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Dynamic Equilibrium Economies: A Framework for Comparing Models and Data

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We propose a constructive, multivariate framework for assessing agreement between (generally misspecified) dynamic equilibrium models and data, which enables a complete second-order comparison of the dynamic properties of models and data. We use bootstrap algorithms to evaluate the significance of deviations between models and data, and we use goodness-of-fit criteria to produce estimators that optimize economically-relevant loss functions. We provide a detailed illustrative application to modelling the U.S. cattle cycle.

1. INTRODUCTION

Dynamic equilibrium models are now used routinely in many fields. Such models, for example, have been used to address a variety of macroeconomic issues, including business-cycle fluctuations, economic growth, and the effects of government policies.[1] Additional prominent fields of application include international economics, public economics, industrial organization, labour economics, and agricultural economics.[2]

At present, however, many important questions regarding the empirical implementation of dynamic equilibrium models remain incompletely answered. The questions fall roughly into two methodological groups. The first group involves issues related to assessing model adequacy, and the second involves issues related to model estimation. We contribute to an emerging literature that has begun to deal with both issues, including Watson (1993), King and Watson (1992, 1996), Canova, Finn and Pagan (1994), Kim and Pagan (1994), Pagan (1994), Leeper and Sims (1994), Cogley and Nason (1995), and Hansen, McGrattan

1. Among many others, see Kydland and Prescott (1982), Hansen (1985), Christiano and Eichenbaum (1995), and Rotemberg and Woodford (1996, 1997) (business cycles), Lucas (1988), Jones and Manuelli (1990), Rebelo (1991), and Greenwood, Hercowitz and Krusell (1997) (growth), and Lucas (1990), Cooley and Hansen (1992), and Ohanian (1997) (policy effects).

2. Among many others, see Backus, Kehoe and Kydland (1994) (international economics), Auerbach and Kotlikoff (1987) (public economics), Ericson and Pakes (1995) (industrial organization), Rust (1989) (labour economics), and Rosen, Murphy and Scheinkman (1994) (agricultural economics).

and Sargent (1997). A 1996 *Journal of Economic Perspectives* symposium (Kydland and Prescott (1996), Sims (1996), Hansen and Heckman (1996)) focused on these issues, and two important messages emerged: (1) dynamic equilibrium models, like all models, are intentionally simple abstractions and therefore should not be construed as the true data generating process; and (2) formal methods should be developed and used to help us assess the models more thoroughly. In this paper, we take a step toward meeting those goals.

Some parts of our framework are new, while others build on earlier work in interesting ways. In many respects, our work begins where Watson (1993) ends. With an eye toward future research, Watson notes that "... one of the most informative diagnostics ... is the plot of the model and data spectra," and he recommends that in the future researchers "present both model and data spectra as a convenient way of comparing their complete set of second moments." Our methods are based directly on comparison of model and data spectra; they can be used to assess the performance of a model (for a given set of parameters), to estimate model parameters, and to test hypotheses about parameters or models. To elaborate, our approach is:

A. Frequency-domain and multivariate. Working in the frequency domain enables decomposition of variation across frequencies, which is often useful, and the multivariate focus facilitates simple examination of cross-variable correlations and lead-lag relationships at the frequencies of interest.
B. Based on a full second-order comparison of model and data dynamics, in contrast to a common approach used in the business cycle literature, in which only a few variances and covariances from the model economy and the actual economy are compared. The spectrum provides a complete summary of Gaussian time series dynamics and an approximate summary of non-Gaussian time series dynamics.
C. Based on the realistic assumption that all models are misspecified. We regard all of the models we entertain as false, in which case traditional statistical methods lose some of their appeal.
D. Graphical and constructive. The framework permits one to assess visually and quickly the dimensions along which a model performs well, and the dimensions along which it performs poorly.
E. Based on a common set of tools that can be used by researchers with potentially very different objectives and research strategies. The framework can be used to evaluate strictly calibrated models, and it can also be used formally to estimate and test models.
F. Designed to facilitate statistical inference about objects estimated from data, including spectra, goodness-of-fit measures, model parameters, and test statistics. Bootstrap methods play an important role in that regard; we develop and use a simple nonparametric bootstrap algorithm.
G. Mathematically convenient. Under regularity conditions, the spectrum is a bounded continuous function, which makes for convenient mathematical developments.

