll}^s]$ . We use time indices in policy and pricing functions to identify the date of the beliefs that match the corresponding AUOP.

It is important to note that the recursive AU equilibrium solution is not the same as full Bayesian optimization, which takes into account projections of the effect of future  $\kappa$  realizations on the evolution of beliefs. This is generally of limited tractability, because it requires a large state space that includes at each date  $t$  the four-dimensional vector  $n_t$  with all permutations of the regime-switching counters that can be observed up to that date.<sup>15</sup> Cogley and Sargent (2008a) show, however, that the optimal plans and asset prices obtained using AU are accurate approximations to those obtained with full Bayesian optimization, even in an environment with incomplete markets, CRRA preferences, and large regime-switching income shocks.<sup>16</sup>

Solving the sequence of AUOPs is akin to solving a sequence of equilibrium policy and pricing functions, one for each set of beliefs obtained at each date  $t = 1, \dots, T$ . This is still a demanding computational problem, particularly because of the non-linearity induced by the occasionally binding collateral constraint, but less vulnerable to the curse of dimensionality than the full Bayesian problem. Consider the date- $t$  AUOP. Agents observe  $\kappa_t$  and update their beliefs. Using the posterior means in (15), we construct the date- $t$  transition matrix

$$E_t^s[\kappa' | \kappa] \equiv \begin{bmatrix} E_t[F_{hh}^s] & 1 - E_t[F_{hh}^s] \\ 1 - E_t[F_{ll}^s] & E_t[F_{ll}^s] \end{bmatrix}.$$

The solution to the date- $t$  AUOP is then given by policy functions  $(b'_t(b, z, \kappa), c_t(b, z, \kappa), \mu_t(b, z, \kappa))$  and a pricing function  $q_t(b, z, \kappa)$  that satisfy conditions (11)–(14) rewritten in recursive form:

$$u'(c_t(b, z, \kappa)) = \beta R \left[ \sum_{z' \in Z_{\kappa'}} \sum_{\kappa' \in \{\kappa^h, \kappa^l\}} E_t^s[\kappa' | \kappa] \pi(z' | z) u'(c_t(b', z', \kappa')) \right] + \mu_t(b, z, \kappa) \quad (16)$$

$$q_t(b, z, \kappa) (u'(c_t(b, z, \kappa)) - \mu_t(b, z, \kappa) \kappa) = \beta \left[ \sum_{z' \in Z_{\kappa'}} \sum_{\kappa' \in \{\kappa^h, \kappa^l\}} E_t^s[\kappa' | \kappa] \pi(z' | z) u'(c_t(b', z', \kappa')) (z' g'(1) + q_t(b', z', \kappa')) \right] \quad (17)$$

<sup>15</sup> Intuitively, the AU approach captures the risk of fluctuations in future  $\kappa$ 's but not the uncertainty about future changes in their transition probabilities, while the Bayesian optimization captures both.

<sup>16</sup> The accuracy of their approximation begins to deteriorate with CRRA coefficients above 5. In our calibration it is set at 2, but since their model does not have a collateral constraint, our approximation may not be as accurate.



$$c_t(b, z, \kappa) = zg(1) - \frac{b'_t(b, z, \kappa)}{R} + b \quad (18)$$

$$\frac{b'_t(b, z, \kappa)}{R} \geq -\kappa q_t(b, z, \kappa)1 \quad (19)$$

It is critical to note that solving the date- $t$  AUOP means solving for a full set of optimal plans over the entire state space  $(b, z, \kappa)$  and conditional on date- $t$  beliefs. Thus, we are solving for the optimal plans agents conjecture they would make over the infinite future acting under those beliefs. For characterizing the “actual” equilibrium dynamics to match against data, however, the solution of the date- $t$  AUOP determines optimal plans for date  $t$  only. For example, the equilibrium decision rules for bond holdings that the model predicts for  $t = 1, \dots, T$  is obtained by chaining the relevant decision rules as follows:  $b_2 = b'_1(b, z, \kappa)$ ,  $b_3 = b_2(b, z, \kappa)$ ,  $\dots$ ,  $b_{T+1} = b'_T(b, z, \kappa)$ . This is crucial because beliefs change as time passes, and each subsequent  $\kappa_t$  is observed, which implies that the policy and pricing functions that solve each AUOP also change. Thus, history matters for the full solution of the model because different histories  $\kappa^t$  yield different sequences of beliefs, and hence different AUOP solutions. If at any two dates  $t$  and  $t+j$  we give the agents the same values for  $(b, z, \kappa)$ , they in general will not choose the same bond holdings for the following period because  $E_t^s[\kappa'|\kappa]$  and  $E_{t+j}^s[\kappa'|\kappa]$  will differ.

We can now define the model's recursive AU equilibrium as follows.

**Definition.** Given a history of realizations  $\kappa^T$ , a recursive AU competitive equilibrium for the economy is given by a sequence of policy functions  $[b'_t(b, z, \kappa), c_t(b, z, \kappa), \mu_t(b, z, \kappa)]_{t=1}^T$  and pricing functions  $[q_t(b, z, \kappa)]_{t=1}^T$  such that: (a)  $b'_t(b, z, \kappa)$ ,  $c_t(b, z, \kappa)$ ,  $\mu_t(b, z, \kappa)$  and  $q_t(b, z, \kappa)$  solve the date- $t$  AUOP conditional on  $E_t^s[\kappa'|\kappa]$ ; and (b)  $E_t^s[\kappa'|\kappa]$  is the transition probability matrix of  $\kappa$  produced by the date- $t$  posterior density of  $F^s$  determined by Eq. (A.4) in Appendix A.

### 3. Quantitative analysis

In this section, we examine the model's quantitative predictions for the following experiment: at  $t=1$ , financial innovation begins with the first realization of  $\kappa^h$ , and the  $\kappa^h$  regime continues for  $J$  periods. At  $t=J+1$  the first realization of  $\kappa^l$  occurs, and this regime continues until date  $T$ .

#### 3.1. Baseline calibration

The functional forms for preferences and technology are standard:  $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$  and  $g(l_t) = l_t^\alpha$ . The calibration requires setting values for the parameters  $(\alpha, \beta, \sigma, R)$ , the Markov process for  $z$ , and the parameters of the learning setup  $(\kappa^h, \kappa^l, n_0^{hh}, n_0^{hl}, n_0^l, n_0^h, J$  and  $T)$ . The baseline calibration is set to U.S. data but later we conduct sensitivity analysis to evaluate the robustness of the results.

We calibrate the model to a quarterly frequency at annualized rates. The date  $t=1$  is set to 1998Q1, because 1998 is in the midpoint of the period covered by the changes in financial regulation and introduction of new instruments (see Fig. 2), and it is also the year in which RMBS spreads rose sharply, from 48 to 126 basis points between end-1997 and end-1998. We date the start of the financial crisis ( $t=J+1$ ) as of 2007Q1, to match the early stages of the subprime mortgage crisis in the Fall of 2006. This is also in line with the observation that the net fraction of banks reporting tighter standards for mortgage loans jumped significantly to 16% in 2007Q1. The experiment ends in 2008Q4. These timing assumptions imply  $\kappa = \kappa^h$  from 1998Q1 to 2006Q4 and  $\kappa = \kappa^l$  from 2007Q1 to 2008Q4. Thus, the learning period has a total length of  $T=44$  quarters, in which the first 36  $\kappa$  realizations are  $\kappa^h$  ( $J=36$ ) and the remaining 8 are  $\kappa^l$ .

The values of  $(\alpha, \beta, \sigma, R)$ ,  $\kappa^l$ , and the Markov process for  $z$  are set either using U.S. data averages from the pre-financial innovation era (before 1998) or targeting moments of the stochastic steady state of a variant of the model that represents that era to match cyclical properties of U.S. data. In the pre-financial-innovation model, there is only one financial regime with  $\kappa = \kappa^l$ , which we assume to be binding, and hence the only source of uncertainty are TFP shocks.

The real interest rate is set to 2.7% annually, which is the average ex post real interest rate on U.S. three-month T-bills during the period 1980Q1–1997Q4. Mean output is normalized to 1 (since  $L=1$  and the unconditional mean of  $z$  also equals 1). The TFP process is set to approximate an AR(1) process  $\ln(z_t) = \rho \ln(z_{t-1}) + e_t$  fitted to HP-filtered real U.S. GDP per capita using data for the period 1965Q1–1997Q4. The estimation yields  $\rho = 0.878$  and  $\sigma_e = 0.00663$ , which imply a standard deviation of TFP of  $\sigma_z = 1.39\%$ . We use Tauchen and Hussey's (1991) quadrature method to construct a Markov approximation to this process with a vector of 9 realizations. The transition probability matrix and realization vector are available on request. The value of  $\sigma$  is set to  $\sigma = 2.0$ , the standard value in quantitative DSGE models.

