"Common Factors in Commodity Futures Curves" by Karstanje, van der Wel, and van Dijk (2015)

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This paper is about modeling **commodity futures curves**

**Commodity:** A standardized good, which is traded in bulk and whose units are interchangeable

- Natural gas, Coffee, Copper

**Futures contract:** Commitment to a transaction on a future date at a prearranged price

- Pay $80 for 1 barrel of oil 1 year in the future
Background: Why are commodity futures curves interesting?

- Futures important for commodities producers to hedge price risks
- Investors use commodities for diversification purposes
This paper

- Investigate **commonality** among factors driving many commodity futures curves
  - Comovement in price levels
  - Comovement in curve shapes

- **24 commodities** separated into **5 sectors**
  - Energy, Metals, Softs, Grains, and Meats
What the authors do

- **Dynamic Nelson Siegel** framework as in *Diebold and Li (2006)*
  - Decomposition into level, slope, curvature

- **Common (market and sector-specific) components** drive individual commodity factors as in *Diebold, Li, and Yue (2008)* and *Kose, Otrok, and Whiteman (2003)*
Preview of main results

- Important **common components** in level, slope, and curvature factors
  - On average $\approx 60\%$ of variation for level factor
  - On average $\approx 70\%$ of variation for slope and curvature factors
- Commonality mostly driven by **sector-specific components** as opposed to market components
  - On average $\approx 40\%$ of variation for level factor
  - On average $\approx 50\%$ of variation for slope and curvature factors
- Many other results that I will not go into today...
  - Economic interpretation of factors
  - Time-variation in importance of common components
Model
Model: Measurement equation

\[ f_{i,t}(\tau) = l_{i,t} + s_{i,t} \left[ \left( \frac{1 - \exp^{-\lambda_i \tau}}{\lambda_i \tau} \right) - \left( \frac{1 - \exp^{-\lambda_i}}{\lambda_i} \right) \right] + \]
\[ c_{i,t} \left[ \left( \frac{1 - \exp^{-\lambda_i \tau}}{\lambda_i \tau} - \exp^{-\lambda_i \tau} \right) - \left( \frac{1 - \exp^{-\lambda_i}}{\lambda_i} - \exp^{-\lambda_i} \right) \right] + \]
\[ \kappa_i \cos \left( \omega \left( g_i(t, \tau) - \theta_i \right) \right) + \nu_{i,t}(\tau) \]

\[ \nu_{i,t}(\tau) \sim iid \ N(0, \sigma_{\nu,i}^2) \]

- \( f_{i,t}(\tau) \): Futures price for commodity \( i \) at time \( t \) with maturity \( \tau \)
- \( l_{i,t}, s_{i,t}, c_{i,t} \): level, slope, curvature
- \( \lambda_i \): decay parameter
- \( \kappa_i \cos \left( \omega \left( g_i(t, \tau) - \theta_i \right) \right) \): seasonal adjustment term
Factor loadings (from Diebold and Li, 2006)
Model: Common components

\[ l_{i,t} = \alpha_i^L + \beta_i^L L_{\text{market},t} + \gamma_i^L L_{\text{sector},t} + L_{i,t} \]
\[ s_{i,t} = \alpha_i^S + \beta_i^S S_{\text{market},t} + \gamma_i^S S_{\text{sector},t} + S_{i,t} \]
\[ c_{i,t} = \alpha_i^C + \beta_i^C C_{\text{market},t} + \gamma_i^C S_{\text{sector},t} + C_{i,t} \]

- $X_{\text{market},t}$: Market component
- $X_{\text{sector},t}$: Sector-specific component
- $X_{i,t}$: Individual commodity component
Model: Transition equation

\[
\begin{pmatrix}
\Delta L_{y,t} \\
S_{y,t} \\
C_{y,t}
\end{pmatrix} = \Phi
\begin{pmatrix}
\Delta L_{y,t-1} \\
S_{y,t-1} \\
C_{y,t-1}
\end{pmatrix} + 
\begin{pmatrix}
\eta_{y,t}^L \\
\eta_{y,t}^S \\
\eta_{y,t}^C
\end{pmatrix}
\]

\[\eta_{y,t}^X \sim iid \ N(0, \sigma_{\eta,X,y}^2)\]

- \(y = \{\text{market, sector, idiosyncratic}\}\)
- \(\Phi\) is diagonal
- **Note:** level factor modelled as nonstationary
Estimation
Data

- 24 commodities separated into 5 sectors: Energy, Metals, Softs, Grains, and Meats

- Choose commodities based off of inclusion in S&P Goldman Sachs Commodity Index

- Monthly data from Jan 1995 to Sept 2012

- Heterogeneity in data availability between commodities and across time
Data example: natural gas
Is the Dynamic Nelson Siegel framework appropriate?

- Yield curve naturally decomposes into **level, slope, and curvature**

- Dynamic Nelson Siegel model has level, slope, and curvature factors

- **Question:** Does the same hold true for commodity futures curves?
Examples of futures price curve

(c) WTI crude oil January 2000

(d) WTI crude oil November 2008
Examples of yield curve (taken from Diebold and Li, 2006)
Answer: As a statistical approximation... YES!

