Real-Time Forecast Evaluation of DSGE Models with Stochastic Volatility

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Motivation

The emerging popularity of DSGE forecasting calls for performance evaluation.

The small existing DSGE forecast evaluation literature focuses mostly on point forecasts and suggests that:

- DSGE point forecasts are as good as VAR's.

Well, OK, but...

Typically, models in the DSGE forecast evaluation literature are analyzed with:

- Linearized solutions
- Gaussian shocks
- Constant volatility

Road Map

Small-scale DSGE model for GDP growth, inflation, and the policy rate

- Linearized state transition equation
  (Constant vol, stochastic vol, deterministic vol)

- Bayesian estimation and forecasting
  (Totally standard)

- Measurement equation and U.S. data, 1964-2011

- Evaluation of point, interval, and density forecasts, 1992-2011
  (Real-time, expanding-sample, vintage data)

DSGE Model and Implied Transition

Builds on Del Negro and Schorfheide (2013)

- Euler equation, new-Keynesian Phillips curve, monetary policy rule, time-varying target inflation rate
- 4 exogenous shocks: technology, government spending, monetary policy, target inflation rate (z_t, g_t, m_t, π_t^*)

State transition equation:

$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta)$$

where

$$s_t = [y_t, y_{t-1}, c_t, \pi_t, R_t, z_t, g_t, m_t, \pi_t^*]'$$

ϵ_t are innovations

θ are parameters

Constant Volatility
Linearized Transition

Linearization-based solution methods produce linear/Gaussian state space representations with transition equation

$$s_t = H(\theta)s_{t-1} + R(\theta)\epsilon_t$$

$$\epsilon_t \sim iid\mathcal{N}(0, Q(\theta))$$

$$Q(\theta) = diag[\sigma_z^2,\ \sigma_g^2,\ \sigma_m^2,\ \sigma_{\pi^*}^2]$$

regardless of whether the original model shocks have stochastic volatility

- Could adopt higher-order solution methods
- Could simply add stochastic volatility to the linearized transition, as in Justiniano and Primiceri (2008)

Stochastic Volatility
Linearized Transition

$$s_t = H(\theta)s_{t-1} + R(\theta)\epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0, Q_t(\theta))$$

where

$$Q_t(\theta) = diag[e^{2h_{z,t}},\ e^{2h_{g,t}},\ e^{2h_{m,t}},\ \sigma^2_{\pi^*}]$$

$$h_{i,t} = \rho_{\sigma_i} h_{i,t-1} + \nu_{i,t}$$

$$\nu_{i,t} \sim iid\mathcal{N}(0,\ s_i^2),$$

for $i = z, g, m$

- Conditionally linear / Gaussian system
- We consider two cases:
  - "SV-AR": $\rho_{\sigma_i} \in (-1, 1)$ for $i = z, g, m$
  - "SV-RW": $\rho_{\sigma_i} = 1$ for $i = z, g, m$

Deterministic Volatility / Structural Break Linearized Transition

$$s_t = H(\theta)s_{t-1} + R(\theta)\epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0, Q_t(\theta))$$

where

$$Q_t(\theta) = diag[\sigma^2_{z,t},\ \sigma^2_{g,t},\ \sigma^2_{m,t},\ \sigma^2_{\pi^*}]$$
$$\sigma_{i,t} = \begin{cases} \sigma_{i,0} & \text{if } t \leq 1984Q4 \\ \sigma_{i,1} & \text{if } t > 1984Q4 \end{cases}$$

for $i = z, g, m$

"DV-SB"

Estimation and Forecasting

Estimation: MCMC posterior simulator

Forecasting: Draw repeatedly from the posterior predictive pdf:

$$p(Y_{T+1:T+h}|Y_{1:T})$$

- Point forecasts – posterior mean
- Interval forecasts – shortest length connected posterior interval
- Density forecasts – full posterior

