Estimating and Understanding High-Dimensional Dynamic Stochastic Econometric Models

(For Volatility, Derivatives, and More...)

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May 3, 2017

High Dimensionality

- Macro
- Finance
- Everywhere...

Some Recent Work

Macro:
Diebold, F.X. and Yilmaz, K. (2015), "Measuring the Dynamics of Global Business Cycle Connectedness," in S.J. Koopman and N. Shephard (eds.), *Unobserved Components and Time Series Econometrics: Essays in Honor of Andrew C. Harvey*, Oxford University Press, 45-89.

Financial:
Demirer, M., Diebold, F.X., Liu, L. and Yilmaz, K. (2017), "Estimating Global Bank Network Connectedness," *Journal of Applied Econometrics*, in press.

Blend:
Diebold, F.X., Liu, L. and Yilmaz, K. (2017), "Commodity Connectedness," Manuscript.

A Very General Environment

$$x_t = B(L)\,\varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

Perhaps $dim(x) = 5$, or 50, or 50000, or 5000000, or ...

Many Interesting Issues / Choices

- x objects: Returns? **Return volatilities**? Return correlations?
- x universe: **How many and which ones**?
- x frequency: **Daily**? Monthly? Quarterly?

- Approximating model: **VAR**? Structural? DFM?
- Estimation: Classical? Bayesian? **Hybrid**?
  - Selection: Information criteria? Stepwise? **LASSO**?
  - Shrinkage: BVAR? Ridge? **LASSO**?
  - **Static** vs. **dynamic** (rolling, expanding, TVP modeling)?
- Identification: **Mechanical** (e.g. Cholesky)? SVAR? DSGE?
- Understanding I: Visualization via **network graphs**
- Understanding II: Summarization via **network degree distributions**

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Estimating the VAR: Regularization

Constrained estimation:

$$\hat{\beta} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|^q \right)$$

Concave penalty functions non-differentiable at the origin produce selection. Convex penalties produce shrinkage (e.g., $q = 2$ is ridge)

$q = 1$ is LASSO (concave and convex, selects and shrinks):

$$\hat{\beta}_{LASSO} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i| \right)$$

– Immediately useful for forecasting

Understanding the VAR: Variance Decomposition (Low Dimensional, Old Days, Circa 1980-2010)

– Parameters are not directly revealing

– So examine variance decomposition matrix $D = [d_{ij}]$

– d_{ij} answers a key question: *What fraction of the future uncertainty faced by variable i is due to shocks from variable j?*

– Consider $dim(x) = 5$:

D

	x_1	x_2	...	x_5
x_1	$d_{1,1}$	$d_{1,2}$	$\cdots$	$d_{1,5}$
x_2	$d_{2,1}$	$d_{2,2}$	$\cdots$	$d_{2,5}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_5	$d_{5,1}$	$d_{5,2}$	$\cdots$	$d_{5,5}$

But what if $dim(x) = 5000$?

D

	x_1	x_2	...	x_{5000}
x_1	$d_{1,1}$	$d_{1,2}$	$\cdots$	$d_{1,5000}$
x_2	$d_{2,1}$	$d_{2,2}$	$\cdots$	$d_{2,5000}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_{5000}	$d_{5,1}$	$d_{5,2}$	$\cdots$	$d_{5,5000}$

– Classical interpretive tools are themselves now totally unworkable

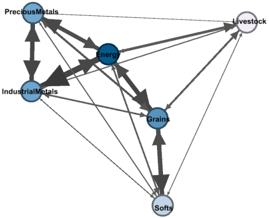
D above has 250,000,000 entries!

A New Approach to Understanding the VAR, I: Variance Decomposition Summarization Via the Network Degree Distribution (Connectedness Perspective)

			D		
	x_1	x_2	...	x_N	From Others
x_1	d_{11}	d_{12}	$\cdots$	d_{1N}	$\sum_{j\neq 1} d_{1j}$
x_2	d_{21}	d_{22}	$\cdots$	d_{2N}	$\sum_{j\neq 2} d_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}	d_{N2}	$\cdots$	d_{NN}	$\sum_{j\neq N} d_{Nj}$
To Others	$\sum_{i\neq 1} d_{i1}$	$\sum_{i\neq 2} d_{i2}$	$\cdots$	$\sum_{i\neq N} d_{iN}$	$\sum_{i\neq j} d_{ij}$

Connectedness: Pairwise, total "from," total "to," system-wide

A New Approach to Understanding the VAR, II: Variance Decomposition Visualization Via the Network Graph



- Node shading/thickness: Total directional connectedness “to others”
- Node location: Average pairwise directional connectedness
- Link thickness: Average pairwise directional connectedness
- Link arrow sizes: Pairwise directional “to” and “from”