As noted in the Introduction, we measure  $b$  using the net credit market assets of U.S. households and non-profit organizations in the *Flow of Funds* data set, and  $ql$  using the value of residential land estimated by Davis and Heathcote (2007). The 1980Q1–1997Q4 average ratios of these variables relative to GDP were  $\bar{b} = -0.316$  and  $\bar{q}l = 0.49$ . The household leverage ratio is the ratio of net credit market assets to the value of residential land, and hence we set the value of  $\kappa^l$  by combining the 1980Q1–1997Q4 average of this leverage ratio with the value of  $R$ :  $\kappa^l = 0.647/1.027 = 0.63$ . The fact that net credit assets and land values were fairly stable prior to 1998 (see Fig. 1) supports the idea of using this constant value of  $\kappa^l$  to characterize the pre-financial-innovation regime, and the fact that by the end of 2010 the median downpayment on

**Table 1**  
Baseline parameter values.

$\beta$	Discount factor (annualized)	0.91
$\sigma$	Risk aversion coefficient	2.0
$c$	Consumption–GDP ratio	0.668
$A$	Lump-sum absorption	0.322
$r$	Interest rate (annualized)	2.702
$\rho$	Persistence of endowment shocks	0.878
$\sigma_e$	Standard deviation of TFP shocks	0.007
$\alpha$	Factor share of land in production	0.026
$L$	Supply of land	1.0
$\kappa^h$	Value of $\kappa$ in the high securitization regime	0.926
$\kappa^l$	Value of $\kappa$ in the low securitization regime	0.629
$F_{hh}^a$	True persistence of $\kappa^h$	0.964
$F_{ll}^a$	True persistence of $\kappa^l$	0.964
$n_0^h, n_0^l$	Priors	0.014

conventional mortgages bounced back to what it was a decade earlier (see February 16, 2011 *Wall Street Journal*) supports the idea of setting  $\kappa^l$  in the new regime to be the same as in the pre-financial-innovation era.

The GDP ratios in the resource constraint at the average of the stochastic stationary state of the pre-financial innovation model must match the observed U.S. data averages from the pre-financial-innovation period. As noted above, the average bond holdings–GDP ratio in the data was  $\bar{b} = -0.316$ . For the consumption–GDP ratio, the data shows a slight trend, so we use the last observation of the pre-financial-innovation regime (1997Q4).<sup>17</sup> This implies  $\bar{c} = 0.669$ . To make these  $\bar{b}$  and  $\bar{c}$  values consistent with the model's average resource constraint, we need to take into account the fact that investment and government absorption are included in the data but not in the model. To adjust for this discrepancy, we introduce a fixed, exogenous amount of autonomous spending  $A$ , so that the long-run average of the resource constraint is  $1 = \bar{c} + A - \bar{b}(R-1)/R$ . Given  $\bar{b} = -0.316$ ,  $\bar{c} = 0.669$  and  $R = 1.027$  it follows that  $A = 1 - \bar{c} + \bar{b}(R-1)/R = 0.322$ .

The values of  $\alpha$  and  $\beta$  are set by solving the pre-financial innovation model repeatedly until we hit these two calibration targets (keeping all the other parameter values we have set up to this point unchanged): the standard deviation of consumption relative to output over the 1980Q1–1997Q4 period (0.8), and the condition that arbitrages the returns on land and bonds at the observed average of the value of residential land to GDP ( $\bar{q}^l = 0.49$ ) for the same period. This condition follows from (11) and (12) and implies  $\alpha = (ql/zl^s)[R-1 + \beta^{-1}(1-\beta R)(1-\kappa^l)]$ .<sup>18</sup> The model hits the targets with  $\alpha = 0.026$  and  $\beta = 0.91$ .

Notice that  $\beta R = 0.934 < 1$ . This is important because it ensures the existence of an ergodic distribution of bonds given that asset markets are incomplete. Intuitively, this occurs because of the interaction between the precautionary savings motive, which pushes for increasing bond holdings, and the incentive to tilt consumption towards the present, and hence reduce bond holdings, because  $\beta R < 1$ . Consumption tilting and precautionary savings will also play a key role later in the analysis of the dynamics induced by financial innovation.

The parameters that remain to be calibrated ( $\kappa^h$  and the  $n_0^{ij}$  counters) are independent of the rest of the calibration strategy, because they are unrelated to the pre-financial-innovation regime. The value of  $\kappa^h$  matches the 2006Q4 leverage ratio, hence  $\kappa^h = 0.926$ . This is the largest leverage ratio attained in the new financial regime just before the crisis. Note, however, that  $\kappa^h$  does not always bind in the new regime. First, as the economy moves from the pre-financial-innovation regime with  $\kappa^l$  binding, agents build debt over time, and hence the equilibrium leverage ratio does not hit its new ceiling immediately. Second, the new regime features two possible realizations of  $\kappa$  that are occasionally binding, so  $\kappa^h$  only binds with some probability (Table 1).

The calibration of the initial priors with the  $n_0^{ij}$  counters is critical because, as shown in Section 2, together with the history of realizations of  $\kappa$ , they drive the magnitude and speed with which optimism and pessimism fluctuate. Thus, the calibration of the priors imposes quantitative discipline on how much the model is allowed to rely on these fluctuations for explaining credit booms and busts. For simplicity, we impose the symmetry condition  $n_0 = n_0^h = n_0^{hh} = n_0^l = n_0^{ll}$ , so that there is only one counter to calibrate.

We set  $n_0$  so that, conditional on all of the parameter values, the model matches an estimate of the observed excess return on land relative to the risk free rate for 1998Q2, which corresponds to  $E_1^s[R_2^q - R]$ . On this date, the credit constraint does not bind, because  $\kappa = \kappa^h$  and at  $t=0$  the economy was in the pre-financial innovation regime with  $\kappa^l$  binding, but agents believe that a switch to  $\kappa^l$  at  $t=2$  can occur with probability  $E_1[F_{hl}^s]$ . This causes  $n_0$  and  $E_1^s[R_2^q - R]$  to be positively related, because the optimism built into the date-1 beliefs bumps up the conditional covariance between  $R_2^q$  and  $\lambda_2$ , and then condition (9) implies a higher  $E_1^s[R_2^q - R]$  even if the collateral constraint does not bind at  $t=1$ . The data proxy for  $E_1^s[R_2^q - R]$

<sup>17</sup> Consumption and GDP data are from the *International Financial Statistics* of the IMF.

<sup>18</sup> This uses the fact that  $E[R^q] \approx R$  in the stochastic steady state of the pre-financial-innovation era, because the regime with only  $\kappa^l$  yields a collateral constraint that is almost always binding and with a negligible excess return.

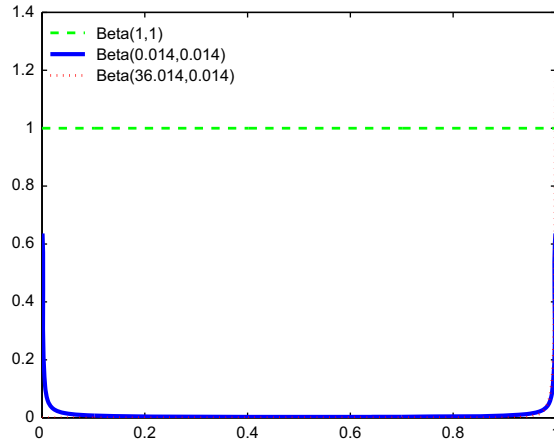


Fig. 4. Distributions of beliefs with different priors.

is the 1998Q2 option-adjusted spread on the Fannie Mae RMBS with 30-year maturity over the T-bill rate, which the *Bloomberg* data service reports at 48 basis points. The model matches this spread with  $n_0 = 0.014$ .<sup>19</sup>

Two caveats about this approach to calibrate  $n_0$ : First, we used “option-adjusted” spreads that are adjusted for prepayment risk, without this adjustment we would have larger spreads. We chose this measure because we do not explicitly model prepayment risk (the unadjusted spread was 110.2 basis points). Second, since the calibration is at a quarterly frequency, we would like to use quarterly excess returns. However, RMBSs do not have such short-term maturities, because the underlying assets tend to be long-term mortgages. Still, using the 30-year RMBS spread is useful because it actually makes it harder for the model to generate optimism than spreads for a lower maturity. This is because 30-year RMBSs generally have higher spreads than securities with a quarterly maturity, and higher spreads are associated with higher  $n_0$ , which weakens the mechanism generating optimism and pessimism in the learning process.

To analyze further the implications of the calibration of the initial priors, Fig. 4 shows the density functions of  $F_{hh}^s$  and  $F_{ll}^s$  for three different  $(n_0^h, n_0^l)$  pairs.  $Beta(0.014, 0.014)$  from the baseline calibration yields priors with a U-shaped, symmetric distribution with most of the mass around 0 and 1, a mean of 0.5 and a variance of 0.24. Thus, agents conjecture that there are four most likely scenarios before the first realization of  $\kappa$  is observed: (a)  $F_{hh}^s \approx 1$ ,  $F_{ll}^s \approx 1$ ; (b)  $F_{hh}^s \approx 1$ ,  $F_{ll}^s \approx 0$ ; (c)  $F_{hh}^s \approx 0$ ,  $F_{ll}^s \approx 1$ ; (d)  $F_{hh}^s \approx 0$ ,  $F_{ll}^s \approx 0$ . After observing the first few realizations of  $\kappa^h$ , however, the agents can rule out (c) and (d). By contrast,  $Beta(1, 1)$ , which assumes that at least one observation of switch and continuation of each  $\kappa$  regime has been observed, has the same mean of 0.5, but the distribution is uniform over the (0,1) interval and it has a lower variance (0.083).

Fig. 4 also plots the  $Beta(36.014, 0.014)$  distribution, which represents the beliefs about  $F_{hh}^s$  at period 36 of the financial innovation experiment. Since agents have observed 36 transitions from  $\kappa^h$  to  $\kappa^h$ , their beliefs are sharply skewed to the right. This is another way to illustrate the high degree of optimism that the learning process creates in the initial optimistic phase.

Consider next how the calibrated initial priors influence the evolution of  $E_t[F_{hh}^s]$  and  $E_t[F_{ll}^s]$  as the sequence of realizations of  $\kappa$  is observed. The initial spell of  $\kappa^h$  leads agents to become optimistic about the persistence of this regime very quickly. With the baseline  $Beta(0.014, 0.014)$ ,  $E_t[F_{hh}^s]$  jumps to about 0.986 in just one quarter, while with  $Beta(1, 1)$  the buildup of optimism is more gradual, but still after 8 quarters  $E_t[F_{hh}^s]$  approaches 90%. This rapid adjustment of beliefs also occurs with the surge of pessimism that follows the first observation of  $\kappa^l$  in period 37: with  $Beta(0.014, 0.014)$ , agents update  $E_t[F_{ll}^s]$  from 0.5 to almost 1 in period 37, and with  $Beta(1, 1)$  the change is slower but again by period 44,  $E_t[F_{ll}^s]$  approaches 0.9.