All of the classical ideas of the business-cycle analysis discussed, for example, by Lucas (1977) have spectral analogues, ranging from univariate persistence (typical spectral shape) to multivariate issues of comovement (coherence) and lead-lag relationships (phase shifts) at business-cycle frequencies. We highlight these links and draw upon the business-cycle literature for motivation in the methodological Sections 2 and 3. The methods we develop, however, are not wed to macroeconomics in any way; rather, they can be used

in a variety of fields. Therefore, to maintain balance and to introduce researchers in different areas to the use of our framework, we apply our methods to a simple and accessible, yet rich, *micro*economic model in Section 4. We conclude in Section 5.

2. ASSESSING AGREEMENT BETWEEN MODEL AND DATA

Our basic strategy is to assess models by comparing model spectra to data spectra. Our goal is the provision of a graphical framework that facilitates visual comparisons of model spectra to interval estimates of data spectra. We compute model spectra exactly (either analytically or numerically); thus, they have no sampling uncertainty. Sampling error does, however, affect the sample data spectra, which are of course just estimates of true but unknown (population) data spectra. We exploit well-established procedures for estimating spectra, and we develop and use bootstrap techniques to assess the sampling uncertainty of estimated spectra.[3]

Estimating spectra

Consider the N-variate linearly regular covariance stationary stochastic process

$$y_t = \mu + B(L)\varepsilon_t = \mu + \sum_{i=-\infty}^{\infty} B_i \varepsilon_{t-i},$$

$$E(\varepsilon_t \varepsilon_s') = \begin{cases} \Omega & \text{if } t = s, \\ 0 & \text{otherwise,} \end{cases}$$

where $E(\varepsilon_t) = 0$, $B_0 = I$, and the coefficients are square summable (in the matrix sense). The autocovariance function is $\Gamma(\tau) = \sum_{i=-\infty}^{\infty} B_i \Omega B_{i+\tau}'$ and the spectral density function is

$$F(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \Gamma(\tau) e^{-i\omega\tau}, \qquad -\pi < \omega < \pi.$$

Consider now a generic off-diagonal element of $F(\omega)$, $f_{kl}(\omega)$. In polar form, the cross-spectral density is $f_{kl}(\omega) = \mathrm{ga}_{kl}(w) \exp[i\, \mathrm{ph}_{kl}(\omega)]$, where $\mathrm{ga}_{kl}(\omega) = [\mathrm{re}^2(f_{kl}(\omega)) + \mathrm{im}^2(f_{kl}(\omega))]^{1/2}$ is the gain or amplitude, and where $\mathrm{ph}_{kl}(\omega) = \arctan\{\mathrm{im}(f_{kl}(\omega))/\mathrm{re}(f_{kl}(\omega))\}$ is the phase. As is well known, the gain tells how the amplitude of y_l is multiplied in contributing to the amplitude of y_k at frequency ω, and phase measures the lead of y_k over y_l at frequency ω. (The phase shift in time units is $\mathrm{ph}(\omega)/\omega$.) We shall often find it convenient to examine coherence rather than gain, where the coherence is defined as $\mathrm{coh}_{kl}(\omega) = \mathrm{ga}_{kl}^2(\omega)/(f_{kk}(\omega) f_{ll}(\omega))$, which measures the squared correlation between y_k and y_l at frequency ω.

