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ABSTRACT

How high can public debt rise without compromising fiscal solvency? We answer this question using a stochastic ability-to-pay model of sovereign default in which risk-neutral investors lend to a government that displays "fiscal fatigue," because its ability to increase primary balances cannot keep pace with rising debt. As a result, the government faces an endogenous debt limit beyond which debt cannot be rolled-over. Using data for 23 advanced economies over 1970–2007, we find evidence of a fiscal reaction function with these features, and use it to compute "fiscal space," defined as the difference between projected debt ratios and debt limits.

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I. INTRODUCTION

A key question confronting the world economy today is whether advanced economies have room for fiscal maneuver—"fiscal space"—or whether they need urgent fiscal adjustment for debt sustainability. Financial bailouts, stimulus spending, and lower revenues in the Great Recession have all contributed to produce some of the highest ratios to GDP of public debt and primary deficits seen in advanced economies in the past forty years (Figure 1). Moreover, many of these countries are expected to continue facing large financing needs over the coming years, while credit markets are already concerned about the ability of some of them (most notably Greece, Ireland, Portugal, and Spain) to service their debts. Whether such countries have any "fiscal space" remaining is thus a pressing question.

In this paper, we develop a new framework for assessing debt sustainability in advanced economies. Specifically, we seek to determine a "debt limit" beyond which fiscal solvency is in doubt. We then define fiscal space as the distance between the current debt level and this debt limit. Our model incorporates a sovereign borrower that follows a fiscal reaction function in response to changes in debt, and risk-neutral creditors who arbitrage the expected return on government debt with the safe interest rate, taking account of the possibility that the government may default because it is unable to repay. In this setting, rising default risk widens the risk premium, and a larger risk premium implies a higher debt service burden and therefore a greater probability of default.

Our analysis begins from the premise that governments in advanced economies usually behave responsibly, increasing primary (i.e., non-interest) surpluses in response to rising debt service so as to stabilize the public debt-to-GDP ratio at a reasonable level. This is an empirically relevant premise consistent with the findings of Bohn (2008) for the United States, and Mendoza and

Ostry (2008) for other industrial countries. Large shocks—such as wars or the fiscal fallout of financial crises—may cause temporary deviations from this (implicit or explicit) primary balance "rule." As long as the subsequent increase in the primary balance is sufficient to offset the higher interest bill, however, the debt ratio will again converge to its long-run value.

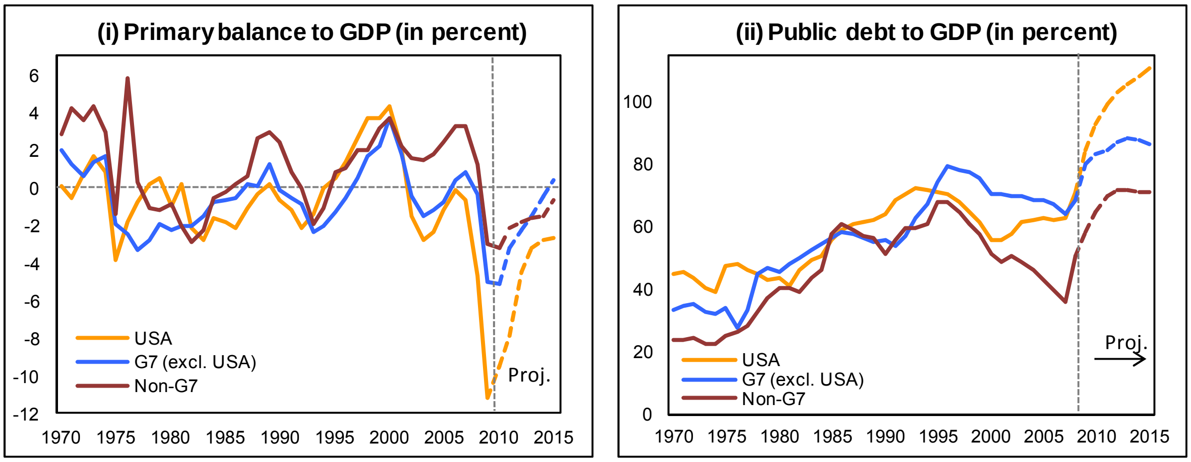
Of course, it cannot literally be true that the primary balance would *always* increase enough to offset the interest bill, because at sufficiently high levels of debt this would require primary balances that exceed GDP.[1] If the primary balance does not keep pace with higher interest payments as debt rises, then—even assuming a constant interest rate so as to abstract from the endogeneity of the risk premium on government debt—there will be a debt level above which the debt dynamics become explosive and the government will necessarily default. In fact default will occur before this point because the rising risk premium—as default becomes imminent—exacerbates the debt dynamics. In particular, as the probability of default rises, so will the risk premium, making it less likely that the primary surplus will suffice to meet the interest bill, and raising the probability of default further.[2] Eventually, the "fixed point" problem of a higher probability of default leading to a larger risk premium, in turn leading to a higher probability of default, has no solution at a finite interest rate. At this point (which we term the *debt limit*) the government loses market access, is unable to rollover its debt, and is forced to default. To

[1] Empirically, we find that while fiscal effort is generally increasing in the debt level, it eventually peters out as it becomes increasingly difficult to keep raising taxes or cutting non-interest expenditures.

[2] There is a large body of literature on sovereign default, though most of these papers consider strategic default by developing country governments (see the survey by Eaton and Fernandez (1995) and the recent quantitative studies along the lines of Arellano (2008) and Aguiar and Gopinath (2006)). Here, we model default as a problem of inability-to-pay, triggered by an inability to rollover debt in the face of rising interest payments and stochastic shocks to the primary balance. We believe that inability-to-pay rather than strategic default is more likely to be relevant for analyzing public debt in advanced economies (see also Cottarelli et al., 2010).

determine this point, we solve simultaneously for the probability of default, the interest rate faced by the sovereign, and the debt limit.

Figure 1. Primary Balance and Public Debt of Advanced Economies



Source: IMF's WEO database.
*Median value for the corresponding year; G7 include: Canada, France, Germany, Italy, Japan, United Kingdom, and United States; Non-G7 advanced countries in the sample include: Australia, Austria, Belgium, Denmark, Finland, Greece, Korea, Iceland, Ireland, Israel, Netherlands, New Zealand, Norway, Portugal, Spain, and Sweden.

Our theoretical framework, motivated by Bohn (1998, 2008), thus departs fundamentally from earlier work in pinning down the concept of debt limit. While Bohn shows that a sufficient condition for the government to satisfy its intertemporal budget constraint is that the primary balance always reacts positively to lagged debt, this can be thought of as a weak sustainability criterion that does not, for example, rule out an ever increasing debt-to-GDP ratio (and thus the need for a primary surplus that eventually exceeds GDP).[3] A stricter sustainability criterion, which we adopt here, is that public debt should be expected to converge to some *finite* proportion of GDP. If the primary balance is always a constant proportion of lagged debt, then a sufficient condition for this stricter definition is that the responsiveness of the primary balance be greater

[3] Bohn (2007) shows that satisfying the government budget constraint requires only that there exist *some* (arbitrarily high but finite) degree of differencing at which the time series of the debt-to-GDP ratio becomes stationary—which is always satisfied in the data. For this reason, he concludes that this sustainability criterion (and the associated stationarity tests on which it rests) is uninteresting, and suggests that examining the behavioral response of the primary balance may be a more fruitful way of establishing debt sustainability; this is the tack taken here.

than the interest rate-growth rate differential.[4] But once we allow for the possibility of "fiscal fatigue" whereby the primary balance eventually responds more slowly to rising debt than the interest rate-growth rate differential, there will in general be a finite debt limit.

Applying our framework empirically to a sample of 23 advanced economies over the period 1970–2007, we find strong support for the existence of a non-linear relationship between the primary balance and (lagged) public debt that exhibits the fiscal fatigue characteristic. Specifically, the relationship is well approximated by a cubic function: at low levels of debt there is no, or even a slightly negative relationship between the primary balance and debt. As debt increases, the primary balance also increases, but the responsiveness eventually weakens, and then actually decreases at very high levels of debt. This relationship is robust to the addition of a long list of conditioning variables and to a variety of estimation techniques.[5]

Combining the empirical estimates of the primary balance reaction function with actual interest rate data or with endogenous interest rates obtained from the model, we gauge each country's debt limit and corresponding fiscal space. Our results indicate that Greece, Italy, Japan, and Portugal have the least fiscal space, with Iceland, Ireland, Spain, the United States, and the United Kingdom also constrained in their degree of fiscal maneuver. By contrast, Australia, Korea, New Zealand, and the Nordic countries appear to have the most fiscal space to deal with unexpected shocks.

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[4] Intuitively, the debt-to-GDP ratio grows autonomously at a rate given by the interest rate-output growth rate differential; if the response of the primary balance to rising debt is stronger than this differential, then the primary adjustment will offset the autonomous dynamics and the debt ratio will converge to a finite ratio.

[5] While we believe this paper is the first to explore the debt sustainability implications of a non-linear response of primary balance to rising debt, we are not the first to find a non-linear response. For example, using a quadratic specification for the United States (over 1916–1995 and 1792–2003), Bohn (1998, 2008) considers how fiscal behavior varies with debt levels, but finds that the primary surplus is *more* responsive to increases in debt at higher debt levels. By contrast, Abiad and Ostry (2005) and Mendoza and Ostry (2008), who look at international evidence (and a more recent sample), find that the response of primary balance to debt weakens at higher debt levels.

Our contribution to the literature is thus three-fold. First, we provide a simple, intuitive definition of debt limit (and corresponding fiscal space) that has the reasonable properties of increasing in the country's average fiscal effort and decreasing in the interest rate-growth rate differential. Second, our model can help explain why governments can suddenly lose financing access and why revisions to market sentiment about a possible shock to the primary balance (even if the shock is not realized) can abruptly push a country into a situation of unsustainable debt dynamics. Third, we provide empirical estimates of available fiscal space for advanced economies, and our approach can quantify the extent to which policy and institutional changes could help to increase the available space.

In what follows, Section II develops the theoretical framework and derives the debt limit. Section III presents estimation results for the fiscal reaction function, and reports fiscal space estimates for our sample of countries. Section IV concludes.

II. Theoretical Framework

2.1 Sovereign Debtor and Creditors

We consider a credit relationship between a sovereign borrower and a large number of atomistic lenders. The government budget constraint is standard:

$$d_{t+1} - d_t = (r_t - g)d_t - s_{t+1} \tag{1}$$

where d is one-period debt (as a share of GDP) at the end of the period, g is the growth rate of real GDP which is assumed to be exogenous and constant, s is the primary balance (in percent of GDP), and r_t is the real interest rate on debt contracted in period t and due in period t+1. The interest rate is endogenous and thus greater than or equal to the risk-free interest rate, r^*, which we assume to be exogenously given (in equilibrium, the interest rate will be an increasing function of the probability of default, as shown below).

We make three assumptions regarding the behavior of the sovereign debtor and its creditors. In the remainder of this Section and in Appendix I, we show that these assumptions are sufficient to guarantee the existence of our debt limit concept.