It is important to note that neither  $Beta(0.014, 0.014)$  nor  $Beta(1, 1)$  bias the initial beliefs in favor of optimism or pessimism, because both yield initial beliefs with a 0.5 mean. This differs from the approach followed by Cogley and Sargent (2008b), who studied the implications of inducing a pessimism bias in the priors. Moreover, even though the changes in  $E_t[F_{hh}^s]$  between periods 1 and 36 are small, they still imply nontrivial changes in the agent’s perceived risk of the financial environment. For  $t=1$ , the baseline model yields  $E_1[F_{hh}^s] = 0.986$ , which implies that the perceived mean duration of  $\kappa^h$  is 71 quarters and the coefficient of variation is 5.5%. In contrast, for  $t=36$  we have  $E_{36}[F_{hh}^s] = 0.999$ , which is close to 0.98, but implies a much higher mean duration of  $\kappa^h$  (1000 quarters) and much lower coefficient of variation (1.3%). As we show below, this reduces incentives for precautionary savings significantly. These effects are also very non-linear, with the mean duration exploding to  $\infty$  as  $E_t[F_{hh}^s]$  approaches 1 from below.

At this point we have calibrated all of the parameters that are needed for solving the model. Notice in particular that the true probability matrix  $F^a$  is not needed. Still, calibrating  $F^a$  is necessary if we want to compare the solutions of the learning

<sup>19</sup> An alternative calibration strategy would be to set  $n_0$  to match the observed increase in household debt. This yields  $n_0 < 0.014$ , which strengthens the buildup of optimism and the surge in land prices and debt in the optimistic phase.

model against the standard RE solution. We calibrate  $F_{hh}^a$  so that the mean duration of high-leverage regimes is in line with the estimated duration of credit boom episodes in industrial economies, which [Mendoza and Terrones \(2012\)](#) estimated at about 7 years. This implies  $F_{hh}^a = 0.964$ . With this calibration of  $F_{hh}^a$  and conditional on observing  $\kappa^h$  at date 1, the probability of observing  $\kappa^h$  the following 35 periods is 0.24. Thus, the true probability of observing the spell of  $\kappa^h$  in the baseline calibration of our financial innovation experiment, which produces substantial optimism, is about 1/4. We assume a symmetric process by setting  $F_{ll}^a = 0.964$ .

An interesting implication of this calibration of the true  $\kappa$  process is that it makes the model consistent with the “this-time-is-different” argument of [Reinhart and Rogoff \(2009\)](#). In the long-run, agents’ beliefs about the mean duration and frequency of credit cycles match the data, but in the early stages of financial innovation they are unduly optimistic.

### 3.2. Quantitative findings

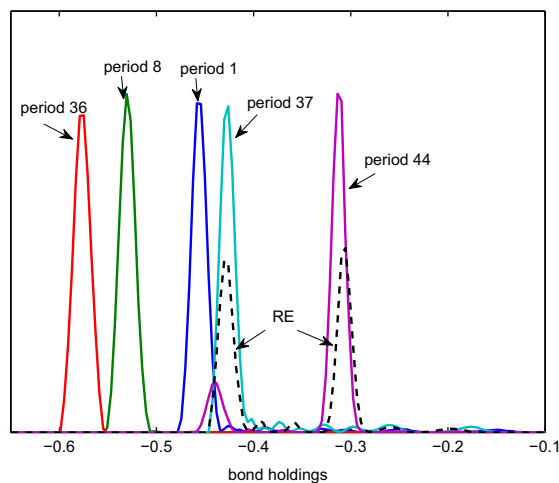
We discuss four sets of numerical results: long-run distributions of bond positions, forecast functions of macroeconomic aggregates, average changes in these aggregates at the “turning points” of the learning experiment, and expected excess returns. We compare the results of the baseline learning model (BL) with the RE model (i.e., a model with the collateral constraint but without the learning friction) and with a fixed land valuation-learning (FVL) model, in which land in the collateral constraint is valued at a constant price set to the long-run average (i.e., a model with the learning friction but without the Fisherian amplification channel). In this case, the collateral constraint becomes  $b_{t+1}/R \geq -\kappa_t \bar{q} l_{t+1}$ , where  $\bar{q} = 0.456$ .

#### 3.2.1. Ergodic distributions

[Fig. 5](#) plots conjectured ergodic distributions of  $b$  as of  $t=1, 8, 36, 37$  and  $44$  in the BL model and the true ergodic distribution of the RE model (see Appendix C for details). The former are conjectures because the actual ergodic distribution of the BL model is the same as in the RE model, since in the long-run agents learn  $F^a$ . In contrast, the conjectured ergodic distributions are the agents’ projections of what the long-run equilibrium would look like using their beliefs as of each date. Plotting these distributions together is useful for illustrating the impact of the optimism and pessimism driving the model’s dynamics on the agents’ willingness to borrow.

Consider first the conjectured distribution for  $t=1$ . Recall that the mean of bond holdings pre-financial-innovation was  $-0.31$ . Hence, already by period 1 agents conjecture that the support of the long-run distribution of bonds will shift sharply to the left (i.e., support higher debt levels). But comparing the period-1 distribution with the new RE distribution for the regime-switching  $\kappa$  process post-financial-innovation, it is clear that agents are also projecting to be saving much less than they eventually will in the new stochastic steady state. The RE distribution has the typical bimodal shape of a two-point regime-switching process with high persistence. In this case, agents are assessing the risk of the financial environment correctly, and in particular they are aware that long spells of *both*  $\kappa$  regimes are possible.

Compare now the RE ergodic distribution with the conjectured ergodic distribution for period 36 in the BL model. Large debt ratios (bond holdings in the interval  $[-0.59, -0.54]$ ) are *never* a long-run equilibrium outcome in the RE model, but they take most of the mass of the long-run distribution of bond holdings that is projected on the basis of the agents’ period-36 beliefs. Something similar happens much earlier because, as shown before, it takes observing only the first few realizations of  $\kappa^h$  for agents to turn very optimistic. By period 8 agents already conjecture that debt positions in the  $[-0.55, -0.51]$  range are most likely long-run equilibria, while in the RE ergodic distribution they have zero probability.



**Fig. 5.** Ergodic distributions of bond holdings.

As optimism builds, the highest debt conjectured to have positive long-run probability rises, and the mass of probability assigned to debt levels larger than the largest debt under rational expectations also rises. This process peaks at the peak of the optimistic phase in date 36. During this phase, agents are willing to “overborrow” (take on more debt at the averages of the conjectured ergodic distributions of  $b$ ) than what is ever optimal in the RE model, and “undersave” (build less precautionary savings, or conjecture they can attain a lower average of  $b$ ) than what is optimal in the RE model. When the first realization of  $\kappa^l$  hits and the pessimistic phase starts, the opposite effects take over and they peak at date 44. By then, agents are “underborrowing” and “oversaving” substantially. However, they have learned from their experience that shifts to  $\kappa^h$  are possible, so the period-44 conjectured distribution is two-peaked.

### 3.2.2. Forecast functions

Forecast functions are useful for illustrating the model's equilibrium dynamics. These forecast functions use the sequence of beliefs and decision rules of each AUOP to trace the equilibrium path of the expected values of the endogenous variables. Intuitively, the algorithm that computes the forecast functions uses a law of motion for the evolution of the probability of the economy being in each triple  $(b, z, \kappa)$  from  $t=0$  to 44. This law of motion can be computed for any triple of initial conditions, but we are interested in the triple that approximates the state of the U.S. economy in 1997Q4. Thus, we start at date 1 with all the probability concentrated in  $(b_1, z_1, \kappa^h)$  where  $b_1 = -0.363$  (net credit assets as a share of GDP in 1997Q4) and  $z_1 = 1$ . Then, for each subsequent date, the value of  $\kappa$  is set to the corresponding realization in the  $\kappa^T$  sequence, the transition probabilities across values of  $z$  are given by the Markov process of  $z$ , and the transitions across values of  $b$  are governed by the sequence of policy functions  $b_t'(b, z, \kappa)$  from the AUOPs. The procedure is similar to the standard forecast functions of a RE model, except that the policy function is time-varying because it varies with each set of beliefs in the sequence  $\{f(F^s|\kappa^t)\}_{t=1}^{44}$  (see Appendix C for details).

Fig. 6 plots the forecast functions for the choice of bond holdings as a share of output ( $b'/y$ ), consumption, the price of land, and the savings rate ( $GSF/y$ ) as percent deviations from their long-run means in the BL, RE and FVL models. The solid (blue) lines correspond to the BL model, the dashed (green) lines are for the FVL model, and the dotted (red) lines represent the RE model. Note that even the RE model generates some dynamics in this exercise, because the initial condition  $b_1$  is not the long-run mean of the new financial regime with stochastic  $\kappa$ , and also because we are using a particular time series of realizations of  $\kappa$  (instead of averaging across possible  $\kappa$  realizations at each date  $t$ ).