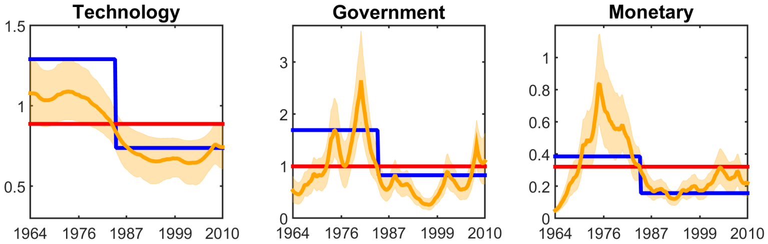
Measurement and Data

Measurement equation:

$$\begin{pmatrix} \Delta GDP_t \\ INF_t \\ FFR_t \\ INF_t^e \end{pmatrix} = D(\theta) + Z(\theta)\, s_t$$

- GDP growth, inflation, federal funds rate, 10-year survey inflation expectations
- Vintage data set constructed by Del Negro and Schorfheide (2013) and Edge and Gürkaynak (2010)
- Expanding-sample estimation; each vintage starts 1964Q2, final vintage ends in 2011.Q2 ("actuals")
- Forecasts generated for January, April, July, and October, starting for 1991Q4

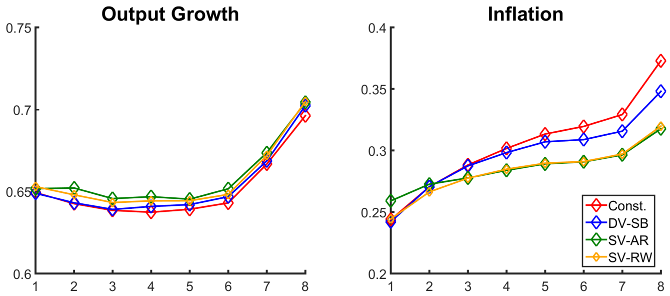
Posterior Mean Structural Shock Volatilities, Based on Final Data Vintage, Constant vs. SV-RW vs. DV-SB



- Constant volatility
- Stochastic volatility with 80 percent credible band (SV-RW)
- Deterministic volatility (structural break) (DV-SB)

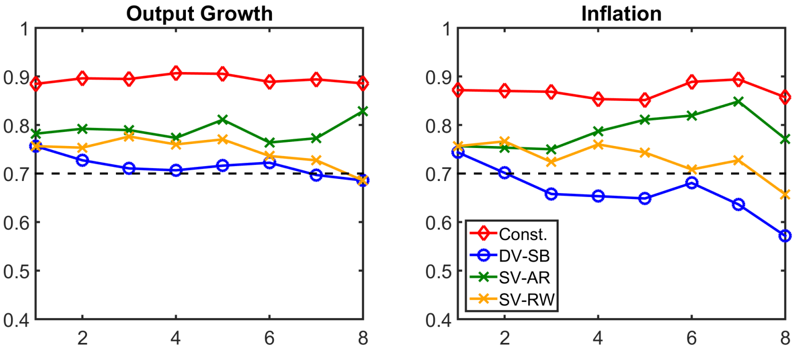
Results: Point Prediction

Relative Point Forecast Evaluation: RMSEs

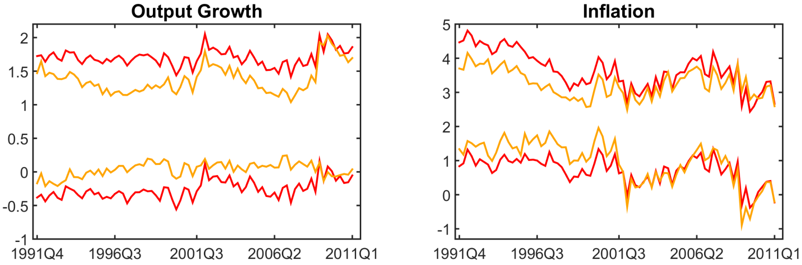


Results: Interval Prediction

Relative Interval Forecast Evaluation: Coverage Rates of 70% Interval Forecasts, $h = 1, \ldots, 8$



Relative Interval Forecast Evaluation: Lengths of 70% Interval Forecasts, $h = 1$



- Red: Constant
- Yellow: SV-RW

Absolute Interval Forecast Evaluation: The Hit Sequence (Christoffersen)

$$H_{i,t+h,t}^{(1-\alpha)} = \begin{cases} 1 \text{ if } y_{i,t+h} \in I_{t+h,t}^{1-\alpha}(y_i) \\ \\ 0 \text{ otherwise} \end{cases}$$