Assumption I. Fiscal Reaction Function with Fiscal Fatigue

The government is committed to follow a fiscal reaction function:

$$s_{t+1} = \mu + f(d_t) + \varepsilon_{t+1} \tag{2}$$

where μ captures all systematic determinants of the primary balance other than lagged debt, $f(d)$ is the response of the primary balance to lagged debt, which is a continuous function with the properties defined below, and ε is an i.i.d. shock to the primary balance with the distribution function $G(\varepsilon)$ defined over the finite support $[-\overline{\varepsilon}, \overline{\varepsilon}]$ with $\overline{\varepsilon} > 0$. We assume that $G(\varepsilon)$ satisfies standard properties such that $G'(\varepsilon) = G'(-\varepsilon) \le G'(0)$, $G''(\varepsilon < 0) \ge 0$, and $G''(\varepsilon > 0) \le 0$. Assuming a finite support is plausible since the primary deficit or surplus cannot exceed 100 percent of GDP (and, in practice, seldom exceeds more than a few percentage points of GDP).

To capture the idea of fiscal fatigue, the function $f(d)$ is assumed to be continuously differentiable and have the property that there exists a debt ratio $d^m > \overline{\varepsilon}$ such that:

$$\mu + f(d^m) - \overline{\varepsilon} \ \ge\ (r^* - g)d^m \quad \text{and} \quad f'(d) \ <\ r^* - g \quad \forall d > d^m \tag{3}$$

Hence, at d^m and with the worst primary balance shock, debt is non-increasing, and for any higher debt ratio, the response of primary balance is lower than the growth-adjusted interest rate.

Assumption II. Default due to Inability to Pay

The government defaults if and only if debt exceeds the debt limit, $\bar{d}$, where the latter is defined as the maximum debt level at which the government can roll-over its maturing debt and finance the primary deficit at a *finite* interest rate. The default rule of the sovereign thus takes the form:

$$D_t = \begin{cases} 1 & \text{if } d_t > \bar{d} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where D is an indicator equal to one if the government defaults, and zero otherwise.[6]

Assumption III. Risk-neutral Creditors

Creditors are atomistic and risk-neutral, who lend to the government under the following assumptions:

(i) There is less than unit probability that government debt (as a proportion of GDP) is on an explosive path (i.e., increasing without bound regardless of all future realizations of ε).

(ii) There exists a *finite* interest rate that compensates risk-neutral lenders for the (endogenous) risk of default:

$$1 + r^* = (1 - p_{t+1})(1 + r_t) + p_{t+1}\theta(1 + r^*), \tag{5}$$

where p_{t+1} is the probability of default in the next period (when current debt matures), and θ is an assumed recovery value in the event of default.[7] This arbitrage condition implies the standard

[6] In principle, debt limit could vary over time with the government's ability-to-pay. The model could be modified easily to incorporate time varying ability-to-pay as long as μ is non-stochastic.

[7] The permissible range of the recovery value needs to be bounded above at a level "sufficiently" less than unity to ensure that the government has the ability to pay the recovery value in the event of default. A convenient and sufficient upper-bound is to assume that $0 \le \theta \le \bar{\theta} = 1 - \bar{\varepsilon} / (1 + r^*)d^m$. This assumption ensures that the debt limit under uncertainty will lie below the debt limit under no uncertainty (see Appendix I), but still allows for large

(continued…)

result from sovereign default models that, as long as the default probability is positive but less than unity, the default risk premium is a positive, increasing, and convex function of default probability.

(iii) If there are multiple interest rates that satisfy the above arbitrage condition, then creditors are assumed to choose the lowest such interest rate.[8]

2.2 Rational Expectations Equilibrium and Debt Limit

The rational expectations equilibrium of the model is defined by sequences of interest rates and public debt such that the government satisfies its budget constraint, its reaction function and its default rule, and lenders satisfy their arbitrage condition. The key concept we seek to establish and analyze next is the debt limit that characterizes the largest debt at which the government can borrow at a finite interest rate. To this end, we first characterize the model's equilibrium default probability, and establish the existence of well-defined bounds within which the debt limit is determined. Appendix I provides a full derivation of the debt limit and its properties.

The probability that the government defaults next period is simply the probability that debt exceeds the debt limit, which is given by:

$$p_{t+1} \; = \; \mathrm{pr}[\, d_{t+1} > \bar{d} \;] \; = \; \mathrm{pr}[\, \varepsilon_{t+1} < H_t \;] \; = \; G(H_t)\,, \qquad (6)$$

where $H_t = (r_t - g)d_t - \mu - f(d_t) - (\bar{d} - d_t)$. The creditors' arbitrage condition can be rewritten as:

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recovery values. For example, $\bar{\theta}$ is above 90 percent if $\bar{\varepsilon}$ and d^m are 5 percent and 60 percent of GDP, respectively.

[8] In case of two interior solutions, the higher one is not sensible such that the interest rate (and the associated default probability) falls, not rises, as debt increases. See Appendix I.

$$1+r_t = (1+r^*)\left[\frac{1-\theta p_{t+1}}{1-p_{t+1}}\right]. \quad (7)$$

Exploiting the time recursive structure of the problem to drop time subscripts, we combine (6) and (7) to obtain default probability (for a given $\bar{d}$) as the solution to the fixed-point problem:

$$p = z(p; d, \bar{d}), \quad (8)$$

where

$$z(p; d, \bar{d}) = \begin{cases} 0 & \text{if } H(p; d, \bar{d}) \leq -\bar{\varepsilon} \\ 1 & \text{if } H(p; d, \bar{d}) > \bar{\varepsilon} \\ G(H(p; d, \bar{d})) & \text{otherwise} \end{cases}$$

$$H(p; d, \bar{d}) = [(r^*-g)d - \mu - f(d)] - (\bar{d} - d) + (1-\theta)(1+r^*)d[p/(1-p)]$$

It is possible to show that there always exists at least one corner solution to (8), which is $p = 1$. There may be multiple interior solutions to (8), but Assumption III (iii) ensures a unique equilibrium, given by the lowest interior solution.

Bounds on the debt limit

Next, we establish that $\bar{d}$ exists within well-defined upper and lower bounds. The *upper bound*, $\bar{d}_2$, is given implicitly by the largest root of the following equation (see Appendix I):

$$\mu + f(\bar{d}_2) + \bar{\varepsilon} = (r^*-g)\bar{d}_2 \quad (9)$$

The left-hand-side of this expression is the best primary surplus (i.e., the surplus under the best realization of the shock) that the government can achieve at a debt ratio $\bar{d}_2$. The right-hand-side is the best (i.e., the lowest) effective interest payment the government would need to make, which assumes that creditors charge the risk-free interest rate (despite the default risk). By (3), as debt increases beyond $\bar{d}_2$, the primary surplus increases at a slower rate than $(r^* - g)$. Hence, if the debt ratio ever exceeds $\bar{d}_2$, the primary balance would not suffice (even under the best of circumstances) to cover the interest payment, and debt dynamics would become explosive, thus violating Assumption III (i) and triggering default.

The *lower bound*, $\bar{d}_1$, is obtained as the largest root of the following equation:

$$\mu + f(\bar{d}_1) - \bar{\varepsilon} = (r^* - g)\bar{d}_1 \tag{10}$$

which states that at $\bar{d}_1$, the worst primary surplus (i.e., the surplus under the worst realization of the shock) suffices to cover the effective interest payment when creditors charge the risk-free interest rate. Since debt is non-increasing at $\bar{d}_1$ for all possible realizations of the shock, there is no risk of default and, therefore, creditors would charge the risk-free interest rate. Since $\bar{d}_1$ is equal to the annuity value of the worst realization of the primary balance evaluated at the risk free rate, it corresponds to the "natural debt limit", defined in the macro literature on savings under incomplete markets as the largest debt that the sovereign could take if it wants to ensure that it will *never* default (in the next period or at any point in the future), even if the primary balance remains at its worst realization forever. Appendix I shows that $\bar{d}_2 > \bar{d}_1 \geq d^m$.

Determination of the Debt Limit

To determine $\bar{d}$, first we note that, for a given debt limit within the bounds defined above, there is a threshold debt ratio $\hat{d} < \bar{d}$ that corresponds to the maximum debt ratio that yields the corner solution $p = 0$ to (8)—i.e., the largest root of $z(0; d, \bar{d})$. Thus, $\hat{d}$ guarantees that the government will be able to repay next period, and the range of debt ratios $(\hat{d}, \bar{d}]$ is the range in which debt with default risk is traded. By construction, the default probability in the next period is zero for $d \le \hat{d}$, hence the interest rate charged by the market will be the risk-free rate by Assumption III (iii).[9] It is readily verified that $\hat{d} \ge \bar{d}_1$ because $\hat{d}$ only guarantees ability to repay one period ahead, while $\bar{d}_1$ guarantees repayment for any future sequence of realizations of shocks (see Appendix I for the proof).

As debt rises above $\hat{d}$, the market charges a positive risk premium, and the interior solution to (8) determines the equilibrium probability of default and interest rate. We denote the interior solution (the lowest one in case of multiple solutions) by $p^* = p^*(d, \bar{d}) \in (0,\ 1)$. Note that p^* and the associated finite interest rate are a legitimate equilibrium solution only for $d > \hat{d}$ because otherwise the corner solution $p = 0$ and the risk-free interest rate must be the equilibrium solution by Assumption III (iii). Moreover, the debt limit must be larger than $\hat{d}$ if p^* exists because it is defined as the maximum level of debt the government can roll-over at a finite interest rate. Assuming that p^* exists, it can be easily shown that:

$$\partial p^* / \partial d \; > \; 0 \quad \text{and} \quad \partial p^* / \partial \bar{d} \; < \; 0 \tag{11}$$

which makes intuitive sense: the default probability rises with actual debt but falls as the debt limit rises.

Given the government's default rule, in the rational expectations equilibrium, the debt limit must satisfy the following two equilibrium conditions simultaneously:

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[9] Given the nonlinearity of $f(d)$, $p = 0$ may not be supported as the fixed point solution at some levels of debt below $\hat{d}$. Nevertheless, the market will charge the risk-free interest rate because it foresees that the same interest rate is consistent with higher debt levels.

$$\begin{array}{ll} \text{(i)} & p^* \text{ must exist for } d \in (\hat{d}, \bar{d}] \\ \text{(ii)} & p^* \text{ must not exist for } d > \bar{d} \end{array} \tag{12}$$

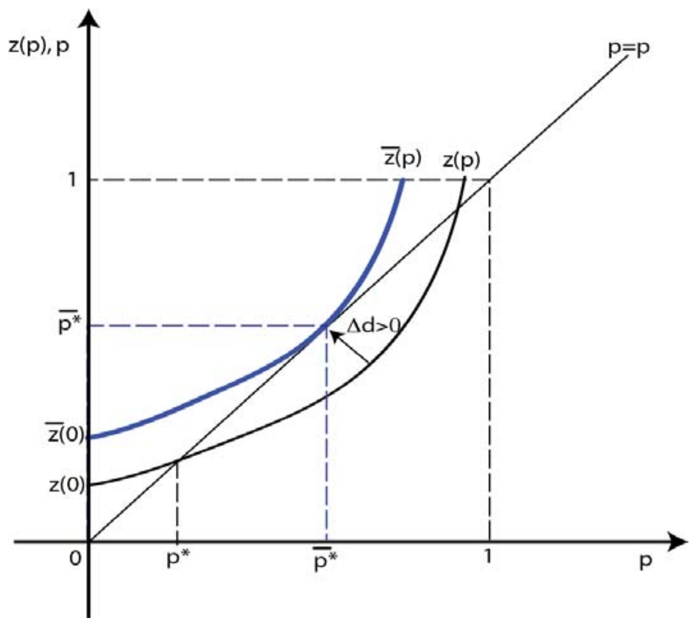
The first condition implies that if debt remains at or below the debt limit, there are always risk-neutral lenders who are willing to lend to the government at a positive finite risk premium. The second implies that $p = 1$ must be the only solution if debt exceeds the debt limit. In this case, no finite risk premium can fully compensate for the associated default risk, and as a result, the government effectively faces an infinite interest rate (or a complete loss of market access).[10] Since the primary balance is finite given that the debt limit and the support of the shock are both finite, debt dynamics become explosive and the government necessarily defaults whenever debt exceeds the debt limit—which is consistent with the assumed default rule. These two conditions essentially pin down the debt limit at equilibrium.