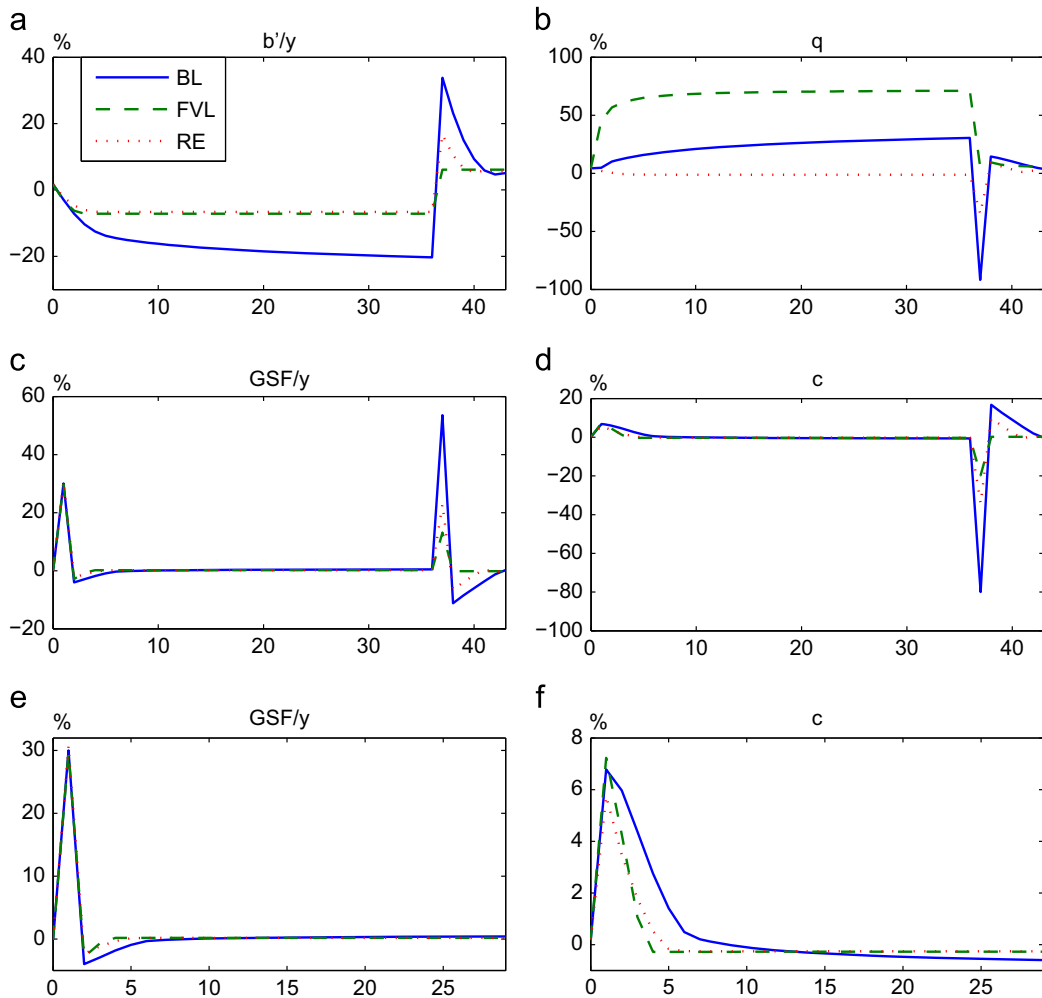
The forecast functions for bonds in Panel (a) show that during the optimistic phase agents consistently borrow more in the BL model than in both the RE and FVL models. In the first two periods after financial innovation is introduced, the three models predict similar debt dynamics, but after that the optimism and the financial amplification feedback loop at work in the BL model produce a much larger decline in bond holdings, while the bond dynamics in the RE and FVL models are similar.<sup>20</sup>  $b'/y$  declines by as much as 21 percentage points below the long-run average at the peak of optimism of the BL model in period 36. These dynamics are in line with the downward trend in household debt observed in the data (see Fig. 1). Interestingly, the combination of the learning friction and the Fisherian amplification delivers a much stronger decline in assets than the alternatives that retain only one of the two mechanisms. In the RE model there is no buildup of optimism to push for overborrowing, and in the FVL model there is no endogenous feedback from higher land prices into higher collateral and thus higher borrowing ability.

The switch to the pessimistic phase in period 37 causes a large correction in bond holdings, which jump about 54 percentage points in the BL model. An adjustment of this magnitude is an equilibrium outcome, despite CRRA preferences and incomplete markets, because the arrival of the first realization of  $\kappa^l$  at date 37 is akin to a large, unexpected shock, in the sense that by date 36 agents believed that  $\kappa^h$  was almost absorbent ( $E_{40}[F_{hh}^s] \approx 1$ ). Part of this debt correction is exogenous, because of the fall in  $\kappa$ , but part is the endogenous outcome of the interaction of Fisherian amplification and pessimistic beliefs. In fact, the  $\kappa$  shock triggers a large Fisherian deflation and its interaction with the pessimistic beliefs yields the largest debt correction across the three models. Bond holdings also jump up in the RE and FVL models, because of the switch from  $\kappa^h$  to  $\kappa^l$  in a state in which the collateral constraint was binding. But the adjustments are much smaller. The debt reversal in the RE case is about half the size of that in the learning model, and it reflects the effect of the Fisherian mechanism in the absence of a switch to pessimistic beliefs. The FVL model yields the smallest correction, which isolates the effect of the switch to pessimistic beliefs without Fisherian amplification.

As agents overborrow during the optimistic phase in the BL model, they also bid more aggressively for the risky asset. This increases the price of land significantly, as shown in Panel (b) of Fig. 6. This contrasts with the RE case, in which the price of land declines slightly relative to the pre-financial-innovation price. This occurs because the price of land in the RE model is falling to a lower long-run average in the financial innovation regime. In turn, the mean price of land in the RE model (with stochastic  $\kappa$ ) is lower than in the pre-financial-innovation regime (with constant  $\kappa^l$ ) because, even though agents know the true distribution of  $\kappa$ , they now face uncertainty about  $\kappa$ . Hence, financial innovation implies not only a higher mean  $\kappa$  but also a higher variance of  $\kappa$ . The former enables the agents to borrow more, and therefore demand more of the risky asset and bid up its price, but the latter gives them an incentive to hold less of the risky asset, because the new financial environment is riskier and they are risk averse. We find that, if the gap between  $\kappa^h$  and  $\kappa^l$  is small, the “mean effect” dominates leading to higher land prices in the RE model, but as the gap

<sup>20</sup> This occurs because (a) in the FVL model the price is fixed at the long-run average of the RE model, and (b) the RE price displays very small deviations from its long-run average. As a result, and since the values of  $\kappa$  are the same in both models, the debt allowed by the collateral constraint in both models is about the same.





**Fig. 6.** Forecast functions. *Notes:* Forecast functions are plotted as percent deviations from their long run means in the rational expectations scenario. GSF/y is calculated as  $((b'/R) - b)/y$ . Date-0  $b'/y$  is the 1997:Q4 observation from data (since debt data are end of period basis), so that the date-1  $b'/y$  is the first endogenous choice of  $b'$  under  $\kappa^h$ , given an initial state determined by the data point from 1997:Q4. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

widens, the “variance effect” becomes stronger and the mean land price in the RE model is lower than in the pre-financial-innovation equilibrium (as is the case in the baseline calibration).

The FVL model generates a larger asset price boom during the optimistic phase and a smaller price crash compared with the other two models. This is because the FVL model rules out the Fisherian amplification by construction, and hence at date 37 the downward spiral on land prices, collateral values, and debt that is at work in the other two models are not active here. Moreover, the fixed land valuation for collateral also serves as a limited asset price guarantee, which produces a larger price boom during the optimistic phase than in the learning and RE models. The guarantee is limited because it applies only for the valuation of land used as collateral. Accordingly, the FVL model produces a smaller reversal in debt in period 37, as agents are able to borrow more than in the other two models because of the constant collateral price. For the same reason, the larger land price increase in the optimistic phase does not feed back into a large debt expansion.

Panels (c) and (d) of Fig. 6 show the forecast functions for the savings rate and consumption. Because of the large adjustments that occur at date 37, Panels (e) and (f) “zoom-in” on the dynamics of these variables in the first 30 periods. To understand the consumption dynamics, consider first what these dynamics would look like in a perfect-foresight model where we switch from the constrained pre-financial innovation steady state with  $\kappa^l$  to a hypothetical financial innovation deterministic steady state for a regime with  $\kappa^h$ . These two steady states are corner solutions because  $\beta R < 1$ , and hence the steady state of bonds is  $b = -\kappa q(\kappa)$ , where  $q(\kappa)$  is the steady state land price, which is increasing in  $\kappa$ .<sup>21</sup> Thus, the increase in  $\kappa$  yields a lower steady state for  $b$  (higher debt) because both  $\kappa$  and  $q(\kappa)$  increase. But higher steady state debt means lower

