Discussion of Mueller and Watson:

"Measuring Uncertainty about Long-Run Predictions"

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(and the Penn Friday Reading Group)

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Really Technical, Really Fun, Really Useful

g_j and g_k are real valued, (6) can be rewritten as $E(\eta_{j,T}\eta_{k,T}) \to 2\int_0^\infty S(\omega)w_{jk}(\omega)d\omega$, where $w_{jk}(\omega) = \mathrm{Re}[\left(\int_0^{1+r} g_j(s)e^{-\mathrm{i}\omega s}ds\right)\left(\int_0^{1+r} g_k(s)e^{\mathrm{i}\omega s}ds\right)]$. With $g_j = \sqrt{2}\cos(\pi js)$, a calculation shows that $w_{jk}(\omega) = 0$ for $1 \le j, k \le q$ and $j+k$ odd, so that $E(X_jX_k) = 0$ for all odd $j+k$,

$$\begin{aligned}
E(\eta_{j,T}\eta_{k,T}) &= T^{-2\alpha}\sum_{s,t=1}^{\lfloor(1+r)T\rfloor}\left(\int_{-\pi}^{\pi}e^{-\mathrm{i}\lambda(t-s)}R(\lambda)d\lambda\right)\tilde{g}_{j,t}\tilde{g}_{k,s} \\
&= T^{-2\alpha}\int_{-\pi}^{\pi}R(\lambda)\left(\sum_{s=1}^{\lfloor(1+r)T\rfloor}\tilde{g}_{k,s}e^{\mathrm{i}\lambda s}\right)\left(\sum_{t=1}^{\lfloor(1+r)T\rfloor}\tilde{g}_{j,t}e^{-\mathrm{i}\lambda t}\right)d\lambda \\
&= T^{1-2\alpha}\int_{-T\pi}^{T\pi}R(\omega/T)\left(T^{-1}\sum_{s=1}^{\lfloor(1+r)T\rfloor}\tilde{g}_{k,s}e^{\mathrm{i}\omega(s/T)}\right)\left(T^{-1}\sum_{t=1}^{\lfloor(1+r)T\rfloor}\tilde{g}_{j,t}e^{-\mathrm{i}\omega(t/T)}\right)d\omega
\end{aligned}$$

transform the sample data $\{x_t\}_{t=1}^T$ into the weighted averages $(\overline{x}_{1:T}, X_T)$, with $X_T = (X_T(1), \ldots, X_T(T-1))'$, and where $X_T(j)$ is the jth cosine transformation

$$X_T(j) = \int_0^1 \Psi_j(s)x_{\lfloor sT\rfloor+1}ds = \iota_{jT}T^{-1}\sum_{t=1}^{T}\Psi_j\left(\frac{t-1/2}{T}\right)x_t \qquad (3)$$

with $\Psi_j(s) = \sqrt{2}\cos(j\pi s)$ and $\iota_{jT} = (2T/j\pi)\sin(j\pi/2T) \to 1$. We make two remarks about this transformation. First, because the Ψ_j weights add to zero, $X_T(j)$ is invariant to location shifts of the sample. Second, the transformation isolates variation in the sample

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Goal

Growth rate x_t, $t = 1, ..., T$

Estimate $\bar{x}_{T+1:T+h} = \frac{1}{h}\sum_{t=1}^{h} x_{T+t}$ for large h

Find A s.t. $P(\bar{x}_{T+1:T+h} \in A) = 1 - \alpha$

Important MW Advances

– Eliminate effects of
high-frequency misspecification

– Incorporate effects of
low-frequency parameter-estimation uncertainty

Methods in Search of Applications?

Maybe, but so what?
The methods are great, and there will be plenty of applications.

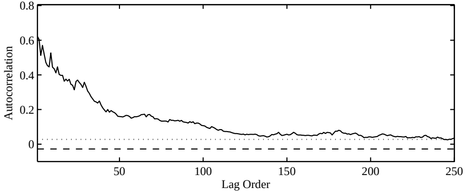
Consider:

– Piketty-type questions

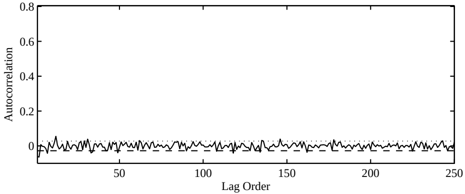
– The equity premium in the 21^{st} century

– Realized asset return volatility
(High frequency, and undeniable **long memory**)