Denoting the interior solution at $d = \bar{d}$ by $\bar{p}^* = p^*(\bar{d}, \bar{d})$, condition (ii), together with the first result in (11), imply that $\bar{p}^*$ must be the maximum interior solution. Figure 2 illustrates this point. For a given debt limit and some initial debt ratio below the limit (and above $\hat{d}$), there would in general be two (or more) interior solutions to (8), the lowest of which is p^*. As d increases, p^* also increases while the other (upper) interior solution decreases. As d increases further, the two interior solutions converge to each other before collapsing into a single interior solution, which occurs at the tangency point between $z(p; d, \bar{d})$ and the 45 degree line. The interior solution at this tangency point is the maximum interior solution that the given debt limit can sustain: any further increase in debt will lead to the corner solution $p = 1$, resulting in an

[10] Flood and Marion (2006) follow a similar approach to analyze how emerging market sovereign borrowers can be shut out of international capital markets in a relatively short period of time.

infinite interest rate. But this is the very definition of debt limit. At the tangency point, therefore, actual debt must coincide with $\bar{d}$ or, equivalently, $\bar{p}^*$ is determined by the tangency point.

Figure 2. Determination of the Probability of Default



The debt limit is unique, and fully determined by the risk-free interest rate (adjusted for growth), the recovery value, and the support of the shock to the primary balance. The interest rate is finite at $\bar{d}$ because the associated probability of default, $\bar{p}^*$, is strictly less than unity. If debt were to increase beyond $\bar{d}$, the equilibrium default probability jumps to unity resulting in an infinite interest rate (or complete loss of market access for the government). As a result, the primary balance is insufficient (even with the best realization of the shock) to offset the explosive autonomous debt dynamics, and the government necessarily defaults for ability-to-pay reasons.

The debt limit obtained here is similar to a well-known result from Eaton and Gersovitz (1981) showing that risk neutral lenders impose a rationing limit at the debt threshold in each period above which, for all possible income realizations in the next period, the borrower would default.

The difference is that in their setting the default probability is determined by a strategic default decision rule, while here it is purely determined by one-period-ahead ability to pay.

2.3 A Deterministic Example

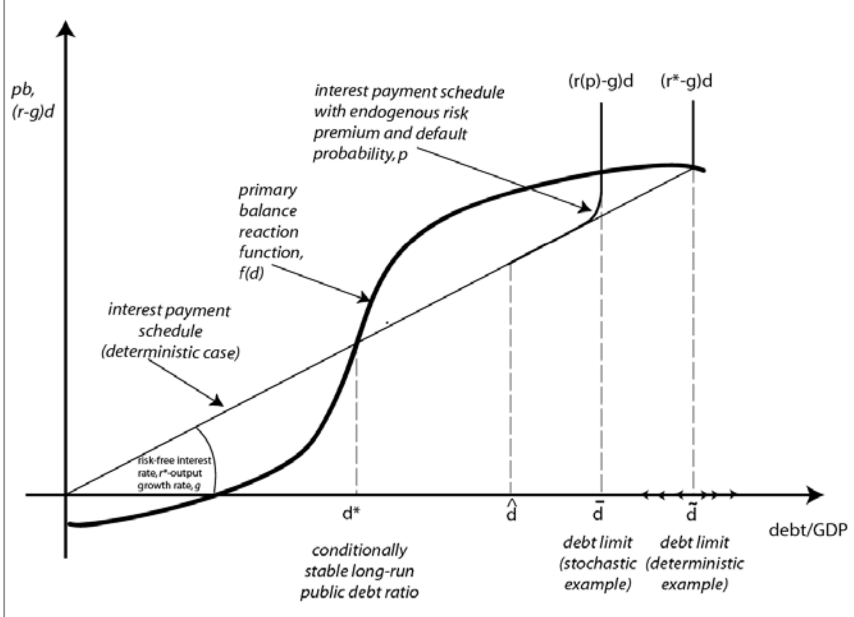
It is instructive to solve a deterministic example where there are no shocks to the primary balance (i.e., $\bar{\varepsilon} = 0$). Denoting by $\tilde{d}$ the debt limit, in the deterministic case, $z(p; d, \tilde{d})$ can take only two discrete values—0 or 1—implying that the fixed-point solution to (8) is characterized by two corner solutions: $p = 0$ or $p = 1$. Given this result, the debt limit is determined by the following two equilibrium conditions: (i) $p = 0$ must be the equilibrium solution for $d \leq \tilde{d}$, and (ii) $p = 1$ must be the equilibrium solution for $d > \tilde{d}$. As shown in Appendix I, these conditions yield the debt limit as the largest root of the following equation:

$$\mu + f(\tilde{d}) = (r^* - g)\tilde{d} \tag{13}$$

The deterministic example is useful because the equilibrium debt, debt limit, and interest rate schedule can be illustrated by means of a simple diagram. In Figure 3, the thick black curve represents the nonlinear fiscal reaction function, which satisfies the properties defined in Assumption III, except that there is no stochastic shock to the primary balance (such shocks would imply upward or downward shifts of the curve). In general, there will be two stationary equilibria (ignoring the intersection that would occur at $d < 0$). The first is given by the lower intersection between $\mu + f(d)$ and the $(r^* - g)d$ schedule. This lower intersection, denoted d^*, is the long-run public debt ratio to which the economy converges conditionally (as long as debt does not cross the limit $\tilde{d}$). It is readily verified that this equilibrium is dynamically stable.

The second stationary equilibrium corresponds to the debt limit implied by the model, and is given by the higher intersection between $\mu + f(d)$ and the $(r^* - g)d$ schedule. Thus, convergence to *d** is only conditional, because if debt were to ever exceed the higher equilibrium, then it would not return to *d**. If debt were to exceed this limit, there is no finite interest rate that would compensate creditors for the probability of default (which is unity). To see this, note that beyond the debt limit the primary balance is insufficient to cover the interest payment—even at the risk-free interest rate—so the debt ratio would increase without bound. Therefore, the interest rate becomes infinite beyond the debt limit, with the government effectively losing market access and unable to roll over its debt. Thus, in the deterministic case, the probability of default is zero up to the debt limit, $\tilde{d}$, jumping to unity thereafter. Hence, the deterministic case is degenerate, with $\bar{d}_1 = \hat{d} = \tilde{d} = \bar{d}_2$.

Figure 3. Determination of Debt Limit in Deterministic Example



In the stochastic model, the equilibrium interest rate schedule, $r = r(d, \bar{d})$, can be derived from (7) and (8) once the debt limit has been identified. The interest rate schedule equals the risk-free rate to the point $\hat{d}$ after which it is convex and bends upwards, becoming vertical as debt exceeds its limit $\bar{d}$ (as in Figure 3).[11] The convex segment of the interest rate schedule occurs over a short interval less than the support of the shock $\bar{\varepsilon}$. This result obtains because, with one-period debt, the probability of default is zero until debt is within the worst possible overall deficit from the debt limit. Since $\hat{d}$ lies in the region where the overall balance is in surplus if there were no shock to the primary balance, the worst possible overall deficit is less than the support of the shock. While the relationship is more complicated with multi-period debt, this result suggests that debt levels initially considered safe could suddenly become unsustainable particularly if $\bar{\varepsilon}$ is small. In general, the precise shape of the interest rate schedule depends on the distribution of the shock to the primary balance, the recovery rate in the event of default, and the curvature of the primary balance reaction function.

2.4 Properties of the Debt Limit

The debt limit obtained above has four important properties (here we give a heuristic treatment based on Figure 3; Appendix I provides a more formal treatment). First, from Figure 3, a decrease in the economy's output growth rate or an increase in the risk-free interest rate rotates the $(r^*-g)d$ schedule counter-clockwise, reducing the $\bar{d}$ and $\tilde{d}$ (while raising the long-run debt ratio to which the economy conditionally converges, d^*). Second, greater willingness to undertake fiscal adjustment (an upward shift of the intercept of the fiscal reaction function or the steepening of this schedule as a function of lagged debt) increases the debt limit (while reducing

[11] In the general stochastic case, the point at which the interest payment schedule becomes vertical (i.e., the debt limit) can occur below, at, or above the point at which the interest rate schedule intersects $\mu+f(d)$, depending upon the precise shape of the fiscal reaction function, the support of the shock, and the recovery rate in the event of default. For most parameter values (and all empirically relevant cases considered below), this point will be below the intersection (as drawn in Figure 3). In the deterministic case, the point at which the interest payment schedule becomes vertical is necessarily where the risk-free interest payment schedule intersects the fiscal reaction function.

d^*). Third, the equilibrium at the debt limit is dynamically unstable: a positive (and even moderately negative) shock to the primary balance at that point would likely bring the debt back toward the long-run debt ratio, d^*; a (large) negative shock will push the government to default.

The fourth property is less obvious. In general, a re-evaluation of the support of shocks to the primary balance (e.g., a symmetric widening of the support $\bar{\varepsilon}$) leads to an immediate steepening of the interest rate schedule and a lowering of the debt limit, even if the shock has not been realized.[12] Since uncertainty implies both downside risk and upside potential, it may not be obvious why uncertainty should reduce the debt limit. The reason is, given the nature of a debt contract, upside potential cannot fully offset downside risk. Large positive shocks to the primary balance yield no additional return to creditors (because the interest rate is predetermined) whereas large negative shocks can cause default. Therefore, downside risk matters more than upside potential even if creditors are risk neutral (*a fortiori* if they are risk averse). If debt were close to $\bar{d}$, this could result in a formerly sustainable level of debt becoming unsustainable, triggering default. In this model, therefore, a data revision that leads to higher debt being reported to the market could have two effects—in Figure 3, a rightward jump of the debt level reflecting the new debt that is being recognized and, if this triggered a revision in market perceptions about the support of the shocks to the primary balance, a counter-clockwise rotation of the interest rate schedule. Both effects would bring the government closer to default—a phenomenon that resonates with the recent experience of some southern European countries.