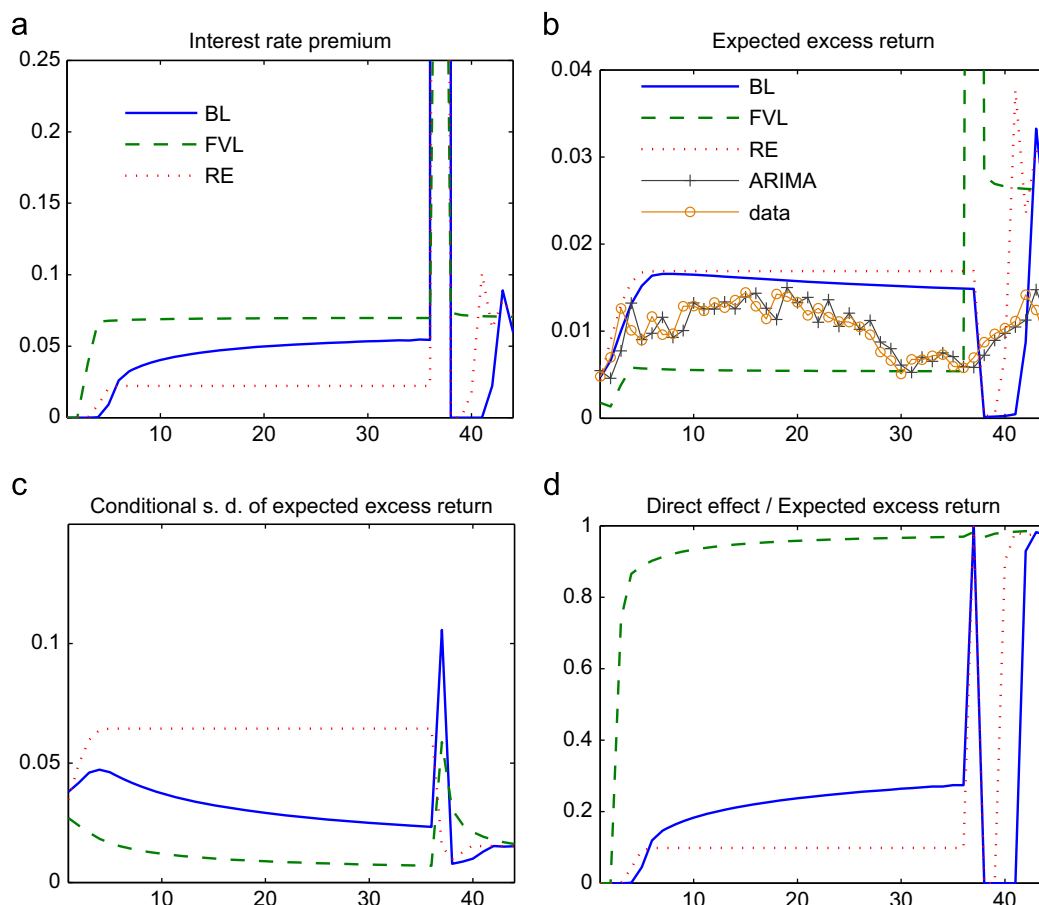
<sup>21</sup> The steady state price is  $q(\kappa) = (\alpha\beta)/[\beta(R-1) + (1-\beta R)(1-\kappa)]$ .

steady state consumption, since the nonfinancial wealth of the economy is invariant to changes in  $\kappa$  and the debt has to be serviced. Thus, financial innovation tilts the time profile of consumption. On impact, when  $\kappa$  first rises, and for a few periods after that, consumption rises above the pre-financial-innovation level as the collateral constraint is relaxed, but then it drops monotonically until it reaches its new steady state below the pre-financial-innovation level. This consumption tilting effect is also at work in the stochastic model, but is weaker because of the precautionary savings motive, which implies a smaller rise in debt and a smaller consumption drop.

Now consider the consumption dynamics of the BL, RE and FVL models in Panels (c) and (e). The fact that the dynamics for the first 36 periods are similar in all three models indicates that the consumption tilting effect dominates these dynamics. This is because consumption converges quickly to its new long-run average (which is identical in the BL and RE models, and very similar in the FVL model). There is over-consumption in the BL model relative to the RE and FVL models in the early stages after the switch to  $\kappa^h$ , because of the larger increase in debt (i.e., decline in bond holdings). In the first two periods, consumption is about the same in all three models, but the overconsumption in the BL model is clear between the 3rd and 10th periods. After period 10, however, the dynamics driven by consumption tilting dominate in all three models. Consumption then remains smooth (as we are averaging across TFP and keeping  $\kappa$  constant at  $\kappa^h$ ), until we arrive at date 37 and  $\kappa$  switches to  $\kappa^l$ .

At date 37, as explained earlier, the  $\kappa$  switch is akin to a large, unexpected shock in the BL and FVL models. In the BL model, which also has the Fisherian amplification, this produces a dramatic collapse in consumption. This is in line with the findings in [Mendoza \(2010\)](#) and [Mendoza and Smith \(2006\)](#), showing that in Fisherian amplification models there are equilibria outside the ergodic distribution of wealth, where the economy could land as a result of unexpected shocks, in which the impact response of consumption can be around  $-80\%$ . In those models, however, precautionary savings and perfect information about the Markov processes of shocks rule out consumption drops of that magnitude from the equilibrium dynamics, while in our model the learning friction allows us to support them as short-run AU equilibria.

The RE and FVL models also produce large consumption declines when the economy switches to  $\kappa^l$ , but both are much smaller than in the BL model. In the RE model this is again because precautionary savings and the lack of overborrowing



**Fig. 7.** Asset price dynamics. *Notes:* The interest rate premium in period 37 is 46.18, 1.99, and 0.80 in the BL, RE, and FVL models respectively. “Data” in top right panel corresponds to actual option-adjusted RMBS spreads (see calibration for details). “ARIMA” in the same panel shows the one-period-ahead excess return forecast of an ARIMA(1,1,1) model of the RMBS spreads estimated over 1998Q1–2008Q4.

prevented a large accumulation of debt in the optimistic phase. In the FVL model the smaller consumption drop occurs because there is no Fisherian deflation of collateral, which yields the smallest corrections in debt and consumption.

Fig. 7 shows the dynamics of key asset pricing variables. Panel (a) plots the implicit endogenous interest rate premium defined by  $[u'(c_t)/\beta E_t^s[u'(c_{t+1})]] - R$ , which is equal to  $\mu_t/\beta E_t^s[u'(c_{t+1})]$ , and thus is also a measure of the shadow value of the collateral constraint. If the constraint does not bind, there is no premium, and when it binds the premium rises as the constraint becomes more binding. The dynamics of the interest premium are in line with our previous argument stating that the constraint becomes nonbinding when financial innovation starts, and then begins to bind after some time. In the BL model, the constraint begins to bind after period 5. Then the premium rises monotonically, at a decreasing rate, to converge to about 5.5% at  $t=36$ . The FVL model generates a larger premium of up to 7%, while the RE model generates a premium of just above 2% in the optimistic phase. This is natural because in the FVL model rising land prices do not relax the borrowing limit, and in the RE model the constraint is less binding because individuals desire to save more with rational expectations than with optimistic beliefs.

When the switch to the pessimistic phase takes place at date 37, there is a large jump in interest premia as the collateral constraint becomes tightly binding. This jump is in part due to the exogenous shift in  $\kappa$ , but in the BL and RE models it is also heavily influenced by the endogenous dynamics driven by the Fisherian amplification and, in the BL model, the surge in pessimism. Consequently, the jump in the interest premia is the largest in the BL model, followed by the RE model, and the FVL model last. After the crisis at date 37, however, the constraint becomes nonbinding for 4 periods in the BL model and for 1 period in RE. Afterwards the interest premia become positive again in all three models.

The dynamics of  $E_t^s[R_{t+1}^q - R]$  and  $\sigma_t^s(R_{t+1}^q)$  are plotted in Panels (b) and (c) respectively. Panel (b) also shows two data proxies for actual expected excess returns on RMBS to gauge the model's performance at tracking them. One proxy is just the actual data on option-adjusted spreads referred to earlier. The second proxy is a one-step-ahead forecast from a univariate time-series model (identified as an ARIMA(1,1,1)). The excess returns rise sharply in the BL and RE models during the optimistic phase, and show a much smaller increase in the FVL model. The BL and RE models are also in line with the sharp increase in actual expected excess returns between 1998 and 2004 ( $t=0, \dots, 24$  in the chart). After that, BL is qualitatively consistent in predicting a decline in expected excess returns, but the drop in the data was larger.

The variability of returns in Panel (c) falls sharply during the optimistic phase in the BL and FVL models, and rises in the RE model. Given this and the rise in expected excess returns in Panel (b), we can infer the dynamics of the Sharpe ratio ( $S_t \equiv E_t^s[R_{t+1}^q - R]/\sigma_t^s(R_{t+1}^q)$ ), which would show that the compensation for taking risk rises gradually in all three models. However, the rise in the BL and FVL models (from near 0 at  $t=1$  to around 2/3rd at  $t=36$ ) is larger and more gradual than in the RE model (for which  $S_{36}$  peaks at about 1/3rd). Moreover, plots (b) and (c) imply that the gradual increase in the Sharpe ratios in the FVL and BL models is driven by the gradual declines in  $\sigma_t^s(R_{t+1}^q)$  during the optimistic phase, since  $E_t^s[R_{t+1}^q - R]$  remains largely stable. Thus, the buildup of optimism in the two models with imperfect information contributes significantly to reduce the perceived risk of land and increase the compensation for risk-taking, putting upward pressure on asset prices. Note, however, that even though the Sharpe ratios of the BL and FVL models are similar during the optimistic phase, both  $E_t^s[R_{t+1}^q - R]$  and  $\sigma_t^s(R_{t+1}^q)$  are higher in the BL model. In contrast, excess returns show fairly similar behavior across the RE and BL models. This is in line with the previous result showing that FVL yields the largest land price increase because of the implicit guarantee for collateral valuation it provides.

Panel (d) shows the direct effect of the borrowing constraint on expected excess returns (see Eq. (9)) expressed as a ratio of the latter. Looking at Panels (a) and (d), we find that the model in which the collateral constraint binds the most and produces the largest interest rate premia (FVL) is also the model in which the direct effect contributes the most to the excess returns (more than 90% by period 36). In the BL model, the direct effect rises gradually to reach about 30% by period 36. In the RE model, the contribution remains stable at about 10% from periods 5–36. The contribution of the direct effect grows very large in all three models when the first switch to  $\kappa^l$  occurs, as the credit constraint becomes very binding, and after that it remains large in the FVL model and falls back to zero in the other two models as debt is adjusted sharply, but then it rises again as the constraint becomes very binding.

