Asset Return Volatility, High-Frequency Data, and the New Financial Econometrics

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Econometric Society, Melbourne
Who Uses Volatility Models, and Why?

- Asset pricing

- Portfolio allocation (incl. direct vol positions)

- Risk management (incl. hedging)
Financial Asset Return Data

- Volatility clustering
- Fat tails
- Convergence to normality under temporal aggregation
Generation I: “GARCH Volatility”

Background:


*Measuring and Forecasting Financial Market Volatilities and Correlations*
GARCH Process

\[ r_t | \Omega_{t-1} \sim N(0, h_t) \]

\[ h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1} \]
Basic Structure and Properties

Time variation in volatility and prediction-error variance

Unconditional symmetry and leptokurtosis

Convergence to normality under temporal aggregation

ARMA representation in squares

GARCH(1,1) and exponential smoothing

Easy estimation and testing
Variations

Asymmetric response and the leverage effect

Volatility components, Long memory, Regime switching

Fat-tailed conditional densities

GARCH-M and time-varying risk premia

Multivariate
Onward...

- Volatility from parametric models
- Volatility from options prices
- Volatility from direct indicators

\[ r_t^2 \]

\[ |r_t| \]

*Useful, but problems remain...*
Generation II: Realized Volatility

Estimate volatility by summing intra-period squared returns

Important early work:

• French, Schwert & Stambaugh (1987)
• Schwert (1989, 1990)
New Developments

• Provide rigorous foundations

• Direct characterization of marginal and conditional distributions
  • Multivariate analysis

• Direct modeling and forecasting
Plan

- Theory
- Data
  - Statics: the marginal distribution of volatility
  - Dynamics: the conditional distribution of volatility
    - The distribution of standardized returns
  - Modeling and Forecasting
    - New developments
Theory

\[ dp_t = \sigma_t \, dW_t \]

\[ r_{(m),t} \equiv p_t - p_{t-1/m} = \int_{0}^{1/m} \sigma_{t+\tau} \, dW_{t+\tau} , \quad t = 1/m, 2/m, ... \]

\[ \sigma^2_{t,h} \equiv \int_{0}^{h} \sigma^2_{t+\tau} \, d\tau \]

\[ \operatorname{plim}_{m \to \infty} \Sigma_{j=1,..,mh} r^2_{(m),t+j/m} = \sigma^2_{t,h} \]

Extensions: multivariate, jumps
Some Background


Data

Construction of 5-minute DM/$ and Yen/$ returns...

• Average of log bid and log ask, interpolated to 5-minute
  • Exclude weekends
    • Exclude fixed and variable holidays
    • Exclude days with data feed shutdown
Construction of Daily Realized Volatilities and Correlations

\[ \text{vard}_t = \sum_{j=1,\ldots,288} (\Delta \log \text{D}(288, t-1+j/m))^2 \]

\[ \text{vary}_t = \sum_{j=1,\ldots,288} (\Delta \log \text{Y}(288, t-1+j/m))^2 \]

\[ \text{cov}_t = \sum_{j=1,\ldots,288} \Delta \log \text{D}(288, t-1+j/m) \cdot \Delta \log \text{Y}(288, t-1+j/m) \]

\[ \text{stdd}_t = \text{vard}_t^{1/2}, \quad \text{stdy}_t = \text{vary}_t^{1/2} \]

\[ \text{lstdd}_t = \frac{1}{2} \log(\text{vard}_t), \quad \text{lstdy}_t = \frac{1}{2} \log(\text{vary}_t) \]

\[ \text{corr}_t = \text{cov}_t / (\text{stdd}_t \cdot \text{stdy}_t) \]
The Distribution of Volatility is Lognormal
Distributions of Realized Volatilities and Correlation

Deutschemark / Dollar Volatility

Yen / Dollar Volatility

Yen / Deutschemark Volatility
The Dynamics of Realized Volatility are Highly Persistent
No Unit Roots, but Clear Long-Memory

<table>
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<th>$lstdd_t$</th>
<th>$lstdy_t$</th>
<th>$corr_t$</th>
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<tr>
<td>$ADF$</td>
<td>-6.370</td>
<td>-7.817</td>
<td>-5.589</td>
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<tr>
<td>$d$</td>
<td>0.421</td>
<td>0.448</td>
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Autocorrelation Functions