[12] The exception to this is when, in the stochastic case, the interest payment schedule becomes vertical above the point at which it intersects $\mu+f(\mathrm{d})$, which only occurs if the recovery value is sufficently high. In this case, at the debt limit, the interest payment is greater than the expected primary balance and, assuming $G(\varepsilon)$ is symmetric, the probability of default exceeds 0.5. This is akin to "gambling for redemption": default can be avoided only if a positive shock occurs. Since any negative shock, *regardless of size*, would result in default while only a sufficiently large positive shock helps avoid default, a mean preserving increase in the support of ε would (in this case) make default less likely, and raise the debt limit (see Appendix I); however, as noted above, for most parameter values, this case does not obtain.

III. EMPIRICAL IMPLEMENTATION

This section applies the theoretical framework developed above to a sample of 23 advanced economies using data for the period 1970–2007. The data includes gross public debt and the primary fiscal balance as shares of GDP as well as a set of control variables discussed below; data details are provided in Appendix II.[13] We proceed in three steps: (i) estimate the primary balance reaction function; (ii) determine the appropriate interest rate-growth rate differential; and (iii) calculate each country's debt limit, and associated fiscal space (defined as the difference between current debt ratios and the computed debt limit).

A. Primary Balance Reaction Function

A first question is whether the "fiscal fatigue" behavior postulated above holds in the simple bivariate relationship between the primary balance and lagged debt. The scatter plot in Figure 4 suggests that the nonlinear fiscal fatigue behavior is indeed plausible: at very low debt ratios, there is little (or even a slightly negative) relationship between lagged debt and primary balance. As debt increases, the primary balance rises but the responsiveness eventually begins to weaken, and then actually decreases at very high levels of debt (though the downward sloping portion is driven by relatively few observations). The fitted line in the Figure is based on the empirical analysis below, which confirms that fiscal fatigue is a robust, statistically significant feature of the data that holds even controlling for other determinants of the fiscal reaction function.

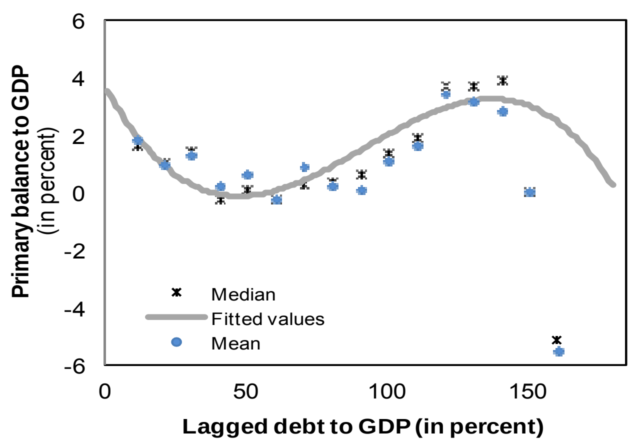
The estimation results for the fiscal reaction function relating primary balances to lagged debt (allowing for a cubic function to capture the two inflexion points in the curvature of the response) and various control variables, as well as country-specific fixed effects, are presented in

[13] We use gross rather than net debt since the latter is not always comparable across countries due to different definitions or treatment of asset components, particularly, those of the social security system.

Table 1. Estimates for two sample periods (1970–2007; 1985–2007) are reported, where data availability for the latter period allows for the inclusion of a richer set of structural variables as determinants of the primary balance.

Figure 4. Primary Balance and Public Debt, 1970-2007

(In percent of GDP)



Source: IMF's WEO database and staff estimates.

Following the literature (for example, Roubini and Sachs, 1988; Gali and Perotti, 2003; Abiad and Ostry, 2005; Bohn, 2008; and Mendoza and Ostry, 2008), we include a range of variables such as the output gap to control for the effect of business cycles, the government expenditure gap to measure the effect of temporary fluctuations in government outlays (such as military spending), fuel and non-fuel commodity prices, trade openness, and the average inflation rate in the previous three years to examine the possible effects of inflation (such as bracket-creep effects, or, greater fiscal effort to counter the effects of higher interest rates accompanying higher inflation) on the fiscal balance.

The institutional variables consist of a political stability index—which is a composite measure of the institutional capacity as well as government stability in a country—with higher values indicating lower risk; a fiscal rules index, which indicates if a country has any type of fiscal rule (balanced budget, expenditure, revenue, or debt) in a given year; and a dummy variable for IMF program arrangement as a proxy for international influence on fiscal behavior. We also take into account the demographic structure, and include the current and projected age dependency ratios.

Estimation of the fiscal reaction function raises some econometric issues stemming from the dependence of lagged debt on past values of the primary balance. From the debt dynamics equation, lagged debt is necessarily correlated with the unobserved country specific determinants of the primary balance—that is, countries able to generate higher primary balances on average because of favorable fixed effects, would tend to have lower levels of public debt. This negative association between debt and time invariant country specific features, if not properly accounted for, could generate a downward bias in the estimated response of primary balance to lagged debt. In our estimation, we control for this source of endogeneity bias through the inclusion of country fixed effects in the fiscal reaction function. Furthermore, to the extent that there is persistence in the error term (ε), the debt dynamics would render lagged debt in equation (2) endogenous—even when country fixed effects are included—and induce a downward bias in its estimated coefficient. To address this potential bias, we allow for serial correlation in the error term—modeling ε as an AR(1) process (that is, $\varepsilon_{t+1} = \alpha\varepsilon_t + v_t$).

The results presented in Table 1 indicate that the coefficients of the cubic functional form—capturing the increasing but slowing response of the primary balance to lagged debt—are statistically significant. In fact, based on the estimated coefficients, the marginal response of primary balance to lagged debt starts to decline at debt levels around 90-100 percent of GDP, becoming negative as the debt ratio approaches about 150 percent of GDP. This is in contrast to the findings of Bohn for the United States, which show that fiscal effort increases with the debt level, but supports the results of Mendoza and Ostry (2008) who find that sustainability is less assured in advanced economies when public debt is high than when it is moderate. The estimated response of primary balance at moderate debt levels (in the range of 0.02–0.06 percentage points) is also consistent with the results of the latter, but the empirical approach adopted here is preferable because instead of an ad hoc classification of countries into high and low debt groups, the cubic specification allows more flexibility, and helps to pin down the debt range where economies risk moving into the insolvency territory.

The estimated coefficients of other determinants included in the fiscal reaction function are also plausible and broadly in line with previous studies. For example, the primary balance responds positively to the output gap. Temporary increases in government outlays as captured by the government expenditure gap variable, affect the primary balance negatively. More open economies in terms of international trade activity demonstrate better fiscal performance, as do countries with stronger institutional quality (as proxied by the political stability variable), and fuel exporters when oil prices rise.[14] The nonlinearity of the fiscal reaction function and the response of the primary balance to other structural and economic determinants are robust to a variety of different estimation methods discussed in Appendix II.

Our empirical analysis, while including fixed effects for individual countries to capture differences in average primary balances, is based on the panel estimates of the primary balance reaction function to lagged debt. As with any panel estimation, this raises questions about slope homogeneity. But the issue is more complex here because the cubic "fiscal fatigue" behavior is postulated to hold over the entire range of debt (from 0 to about 180 percent of GDP), whereas

[14] Norway is the only country classified as an oil exporter in the sample.

individual countries' debt ratios may be observed over only a portion of that range. Our analysis for countries that, in the sample, we do not observe at high debt ratios must therefore rest on the assumption that these countries—if their debt were to increase—would behave similarly (in terms of fiscal fatigue) as countries that we do observe at high debt ratios.

While this is—and can only be—a conjecture, three observations suggest that it is a reasonable conjecture. First, our sample consists only of advanced economies, which share similar characteristics, and is therefore relatively homogeneous. Second, the contrary assumption—that some countries would never exhibit fiscal fatigue, and would always increase the primary balance in line with rising debt—seems implausible, if only because the primary surplus cannot exceed GDP. Third, slope homogeneity across countries cannot be rejected (in almost every case) over the debt ranges (low-to moderate; moderate-to-high) in which they are observed. Specifically, when observed at low-to-moderate debt ranges, countries (including those that eventually have high debt) react similarly with a positive, linear coefficient to lagged debt (see Appendix II for detailed results).[15] Likewise, when observed at moderate-to-high debt ranges, countries (including those that, in the sample, have some low debt observations) exhibit similar fiscal fatigue as captured by the cubic specification. In other words, countries in the sample behave similarly at common debt ratios at which they are observed—providing some confidence that they would behave similarly at debt ratios at which (some) are not observed.[16]

[15] Of course, countries do differ in their average primary balances, as captured by the country fixed effects and other structural determinants of the primary balance.

[16] This observation is in line with the strong evidence of homogeneity (for 42 out of 56 industrial and developing countries) of the fiscal response coefficient reported in Mendoza and Ostry (2008).

Table 1. Estimation Results for the Fiscal Reaction Function[1]

Sample	1970-2007		1985-2007	
Specification	(1)	(2)	(3)	(4)
Lagged debt	-0.2080***	-0.2249***	-0.0805	-0.0864
	(0.059)	(0.061)	(0.076)	(0.070)
Lagged debt_square	0.0032***	0.0034***	0.0016*	0.0017*
	(0.001)	(0.001)	(0.001)	(0.001)
Lagged debt_cubic	-0.00001***	-0.00001***	-0.00001*	-0.00001**
	(3.0e-06)	(3.0e-06)	(3.0e-06)	(3.0e-06)
Output gap	0.4974***	0.4910***	0.4849***	0.4408***
	(0.047)	(0.046)	(0.053)	(0.053)
Government expenditure gap	-0.1847***	-0.1837***	-0.1831***	-0.1826***
	(0.047)	(0.045)	(0.052)	(0.047)
Trade openness		0.0908*		0.1461***
		(0.050)		(0.054)
Inflation		3.4005		4.6201**
		(2.519)		(2.008)
Oil price[a]		8.7747***		9.5288***
		(3.216)		(3.244)
Age dependency				-0.0717
				(0.101)
Future age dependency				-0.0154
				(0.067)
Nonfuel commodity price[a]				3.0049
				(8.362)
Political stability				0.0678**
				(0.030)
IMF arrangement				-1.1421
				(0.999)
Fiscal rules				0.3000
				(0.347)
Observations	642	642	496	491
Number of countries	23	23	23	23
R-squared	0.282	0.316	0.304	0.405
AR (1) coefficient	0.791	0.760	0.819	0.749

Source: Authors' estimates.

[1]Dependent variable is general government primary balance to GDP (in percent); in all specifications, country-specific fixed effects included, and error term assumed to follow an AR (1) process; robust standard errors reported in parentheses; ***, **, and * denote

[a] Applies to oil and nonoil commodities exporters only.