The relatively small contribution of the direct effect to mean excess returns in the optimistic phase of the BL and RE models, coupled with the nontrivial mean excess returns, indicates that the indirect effects operating via  $\text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)/E_t^s(\lambda_{t+1})$  also play an important role. Moreover, in the very early stages of this phase, when the collateral constraint does not bind, mean excess returns in the RE and BL models increase *only* because  $\text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)/E_t^s(\lambda_{t+1})$  is becoming more negative. Thus, the perceived risk of land falls because both  $\sigma_t^s(R_{t+1}^q)$  and  $\text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)/E_t^s(\lambda_{t+1})$  are falling. In addition, the fact that in the early stages of the experiment the only driving force of rising excess returns is the fall in  $\text{cov}_t^s(\lambda_{t+1}, R_{t+1}^q)/E_t^s(\lambda_{t+1})$  is consistent with our previous remark stating that the undervaluation of risk is the only mechanism at work when we calibrate  $n_0$  to match the observed 48 basis points RMBS spread at  $t=1$ . The direct effect of the borrowing constraint is not at work because the constraint is not binding.

### 3.2.3. Turning points

Table 2 lists changes in average bond–output ratios and land prices, calculated with the data of the forecast functions, at the key turning points: the *peak of optimism* at  $t=36$  relative to the pre-financial-innovation initial conditions, and at the end of the learning experiment ( $t=44$ ) relative to the peak of optimism (which we label as *financial crisis*). The figures are differences in the levels of  $b/y$  and  $q$  projected by the forecast functions, but not expressed in deviations from long-run means (as was the case in the plots of Fig. 6).

**Table 2**  
Average changes at the turning points.

Period	(1) Data	(2) RE	(3) FVL	(4) BL
<i>Peak of optimism:</i>				
$E[(b/y)_{36} - (b/y)_0]$	-0.334	-0.065	-0.071	-0.213
$E[(ql/y)_{36} - (ql/y)_0]$	0.267	-0.025	0.306	0.131
<i>Financial crisis:</i>				
$E[(b/y)_{44} - (b/y)_{36}]$	0.023	0.122	0.133	0.262
$E[(ql/y)_{44} - (ql/y)_{36}]$	-0.149	0.013	-0.301	-0.130

Note: Data column reports the difference between 2006Q4 and 1997Q4 observations (top panel) and between 2008Q4 and 2006Q4 observations (bottom panel). Period 0 in all three model scenarios corresponds to the 1997Q4 data observations, which are the initial conditions.

This table illustrates two main results. First, the BL model explains a significant part of the increases in debt and land prices before the financial crisis. Second, the BL model generates significantly higher debt in the optimistic phase than the RE or FVL models, and a much larger land price increase than the RE model.

The BL model explains 64% of the increase in household debt observed in the data ( $b/y$  falls by 21.3 percentage points vs. 33.4 in the data). Moreover, the decline in bond holdings in the BL model is about 14 percentage points of GDP larger than in the RE or FVL models. The comparison with the RE model shows again that, when Fisherian amplification and optimistic beliefs interact, financial innovation produces significant overborrowing. The comparison with the FVL model shows, also in line with our previous findings, that the interaction of those two forces has significant quantitative implications for the size of the credit boom.

Comparing changes in land prices, the BL model accounts for 49% of the land price boom observed in the data (the increase in  $q$  in the model reaches 13 percentage points at date 36, vs. 26.7 in the data). As noted in the comparison of forecast functions, the RE model yields a slight fall in  $q$  and the FVL model generates a larger price increase than the BL model.

Consider now the changes in bond holdings and land prices during the financial crisis. The BL model generates a large debt reversal of 26 percentage points (and this after an even larger reversal between periods 36 and 37, as shown in Fig. 6). By contrast, in the data the correction was only 2.3 percentage points. The model clearly overestimates the reversal in debt, but part of the discrepancy is due to the fact that bonds in the model are one-period bonds while the average maturity of household debt is significantly higher, particularly for mortgages. As a result, the switch to  $\kappa^l$  leads to an abrupt decline in debt in the model, while in the data this has an effect that is spread over time. Indeed, as shown in the top panel of Fig. 1, the reversal in the household debt ratio has continued, and by 2011 it had increased by about 10 percentage points of GDP.

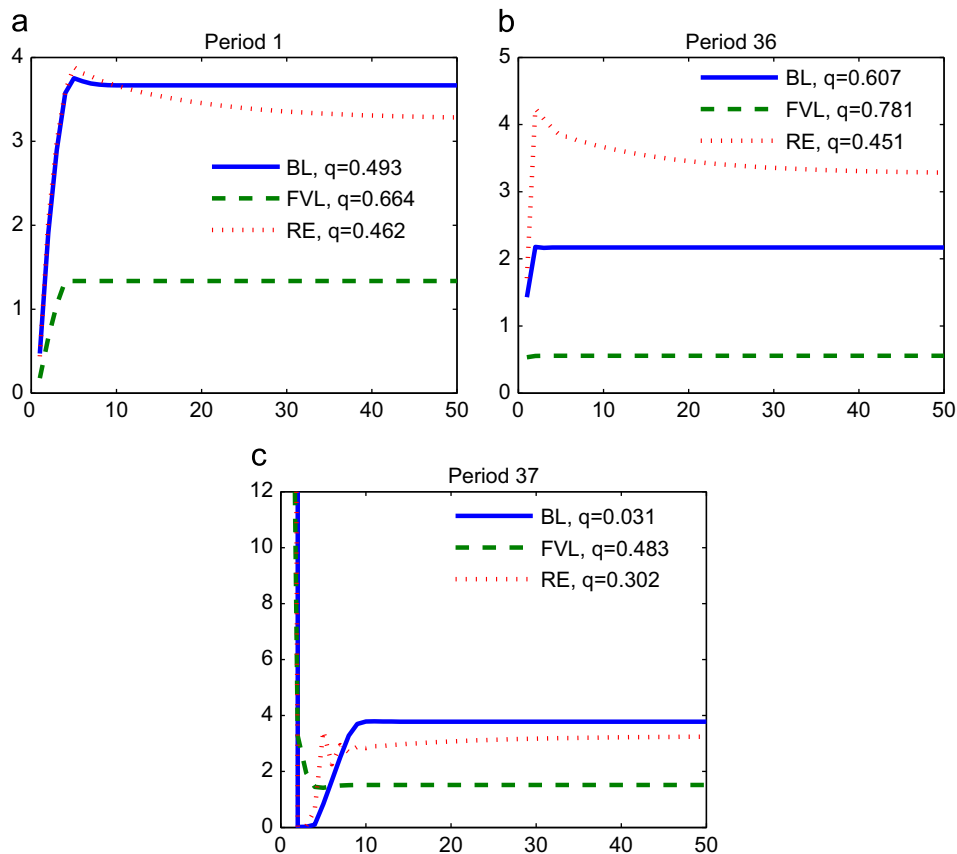
The BL does a nice job at matching the observed decline in land prices during the financial crisis (13 and 14.9 percentage points in model and data respectively). This is after an initial price collapse between periods 36 and 37 that is significantly larger. In contrast, the FVL model produces a larger price decline, about twice as large as in the data, and the price change in the RE model is again small and in the opposite direction from the BL and FVL models.

### 3.2.4. Projected excess returns on land

Next we investigate the projections of future excess returns that underlie the discounting of land dividends for the computation of  $q_t$  using Eq. (10) at key dates in the model's dynamics. Looking at these projections illustrates further the agents' changing beliefs about the riskiness of land. Fig. 8 plots expected excess returns for up to 50 periods ahead of  $t=1$ , 36, and 37 (in Panels (a), (b) and (c) respectively). These are expectations formed with the beliefs and decision rules as of periods 1, 36 and 37. In each scenario, we set the initial state of nature so that  $b$  is at the mean bond holdings predicted by the forecast function in Fig. 6 for the corresponding date,  $\kappa$  to its corresponding value in the history  $\kappa^t$ , and TFP to its mean value.

Focusing on expected excess returns projected as of  $t=1$  in Panel (a), the excess returns in the RE model exceed slightly those of the BL setup up to the 10th period, and afterwards the BL model projects slightly higher returns. This pattern justifies the result showing that the land price at date 1 is slightly lower in the RE model (because agents in the RE model expect relatively higher excess returns in the first 10 periods, which carry more weight in discounting dividends). The FVL model yields expected excess returns that lie significantly below both the RE and BL models, and this is consistent with the sharply higher date-1 land price produced by the FVL model. The FVL model has lower excess returns because the removal of the Fisherian amplification weakens the effects of the collateral constraint on excess returns shown in Eq. (9).

As agents reach the peak of the optimistic phase after observing  $\kappa^h$  for 36 periods, expected excess returns ahead of date 36 (Panel (b)) are significantly lower than they were predicted to be ahead of date 1 in the two models that incorporate the learning friction. As explained in Section 2, this is because once the constraint binds at  $\kappa^h$ , land returns are lower in the states with  $\kappa^h$  than with  $\kappa^l$ , and optimistic agents assign higher probabilities to the former than the true probabilities. Given lower projected excess returns, these two models also produce sharply higher land prices at date 36 than at date 1. Moreover, comparing now the projected future returns paths in the three models as of date 36 itself, projected returns in the BL model are significantly smaller than in the RE model, and the FVL model predicts even smaller excess returns. This is because in the FVL model beliefs turn as optimistic as in the learning model, but the removal of the Fisherian mechanism reduces land risk premia.



**Fig. 8.** Expected excess returns 50 periods ahead. *Notes:* Expected returns conditional on the bond holdings predicted for each initial date by the forecast functions of Fig. 6, the mean value of TFP ( $z=1$ ), and the value of kappa indicated in the history of realizations for each date  $t$ . The one period ahead expected excess return in Panel (c) is 1433, 69, and 43% in baseline, RE, and FVL scenarios respectively.

