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Assessing point forecast accuracy by stochastic loss distance

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ABSTRACT

We explore the evaluation (ranking) of point forecasts by a "stochastic loss distance" (*SLD*) criterion, under which we prefer forecasts with loss distributions $F(L(e))$ "close" to the unit step function at 0. We show that, surprisingly, ranking by *SLD* corresponds to ranking by expected loss.

1. Introduction

Applied econometricians and others often want to evaluate the accuracy of competing point forecasts. Invariably they evaluate accuracy by expected loss, $E(L(e))$, where e is forecast error and the loss function $L(e)$ satisfies $L(0) = 0$ and $L(e) \geq 0$, $\forall e$. In this paper we begin differently, evaluating forecasts by the distance between the cdf of $L(e)$ and the cdf of a perfect forecast.

2. Ranking forecasts by stochastic loss distance

We call our criterion "stochastic error distance" (*SLD*). We first define it precisely, and then we relate it to the conventional expected loss criterion.

2.1. The SLD criterion

We rank forecasts by a "stochastic loss distance" (*SLD*) criterion, under which we prefer the forecast whose loss distribution $F(L(e))$ has the smallest distance from the unit step function at 0,

$$F^*(L(e)) = \begin{cases} 0, & L(e) < 0 \\ 1, & L(e) \geq 0. \end{cases}$$

$F^*(\cdot)$ is the obvious reference loss distribution because nothing can dominate a benchmark forecast whose errors consistently achieve zero loss; that is, a forecast whose errors achieve $F(L(e)) = F^*(L(e))$.

Hence we rank forecasts by the area,

$$SLD(F, F^*, L(e)) = \int_0^\infty \left| F(L(e)) - F^*(L(e)) \right| dL(e) \quad (1)$$

$$= \int_0^\infty [1 - F(L(e))] \, dL(e), \quad (2)$$

where smaller $SLD(\cdot)$ is better. We illustrate the *SLD* idea in Fig. 1; we prefer forecasts whose error loss distribution has small shaded *SLD* area.

2.2. Forward to the past: the relationship between SLD(L(e)) and E(L(e))

We begin with a lemma.

Lemma 2.1. *For random variable x with cdf $F(x)$, if $E(|x|) < \infty$,* $\lim_{c\to\infty} c(1 - F(c)) = 0$.

Proof. We have

$$c(1 - F(c)) = cP(X > c) = c\int_c^\infty dP(x) = \int_c^\infty c \, dP(x)$$

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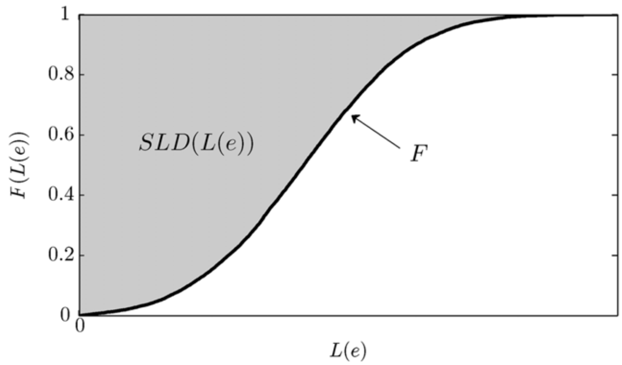

Fig. 1. Error-loss cdf, $F(L(e))$, and its stochastic loss distance, $SLD(L(e))$.

$$\leq \int_c^{\infty} x \, dP(x) \quad \text{(replacing } c \text{ with } x\text{)}$$
$$= \int_0^{\infty} x \, dP(x) - \int_0^{c} x \, dP(x).$$

But this converges to zero as $c \to \infty$, because

$$\int_0^{\infty} x \, dP(x) \leq \int_{-\infty}^{\infty} |x| \, dP(x) < \infty. \quad \blacksquare$$

Now let us proceed to characterize the relationship between $SLD(L(e))$ and $E(L(e))$.

Proposition 2.2 (*Equivalence of Stochastic Loss Distance and Expected Loss*). *Let $L(e)$ be a forecast-error loss function satisfying $L(0) = 0$ and $L(e) \geq 0, \forall e$, with $E(|L(e)|) < \infty$.*[1] *Then*

$$SLD(L(e)) = \int_0^{\infty} [1 - F(L(e))] \, dL(e) \tag{3}$$
$$= E(L(e)), \tag{4}$$

where $F(L(e))$ is the cumulative distribution function of $L(e)$. That is, SLD equals expected loss for any loss function and error distribution.

Proof. To evaluate $E(L(e))$ we integrate by parts:

$$\int_0^{c} L(e) f(L(e)) \, dL(e)$$
$$= -L(e)[1 - F(L(e))]|_0^c + \int_0^{c} [1 - F(L(e))] \, dL(e)$$
$$= -c(1 - F(c)) + \int_0^{c} [1 - F(L(e))] \, dL(e).$$

Now letting $c \to \infty$ we have

$$E(L(e)) = \int_0^{\infty} L(e) f(L(e)) \, dL(e)$$
$$= \lim_{c \to \infty} -c(1 - F(c)) + \int_0^{\infty} [1 - F(L(e))] \, dL(e)$$
$$= 0 + \int_0^{\infty} [1 - F(L(e))] \, dL(e) \quad \text{(by Lemma 2.1)}$$
$$= SLD(L(e)). \quad \blacksquare$$

To the best of our knowledge, this result has not appeared in the forecast evaluation literature. It does appear, however, in different guise in the hazard and survival modeling literature, in whose jargon "expected lifetime equals the integrated survival function".[2]

3. Concluding remarks

Our result is clearly negative: The obvious and intuitive approach of ranking forecasts by $SLD(L(e))$ unfortunately takes us nowhere relative to ranking by $E(L(e))$, because $SLD(L(e))$ *is* $E(L(e))$.

One route forward is to consider generalized notions of stochastic loss distance. Consider, for example, a generalized weighted stochastic loss distance (*GWSLD*),

$$GWSLD(F, F^*, L; p, w) = \int \left| F(L(e)) - F^*(L(e)) \right|^p \times w(L(e)) \, dL(e), \tag{5}$$

where neither the exponent p nor the weighting function $w(\cdot)$ need be 1. Ranking forecasts by *GWSLD* no longer corresponds to ranking by expected loss. But the *GWSLD* approach lacks simplicity and appeal, insofar as it is not obvious why one would generally want $p \neq 1$ and/or $w(\cdot) \neq 1$.

A second route is to abandon the loss function, and hence expected loss minimization, entirely, focusing instead on stochastic dominance. Building on important earlier work of Linton et al. (2005), for example, Corradi and Swanson (2013) and Lee et al. (2014) consider first-order stochastic dominance. Unfortunately, however, first-order stochastic dominance is such a strong criterion that it is unlikely that one forecast's $L(e)$ would ever first-order stochastically dominate another's. But weaker (higher-order) notions of stochastic dominance may merit exploration for forecast accuracy comparisons.

Finally, one can stay with $E(L)$ minimization but use ideas closely related to *SLD* to suggest an appropriate loss function L. We do so in a companion paper, Diebold and Shin (2014).

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[1] In an abuse of notation, we use "$L(e)$" to denote either the loss random variable or its realization. The meaning will be clear from context.

[2] See, for example, Neumann (1999).