B. Fiscal Space

To perform our calculations of fiscal space, we also need estimates of the relevant interest rates on public debt. We consider two approaches to obtain these estimates. The first, which we label "market approach," is to use current (or projected) market interest rates on government debt, based on the assumption that the market rate reflects the perceived probability of default. Table 2 reports the interest rate-growth rate differentials obtained using two variants of this approach. The first variant is the historical average (over the past ten years) of the implied nominal interest rate on government debt (interest payments divided by end-period debt) relative to the growth rate of nominal GDP. The second variant replaces historical averages with International Monetary Fund's projections of long-term government bond yields and for GDP growth obtained from IMF (2010). With minor exceptions, projected interest rate-growth differentials are considerably less favorable than historical ones, reflecting the expectation of both higher interest rates and lower real GDP growth rates (Table 2).[17]

The use of (historical or projected) market interest rates may overestimate the true debt limit implied by the model because it ignores the fact that the interest rate will rise sharply as debt approaches its limit and default risk increases. An alternative approach, therefore, is to calculate interest rates endogenously, which provide us with a framework to estimate the rising risk of default as debt approaches its limit. We label this the "model approach." Its main drawback is that it requires various assumptions about the risk-free interest rate, the distribution (and support) of the shocks to the primary balance, and the recovery rate in the event of default. We assume a triangular distribution for the shocks to primary balance, calibrated for each country based on the

[17] This may in turn reflect the worse public debt outlook—on average, public debt increased from 60 percent of GDP on the eve of the crisis (end-2007) to almost 75 percent of GDP by end-2009, and, more strikingly, is projected to continue rising over the next five years.

residuals obtained from the primary balance reaction function in the estimation sample, and assume a recovery rate of 90 percent in the event of default.[18]

Table 2. Public Debt and Interest Rate-Growth Rate Differential (in percent)

Country	Debt/GDP			Interest Rate-Growth Rate Differential	
	2007	2009	2015 1/	Historical 2/	Projected 1/
Australia	9.4	15.5	20.9	0.1	1.2
Austria	59.5	67.3	77.3	1.4	0.8
Belgium	82.8	97.3	99.9	1.2	2.1
Canada	65.0	82.5	71.2	1.7	0.4
Denmark	34.1	47.3	49.8	3.2	0.1
Finland	35.2	44.0	76.1	0.0	1.4
France	63.8	77.4	94.8	0.8	0.5
Germany	65.0	72.5	81.5	2.6	1.5
Greece	95.6	114.7	158.6	-1.5	2.2
Iceland	29.3	105.1	86.6	-1.4	4.1
Ireland	24.9	64.5	94.0	-5.8	3.2
Israel	78.1	77.8	69.9	0.1	0.2
Italy	103.4	115.8	124.7	1.4	1.7
Japan	187.7	217.7	250.0	2.0	1.0
Korea	29.6	32.6	26.2	-0.7	-2.3
Netherlands	45.5	59.7	77.4	0.5	0.6
New Zealand	17.4	26.1	36.1	1.1	2.5
Norway	58.6	53.6	53.6	-3.4	-0.7
Portugal	63.6	77.1	98.4	-0.6	2.2
Spain	36.1	55.2	94.4	-2.4	2.6
Sweden	40.5	40.9	37.6	-0.5	-0.7
United Kingdom	44.1	68.2	90.6	0.4	1.3
United States	62.1	83.2	109.7	0.3	1.6
Median	58.6	68.2	81.5	0.3	1.3
Mean	57.9	73.7	86.1	0.0	1.2

Source: IMF's WEO database.
1/ IMF (2010) Projections. Interest rate-growth rate differential is based on the long-term government bond yield (average for 2010-14).
2/ Average of 1998-2007 based on the implied interest rate on public debt.

[18] Benjamin and Wright (2008) estimate the recovery rate for upper-middle income countries to be about 60-100 percent; hence, for advanced economies we assume an average recovery rate of 90 percent. We construct the finite support of the shock ($\bar{\varepsilon}$) by using the country-specific average of the worst five negative residuals. The risk-free interest rate is assumed to be common to all countries in the sample, and is taken as the average over 2003-07 of the effective real interest rate on government debt. It is growth adjusted by using the five-year country-specific average of the IMF projected real GDP growth rate.

Point estimates

Table 3 reports the debt limits, $\bar{d}$, as well as the long-run debt ratio to which each country's public debt conditionally converges, d^*, calculated using the country's predicted primary balance obtained from the results for the recent period (1985–2007) reported in Table 1 (column 4), and historical, market and model-generated interest rates.[19] While there is, of course, significant variation across countries (reflecting the country specific fixed effects and interest-growth rate gaps), d^* ranges between 0 to about 100 percent of GDP—with a median of 50 percent of GDP—using historical interest rates, and the estimated $\bar{d}$ ranges between 150 to 250 percent of GDP with a median of 190 percent. Since the projected interest rate-growth rate differentials are generally less favorable than the historical experience, the corresponding median d^* is 57 percent of GDP and the median $\bar{d}$ is 185 percent of GDP.

The estimated d^* and $\bar{d}$ obtained from the model-implied interest rate—reported in the last two columns of Table 3—are similar to the estimates based on market interest rates—though in most cases somewhat lower (consistent with the intuition that, especially for countries with lower debt levels, the market-based interest rate will likely overestimate the available fiscal space). In a few cases (Greece, Iceland, Italy, Japan, Portugal) and depending on the interest rate used, no estimate of d^* (or of $\bar{d}$) is reported. This is because, given these countries' estimated primary balance reaction function and assumed interest rate-growth rate differential, public debt would not be expected to converge to a finite steady state debt ratio (it follows that there is no maximum debt level below which convergence occurs, so it is not meaningful to calculate $\bar{d}$

[19] For the calculation of $\bar{d}$ and d*, output and government expenditure gaps are assumed to be closed. We also compute fiscal space estimates based on the 1970–2007 sample, and find the results to be very similar.

either).[20] For Italy and Japan, d^* does not exist even using the historical interest rate-growth rate differential, implying that their debt ratios had not been on a convergent path even prior to the crisis (and indeed, especially for Japan, the public debt ratio has been rising steadily over much of the sample period).[21] For Iceland, there is a finite long-run debt ratio using either the historical or the model-generated interest rate, but not using projected market interest rates. For Greece and Portugal, d^* exists only using the historical interest rate-growth rate differential—not at market or model-implied interest rates.

It is worth noting however that these fiscal space estimates are based on the projected values of the set of structural variables included in $f(d)$. Any change in these values would shift the predicted fiscal reaction function, and hence affect the debt limit and the corresponding fiscal space estimates. This is a particularly useful feature of our approach since it allows us to quantify the impact of any structural changes on fiscal sustainability. Thus, for example, using the estimates reported in column (4) of Table 1, and assuming that the institutional capacity (that is, the political stability variable) in all countries is at the sample maximum, we find that the available fiscal space increases—albeit to different degrees—in almost all economies.[22] Among the countries with no available fiscal space, the most notable increase is for Iceland, which is projected to have positive fiscal space with an improvement in its institutional capacity.[23]

[20] In terms of Figure 3, the fiscal reaction function lies below the interest rate schedule, implying no intersection.

[21] This holds even if net debt rather than gross debt is used in the analysis for Japan.

[22] The highest score for the political stability index in the sample is for Finland. The results for this exercise are available upon request.

[23] For Greece, Italy, Japan and Portugal, $\bar{d}$ increases but remains lower than the projected debt level for 2015.

Table 3. Long-run d^* and $\bar{d}$ Under Alternative Interest Rate-Growth Rate Assumptions[1]

(In percent of GDP), 1985-2007

Country	Debt (end-2015)	Market Interest Rate				Model-implied Interest Rate[2]	
		d^*		$\bar{d}$		d^*	$\bar{d}$
		Historical	Projected	Historical	Projected		
Australia	20.9	0.0	0.0	203.9	193.2	0.0	202.7
Austria	77.3	63.9	54.3	179.7	187.3	55.1	170.7
Belgium	99.9	60.3	76.3	182.0	168.4	53.7	172.0
Canada	71.2	110.8	82.6	152.3	181.1	75.2	173.1
Denmark	49.8	0.0	0.0	175.7	208.7	0.0	195.9
Finland	76.1	0.0	0.0	200.4	184.5	0.0	167.0
France	94.8	94.8	89.8	170.9	176.1	92.7	159.7
Germany	81.5	94.5	71.0	154.1	175.8	63.6	170.0
Greece	158.6	80.5	…	196.5	…	…	…
Iceland	86.6	0.0	…	213.5	…	0.0	157.3
Ireland	94.0	0.0	90.7	245.7	149.7	42.9	157.6
Israel	69.9	79.7	82.1	184.8	182.4	65.0	183.9
Italy	124.7	…	…	…	…	…	…
Japan	250.0	…	…	…	…	…	…
Korea	26.2	0.0	0.0	217.2	229.2	0.0	220.3
Netherlands	77.4	50.2	50.7	190.5	190.1	58.0	168.7
New Zealand	36.1	0.0	0.0	201.0	186.4	0.0	197.6
Norway	53.6	0.0	0.0	263.2	249.2	0.0	233.5
Portugal	98.4	77.1	…	191.6	…	…	…
Spain	94.4	0.0	94.8	218.3	153.9	70.2	168.4
Sweden	37.6	0.0	0.0	203.5	204.9	0.0	167.8
United Kingdom	90.6	79.6	94.9	182.0	166.5	75.5	166.0
United States	109.7	78.7	101.2	183.3	160.5	77.6	173.1
Median	81.5	50.2	62.6	191.6	183.4	53.7	170.7
Mean	86.1	41.4	49.3	195.7	186.0	38.4	179.2

Source: IMF's WEO database and authors' calculations.

[1] $\bar{d}$ is the debt limit, above which debt grows without bound given the country's historical primary balance behavior; d^* is the long-run average debt ratio to which the economy converges conditional on not exceeding $\bar{d}$; … indicates that given the fiscal reaction function and the interest rate-growth differential, the public debt dynamics are not on a sustainable path to converge to a finite stable steady state debt ratio; 0 indicates that convergence is achieved at a negative d^*, implying a positive asset position. All results are based on the estimated fiscal reaction function reported in Table 1 (column 4).

[2] The estimates of $\bar{d}$ are obtained assuming a recovery rate of 90 percent in the event of default.

Likelihood of fiscal space

As mentioned above, our definition of fiscal space is simply the difference between the debt ratio projected for 2015 and the debt limit, $\bar{d}$. But this difference gives the point estimate, which is a complicated function of the underlying parameter estimates of the fiscal reaction function, and even modest uncertainty in the estimates of the primary balance could potentially translate into significant differences in debt limits.[24] To take account of this uncertainty, the model is simulated allowing stochastic variation in the estimated coefficients of the primary balance reaction function.[25] Table 4 reports the estimates of fiscal space (i.e., the difference between 2015 projected debt levels and the debt limit) in terms of the *probability* that a country has a given amount (0, 50, or 100 percent of GDP) of remaining fiscal space, with cells shaded dark or light grey if the probability is less than 50 percent, or between 50 and 85 percent, respectively.

Consistent with the discussion above, the probability that Greece, Iceland, Italy, Japan, and Portugal have additional fiscal space is low—though in a few cases, depending on the interest rate used, as high as 50 percent. Next are Ireland and Spain, where the probability that these countries have at least some additional fiscal space is around 70 percent (rising to more than 80 percent for Spain using the model-implied rather than the current interest rate). For the United States and the United Kingdom, the probability of any remaining fiscal space is around 70–80 percent. For the other countries, the estimated probability of at least some additional fiscal space is greater than 85 percent. Intuitively, the estimates of fiscal space depend on the debt level

[24] To put this in perspective, if the primary balance that a country can run is 1 percent of GDP larger than estimated, then with an interest rate-growth rate differential of 1 percent per year, $\bar{d}$ increases by 100 percent of GDP.