At date 37, when the switch to  $\kappa^l$  takes place, the ordering of projected excess returns across RE and BL models reverses (Panel (c)). Projected excess returns for period 38 are very high in all three models, because they reflect the strong direct effect of the borrowing constraint tightening sharply as  $\kappa$  switches. This direct effect includes both an exogenous factor, simply because of the switch to  $\kappa^l$ , and an endogenous factor, because of the surge in  $\mu_{37}$  (the direct effect in the right-hand-side of (9) for the excess return at  $t=38$  expected as of  $t=37$  is given by  $(1-\kappa^l)\mu_{37}$ ). Moreover, this direct effect is the strongest in the BL model that combines both learning and Fisherian amplification, followed by the RE model, and with the FVL model last. This is also in line with the size and ordering of interest rate premia displayed in period 37 in Fig. 7.

After the initial severe tightening of the borrowing constraint, and the abrupt debt adjustment that follows, the borrowing constraint is not projected to bind in the BL and RE models for a couple of periods, before enough debt is built up to make the constraint bind again. In the FVL model the constraint is projected to remain binding, but still the debt adjustment reduces the tightness of the constraint sharply and hence the projected returns. Beyond the adjustment phase of the first 10 periods, projected returns in the BL model exceed those of the other two models, and those of the FVL model are sharply lower. This pattern is consistent with the results showing that at date 37 the value of  $q$  is the highest in the FVL model, followed by the RE model, and with the BL model price sharply lower.

It is also interesting to note that during periods 2–7 ahead of date 37, the projected excess returns of the RE model exceed those of the BL model. This reflects the fact that the pessimistic expectations of the BL model result in a slower build up of debt, so that the collateral constraint is expected to start binding a period later than under RE, and then to bind with lower shadow values than under RE until period 10. However, since as of date 37 beliefs still favor overborrowing over the long run, relative to rational expectations (compare the projected long-run debt distribution of bonds for period 37 with the ergodic RE distribution in Fig. 5), agents project that the borrowing constraint will eventually become more binding in the BL model than in the RE model, and hence they project that land returns will converge to a higher level.

### 3.2.5. Sensitivity analysis

We now conduct a sensitivity analysis to study how changing the model's key parameters alters our main findings. To simplify the exposition, we focus only on the turning point effects. We examine first in Table 3 various scenarios changing the initial priors. Then we study in Table 4 changes in  $\kappa^h$ ,  $\kappa^l$ ,  $\beta$  and  $R$ . The second column of both tables shows the results for



**Table 3**  
Sensitivity analysis: initial priors.

Period	BL	(1) $n_0 = 1$	(2) $n_0 = 0.01$	(3) $n_0^{hh} = 0.37$ $n_0^{ll} = 0.37$	(4) $n_0^{ll} = 72$ $n_0^{hh} = 1$
$E[(b/y)_{36} - (b/y)_0]$	-0.213	-0.065	-0.222	-0.213	-0.212
$E[(ql/y)_{36} - (ql/y)_0]$	0.131	-0.024	0.140	0.129	0.128
$E[(b/y)_{44} - (b/y)_{36}]$	0.262	0.133	0.271	0.261	0.263
$E[(ql/y)_{44} - (ql/y)_{36}]$	-0.130	-0.003	-0.139	-0.128	-0.131

**Table 4**  
Sensitivity analysis: key parameters.

Period	BL	(1) $R = 1.0098$	(2) $\kappa^l = 0.75$	(3) $\kappa^h = 0.8$	(4) $\beta = 0.95$
$E[(b/y)_{36} - (b/y)_0]$	-0.213	-0.240	-0.228	-0.128	-0.227
$E[(ql/y)_{36} - (ql/y)_0]$	0.131	0.168	0.145	0.122	0.147
$E[(b/y)_{44} - (b/y)_{36}]$	0.262	0.161	0.162	0.177	0.183
$E[(ql/y)_{44} - (ql/y)_{36}]$	-0.130	-0.036	-0.064	-0.122	0.089

the BL model for comparison. In general, the parameterizations that generate larger booms during the optimistic phase also generate larger busts in the financial crisis.

Scenario (1) in Table 3 shows the results obtained by setting uniformly distributed initial priors ( $n_0 = 1$ ). Debt dynamics are qualitatively the same as in the BL model but the debt buildup is smaller (6.5 percentage points vs. 21.3 in BL). Moreover, the price of land falls to a level about 2 percentage points lower in period 36 than in period 1, which differs sharply from the 13 percentage points increase produced by the BL model. This is because, throughout the optimistic phase, the beliefs about the persistence of  $\kappa^h$  with the uniform priors are always lower than in the BL model. As explained before, the initial means of the two priors are the same (0.5), but with the BL priors agents turn optimistic faster and reach a higher level of optimism.

Examining the forecast functions of land prices we found that prices with uniform priors follow a u-shaped trajectory in the optimistic phase, instead of the monotonically increasing path displayed in the BL case. The reason for this is again the more gradual buildup of optimism with the uniform priors, which affects the relative magnitude of the effects of higher mean and higher variance of  $\kappa$  on land prices after financial innovation. In the early years, the effect of higher variance dominates that of higher mean, causing a decline in the price of land, until agents turn optimistic enough. As agents observe more  $\kappa^h$ s, and sufficient optimism builds up, the higher mean dominates the higher variance, but under uniform priors this requires a longer sequence of  $\kappa^h$  than in the baseline case. Thus, if we look at an optimistic phase of more than 36 periods with the uniform priors, we again find that at the peak of the optimistic phase the price of land is higher than in period 1.

The financial crisis effects on land prices and debt are also much weaker under the uniform priors than in the BL scenario, again because of the more gradual adjustment of beliefs (now in the switch to pessimistic beliefs and the buildup of pessimism). Debt adjusts by 13.3 percentage points instead of 26.2, and land prices fall only by 0.3 of a percentage point, instead of 13.

Reducing the initial priors to  $n_0 = 0.01$  in Scenario (2) moves the model further away from uniform priors than in the BL case (which was calibrated to  $n_0 = 0.014$ ). Consequently, when agents observe the first  $\kappa^h$  they turn more optimistic than in the BL case, and hence they borrow more and demand more of the risky asset. This produces larger debt and land price booms. Similar effects are at work, but in the opposite direction, in the pessimistic phase, and hence with  $n_0 = 0.01$  we find a larger correction in debt and a larger drop in land prices in the financial crisis.

In Scenario (3) we increase  $n_0^{hh}$  and  $n_0^{ll}$  so that  $E_0^s[F] = E_0^a[F]$ , which requires  $n_0^{hh} = n_0^{ll} = 0.37$ , while keeping  $n_0^{hl}$  and  $n_0^{lh}$  at the BL calibration values of 0.014 (i.e., in this case the continuation counters and the switching counters differ). The aim here is to start agents off with distributions of priors that have means that happen to be equal to the true persistence parameters of the  $\kappa$  regimes. Since agents do not know that this is the case, however, they still update their beliefs as they observe subsequent realizations of  $\kappa$ . This scenario yields results very similar to the BL model. This is because with  $n_0^{hh} = n_0^{ll} = 0.37$  and  $n_0^{hl} = n_0^{lh} = 0.014$  agents still face significant uncertainty about the true regime-switching structure of the credit regimes, and hence they still turn quite optimistic. In fact, their initial beliefs about  $E^s[F_{hh}]$  are more optimistic in Scenario (3) than in the BL case (in the latter they start with  $E_0^s[F_{hh}] = 0.5$  vs.  $E_0^s[F_{hh}] = 0.964$  in Scenario (3)). This does not, however, translate into significantly more borrowing because these differences in initial optimism become small after a few periods, since the BL model also generates a substantial amount of optimism relatively quickly.

Next we examine a scenario that assumes that the  $\kappa^l$  realizations observed during the pre-innovation period are part of the information set (Scenario (4)). This runs contrary to our assumption that a true structural financial innovation took place, so that any previous information about the financial environment became useless. Since the BL calibration sets  $\kappa^l$  at the value of the pre-financial-innovation era, however, one could argue that  $\kappa^l$  had been observed for a while before financial innovation, and thus

agents could learn from that. Accordingly, Scenario (4) sets the initial priors such that agents take into account the realizations of  $\kappa^l$  during 1980–1997 ( $n_0^l = 72$  observations) and 1 transition from  $\kappa^l$  to  $\kappa^h$  right at the beginning of innovation ( $n_0^{hl} = 1$ ). This scenario is also akin to a formulation in which we could bias the initial priors so as to make agents perceive a much higher probability of continuation of the low-leverage regime.

The priors in Scenario (4) imply  $E_0^s[F_{ll}] = 0.985$  vs.  $E_0^s[F_{ll}] = 0.5$  in the BL model. With the modified priors, agents believe that when or if the economy transits into the  $\kappa^l$  regime, it will stay there for some time. As a result, during the earlier periods of the optimistic phase the conjectured ergodic distributions of bonds are double-peaked with a large mass around the mean conditional on  $\kappa^l$ . This implies that unconditional mean debt is smaller than in the BL model during the optimistic phase. Conversely, we find higher debt levels during the pessimistic phase in Scenario (4) because, having observed one transition from  $\kappa^l$  to  $\kappa^h$ , the agents turn less pessimistic compared to the BL case. Despite these differences in conjectured long-run distributions, the turning-points dynamics reported in Table 3 do not differ much across Scenario (4) and the BL case. This is because, despite the differences in unconditional means, the means conditional on  $\kappa$  do not differ much, and this occurs because the forecast functions used for the turning points use the history  $\kappa^T$  described earlier and we also kept  $n_0^{hl}$  and  $n_0^{hl}$  unchanged. Thus the evolution of  $E_t^s[F_{nh}]$  is also the same and this results in similar dynamics for the first 40 periods in which  $\kappa^h$  is observed.

