Estimating Global Bank Network Connectedness

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Financial and Macroeconomic Connectedness

- Market Risk, Portfolio Concentration Risk (return connectedness)
- Credit Risk (default connectedness)
- Counterparty Risk, Gridlock Risk (bilateral and multilateral contractual connectedness)
- Systemic Risk (system-wide connectedness)
- Business Cycle Risk (local or global real output connectedness)
A Natural Financial/Economic Connectedness Question:

What fraction of the $H$-step-ahead prediction-error variance of variable $i$ is due to shocks in variable $j$, $j \neq i$?

Non-own elements of the variance decomposition: $d_{ij}^H$, $j \neq i$
Reading and Web Materials

Two Papers:


Book:


www.FinancialConnectedness.org
Network Representation: Graph and Matrix

Symmetric adjacency matrix $A$

$A_{ij} = 1$ if nodes $i, j$ linked

$A_{ij} = 0$ otherwise
Network Connectedness: The Degree Distribution

Degree of node $i$, $d_i$:

$$d_i = \sum_{j=1}^{N} A_{ij}$$

Discrete degree distribution on $0, \ldots, N - 1$

Mean degree, $E(d)$, is the key connectedness measure
Network Representation II (Weighted, Directed)

\[ A = \begin{pmatrix}
0 & 0.5 & 0.7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.3 & 0 \\
0 & 0 & 0 & 0.7 & 0 & 0.3 \\
0.3 & 0.5 & 0 & 0 & 0 & 0 \\
0.5 & 0 & 0 & 0 & 0 & 0.3 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \]

“to \( i \), from \( j \)”
Network Connectedness II: The Degree Distribution(s)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_{i^{from}} = \sum_{j=1}^{N} A_{ij}$$

$$d_{j^{to}} = \sum_{i=1}^{N} A_{ij}$$

“from-degree” and “to-degree” distributions on $[0, N-1]$

Mean degree remains the key connectedness measure
## Variance Decompositions as Weighted, Directed Networks

### Variance Decomposition / Connectedness Table

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>...</th>
<th>$x_N$</th>
<th>From Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$d_{11}^H$</td>
<td>$d_{12}^H$</td>
<td>...</td>
<td>$d_{1N}^H$</td>
<td>$\sum_{j \neq 1} d_{1j}^H$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$d_{21}^H$</td>
<td>$d_{22}^H$</td>
<td>...</td>
<td>$d_{2N}^H$</td>
<td>$\sum_{j \neq 2} d_{2j}^H$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$x_N$</td>
<td>$d_{N1}^H$</td>
<td>$d_{N2}^H$</td>
<td>...</td>
<td>$d_{NN}^H$</td>
<td>$\sum_{j \neq N} d_{Nj}^H$</td>
</tr>
</tbody>
</table>

To Others

$$\sum_{i \neq 1} d_{i1}^H \quad \sum_{i \neq 2} d_{i2}^H \quad \cdots \quad \sum_{i \neq N} d_{iN}^H \quad \sum_{i \neq j} d_{ij}^H$$

Total directional connect. “from,” $\mathbf{C}_{i \leftarrow \bullet}^H = \sum_{j=1}^{N} \sum_{j \neq i} d_{ij}^H$: “from-degrees”

Total directional connect. “to,” $\mathbf{C}_{\bullet \leftarrow j}^H = \sum_{i=1}^{N} \sum_{i \neq j} d_{ij}^H$: “to-degrees”

Systemwide connect., $\mathbf{C}^H = \frac{1}{N} \sum_{i,j=1}^{N} \sum_{i \neq j} d_{ij}^H$: mean degree
Relationship to \textit{MES}

\[ \text{MES}_{j|mkt} = \mathbb{E}(r_j | \mathbb{C}(r_{mkt})) \]

- Sensitivity of firm \( j \)'s return to extreme market event \( \mathbb{C} \)
- Market-based "stress test" of firm \( j \)'s fragility

"Total directional connectedness \textit{from}" (from-degrees)

"From others to \( j \)"
Relationship to CoVaR

\[ \text{VaR}^p : \ p = P(r < - \text{VaR}^p) \]
\[ \text{CoVaR}^{p,j|i} : \ p = P\left(r_j < - \text{CoVaR}^{p,j|i} \mid C(r_i)\right) \]
\[ \text{CoVaR}^{p,mkt|i} : \ p = P\left(r_{mkt} < - \text{CoVaR}^{p,mkt|i} \mid C(r_i)\right) \]