VERY simple case (nonstochastic)

\[ f_t(\tau) = s_t + (r - c)\tau \]

- \( f_t(\tau) \): futures price with maturity \( \tau \)
- \( s_t \): spot price
- \( r \): risk free rate
- \( c \): convenience yield - the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery (Brennan, 1991)

**Question:** Could there be a theoretical issue with DNS approximation as maturity goes to \( \infty \)?
Relationship with Schwartz (1997)

- Schwartz (1997) 3-factor model
  - Arbitrage-free model of commodities futures curve
  - Posits stochastic processes for spot price, convenience yield, and risk free rate
  - Uses no-arbitrage restrictions to compute commodities futures curve

- Commodity-by-commodity: estimate individual factor DNS model and 3-factor arbitrage-free model
Comparison of DNS and Schwartz (1997) model estimates

- Orange: DNS level factor
- Blue: Spot price factor
- Average correlation across commodities: 0.67
Comparison of DNS and Schwartz (1997) model estimates

- Orange: DNS slope factor
- Blue: Convenience yield factor
- Average correlation across commodities: 0.76
### Joint model factor estimates

#### Autoregressive parameters

<table>
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<th>(\Delta) Level</th>
<th>Slope</th>
<th>Curvature</th>
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<td><strong>Market</strong></td>
<td>0.11</td>
<td>0.93</td>
<td>0.78</td>
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<td><strong>Energy</strong></td>
<td>0.12</td>
<td>0.79</td>
<td>0.88</td>
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<tr>
<td><strong>Metals</strong></td>
<td>0.12</td>
<td>0.96</td>
<td>——</td>
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<td><strong>Softs</strong></td>
<td>0.45</td>
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<td>0.87</td>
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<td><strong>Grains</strong></td>
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<td>0.88</td>
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<td><strong>Meats</strong></td>
<td>0.05</td>
<td>0.88</td>
<td>0.72</td>
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**Red** numbers are significant at 10% level
## Variance decompositions

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<th>Δ Level</th>
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<tr>
<td>Energy</td>
<td>12.4</td>
<td>61.3</td>
<td>26.3</td>
<td>74.6</td>
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<td>13.1</td>
<td>28.3</td>
<td>44.0</td>
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<td>Metals</td>
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<td>49.1</td>
<td>34.7</td>
<td>17.5</td>
<td>75.6</td>
<td>6.9</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Softs</td>
<td>34.1</td>
<td>12.8</td>
<td>53.2</td>
<td>1.7</td>
<td>26.6</td>
<td>71.7</td>
<td>22.7</td>
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<tr>
<td>Grains</td>
<td>53.5</td>
<td>21.6</td>
<td>25.0</td>
<td>0.1</td>
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<td>25.9</td>
<td>25.4</td>
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<tr>
<td>Meats</td>
<td>7.3</td>
<td>25.2</td>
<td>67.4</td>
<td>7.2</td>
<td>53.3</td>
<td>39.5</td>
<td>15.4</td>
<td>61.8</td>
<td>22.7</td>
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Further comments/extensions
Extension 1: Forecasting exercise

- DNS model has been shown to forecast well (Diebold and Li, 2006), especially when compared to affine term structure models.

- Forecast comparison:
  - Individual-commodity DNS model
  - Schwartz (1997) 3-factor model
  - Benchmark forecasting models (i.e. random walk)
Extension 2: Stochastic volatility

WTI spot price growth

Squared innovations WTI spot price growth

Autocorrelation

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Extension 3: Going beyond the U.S. economy

- Economic interpretation of futures curves in terms of **U.S. macroeconomic variables**
- What about other countries?
- Importers versus exporters of commodities (Peersman and van Robays, 2012)
Extension 4: Real effects of commodities futures curve uncertainty shocks

- How does uncertainty about the shape of commodity futures curves impact the real economy?
  - Overall economy versus commodity sectors
  - Commodity importers versus exporters
  - Monetary policy implications?

- Creal and Wu (2014): Effects of short and long-term interest rate uncertainty on the macroeconomy
Conclusion

- I really like this paper
  - Novel application of DNS methodology
  - **Lots** of clean empirical results
- Opens up a lot of interesting empirical questions (of which I have only listed 4)
(a) Level - market-wide
(a) Slope - market-wide
Economic interpretation of factors

- Theory of normal backwardation (Keynes, 1930)
  - Commodity producers and inventory holders hedge risk by selling commodity futures
  - Requires setting futures prices at discount to encourage speculators to take opposite long position
  - **Empirical implication:** relationship between *hedging pressure* and commodity futures curve

- Theory of storage (Kaldor, 1939, Working, 1949)
  - Relationship between futures price, spot price, inventory storage benefits, and inventory storage costs
  - **Empirical implication:** relationship between *inventory levels* and commodity futures curve
Economic interpretation of factors

Market-wide components:

▶ Δ Level
  ▶ Hedging pressure (+), Equity returns (+), USD exchange rate returns (−)

▶ Slope
  ▶ Business inventories (+), Hedging pressure (+), Leading economic indicators (+), Current economic indicators (−)

▶ Curvature
  ▶ Business inventories (+), Hedging pressure (−), Interest rates (+)
Time-variation in importance of factors

Investigate time-variation in importance of common components

- Weights $\delta^{|k|}$ to each observation
- $\delta = 0.99$
- $k = \ldots, -2, -1, 0, 1, 2, \ldots$
- Centered at $\Theta = 1, 2, \ldots, T$