Summing up, the results in Table 3 illustrates the importance of the initial priors. In particular, what is crucial is that the new financial regime is truly a structural change, in terms of agents having little knowledge about the transition probabilities of  $\kappa^h$ . To be precise, for the model to generate sizable booms and busts, agents having never observed  $\kappa^h$  (i.e., having uninformative priors about the likelihood of transitions from  $\kappa^h$  to  $\kappa^l$ ) is crucial. This is evident in the results for Scenario (1) with uniform priors, which are far less favorable than the BL model and the other three scenarios. In this scenario, agents have “stronger” priors about the persistence of  $\kappa^h$ , because  $n_0 = 1$  implies that they have observed one transition from  $\kappa^h$  to  $\kappa^l$  and also one from  $\kappa^h$  to  $\kappa^h$ . Hence, they can rule out the possibility of  $\kappa^h$  being close to absorbent. In addition, Scenario (4) shows that we can allow the priors about  $\kappa^l$  to be informative (i.e., reflect a high number of observed realizations of that regime), but as long as  $n^{hh}$  and  $n^{hl}$  are low, the magnitude of the boom and crash in debt and land prices remain about the same as in the BL case. The same applies to scenarios (2) and (3), because both of these use initial counters that are far below 1.

We now turn to Table 4 which reports the turning-point effects in scenarios that change  $R$ ,  $\kappa^h$ ,  $\kappa^l$ , and  $\beta$ . Scenario (1) considers a lower value of  $R$ , and is motivated by the observation that interest rates declined at around the same time as the beginning of financial innovation, and remained very low since then. Hence, one may argue that, because of U.S. financial innovation, or because of other forces like the large purchases of U.S. T-bills from China, the real interest rate fell along with the increase in the U.S. agents' ability to borrow.<sup>22</sup> Accordingly, we changed  $R$  to the average interest rate for the period 1998–2008 in the data, which is 0.98% (as opposed to the 1980–1997 average of 2.7% used in the BL case).

With the lower  $R$ , the new long-run means of debt and land price are considerably larger.  $E[b/y]$  goes down from  $-0.36$  in the RE and BL to  $-0.41$ , and the land price increases from 0.45 to 0.53. Intuitively, since  $R$  falls, the asset price needs to go up in equilibrium in order for the expected returns of these two to be equated. In addition to these changes in long-run averages, Scenario (1) in Table 4 shows that lower  $R$  generates larger increases in debt and asset prices than the BL case during the optimistic phase. Lower interest rates support higher asset prices which in turn relax the borrowing constraint allowing the agents to borrow more.<sup>23</sup> Note, however, that in the crisis the correction in debt is smaller, and the fall in the price of land is significantly smaller. Thus, considering falling real interest rates that coincide with financial innovation enlarges the size of the debt and land price booms predicted by the model, but it makes the reversal of both smaller.

Increasing  $\kappa^l$  in Scenario (2) increases the size of booms in debt and land prices slightly. Note that the change only applies to the value of  $\kappa^l$  in the new financial regime, while the pre-innovation  $\kappa$  is still 0.63. The larger debt and price booms occur, even though we still have the same sequence of 36 realizations of  $\kappa^h$  as in the BL case, because agents take into account the fact that with the higher  $\kappa^l$  the low-leverage regime is not as low as in the BL case. This results in both a higher mean and a lower variance of  $\kappa$ , which support both larger debt and higher land prices. The size of the reversals in debt and land prices are smaller because  $\kappa^l$  is higher than in the baseline, and thus allows agents to borrow more than in the BL case, and the resulting direct and indirect effects of the collateral constraint on asset prices are weaker.

Reducing  $\kappa^h$  to 0.8 (Scenario (3)) reduces the size of the debt buildup, because of the tighter credit constraint in the high-leverage regime in this experiment, and reduces also the size of the debt reversal in the crisis, because debt falls from a lower level at the peak of the boom. The land price boom and crash change only marginally.

Finally, Scenario (4) shows results for a higher value of  $\beta$  (0.95 vs. 0.91 in the BL case). The higher discount factor supports higher asset prices, because agents discount the future less. Since the collateral constraint becomes binding early in the optimistic phase, these higher asset prices translate into higher debt levels.<sup>24</sup> Hence, this scenario delivers slightly larger debt and land price booms than in the BL case during the optimistic phase. In contrast, the financial crisis effects with higher

<sup>22</sup> This experiment is also motivated by the aim to explore the role of interest rate changes, since  $R$  in the model is exogenous. In a variant of the BL model with endogenous interest rate, the boom in debt would be weakened by an increase in  $R$ . But in the data the opposite was observed, so instead we examine a scenario in which the exogenous  $R$  changes with financial innovation to match its observed decline.

<sup>23</sup> In calculating turning points we measure booms relative to the 1997Q4 values in the data. If we measure the boom as deviations from long run means, the boom during the optimistic phase would be smaller in this scenario than in the BL case, because the long-run means of debt and land price are higher.

<sup>24</sup> By contrast, in the long run, as the model converges to RE and the constraint does not bind, the higher  $\beta$  brings  $\beta R$  closer to 1, which strengthens precautionary savings incentives and reduces debt.

$\beta$  are weaker than in the BL model, particularly in the case of asset prices (which recover quickly after a decline in period 37 and reach levels higher than pre-crisis by period 44).

#### 4. Conclusion

The U.S. financial crisis was preceded by a decade of fast growth in household debt, residential land prices, and leverage, accompanied by far-reaching financial innovation with the introduction of new instruments and deep changes in the regulatory framework. In this paper, we argued that financial innovation in an environment with imperfect information and credit frictions was a central factor behind the credit and land price booms that led to the crisis, and in the transmission mechanism that drove the crisis itself. To make these points, we examined the interaction among financial innovation, learning, and a Fisherian collateral constraint in a stochastic equilibrium model of household debt and land prices.

We used the model to study the quantitative implications of an experiment calibrated to U.S. data in which financial innovation begins with a switch to a high-leverage regime, but agents do not know the true regime-switching probabilities across high- and low-leverage regimes. Agents are Bayesian learners, however, so in the long-run, after observing a long history of realizations of leverage regimes, they learn the true regime-switching transition probabilities. The collateral constraint introduces Fisher's classic debt-deflation amplification mechanism, providing a vehicle for the waves of optimism and pessimism produced by Bayesian learning to have amplification effects on debt and land prices. In this regard, this paper offers a novel analysis in which the Fisherian feedback loop between debt and asset prices interacts with the formation of beliefs, both of which are at the core of Fisher's original arguments.

In our setup, a buildup of optimism is a natural consequence of financial innovation, because agents start without enough data to correctly evaluate the riskiness of the new environment. Calibrating the leverage regimes to data on the ratio of household debt to residential land values, and the initial priors to the excess returns on the 30-year Fannie Mae RMBS in early 1997, the model predicts that agents would turn very optimistic very quickly between the mid-1990s and the mid-2000s, after observing only a few quarters of the high-leverage regime.

Fisher's financial amplification channel plays an important role because, as optimism builds up and land prices rise, the agents' ability to borrow also grows. Similarly, when optimism turns to pessimism, after the first observation of the low-leverage regime, which we dated at the beginning of 2007, after the start of the sub-prime mortgage crisis in the Fall of 2006, the Fisherian channel amplifies the reversals in debt and asset prices. This occurs because fire-sales of land drive down land prices and reduce the agents' ability to borrow.

The interaction of the learning friction and the Fisherian amplification mechanism generates substantial overborrowing, which accounts for almost two-third of the increase in net debt of U.S. households, and about two-fifth of the boom in residential land prices, observed between 1997 and 2006. Moreover, the model also predicts a credit crunch, a crash in land values, a collapse in consumption and a surge in private savings after the first realization of the low-leverage regime. In contrast, the size of the debt and price booms, and the subsequent collapses, are significantly smaller in variants of the model that remove the learning friction or the Fisherian mechanism.

Our work has important implications for the ongoing debate on financial reforms to prevent future crises. First, since by definition the true riskiness of a truly brand-new financial regime cannot be correctly evaluated when the new regime starts, and little or no data is available on its performance, exposure to the credit boom–bust process we studied in this paper comes along with the potential benefits of financial innovation. Hence, close supervision of financial intermediaries in the early stages of financial innovation is critical.

Second, the interaction of informational frictions and collateral constraints can be critical for the evaluation of macro-prudential financial regulation policies. The credit constraint present in our model features the pecuniary externality typical of the macro-prudential regulation literature, because agents do not internalize the effect of their individual borrowing plans on equilibrium collateral prices, and this leads to “overborrowing” relative to debt levels that would be acquired without the externality. In [Bianchi et al. \(2012\)](#) we use a setup similar to the one presented in this paper to study the interaction between the externality and the underpricing of risk driven by the process of “risk discovery,” and use it to analyze the effectiveness of macro-prudential debt taxes. We find that these taxes can weaken boom–bust cycles if regulators have access to better information than private agents. Conversely, if policymakers are as uninformed as households about how financial markets will perform after radical structural changes, taxes on debt can address overborrowing due to the credit externality, but not due to optimistic beliefs.

Third, the ongoing financial reform process (e.g. Basle III, the Dodd–Frank act) is a new round of radical innovation in capital markets, now tightening the legal and regulatory framework, which will affect the types of securities that will be available and the size of the markets in which they will trade. Hence, agents once again will have to evaluate the riskiness of the new financial environment with beliefs based on imperfect information. As a result, the risk exists that a few years of slow credit growth and poor performance in asset markets can lead to the buildup of pessimistic expectations that will hamper the recovery of financial markets.

#### Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2013.07.001>.

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