Given a sample path $\{y_{1t}, \ldots, y_{Nt}\}_{t=1}^{T}$, we estimate the $N \times 1$ mean vector μ with $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_N)'$. From this point onward, we assume that all sample paths have been centred around this sample mean. We estimate the autocovariance function with $\hat{\Gamma}(\tau) = [\hat{\gamma}_{kl}(\tau)]$ $(k = 1, \ldots, N, l = 1, \ldots, N)$, where $\hat{\gamma}_{kl}(\tau) = 1/T \sum_{\tau=1}^{t-|\tau|} y_{kt} y_{1,t+\tau}$, $\tau = 0, \pm 1, \ldots, \pm(T-1)$. We estimate the spectral density matrix using the Blackman–Tukey lag-window approach in which we replace the sample spectral density function, $\hat{F}(\omega_j) = 1/2\pi \sum_{\tau=-(T-1)}^{(T-1)} \hat{\Gamma}(\tau) e^{-i\omega_j\tau}$ $(\omega_j = 2\pi j/T, j = 1, \ldots, T/2-1)$ with one involving the

3. Alternatively, one could fix the data spectrum, and assess sampling error in the model spectrum by simulating repeated realizations from the model. The two approaches are essentially complementary, roughly analogous to the Wald and Lagrange multiplier testing perspectives. See, for example, Gregory and Smith (1991).

"windowed" sample autocovariance sequence, $F^*(\omega_j) = 1/2\pi \sum_{\tau=-(T-1)}^{(T-1)} \Lambda(\tau)\hat{\Gamma}(\tau)e^{-i\omega\tau}$, where $\Lambda(\tau)$ is a matrix of lag windows. The Blackman–Tukey procedure results in a consistent estimator if we adjust the lag window $\Lambda(\tau)$ with sample size in such a way that variance and bias decline simultaneously. We then obtain the sample coherence and phase at any frequency ω_j by transforming the appropriate elements of $F^*(\omega_j)$.

Assessing sampling variability

A key issue for our purposes is how to ascertain the sampling variability of the estimated spectral density function. To do so, we use an algorithm for resampling from time series data, which we call the Cholesky factor bootstrap, and which is closely related to the Ramos (1988) bootstrap; the main difference is that Ramos proceeds in the frequency domain, whereas we develop the Cholesky factor bootstrap in the time domain. First we compute the Cholesky factor of the sample covariance matrix of the series of interest. We then exploit the fact that, up to second order, the series of interest can be written as the product of the Cholesky factor and serially uncorrelated disturbances, which can be easily bootstrapped using parametric or non-parametric procedures.[4]

First we need some definitions and notation. Let $z_t = (y_{1t}, \ldots, y_{Nt})'$, and let $z = (z_1', z_2', \ldots, z_T')'$. Then $z \sim (1 \otimes \mu, \Sigma)$, where 1 is an N-dimensional column vector of ones, and $\Sigma = \text{Toeplitz}(\Gamma(0), \Gamma(1), \ldots, \Gamma(T-1))$. By symmetry and positive definiteness, we can write $\Sigma = PP'$, where the unique Cholesky factor P is lower triangular. We estimate Σ by $\hat{\Sigma} = \text{Toeplitz}(\hat{\Gamma}(0), \hat{\Gamma}(1), \ldots, \hat{\Gamma}(T-1))$, where $\hat{\Gamma}(\tau) = 1/T \sum_{t=1}^{T-|\tau|} z_t z_{t+|\tau|}'$, $\tau = 0, \pm 1, \ldots, \pm(T-1)$; this ensures that we can write $\hat{\Sigma} = \hat{P}\hat{P}'$, where the unique Cholesky factor $\hat{P}$ is lower triangular. Now let $\{\lambda_{|i-j|}\}_{|i-j|=0}^{T-1}$ be a set of decreasing weights applied to the successive off-diagonal blocks of $\hat{\Sigma}$, and call the resulting matrix Σ^*. Finally, let P^* be the Cholesky factor of Σ^*.

The fact that $z \sim (1 \otimes \mu, PP')$ implies that data generated by drawing $\varepsilon^{(i)} \overset{\text{iid}}{\sim} (0, I_{NT})$ and forming

$$z^{(i)} = \mu_z + P\varepsilon^{(i)},$$

where $\mu_z = 1 \otimes \mu$, will have the same second-order properties as the observed data. In practice we replace the unknown population first and second moments with the consistent estimates described above. Thus, to perform a parametric bootstrap, we draw $\varepsilon^{(i)} \sim N(0, I_{NT})$, we form

