VAR Network Methods for Summarizing and Visualizing High-Dimensional Connectedness

Discussion of Basu, Das, Michailidis, and Purnanandam:

"A System-Wide Approach to Measure Connectivity in the Financial Sector"

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Vector Autoregressions (VAR's)

N-dimensional $VAR(p)$ environment:

$$\Phi(L)x_t = \varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

e.g., 2-dimensional $VAR(1)$:

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right)$$

Understanding Connectedness: Variance Decompositions (Diebold-Yilmaz Tradition)

[Diebold, F.X. and K. Yilmaz (2014), "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *J. Econometrics*, 182, 119-134]

v_{ij} answers a key question:

What fraction of the future uncertainty faced by variable i is due to shocks from variable j?

V

	x_1	x_2	...	x_5
x_1	$v_{1,1}$	$v_{1,2}$	$\cdots$	$v_{1,5}$
x_2	$v_{2,1}$	$v_{2,2}$	$\cdots$	$v_{2,5}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_5	$v_{5,1}$	$v_{5,2}$	$\cdots$	$v_{5,5}$

Financial Connectedness

– Old days: $dim(x) = 5$

– Now: $dim(x) = 50$, or 500, or 5,000, or ...

– Standard estimation methods are now totally unworkable
(Must regularize with shrinkage, selection, hybrid, ...)

– Standard interpretive tools are now totally unworkable
(Must summarize and visualize.)

V

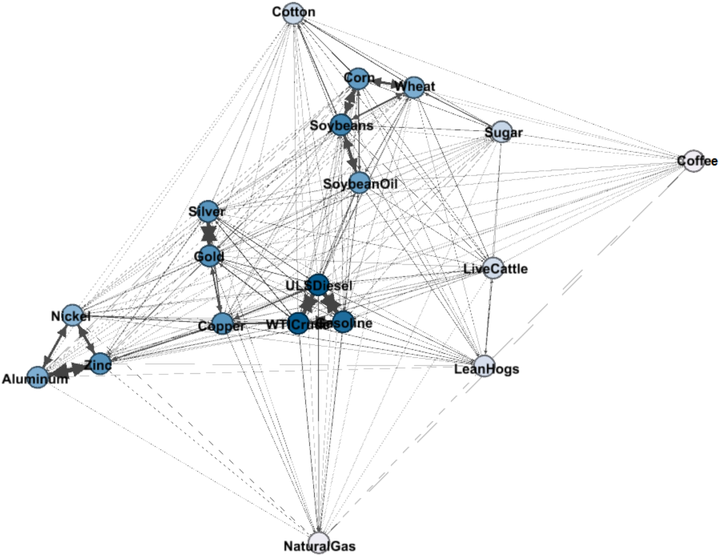
	x_1	x_2	...	x_{5000}
x_1	$v_{1,1}$	$v_{1,2}$	$\cdots$	$v_{1,5000}$
x_2	$v_{2,1}$	$v_{2,2}$	$\cdots$	$v_{2,5000}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_{5000}	$v_{5,1}$	$v_{5,2}$	$\cdots$	$v_{5,5000}$

Variance Decomposition Summarization Via the Network Degree Distribution

	V				
	x_1	x_2	...	x_N	From Others
x_1	v_{11}	v_{12}	$\cdots$	v_{1N}	$\sum_{j \neq 1} v_{1j}$
x_2	v_{21}	v_{22}	$\cdots$	v_{2N}	$\sum_{j \neq 2} v_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	v_{N1}	v_{N2}	$\cdots$	v_{NN}	$\sum_{j \neq N} v_{Nj}$
To Others	$\sum_{i \neq 1} v_{i1}$	$\sum_{i \neq 2} v_{i2}$	$\cdots$	$\sum_{i \neq N} v_{iN}$	$\sum_{i \neq j} v_{ij}$

"pairwise connectedness"
"total connectedness from all others (similar to S-Risk)"
"total connectedness to all others (similar to CoVaR)"
"system-wide connectedness"

Variance Decomposition Visualization Via the Network Graph



Understanding Connectedness: Granger-Sims Causality (Billio et al. Tradition, Including BDMP)

[Billio M., M. Getmansky, A.W. Lo, and L. Pelizzon (2012), “Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors,” *J. Financial Economics*, 104, 535-559.]

g_{ij} answers a key question:

Is the history of x_j useful for predicting x_i, over and above the history of x_i?

G

	x_1	x_2	...	x_5
x_1	$g_{1,1}$	$g_{1,2}$	$\cdots$	$g_{1,5}$
x_2	$g_{2,1}$	$g_{2,2}$	$\cdots$	$g_{2,5}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_5	$g_{5,1}$	$v_{g,2}$	$\cdots$	$g_{5,5}$

Thoughts on BDMP

1. BDMP Improve Importantly on Billio et al.
   - Full VAR rather than many bivariate VAR's
   - Control false discovery rate
   - Network methods for understanding G
2. There are Many Interesting BDMP Issues/Extensions
   - Are returns interesting? Basically serially uncorrelated...
   - What is the relevant causality horizon? Single-step or multi-step?
   - Related, what is the relevant observational frequency?
   - Examine (big) block causality...

Moving Forward (And Backward) I: Going beyond 0-1 G matrix to account for "full" VAR

$$\Phi(L)x_t = \varepsilon_t$$

– Account for all of Φ

Bonaldi, Hortacsu, and Kastl (2013), "An Empirical Analysis of Funding Cost Spillovers in the EURO-Zone With Application to Systemic Risk," Manuscript, Chicago and Princeton.

Moving Forward (And Backward) II: Incorporating Σ

$$\Phi(L)x_t = \varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

– Account for all of Φ ***and*** Σ

Diebold and Yilmaz (2014), "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *J. Econometrics*, 182, 119-134

– G accounts only for Φ ($G = f(\Phi)$)
– V accounts for both Φ and Σ ($V = f(\Phi, \Sigma)$)

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