Under correct conditional calibration of the interval forecast,

$$H_{i,t+1,t}^{(1-\alpha)} \sim iid\ Bernoulli(1-\alpha)$$

Absolute Interval Forecast Evaluation: H Tests, Nominal 70% Intervals, $h = 1$

	Coverage	Independence	Joint
		Output Growth	
Const.	15.1 (0.00)	3.50 (0.06)	18.9 (0.00)
DV-SB	1.23 (0.27)	0.62 (0.43)	2.42 (0.30)
SV-AR	2.66 (0.10)	0.26 (0.61)	3.41 (0.18)
SV-RW	1.23 (0.27)	0.04 (0.85)	1.83 (0.40)
		Inflation	
Const.	12.9 (0.00)	0.10 (0.76)	13.2 (0.00)
DV-SB	0.73 (0.40)	1.10 (0.29)	2.42 (0.30)
SV-AR	1.23 (0.27)	6.43 (0.01)	8.23 (0.02)
SV-RW	1.23 (0.27)	1.90 (0.17)	3.69 (0.16)

Results: Density Prediction

Relative Density Forecast Evaluation: Log Predictive Scores, $h = 1$ Joint Across All Variables

$$\left(\text{Recall } LPS_{t+h,t} = \sum \log p_{t+h,t}(y_{t+h})\right)$$

Const. -6.41

DV-SB -7.22

SV-AR -6.36

SV-RW -6.22

Relative Density Forecast Evaluation: Log Predictive Scores, $h = 1$, Variable-by-Variable

	Output Growth	Inflation
Const.	-1.11	-1.88
DV-SB	**-0.99**	-1.71
SV-AR	-1.04	-1.63
SV-RW	-1.02	**-1.62**

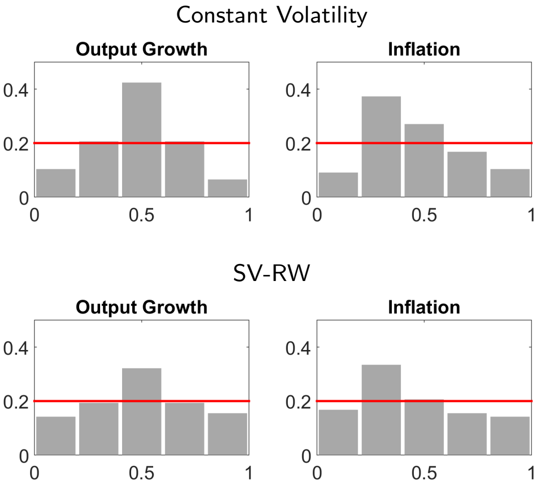
Absolute Density Forecast Evaluation: The Probability Integral Transform (Diebold-Gunther-Tay)

$$PIT_{i,t+h,t} = \int_{-\infty}^{y_{i,t+h}} p_{i,t+h,t}(y)\, dy$$

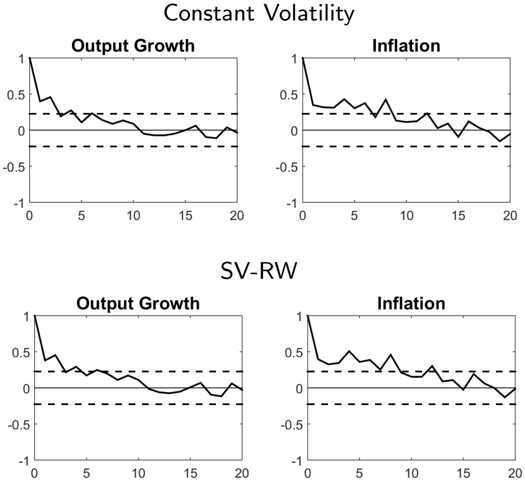
Under correct conditional calibration of the density forecast,

$$PIT_{i,t+1,t} \sim iid\ U(0,1)$$

Absolute Density Forecast Evaluation: *PIT* Histograms, $h = 1$



Absolute Density Forecast Evaluation: *PIT* Correlograms, $h = 1$



Conclusion

- SV actually helps a bit for point forecasting...
- SV looks good for interval forecasts in both relative and absolute terms
- SV looks good for density forecasts in relative terms, but it's still below the bar in absolute terms