$$z^{(i)} = \bar{z} + P^*\varepsilon^{(i)} \sim N(\bar{z}, \Sigma^*),$$

where $\bar{z} = 1 \otimes \bar{y}$, and then we compute both the estimates $F^{*(i)}(\omega_j), j = 1, \ldots, T/2-1, i = 1, \ldots, R$ and confidence intervals. Alternatively, to perform a nonparametric bootstrap, we note that $\varepsilon^{(i)} = P^{-1}(z - \mu_z)$. In practice, we draw $\varepsilon^{(i)}$ with replacement from $P^{*-1}(z - \bar{z})$, we form

$$z^{(i)} = \bar{z} + P^*\varepsilon^{(i)} \sim (\bar{z}, \Sigma^*),$$

from which we compute $F^{*(i)}(\omega_j), j = 1, \ldots, T/2-1, i = 1, \ldots, R$, and then we construct confidence intervals.

4. Note that the Cholesky factor bootstrap will miss nonlinear dynamics such as GARCH—it is designed to capture only second-order dynamics, in identical fashion to standard (as opposed to higher-order) spectral analysis. Users should be cautious in employing our procedures if nonlinearities are suspected to be operative, as may be the case with high-frequency financial data, or with models of prices in which inventory constraints may be important, as in Deaton and Laroque (1995). Such nonlinearities, however, are not likely to be important in non-price data.

The simplicity of the Cholesky factor bootstrap is appealing; it is an intuitive and easily-implemented generic method for generating samples of time-series pseudo-data whose autocovariances match those of the actual data. Moreover, and perhaps surprisingly, the Cholesky factor bootstrap, which has a nonparametric flavour, and alternatives such as the VAR bootstrap (*e.g.*, Canova, Finn and Pagan (1994)), which has a parametric flavour, are in fact closely related. A modern and unifying view, currently the focus of intense research in mathematical statistics, is to interpret various time series bootstraps as sieves (Grenander (1981) whose complexity increases with sample size at a suitable rate.[5] The Cholesky factor bootstrap has a sieve interpretation; the sieve is a spectrum estimated by smoothing a suitably-increasing number of sample autocovariances as the sample size grows. The VAR bootstrap also has a sieve interpretation; the sieve is an estimated autoregression of suitably-increasing length as the sample size grows. Thus, asymptotically in T, both the Cholesky factor and VAR bootstraps can be effective algorithms for generating data with the same second-order properties as an observed sample path. Neither is in general "superior" to the other; the two are best viewed as complements. Both are the subject of ongoing research, as is the "block" bootstrap of Künsch (1989) and Liu and Singh (1992) as modified for spectra by Politis and Romano (1992), as well as the spectral bootstrap of Franke and Härdle (1992) and Berkowitz and Diebold (1997).

We hasten to add, however, that the literature on bootstrapping time series in general—and spectra in particular—is very young and very much unsettled. We still have a great deal to learn about the comparative properties of various bootstraps, both asymptotically and in finite samples, and the conditions required for various properties to obtain. Presently available results differ depending on the specific statistic being bootstrapped, and moreover, only scattered first- and occasionally second-order asymptotic results are available, and even less is known about actual finite-sample performance. With this in mind, we present both theoretical and Monte Carlo analyses of the performance of the Cholesky factor bootstrap in two appendices to this paper, available at http://www.ssc.upenn.edu/~diebold/index.html. In Appendix 1, we establish first-order asymptotic validity, and in Appendix 2, we document good small-sample performance in a simple Monte Carlo experiment.

Constructing confidence tunnels

For notational simplicity we focus on confidence tunnels for univariate spectra; as will be clear, the extension to cross spectra is immediate. If interest centres on only one frequency, we simply use the bootstrap distribution at that frequency to construct the usual bootstrap confidence interval. That is, we find q_T^L, q_T^U such that $P(f^{*(\cdot)}(\omega) \leqq q_T^U) = 1 - \alpha/2$ and $P(f^{*(\cdot)}(\omega) \geqq q_T^L) = 1 - \alpha/2$, where $(1-\alpha)$ is the desired confidence level, "L" stands for lower, "U" stands for upper, the "T" subscript indicates that we tailor the band to the finite-sample size T, and the $(\cdot)$ superscript indicates that we take the probability under the bootstrap distribution. The $(1-\alpha)\%$ two-sided confidence interval is $[q_T^L, q_T^U]$.