[25] In the simulations, random shocks are drawn from the multivariate normal distribution (with mean zero) and the estimated variance-covariance matrix of the coefficients of the fiscal reaction function. In Table 4, the reported probabilities under the projected market (model implied) interest rate are based on 10,000 (1,000) iterations.

projected for 2015 and the debt limit, which depends on the country's historical track record of primary balances. Thus even a country with relatively high debt currently may enjoy additional fiscal space (and thus scope for dealing with unexpected shocks) if, in the past, it has acted fiscally responsibly, adjusting the primary balance once the shock has passed.

In assessing these results, several points should be borne in mind. First, the reported estimates of fiscal space are against projected debt levels in 2015, and do not take account of possible contingent liabilities. Contingent liabilities, such as unfunded pension obligations and potential bank bailouts, by increasing actual debt can worsen our fiscal space estimates substantially. Moreover, the debt limit—on which the fiscal space estimates are predicated—is by no means a "desirable" or "optimal" level of debt. For a variety of reasons, including possible rollover risk, governments will generally want to ensure they do not exhaust their fiscal space and that their debt remains well below its calculated limit. Second, as discussed in the theoretical section, a key feature of the model is that a revision in market estimates of the support of the possible shock to the primary balance—even if the shock does not occur—can trigger a rise in the market interest rate, potentially undermining debt sustainability. Third, though not evident in the reported estimates of fiscal space, simulations to calculate the fiscal space under the model endogenous interest rate suggest that the increase from the risk-less to the risky interest tends to happen quite abruptly as debt reaches its limit. Thus, a country may be able to borrow at low ("risk-free") interest rates even as its public debt increases, and then suddenly find itself facing much steeper interest rates or even shut out of the capital markets. The last two properties resonate with the experience of some Southern European countries in the recent crisis, and underscore the importance of maintaining sufficient fiscal space.

Table 4. Estimated Probability of Given Fiscal Space: 1985-2007

(In percent)

	Projected Market Interest Rate			Model-implied Interest Rate		
	FS > 0	FS > 50	FS >100	FS > 0	FS > 50	FS >100
Australia	99.8	99.5	99.5	99.8	99.8	99.8
Austria	97.9	97.8	75.1	81.4	81.4	38.1
Belgium	95.9	89.7	2.9	95.5	92.0	5.2
Canada	92.2	92.1	70.3	80.9	80.9	57.1
Denmark	100.0	100.0	100.0	100.0	100.0	100.0
Finland	96.2	96.0	69.3	72.8	72.8	37.1
France	88.7	86.6	12.0	65.5	63.1	4.3
Germany	93.0	92.3	35.3	82.6	82.3	25.8
Greece	6.3	0.1	0.1	0.3	0.0	0.0
Iceland	49.1	44.0	5.8	57.9	57.3	20.4
Ireland	66.0	55.9	1.7	60.9	58.8	4.3
Israel	97.1	97.1	80.7	95.1	95.1	81.4
Italy	17.3	1.7	0.2	0.1	0.1	0.1
Japan	0.1	0.1	0.1	0.0	0.0	0.0
Korea	100.0	100.0	100.0	100.0	100.0	100.0
Netherlands	99.3	99.2	83.1	81.0	80.8	35.1
New Zealand	93.3	93.0	92.1	94.5	94.5	94.5
Norw ay	100.0	100.0	100.0	99.9	99.9	99.9
Portugal	34.4	27.1	0.4	27.6	23.8	0.5
Spain	69.9	61.0	1.6	82.6	79.8	6.3
Sw eden	99.9	99.9	99.9	71.3	71.3	70.6
United Kingdom	78.1	75.9	8.9	69.3	68.9	12.1
United States	71.8	52.2	1.2	82.9	71.2	2.8
Median	93.0	92.1	35.3	81.0	79.8	25.8
Mean	75.9	72.2	45.2	69.7	68.4	38.9

Source: Authors' calculations.
Black and grey cells indicate probability less than 50 percent, and betw een 50 and 85 percent, respectively.

IV. CONCLUSION

This paper contributes to the existing literature on fiscal sustainability by proposing a new framework that conceptualizes the notion of fiscal space—defined as the difference between current debt level and a (country-specific) debt limit, where the latter is the debt level beyond which fiscal solvency fails. Our concept of the debt limit, which takes explicit account of fiscal fatigue in a stochastic setting, is intuitively appealing and has several sensible properties. In particular, an improvement in a country's structural characteristics or economic growth rate raises its debt limit, while the occurrence (or recognition of the possibility) of a negative shock could push an otherwise sustainable debt level to the unsustainable territory.

We apply our framework empirically to a sample of 23 advanced economies over 1970–2007, and find robust empirical support for the fiscal fatigue characteristic. Specifically, we find that the marginal response of primary balance to lagged debt is nonlinear, remaining positive at moderate debt levels but starting to decline when debt reaches around 90-100 percent of GDP. Combining these estimates with those of exogenous and endogenous (model-implied) interest rates, we find that the estimated debt limits and corresponding fiscal space vary considerably across countries. For example, the debt limit obtained for countries in the sample ranges between 150 to 250 percent of GDP, while the fiscal space estimates indicate limited or no available fiscal space for Greece, Iceland, Italy, Japan and Portugal, and ample space for Australia, Korea and the Nordic countries.

The probability estimates of positive fiscal space present a somewhat similar picture with several countries including Australia, Korea, and the Nordics projected to have the highest probability of positive fiscal space, and a number of Southern European countries, and Japan and Iceland the lowest. In addition, the United Kingdom and the United States also appear to be constrained in their degree of fiscal maneuver. These estimates however do not take into account liquidity/rollover risks or the possible realization of contingent liabilities. Further, while our model is essentially a one-period model that incorporates an endogenous interest rate, exploring the implications of longer-term debt and/or stochastic endogenous growth are possible avenues for future research. From a policy perspective, however, our probability estimates can be used to flag cases where fiscal consolidation may be urgently needed in order to ensure that debt remains on a sustainable path and that shocks, including data revisions, do not derail sustainability.

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Appendix I. Determination of the Debt Limit

This appendix derives the debt limit formally, and provides mathematical proofs for key results used in the derivation and in the discussion on the properties of the debt limit.

Boundary condition for the debt limit

For notational convenience, we denote $\bar{z}(p) = z(p; \bar{d}, \bar{d})$. Since $\bar{p}^*$ is an interior solution to $p = \bar{z}(p)$, the following boundary condition must hold for $\bar{p}^*$:

(A.1) $$p_L < \bar{p}^* < p_U$$

where $p_L = \bar{z}(0)$ and $p_U = \arg\min \bar{z}^{-1}(1)$. Substituting from (8) yields,

(A.2) $$p_L = G(F(\bar{d})) \quad \text{and} \quad p_U = \frac{\bar{\varepsilon} - F(\bar{d})}{W(\bar{d}) + \bar{\varepsilon} - F(\bar{d})}$$

where

$$F(d) = (r^* - g)d - \mu - f(d)$$
$$W(d) = (1-\theta)(1+r^*)d$$

The assumption on the recovery value (see footnote 7) imposes the following restiction on the value of $W(d)$:

(A.3) $$W(d) \geq (1-\bar{\theta})(1+r^*)d = (d / d^m)\bar{\varepsilon}.$$

In addition, (A.1) implies that $\bar{p}^*$ exists only if $p_U > p_L > 0$, which in turn requires the following inequality conditions:

(A.4) $$-\bar{\varepsilon} < F(\bar{d}) < \bar{\varepsilon} \quad \text{and} \quad (\bar{d} / d^m)\bar{\varepsilon} < W(\bar{d}) < \left(\frac{1 - G(F(\bar{d}))}{G(F(\bar{d}))}\right) \cdot [\bar{\varepsilon} - F(\bar{d})].$$

The first condition in (A.4) can be further reduced to yield,

(A.5) $$\bar{d}_1 \leq \bar{d} < \bar{d}_2$$

where $\bar{d}_1$ and $\bar{d}_2$ are as defined in Section 2.2, and $d^m \leq \bar{d}_1 < \bar{d}_2$ (see Proof 1).

Determination of the debt limit

Stochastic case

According to the equilibrium conditions for the debt limit shown in (12), $\bar{d}$ and $\bar{p}^*$ are determined by solving the following two equations simultaneously:

$$\text{(A.6)} \qquad \begin{array}{ll} \text{(i)} & p = \bar{z}(p) \\ \text{(ii)} & \partial \bar{z}(p) / \partial p = 1 \end{array}$$

The first equation is for the existence of an interior solution at $d = \bar{d}$ while the second equation ensures that the interior solution is the maximum interior solution obtained at the tangency point between $\bar{z}(p)$ and $p = p$ (see Proof 4).

Deterministic case

Substituting from (8), the equilibrium conditions for the debt limit discussed in Section 2.3 can be written as follows:

$$\text{(A.7)} \qquad \text{(i)}\ H(0; d, \tilde{d}) \leq 0 \ \text{ for } d \leq \tilde{d} \quad \text{and} \quad \text{(ii)}\ H(1; \tilde{d}, \tilde{d}) > 0 \ \text{ for } d > \tilde{d}.$$

Since (ii) holds trivially, $\tilde{d}$ is essentially determined by (i). It can be easily shown that $H(0; \tilde{d}, \tilde{d}) = F(\tilde{d})$ where $F(d)$ is as defined in (A.2). Given this result, $\tilde{d}$ is obtained as the largest root of $F(d)$ as shown in (13). It is possible that $H(0; d, \tilde{d}) > 0$ for some $d < \tilde{d}$ due to the nonlinearity of $f(d)$. But the market will charge the risk-free interest rate because it is assured that debt is sustainable up to and including $\tilde{d}$.

Key characteristics of the debt limit

Proof 1: $d^m \leq \overline{d}_1 < \tilde{d} < \overline{d}_2$.

By property (3), $F(d^m) \leq -\overline{\varepsilon}$ and $F'(d) > 0,\ \forall\, d > d^m$. We show for the proof that $F(d^m) \leq F(\overline{d}_1) < F(\tilde{d}) < F(\overline{d}_2)$. By definition, $F(\overline{d}_1) = -\overline{\varepsilon}$, $F(\tilde{d}) = 0$, and $F(\overline{d}_2) = \overline{\varepsilon}$. This completes the proof.

Proof 2: $\hat{d} \geq \overline{d}_1$.

Substituting $p = 0$ into (8) yields the following inequality:

$$B(d) = F(d) + \overline{\varepsilon} - (\overline{d} - d) \leq 0$$

where $d \leq \overline{d}$. Note that $B(\hat{d}) = 0$ by definition, and that by property (3), $B'(d) > 0$ for all $d > d^m$. Since $\overline{d} \geq \overline{d}_1$ and $F(\overline{d}_1) + \overline{\varepsilon} = 0$, it follows that $B(\overline{d}_1) \leq 0$. This completes the proof.

Proof 3: $\partial p^* / \partial d > 0$ and $\partial p^* / \partial \overline{d} < 0$ if *p** exists.