Example: Commodity Return Volatilities

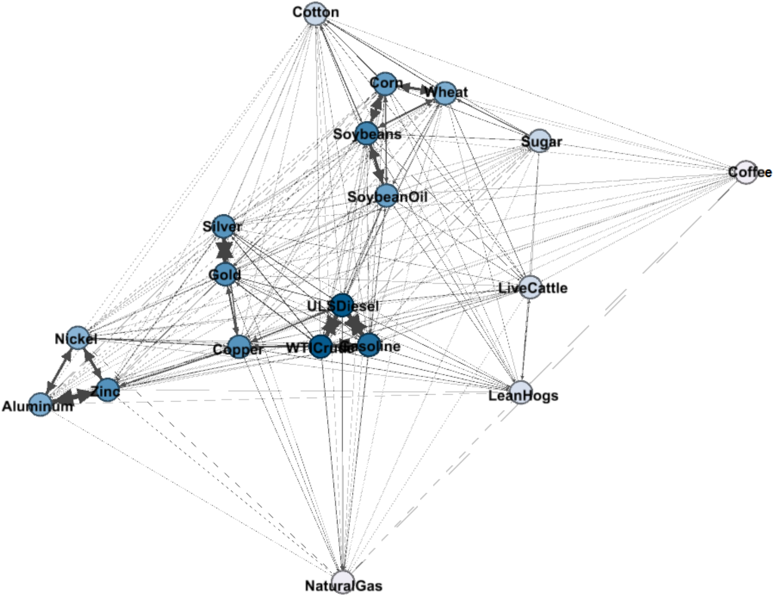
19 sub-indices (based on futures contracts)
underlying the Bloomberg Commodity Price Index:

– 4 energies (crude oil, heating oil, natural gas, unleaded gasoline)
– 2 precious metals (gold, silver)
– 4 industrial metals (aluminum, copper, nickel, zinc)
– 2 livestocks (live cattle, lean hogs)
– 4 grains (corn, soybeans, soybean oil, wheat)
– 3 "softs" (coffee, cotton, sugar)

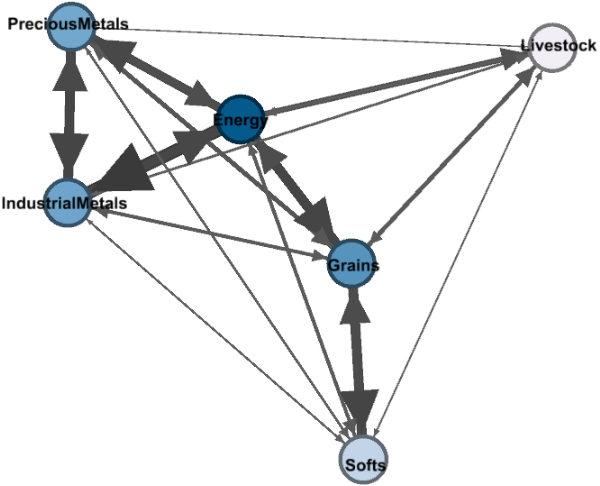
Garman-Parkinson-Klass range-based daily realized volatility

May 2006 - January 2016

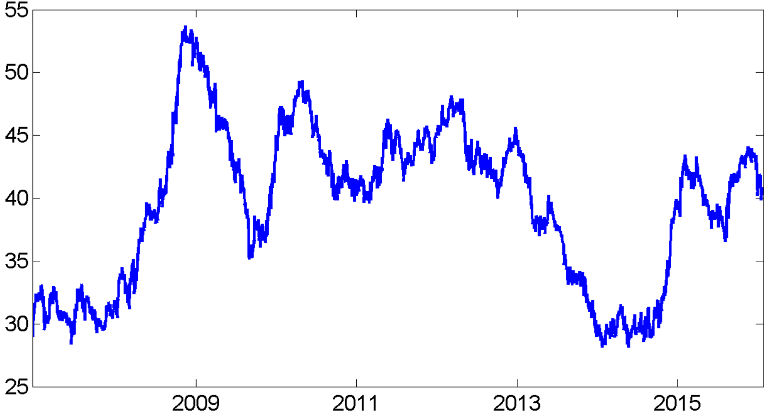
Full-Sample Network Graph



Full-Sample Network Graph, Six-Group Aggregation



Rolling-Sample System-Wide Connectedness



Conclusion

THERE'S NOTHING NEW UNDER THE SUN...

– Standard time-series dynamic econometric modeling

– VAR estimation, forecasting, understanding, ...

...BUT NEW TOOLS ARE REQUIRED FOR BIG-DATA ENVIRONMENTS:

– Regularization methods for estimation

– Network methods for understanding

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