However, one often wants to assess the sampling variability of the entire spectral density *function* over many frequencies (*e.g.*, business-cycle frequencies, or perhaps all frequencies) to learn about the broad agreement between data and model. One approach is to form the pointwise bootstrap confidence intervals described above, and then to "connect the dots." But obviously, a set of $(1-\alpha)\%$ confidence intervals constructed for each of n ordinates will not achieve $(1-\alpha)\%$ joint coverage probability. Rather, the actual

5. See, for example, Bühlmann (1997) and Bickel and Bühlmann (1996).

confidence level will be closer to $(1-\alpha)^n\%$, which holds exactly if the pointwise intervals are independent. A better approach is to use the Bonferroni method to approximate the desired coverage level, by assigning $(1-\alpha/n)\%$ coverage to each ordinate.[6] The resulting "confidence tunnel" has coverage of at least $(1-\alpha)\%$ and therefore provides a conservative estimate of the tunnel.

A third approach to confidence tunnel construction is the supremum method of Woodroofe and van Ness (1967) and Swanepoel and van Wyk (1986), which uses an estimate of the (standardized) distribution of $\sup_{0<\omega_j<\pi}|f^*(\omega_j)-f(\omega_j)|$, $\omega_j=2\pi j/T$, $j=1,\ldots,T/2-1$, to construct a confidence tunnel for the curve. Specifically,

(1) Calculate $f^{*(\cdot)}(\omega_j)$, $\omega_j=2\pi j/T, j=1,\ldots,T/2-1$.
(2) Find c such that:

$$P\left[\sup_{0<\omega_j<\pi}\left(\frac{|f^{*(\cdot)}(\omega_j)-f^*(\omega_j)|}{\sqrt{2/T}f^*(\omega_j)}\right)\leqq c\right]=1-\alpha,$$

where we evaluate the probability with respect to the bootstrap distribution.
(3) Construct the confidence tunnel, $f^*(\omega_j)\pm c\sqrt{2/T}f^*(\omega_j)$,

$$\omega_j=2\pi j/T, \qquad j=1,\ldots,T/2-1.$$

Unlike the Bonferroni tunnels, the supremum tunnels attain asymptotically correct coverage rates even with statistical dependence among ordinates. Little is known, however, about the comparative finite-sample performance of the Bonferroni and supremum tunnels, and the supremum tunnels may require very large samples for accurate coverage, as noted by Hannan (1970, p. 294).

3. ESTIMATION: MAXIMIZING AGREEMENT BETWEEN MODEL AND DATA

Now we consider estimation, together with the related issues of goodness-of-fit and hypothesis testing. To make the discussion as transparent as possible, we first discuss the univariate case, and then we proceed to the multivariate case.

Univariate

Estimation requires a loss function, or goodness-of-fit measure, for assessing closeness between model and data. A strength of our approach is that many loss functions may be entertained; the particular loss function adopted should reflect the user's preferences, as argued by Pagan (1994). In most cases it would seem that a function of the form

$$C_{gw}(\theta)=\int_0^\pi g(f_m(\omega;\theta),f^*(\omega))w(\omega)d\omega,$$

will be adequate. The function g measures the divergence between $f_m(\omega;\theta)$ (the model spectrum) and $f^*(\omega)$ (an estimate of the data spectrum); note that the model spectrum

6. In the univariate case, typically $n=T/2-1$. In the multivariate case, the question arises as to "how wide to cast the net" in forming confidence tunnels. One might view each element of the spectral density matrix in isolation, for example, in which case each of the respective confidence tunnels would use $n=T/2-1$. At the other extreme, one could use $n=N^2(T/2-1)$, effectively forming a tunnel for the entire matrix.

is analytically or numerically computable to any desired degree of accuracy, and that the data spectrum, although unknown, is consistently estimable. We weight this divergence across frequencies by the function $w(\omega)$. In practice, we replace the integral with a sum over frequencies $\omega_j = 2\pi j/T, j = 1, \ldots, T/2 - 1$. Quadratic loss with uniform weighting over all frequencies, for example, corresponds to $g(a, b) = (a-b)^2$ and $w(\omega) = 1$, yielding $C_{gw}(\theta) = \sum_j (f_m(\omega_j; \theta) - f^*(\omega_j))^2$.