Assuming that *p** exists, we first show that

$$\begin{aligned}
&\partial z / \partial d = G'(H) \cdot [1 + (1-\theta)(1+r^*)p/(1-p) + F'(d)] > 0, \\
&\partial z / \partial \overline{d} = -G'(H) < 0, \\
&\partial z / \partial p = G'(H) \cdot W(d)/(1-p)^2 > 0
\end{aligned}$$

where the first inequality follows from the boundary condition (A.5) and Proof 1. Total differentiating (8) and rearranging terms yields,

$$\partial p^* / \partial d = (\partial z / \partial d) / [1 - (\partial z / \partial p)] \quad \text{and} \quad \partial p^* / \partial \overline{d} = (\partial z / \partial \overline{d}) / [1 - (\partial z / \partial p)].$$

For the proof, we show that $\partial z / \partial p < 1$ at $p = p^*$. Since $\partial z / \partial d > 0$ and $z(0; \hat{d}, \overline{d}) = 0$ by definition, it readily follows that $z(0; d, \overline{d}) > 0$ for $d \in (\hat{d},\ \overline{d}]$. This result, together with

$\partial z / \partial p > 0$, ensures that $\partial z / \partial p < 1$ at $p = p^*$ (the lowest interior solution), and $\partial z / \partial p > 1$ at the upper interior solution in case of two interior solutions. This completes the proof.

Proof 4: Tangency condition for the debt limit ($\partial \overline{z}(p) / \partial p = 1$).

The equilibrium condition states that p^* must exist for $d \leq \overline{d}$ and must not exist for $d > \overline{d}$. This condition can be satisfied only if the fixed point solution jumps from p^* to $p = 1$ at the very moment when debt exceeds the debt limit or, equivalently, only if $\partial p^* / \partial d = \infty$ at $d = \overline{d}$. It follows from Proof 3 that $\partial \overline{p}^* / \partial d = \infty$ iff $\partial \overline{z} / \partial p = 1$. This completes the proof.

Proof 5: $\overline{d} < \tilde{d}$.

For the proof, we show that the boundary condition for $W(d)$ in (A.4) is violated if $\overline{d} \geq \tilde{d}$. Suppose that $\overline{d} \geq \tilde{d}$. It follows from Proof 1 that $F(\overline{d}) \geq 0$, which implies that $G(F(\overline{d})) \geq 1/2$. Given these results, it is straightforward to show

$$\left(\frac{1 - G(F(\overline{d}))}{G(F(\overline{d}))} \right) \cdot [\overline{\varepsilon} - F(\overline{d})] < \overline{\varepsilon} < (\overline{d} / d^m) \cdot \overline{\varepsilon} .$$

This completes the proof.

Proof 6: $0 \leq (\overline{d} - \hat{d}) < \overline{\varepsilon}$.

The expression $B(\hat{d}) = 0$ shown in Proof 2 can be rewritten as

$$(\overline{d} - \hat{d}) = F(\hat{d}) + \overline{\varepsilon} .$$

It suffices for the proof to show that $-\overline{\varepsilon} \leq F(\hat{d}) \leq 0$. Proof 1 and 5 immediately imply that $-\overline{\varepsilon} = F(\overline{d}_1) \leq F(\hat{d}) < F(\tilde{d}) = 0$. This completes the proof.

Proof 7: Equilibrium properties of the debt limit.

For the proof, we show that

$$\partial\overline{d} / \partial r^* < 0, \quad \partial\overline{d} / \partial g > 0, \quad \partial\overline{d} / \partial\mu > 0, \quad \text{and} \quad \partial\overline{d} / \partial\overline{\varepsilon} = \begin{cases} \leq 0 & \text{if } \overline{p}^* \leq 0.5 \\ > 0 & \text{otherwise} \end{cases}$$

where in the last inequalities, $\partial\overline{\varepsilon} > 0$ refers to a mean-preserving spread (i.e., a symmetric widening of the support). Total differentiating $p = \overline{z}(p)$ while exploiting the tangency condition for the debt limit yields,

$$\partial\overline{d} / \partial x = -(\partial\overline{z} / \partial x) / (\partial\overline{z} / \partial\overline{d})$$

where $x = \{r^*, \ g, \ \mu, \ \overline{\varepsilon}\}$. It is straightforward to show

$$\partial\overline{z} / \partial r^* > 0, \quad \partial\overline{z} / \partial g < 0, \quad \partial\overline{z} / \partial\mu < 0, \quad \text{and} \quad \partial\overline{z} / \partial\overline{d} > 0.$$

To complete the proof, we show that

$$\partial\overline{z} / \partial\overline{\varepsilon} = \begin{cases} \geq 0 & \text{if } \overline{p}^* \leq 0.5 \\ < 0 & \text{otherwise} \end{cases}.$$

Note that (A.1)-(A.2) imply that $\overline{p}^*$ can exceed 0.5 only if $p_U > 0.5$ or, equivalently, if $W(\overline{d}) < \overline{\varepsilon} - F(\overline{d}) < 2\overline{\varepsilon}$. As discussed in Section 2.4 (footnote 12), this condition would only hold if the recovery value is sufficiently large. Conversely, for any given recovery value (strictly less than unity), this condition is necessarily violated if $\overline{\varepsilon}$ converges to zero (i.e., $\lim_{\overline{\varepsilon} \to 0} p_U = 0$).

Since $\overline{z}(\overline{p}^*) \equiv G(H(\overline{p}^*; \overline{d}, \overline{d})) = \overline{p}^*$ in equilibrium, it is readily verified that $H(\overline{p}^*; \overline{d}, \overline{d}) \leq 0$ if $\overline{p}^* \leq 1/2$ and $H(\overline{p}^*; \overline{d}, \overline{d}) > 0$ otherwise. Then, the assumed properties of $G(\varepsilon)$ ensure that $\tilde{G}(H) \geq G(H)$ if $H \leq 0$, and $\tilde{G}(H) < G(H)$ otherwise, where $\tilde{G}$ is a mean-preserving spread of G. This completes the proof.

APPENDIX II. SENSITIVITY ANALYSIS OF THE ESTIMATED FISCAL REACTION FUNCTION

This appendix examines the robustness of the estimated nonlinear fiscal reaction function.[26]

Estimation methods

The nonlinearity of the fiscal reaction function and the response of the primary balance to other structural and economic determinants are generally robust to different estimation methods. Table A2 present the results of the cubic model obtained through pooled ordinary least squares method (col.1); country fixed effects estimation without assuming an AR(1) error structure (col. 2); country fixed effects estimation with AR(1) disturbances and controlling for time effects (col. 3); and correcting for both serial correlation and the possibility of cross sectional dependence in the error term (col. 4). In each case, the negative cubic term is statistically significant at the 1 percent level; fiscal behavior is estimated to be countercyclical; and the primary balance is estimated to react negatively to temporary increases in government outlays.[27]

Slope Homogeneity

The estimation results for the fiscal reaction function supporting the fiscal fatigue hypothesis are based on a panel dataset, which while allowing for differences in the average primary balance across countries, assumes homogeneity of fiscal response to lagged debt for all countries. A panel data approach is especially useful here because, for most countries in the sample, we do not observe the full range of debt ratios over which the varying primary balance response is

[26] The results presented here pertain to the 1970–2007 sample. Results obtained from the 1985–2007 sample are broadly similar and are available upon request.

[27] In addition, we estimate an alternate dynamic specification of the fiscal reaction function—which includes lagged primary balance as a regressor—to take account of any persistence in the primary balance behavior due to sluggish tax and expenditure adjustment. The obtained results, using Blundell and Bond’s (1998) system Generalized Method of Moments (GMM) estimator, also support the existence of a nonlinear fiscal response to lagged debt.

postulated to occur. This also means, however, that conventional tests for slope homogeneity cannot be applied.[28] Instead, we proceed by testing whether countries, when observed over a common debt range (low-to moderate; moderate-to-high), behave similarly in terms of their reaction to rising debt. To the extent that they do, this gives some confidence that they will behave similarly at debt ratios at which some of them are not observed in the sample.

Specifically, we first restrict the sample to low-to-moderate debt observations (30 to 100 percent of GDP) and test whether the response of primary balance to lagged debt is similar across countries over this range.[29] The results presented in Table A3 indicate that the linear model gives the best fit, and that the primary balance exhibits an increasing response to lagged debt over the considered debt range (column 1). Estimating country-specific slope parameters for the linear debt term, we find that for all countries, the hypothesis of a positive linear slope cannot be rejected (columns 2-24). Since countries in our sample (including those that eventually have high debt) behave in a similar way at lower-to-moderate debt ratios, one could reasonably argue that they may exhibit a similar response at higher debt levels.

Next, we also test if the high debt countries—defined as those for which the maximum observed (lagged) debt to GDP ratio is ever greater than 100 percent of GDP—behave similarly, and exhibit fiscal fatigue. If the hypothesis of a nonlinear response of primary balance to lagged debt cannot be rejected for these countries, one could argue that a similar pattern may also apply to

[28]Testing explicitly for slope homogeneity in our full panel dataset is difficult precisely because the overlap of debt ratios observed across countries over different debt ranges is limited. For example, for Australia and Korea the observed debt range is about 10 to 30 percent of GDP—well below the debt ratio at which the panel estimates suggest that fiscal fatigue is likely to set in.

[29] Specifically, we estimate an equation of the form: $pb_{it} = adebt_{it} + a_i debt_{it} * dummy_i + \mu_i + controlvar_{it} + \epsilon_{it}$, where i=1,2,…,23, $dummy_i$ is a dummy variable for country i, μ_i are country fixed effects, a and a_i are the (average) panel and country specific slope coefficients to be estimated, respectively, and ϵ_{it} is an AR(1) error term. We estimate the equation one by one for each country, and test the null hypothesis of similarity between the country and panel parameter estimates (i.e., $\hat{a}_i$=0).

other advanced economies. In our sample, moderate-to-high debt ratios are observed for seven countries (Belgium, Canada, Greece, Ireland, Israel, Italy, and Japan). Across these countries, there exists some overlap of observations over different debt ranges making it possible to test the applicability of the fiscal fatigue hypothesis to these countries individually.[30]

The results presented in Table A4 (column 1) support the existence of a cubic model for high debt countries. When country-specific debt terms are included in the model (columns 2-8), the null hypothesis of slope homogeneity cannot be rejected for Belgium, Canada and Ireland. For Greece, Israel, and Italy, the joint significance test of parameter estimates indicates a statistically different fiscal response, but one that is consistent with the fiscal fatigue hypothesis. The only country in the sample for which the fiscal fatigue hypothesis does not hold is Japan, for which the response of primary balance to lagged debt is negative and deteriorating over most of the debt range but exhibits a rising trend at the top end of the range.[31]

[30] Specifically, we estimate an equation of the form: $pb_{it} = a_1 debt_{it} + a_2 debt_{it}^2 + a_3 debt_{it}^3 + a_{1i} debt_{it} * dummy_i + a_{2i} debt_{it}^2 * dummy_i + a_{3i} debt_{it}^3 * dummy_i + \mu_i + controlvar_{it} + \epsilon_{it}$, where i=1,2,…,7, $dummy_i$ is a dummy variable for country i, μ_i are country fixed effects, a_{ki} are the potentially heterogeneous slope coefficients to be estimated, and ϵ_{it} is an AR(1) error term. We estimate the equation for each country one by one, and test the joint significance of $\hat{a}_{1i}$, $\hat{a}_{2i}$, and $\hat{a}_{3i}$ in each case.

[31] Excluding Japan from this sample does not have any significant effect on the results—the cubic specification holds for the data, and we find that other high debt countries conform to the cubic behavior.

