VAR Network Methods for Summarizing and Visualizing High-Dimensional Connectedness

Discussion of Basu, Das, Michailidis, and Purnanandam:

“A System-Wide Approach to Measure Connectivity in the Financial Sector”

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Vector Autoregressions (VAR’s)

\(N\)-dimensional \(VAR(p)\) environment:

\[
\Phi(L)x_t = \varepsilon_t
\]

\(\varepsilon_t \sim (0, \Sigma)\)

e.g., 2-dimensional \(VAR(1)\):

\[
\begin{pmatrix}
x_{1t} \\
x_{2t}
\end{pmatrix} = \begin{pmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{pmatrix} \begin{pmatrix}
x_{1t-1} \\
x_{2t-1}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{pmatrix} \sim WN\left(\begin{pmatrix}
0 \\
0
\end{pmatrix}, \begin{pmatrix}
\sigma_1^2 & \sigma_{12} \\
\sigma_{12} & \sigma_2^2
\end{pmatrix}\right)
\]
Understanding Connectedness: Variance Decompositions (Diebold-Yilmaz Tradition)


$v_{ij}$ answers a key question:

*What fraction of the future uncertainty faced by variable $i$ is due to shocks from variable $j$?*

\[
V = \begin{bmatrix}
  x_1 & x_2 & \cdots & x_5 \\
  x_1 & v_{1,1} & v_{1,2} & \cdots & v_{1,5} \\
x_2 & v_{2,1} & v_{2,2} & \cdots & v_{2,5} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
x_5 & v_{5,1} & v_{5,2} & \cdots & v_{5,5}
\end{bmatrix}
\]
Financial Connectedness

- Old days: \( \text{dim}(x) = 5 \)

- Now: \( \text{dim}(x) = 50, \text{ or } 500, \text{ or } 5,000, \text{ or } \ldots \)

- Standard estimation methods are now totally unworkable (Must regularize with shrinkage, selection, hybrid, \ldots)

- Standard interpretive tools are now totally unworkable (Must summarize and visualize.)

\[
\begin{array}{c|cccc}
 & x_1 & x_2 & \ldots & x_{5000} \\
\hline
x_1 & v_{1,1} & v_{1,2} & \cdots & v_{1,5000} \\
x_2 & v_{2,1} & v_{2,2} & \cdots & v_{2,5000} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{5000} & v_{5,1} & v_{5,2} & \cdots & v_{5,5000} \\
\end{array}
\]
Variance Decomposition Summarization
Via the Network Degree Distribution

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$\ldots$</th>
<th>$x_N$</th>
<th>From Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$v_{11}$</td>
<td>$v_{12}$</td>
<td>$\ldots$</td>
<td>$v_{1N}$</td>
<td>$\sum_{j \neq 1} v_{1j}$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$v_{21}$</td>
<td>$v_{22}$</td>
<td>$\ldots$</td>
<td>$v_{2N}$</td>
<td>$\sum_{j \neq 2} v_{2j}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$x_N$</td>
<td>$v_{N1}$</td>
<td>$v_{N2}$</td>
<td>$\ldots$</td>
<td>$v_{NN}$</td>
<td>$\sum_{j \neq N} v_{Nj}$</td>
</tr>
</tbody>
</table>

To Others

$\sum_{i \neq 1} v_{i1}$ \quad $\sum_{i \neq 2} v_{i2}$ \quad $\ldots$ \quad $\sum_{i \neq N} v_{iN}$ \quad $\sum_{i \neq j} v_{ij}$

“pairwise connectedness”
“total connectedness from all others (similar to S-Risk)”
“total connectedness to all others (similar to CoVaR)”
“system-wide connectedness”
Variance Decomposition Visualization Via the Network Graph
Understanding Connectedness: Granger-Sims Causality (Billio et al. Tradition, Including BDMP)


\( g_{ij} \) answers a key question:

*Is the history of \( x_j \) useful for predicting \( x_i \), over and above the history of \( x_i \)?*

\[
\begin{array}{cccccc}
 & x_1 & x_2 & \ldots & x_5 \\
\hline
x_1 & g_{1,1} & g_{1,2} & \cdots & g_{1,5} \\
x_2 & g_{2,1} & g_{2,2} & \cdots & g_{2,5} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_5 & g_{5,1} & v_{g,2} & \cdots & g_{5,5} \\
\end{array}
\]
Thoughts on BDMP

1. BDMP Improve Importantly on Billio et al.
   - Full VAR rather than many bivariate VAR’s
   - Control false discovery rate
   - Network methods for understanding $G$

2. There are Many Interesting BDMP Issues/Extensions
   - Are returns interesting? Basically serially uncorrelated...
   - What is the relevant causality horizon? Single-step or multi-step?
   - Related, what is the relevant observational frequency?
   - Examine (big) block causality...
Moving Forward (And Backward) I:
Going beyond 0-1 $G$ matrix to account for “full” VAR

$$\Phi(L)x_t = \varepsilon_t$$

– Account for all of $\Phi$

Moving Forward (And Backward) II: 
Incorporating $\Sigma$

$$\Phi(L)x_t = \varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

– Account for all of $\Phi$ *and* $\Sigma$


– $G$ accounts only for $\Phi$ ($G = f(\Phi)$)
– $V$ accounts for both $\Phi$ and $\Sigma$ ($V = f(\Phi, \Sigma)$)