- Measures tail-event linkages
- Leading choice of \( C(r_i) \) is a VaR breach

“Total directional connectedness to” (to-degrees)

“From i to others”
Estimating Global Bank Network Connectedness

- Daily range-based equity return volatilities

- Top 150 banks globally, by assets, 9/12/2003 - 2/7/2014
  - 96 banks publicly traded throughout the sample
  - 80 from 23 developed economies
  - 14 from 6 emerging economies

- Market-based approach:
  - Balance sheet data are hard to get and rarely timely
  - Balance sheet connections are just one part of the story
  - Hard to know more than the market
Many Interesting Issues / Choices

- Approximating model: \textbf{VAR}? Structural DSGE?

- Identification of variance decompositions: Cholesky? \textbf{Generalized}? SVAR? DSGE?

- Estimation: Classical? Bayesian? \textbf{Hybrid}?  
  - Selection: Information criteria? Stepwise? \textbf{Lasso}?  
  - Shrinkage: BVAR? Ridge? \textbf{Lasso}?
Selection and Shrinkage via Penalized Estimation of High-Dimensional Approximating Models

\[ \hat{\beta} = \text{argmin}_\beta \sum_{t=1}^T \left( y_t - \sum_i \beta_i x_{it} \right)^2 \quad \text{s.t.} \quad \sum_{i=1}^K |\beta_i|^q \leq c \]

\[ \hat{\beta} = \text{argmin}_\beta \left( \sum_{t=1}^T \left( y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^K |\beta_i|^q \right) \]

Concave penalty functions non-differentiable at the origin produce selection. Smooth convex penalties produce shrinkage. \( q \to 0 \) produces selection, \( q = 2 \) produces ridge, \( q = 1 \) produces lasso.
Lasso

\[
\hat{\beta}_{\text{Lasso}} = \arg\min_{\beta} \left( \sum_{t=1}^{T} \left( y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} \left| \beta_i \right| \right)
\]

\[
\hat{\beta}_{\text{ALasso}} = \arg\min_{\beta} \left( \sum_{t=1}^{T} \left( y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} w_i \left| \beta_i \right| \right)
\]

\[
\hat{\beta}_{\text{Enet}} = \arg\min_{\beta} \left( \sum_{t=1}^{T} \left( y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} \left( \alpha \left| \beta_i \right| + (1 - \alpha) \beta_i^2 \right) \right)
\]

\[
\hat{\beta}_{\text{AEnet}} = \arg\min_{\beta} \left( \sum_{t=1}^{T} \left( y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} \left( \alpha w_i \left| \beta_i \right| + (1 - \alpha) \beta_i^2 \right) \right)
\]

where \( w_i = 1/\hat{\beta}_i^\nu \), \( \hat{\beta}_i \) is OLS or ridge, and \( \nu > 0 \).
Still More Choices (Within Lasso)

- Adaptive elastic net
- $\alpha = 0.5$ (equal weight to $L_1$ and $L_2$)
- OLS regression to obtain the weights $w_i$
- $\nu = 1$
- 10-fold cross validation to determine $\lambda$
- Separate cross validation for each VAR equation.
A Final Choice: Graphical Display via “Spring Graphs”

- Node size: Asset size

- Node color: Total directional connectedness “to others”

- Node location: Average pairwise directional connectedness
  (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness “to” and “from.”)

- Edge thickness: Average pairwise directional connectedness

- Edge arrow sizes: Pairwise directional connectedness “to” and “from”
Dynamic System-Wide Connectedness
150-Day Rolling Estimation Window

![Graph showing system-wide connectedness over years]

- **Total**
- **Cross-Country**
- **Within-Country**

Year range: 2004 to 2013
Conclusions: Connectedness Framework and Results

- Use network theory to understand large VAR’s, static or dynamic
- Directional, from highly granular to highly aggregated (Pairwise “to” or “from”; total directional “to or “from”; systemwide)
- For one asset class (stocks), network clustering is by country, not bank size
- For two asset classes (stocks and government bonds), clustering is by asset type, not country
- Dynamically, there are low-frequency changes in “good” connectedness (due to global financial integration), and high-frequency changes “bad” connectedness (due to crises)
- Most total connectedness changes are due to changes in pairwise connectedness for banks in different countries