Table A1. Variable Definitions and Data Sources

Variable	Description	Source
Dependent variable		
Primary balance to GDP ratio	In percent	IMF's World Economic Outlook (WEO) and OECD databases.
Explanatory variables		
Lagged debt to GDP ratio	In percent	WEO database.
Output gap	Difference between actual and potential (calculated using the Hodrick-Prescott filter) real GDP.	Staff calculations based on WEO database.
Government expenditure gap	Difference between actual and potential (calculated using the Hodrick-Prescott filter) real government consumption spending.	Staff calculations based on WEO database.
Trade openness	Sum of exports and imports to GDP (in percent)	Staff calculations based on WEO database.
Inflation	Three year lagged moving average of CPI inflation.	Staff calculations based on WEO database.
Oil price	Log of (trend) oil price applied to oil exporters only.	Staff calculations based on WEO database.
Nonfuel commodity price	Log of (trend) nonfuel commodity price index applied to nonfuel commodity exporters only.	Staff calculations based on WEO database.
Age dependency ratio	Ratio of the dependent to working age (15-64 years) population.	UN's database. Available online at: http://data.un.org/Data.aspx?q=dependency+ratio+(per+cent)&d=PopDiv&f=variableID%3a42
Future age dependency ratio	Projected age dependency ratio 20 years ahead.	UN's database. Available online at: http://data.un.org/Data.aspx?q=dependency+ratio+(per+cent)&d=PopDiv&f=variableID%3a42
Political stability index	Smaller (larger) values indicating higher (lower) political risk.	International Country Risk Guide (ICRG) dataset.
IMF arrangement	Binary variable equal to one if a country has an IMF support program in a given year, and zero otherwise.	IMF's History of Lending Arrangements database. Available online at http://www.imf.org/external/np/fin/tad/extarr1.aspx.
Fiscal rules	Binary variable equal to one if a country has any type (expenditure, revenue, balanced budget, and debt) of fiscal rule in a given year, and zero otherwise.	IMF's Fiscal Rules database, 1985-2009.

Table A2. Alternative Estimation Methods of the Fiscal Reaction Function

Method	OLS	FE[a]	FE[b]	AR(1)[c]	PCSE[d]
Specification	(1)	(2)	(3)	(4)	(5)
Lagged debt	-0.168***	-0.202**	-0.133	-0.147***	-0.144***
	(0.031)	(0.075)	(0.091)	(0.056)	(0.041)
Lagged debt_square	0.002***	0.00316***	0.00227*	0.00228***	0.00229***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
Lagged debt_cubic	-0.00001***	-0.00001***	-0.00001***	-0.00001***	-0.00001***
	(2.0e-06)	(3.0e-06)	(4.0e-06)	(3.0e-06)	(2.0e-06)
Output gap	0.547***	0.553***	0.476***	0.427***	0.428***
	(0.052)	(0.085)	(0.088)	(0.054)	(0.044)
Govt. expenditure gap	-0.319***	-0.293***	-0.275***	-0.188***	-0.188***
	(0.064)	(0.088)	(0.079)	(0.043)	(0.034)
Trade openness	0.064***	0.125*	0.114	0.00937	0.0313
	(0.010)	(0.062)	(0.093)	(0.069)	(0.048)
Inflation	-1.376	-0.835	3.003	5.995***	5.600***
	(3.531)	(5.133)	(3.665)	(2.109)	(1.759)
Oil price	1.236***	10.560***	10.890***	9.329***	9.611***
	(0.213)	(0.242)	(0.866)	(2.906)	(3.503)
Observations	642	642	642	642	642
R-squared	0.337	0.348	0.466	0.418	0.432
Number of countries	23	23	23	23	23
Country fixed effects	No	Yes	Yes	Yes	Yes
Time effects	No	No	Yes	Yes	Yes
AR rho				0.762	

Source: Staff estimates.

[1]Dependent variable is general government primary balance to GDP (in percent). Estimation pertains to the sample 1970-2007. Robust standard errors reported in parentheses. ***, **, and * denote significance at 1, 5, and 10 percent levels, respectively.

[a] Fixed effects estimation without assuming an AR(1) error structure.

[b] Fixed effects estimation with year effects included without assuming an AR(1) error structure.

[c] Fixed effects estimation with year effects included assuming an AR(1) error structure.

[d] Panel corrected standard errors assuming an AR(1) structure.

Table A3. Estimation results for the fiscal reaction function for low debt observations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
debt	0.032**	0.033**	0.029**	0.034**	0.029*	0.031**	0.032**	0.032**	0.029**	0.029*	0.035**	0.035**	0.032**	0.033**	0.031**	0.024*	0.033**	0.031*	0.031**	0.032**	0.032**	0.035**	0.032**	0.032**
	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.014)	(0.015)	(0.016)	(0.015)	(0.015)	(0.015)	(0.014)	(0.015)	(0.015)
debt × d_{USA}		-0.042																						
		(0.046)																						
debt × d_{UK}			0.152*																					
			(0.079)																					
debt × $d_{Austria}$				-0.076																				
				(0.062)																				
debt × $d_{Belgium}$					0.180																			
					(0.145)																			
debt × $d_{Denmark}$						0.002																		
						(0.047)																		
debt × d_{France}							-0.024																	
							(0.038)																	
debt × $d_{Germany}$								-0.014																
								(0.048)																
debt × d_{Italy}									0.057															
									(0.063)															
debt × $d_{Netherlands}$										0.061														
										(0.051)														
debt × d_{Norway}											-0.083													
											(0.073)													
debt × d_{Sweden}												-0.038												
												(0.047)												
debt × d_{Canada}													0.001											
													(0.044)											
debt × d_{Japan}														-0.021										
														(0.051)										
debt × $d_{Finland}$															0.009									
															(0.073)									
debt × d_{Greece}																0.122**								
																(0.058)								
debt × $d_{Iceland}$																	-0.102							
																	(0.078)							
debt × $d_{Ireland}$																		0.006						
																		(0.034)						
debt × $d_{Portugal}$																			0.052					
																			(0.138)					
debt × d_{Spain}																				0.000				
																				(0.037)				
debt × $d_{Australia}$[1]																					0.756**			
																					(0.369)			
debt × $d_{New Zealand}$																						-0.061		
																						(0.111)		
debt × d_{Israel}																							-0.048	
																							(0.047)	
debt × d_{Korea}[1]																								-0.029
																								(0.100)
Observations	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473	473
R-squared	0.393	0.393	0.397	0.397	0.394	0.393	0.393	0.393	0.395	0.394	0.394	0.393	0.393	0.393	0.393	0.405	0.396	0.393	0.393	0.393	0.393	0.392	0.393	0.393
AR rho	0.696	0.696	0.692	0.692	0.705	0.696	0.696	0.695	0.694	0.697	0.698	0.699	0.696	0.696	0.696	0.688	0.695	0.696	0.696	0.696	0.696	0.700	0.697	0.696

Notes: Dependent variable is primary balance to GDP (in percent). Sample restricted to (lagged) debt observations between 30 and 100 (inclusive) percent of GDP. The estimated equation is: $pb_{it}=a*debt_{it}+a_i*debt_{it}*d_i+\mu_i+controlvar_{it}+\epsilon_{it}$, where i=1,2,…,23, debt is lagged debt ratio, μ_i are country fixed effects, d_i are country dummies, a is the average parameter estimate for the panel, a_i is the slope coefficients for individual countries, and ϵ is an AR(1) error term. Control variables include country-specific fixed effects, output gap, government expenditure gap, tradeopenness, (log of) inflation, and oil prices. Robust standard errors in parentheses. ***,**,* indicate significance at 1, 5, and 10 percent levels, respectively.

1/ The number of observations for Australia and Korea in the restricted sample is only 3 and 1, respectively, as most of the lagged debt observations for these countries are below 30 percent of GDP.

Table A4. Estimation results for the fiscal reaction function for high debt countries

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
debt	-0.152	-0.102	-0.133	-0.144	-0.272	-0.120	-0.207*	-0.176
	(0.117)	(0.120)	(0.119)	(0.123)	(0.264)	(0.117)	(0.106)	(0.122)
debt_square	0.003**	0.002	0.003*	0.003**	0.005	0.002*	0.003**	0.003**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)
debt_cubic	-0.00001**	-0.00001*	-0.00001**	-0.00001**	-0.00001*	-0.00001**	-0.00001***	-0.00001**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
debt × $d_{Belgium}$		0.599						
		(1.699)						
debt_square × $d_{Belgium}$		-0.002						
		(0.017)						
debt_cubic × $d_{Belgium}$		0.000						
		(0.000)						
debt × d_{Italy}			-2.674**					
			(1.037)					
debt_square × d_{Italy}			0.031**					
			(0.012)					
debt_cubic × d_{Italy}			-0.0001**					
			(0.000)					
debt × d_{Canada}				1.138				
				(1.415)				
debt_square × d_{Canada}				-0.018				
				(0.019)				
debt_cubic × d_{Canada}				0.0001				
				(0.000)				
debt × d_{Japan}					0.338			
					(0.276)			
debt_square × d_{Japan}					-0.006*			
					(0.003)			
debt_cubic × d_{Japan}					0.00002*			
					(0.000)			
debt × d_{Greece}						-4.266		
						(3.163)		
debt_square × d_{Greece}						0.055		
						(0.041)		
debt_cubic × d_{Greece}						-0.0002		
						(0.000)		
debt × $d_{Ireland}$							0.923*	
							(0.558)	
debt_square × $d_{Ireland}$							-0.013*	
							(0.008)	
debt_cubic × $d_{Ireland}$							0.0001*	
							(0.000)	
debt × d_{Israel}								-2.606
								(2.040)
debt_square × d_{Israel}								0.021
								(0.018)
debt_cubic × d_{Israel}								-0.0001
								(0.000)
Observations	199	199	199	199	199	199	199	199
R-squared	0.300	0.325	0.315	0.305	0.345	0.324	0.315	0.316
AR rho	0.892	0.895	0.873	0.894	0.863	0.887	0.896	0.896
Joint significance test (p-value)[1]		0.180	0.051	0.562	0.000	0.000	0.353	0.072

Notes: Dependent variable is primary balance to GDP (in percent). Sample restricted to countries where maximum observed (lagged) debt is over 100 percent of GDP. The estimated equation is: $pb_{it}=a1*debt_{it}+a2*debt_{it}^2+a3*debt_{it}^3+a_{1i}*debt_{it}*d_i+a_{2i}*debt_{it}^2*d_i+a_{3i}*debt_{it}^3*d_i+\mu_i+controlvar_{it}+\epsilon_{it}$, where i=1,2,…,23, debt is lagged debt ratio, μi are country fixed effects, di are country dummies, ak (k=1, 2, 3) are the average parameters for the linear, quadratic and cubic terms for the panel, aki are the country-specific parameters for the linear, quadratic and cubic terms, and ϵ is an AR(1) error term. Control variables include country-specific fixed effects, output gap, government expenditure gap, tradeopenness, (log of) inflation, and oil prices. Robust standard errors in parentheses. ***,**,* indicate significance at 1, 5, and 10 percent levels, respectively.

1/Joint significance test of country-specific parameter estimates (a_{ki}).