Deutschemark / Dollar Volatility

Yen / Dollar Volatility

Yen / Deutschemark Volatility
Volatility Forecasts From Long-Memory Models


- **VAR-RV:** \( A(L)(1-L)^{4}(\sigma_t - \mu) = \epsilon_t \)

- **RiskMetrics:** \( \sigma_t^2 = .94 \sigma_{t-1}^2 + .06 r_t^2 \)

- **GARCH(1,1):** \( r_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2) \)

  \[
  \sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2
  \]
### Forecast Evaluation Regressions for Realized Volatilities

**Out-of-Sample, One-Day-Ahead**

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<thead>
<tr>
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<th>$b_0$</th>
<th>$b_1$ (VAR-RV)</th>
<th>$b_2$ (Other)</th>
<th>$R^2$</th>
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<tr>
<td><strong>DM/$</strong></td>
<td></td>
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</tr>
<tr>
<td>VAR-RV</td>
<td>0.02 (.05)</td>
<td>0.99 (.09)</td>
<td>-</td>
<td>.25</td>
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<tr>
<td>RiskMetrics</td>
<td>0.22 (.04)</td>
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<td>0.63 (.08)</td>
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<tr>
<td>GARCH</td>
<td>0.05 (.06)</td>
<td>-</td>
<td>0.85 (.10)</td>
<td>.10</td>
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<tr>
<td><strong>VAR-RV</strong></td>
<td>0.02 (.05)</td>
<td>0.98 (.13)</td>
<td>0.01 (.11)</td>
<td>.25</td>
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<tr>
<td><strong>+ RiskMetrics</strong></td>
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<tr>
<td><strong>VAR-RV</strong></td>
<td>0.02 (.06)</td>
<td>0.98 (.13)</td>
<td>0.02 (.16)</td>
<td>.25</td>
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<tr>
<td><strong>+ GARCH</strong></td>
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Standardized Returns are Approximately Gaussian

Unstandardized Returns

\[ r_t = \sigma_t \, \varepsilon_t \]

Standardized Returns

\[ \varepsilon_t = \frac{r_t}{\sigma_t} \]
Return Distributions

- Deutschemark / Dollar Returns
- Yen / Dollar Returns
- Portfolio Returns
Return Density Forecasts from Lognormal-Normal Mixtures

Recall the lognormal-normal mixture model:

\[ r_t = \sigma_t \cdot \varepsilon_t \]

log-N(0,1)

normal
Out-of-Sample One-Day-Ahead Density Forecast Evaluation
CDF of Probability Integral Transform

Cumulative Density Function

DM/$

Cumulative Density Function

Yen/$

Portfolio

CDF of Probability Integral Transform
Out-of-Sample One-Day-Ahead Density Forecast Evaluation

Autocorrelations of Probability Integral Transform
Realized Volatility and Out-of-Sample GARCH Forecasts
Realized Volatility and Out-of-Sample VAR-RV Forecasts
The Future

I. Risk Management

Regulatory compliance and best practice
Density forecasting, drawdown control, ...

- Microstructure noise: sampling, filtering, ...

- High-dimensional volatility modeling: factor structure, ...
  In progress...
II. Asset Pricing

- Asset pricing: “standard” derivatives...


- Asset pricing: “exotic” derivatives...

III. Portfolio Allocation

- Realized beta

“Realized Beta,” Working paper, University of Pennsylvania, 2005

- Volatility and market timing

Volatility Timing

\[ m \text{ in } \omega_t \omega_t' \Sigma_t^{-1} \omega_t \]

s.t. \[ \omega_t' \mu + (1 - \omega_t' 1) R_f = \mu_p \]

\[ \omega_t^* = \frac{(\mu_p - R_f) \Sigma_t^{-1} (\mu - R_f 1)}{(\mu - R_f 1) \Sigma_t^{-1} (\mu - R_f 1)} \]

Fleming et al. (2001, JF; 2002, JFE):
Utility value of volatility timing:  50 - 200 basis points!
Volatility Timing and *Market* Timing

The Probability of a Positive Return Depends on Volatility

![Graph showing probability distributions for different volatility levels. The graphs compare μ = 0.10 and σ = 0.05 with μ = 0.10 and σ = 0.15. The x-axis represents the return, while the y-axis represents the probability density. The blue vertical line indicates the threshold for a positive return.]
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Volatility as an Asset Class...