Long Memory in S&P 500 Realized Volatility



(Hyperbolic) Autocorrelations of Daily Realized Volatility, V_t



(Zero) Autocorrelations of $(1-L)^{.4}\, V_t$

A Trivially Simple Case (to Build Intuition)

DGP: $x_t \sim iid\ (0,1)$
Large-h asymptotics: $h \to \infty,\ T \to \infty,\ \frac{h}{T} \to r > 0$

$$\sqrt{T}\left(\underbrace{\frac{1}{h}\sum_{t=1}^{h} x_{T+t}}_{\text{object of interest}} - \frac{1}{T}\sum_{t=1}^{T} x_t\right) = \left(\frac{\sqrt{T}}{\sqrt{h}}\frac{1}{\sqrt{h}}\sum_{t=1}^{h} x_{T+t} - \frac{1}{\sqrt{T}}\sum_{t=1}^{T} x_t\right)$$

$$\to_d \left(\frac{1}{\sqrt{r}} z_1 - z_2\right) \sim N\left(0,\ \frac{1}{r} + 1\right)$$

where z_1 and z_2 are independent $N(0,1)$

Now "simply" generalize from $x_t \sim iid(0,1)$ to $\Delta x_t \sim I(0)$

Beyond *iid* I: Historical Information

Construct prediction interval conditional on $\tilde{\mathcal{I}}_T = \{X_{T,1:q}\}$

(1) Joint distribution:

$$\begin{pmatrix} \tilde{\mathcal{I}}_T \\ \bar{x}_{T+1:T+h} \end{pmatrix} \rightarrow_d D$$

(2) Then get conditional distribution:

$$p\left(\bar{x}_{T+1:T+h} | \tilde{\mathcal{I}}_T\right)$$

– For x_t $I(0)$, $\tilde{\mathcal{I}}_T$ is not useful for forecasting $\bar{x}_{T+1:T+h}$ under large-h asymptotics (short memory)

– For x_t $I(d), d \neq 0$, $\tilde{\mathcal{I}}_T$ *is* useful for forecasting $\bar{x}_{T+1:T+h}$ even under large-h asymptotics (long memory)

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Beyond *iid* II: It's About The Low-Frequency Spectrum

Covariance structure of joint limiting distribution D
depends low-frequency spectrum of x_t

MW parameterize as:

$$S(\omega; b, c, d) \propto \left(\frac{1}{\omega^2 + c^2}\right)^d + b^2$$

b captures local level effects
c captures local-to-unity effects
d captures long-memory effects

Standard Methods Exist for Estimating d Consistently and Robustly (e.g., GPH)

As $\omega \to 0$,

$$S(\omega) \propto \frac{1}{\omega^{2d}}$$

or:

$$ln\, S(\omega) \approx c - 2d\, ln\, \omega$$

So run a regression on low-frequency periodogram ordinates:

$$ln\, I(\omega) = \beta_0 + \beta_1\, ln\, \omega + \varepsilon$$

Take $\hat{d} = -\hat{\beta}_1/2$

Monte Carlo

– One route: more thorough Monte Carlo

– Another route: *no* Monte Carlo
(do thorough Monte Carlo in a follow-up paper)

– Presently the Monte Carlo appears to be an afterthought,
with stochastic volatility curiously stirred into the mix

– Include comparison to forecasts from pure long memory model
estimated by GPH, low-frequency Whittle, etc.

Empirics

– For series with long histories it may be possible to do (crude) *checks* of interval calibration.

– Example: We have about 5000 days of S&P 500 realized volatility. Use 1:100 to forecast 101:200 and see whether the realization is in the interval; use 201:300 to forecast 301:400 and see whether the realization is in the interval; etc. At the end, we have many long-horizon interval forecasts and corresponding realizations. Are approximately 95 percent of the realizations in the (alleged) 95 percent intervals?

– Also explore forecasts from pure long memory model estimated by GPH, low-frequency Whittle, etc.

Miscellaneous

– Relationship to general mean-estimation literature

– Importance of location-scale invariance

– Matching CBO is neither here nor there

– b and c strike me as much less important than d

– Indeed priors in MW empirical work put all mass on $b = c = 0$

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The (Massive) Elephant in the Room: **Structural Change**

Imagine standing in 1980 with 30 years of available history, and forecasting average growth over the next 30 years of:

– Cost of a billion floating-point operations

– Sales of typewriters

– Number of checks cleared

– Trading volume in financial derivatives

– Membership in the Soviet Union's Politburo