The goodness-of-fit measure may readily be transformed into an estimation criterion by taking

$$\hat{\theta}_{gw} = \underset{\theta}{\operatorname{argmin}}\ C_{gw}(\theta).$$

The Gaussian ML estimator is asymptotically of this form, for a particular and potentially restrictive choice of g, f^*, and w; it is

$$\underset{\theta}{\operatorname{argmax}} \left(-\tfrac{1}{2} \sum_j \ln f_m(\omega_j; \theta) - \tfrac{1}{2} \sum_j \frac{\hat{f}(\omega_j)}{f_m(\omega_j; \theta)} \right).$$

Note in particular that Gaussian ML involves minimization of spectral divergence with equal weighting across all frequencies.

To compute standard errors and interval estimates for parameters of interest, and to test hypotheses about the elements of $\hat{\theta}_{gw}$, we again use the Cholesky factor bootstrap. We proceed as follows:

(1) At bootstrap replication (i), draw a bootstrap sample of size T using the Cholesky factor algorithm.
(2) Numerically minimize $C^{(i)}_{gw}(\theta)$ to get $\hat{\theta}^{(i)}_{gw}$.
(3) Repeat R times.
(4) Compute standard errors, form interval estimates, implement bias corrections, or test hypotheses using the distribution of $\hat{\theta}^{(i)}_{gw}$, $i = 1, \ldots, R$.

Note that implementation of the Cholesky factor bootstrap does not involve conditioning on the economic model; instead, we generate the bootstrap samples directly from the sample autocovariance matrix of the data. This is important in our environment, in which all models are best regarded as false.

In closing this section, let us elaborate on our allowance for differential weighting by frequency. There are at least two reasons for entertaining this possibility. First, the use of a loss function that weights differentially by frequency may be helpful in dealing with measurement error, which may not contaminate all frequencies equally. In such situations, it may be prudent to downweight those frequencies that are likely to be more contaminated by measurement error. Second, the use of a loss function that weights differentially by frequency may be important in misspecified models. For example, as discussed by Hansen and Heckman (1996), model misspecification may contaminate some frequencies more than others. Examples of this include potential contamination at seasonal frequencies, as in the work of Hansen and Sargent (1993) and Sims (1993). Watson (1993) also advocates the use of differential weighting in parameter estimation, for the same reason, although he does not pursue the matter, and notes that optimizing a loss function at particular frequencies corresponds to constructing an analogue estimator along the lines of Manski (1988).

Multivariate

The multivariate analogue of our earlier loss function is

$$C_{GW}(\theta)=\int_0^{\pi} G(F_m(\omega;\theta), F^*(\omega)) \odot W(\omega)d\omega,$$

where $\odot$ denotes component-by-component multiplication. The multivariate analogue of our earlier univariate quadratic loss function, for example, is $C_{GW}(\theta)=\sum_j \text{tr}\,(D'(\omega_j;\theta)D(\omega_j;\theta))$, where $D(\omega_j;\theta)=F_m(\omega_j;\theta)-F^*(\omega_j)$, $\omega_j=2\pi j/T$, $j=1,\ldots,T/2-1$.

The estimation criterion function has the same form as in the univariate case,

$$\hat{\theta}_{GW}=\underset{\theta}{\text{argmin}}\; C_{GW}(\theta),$$

and the bootstrap approaches to computing standard errors, confidence intervals, and hypothesis testing parallel the univariate case precisely. Furthermore, as expected, the multivariate Gaussian ML estimator emerges as a special and potentially restrictive case; it is

$$\underset{\theta}{\text{argmax}}\left(-\tfrac{1}{2}\textstyle\sum_j \ln|F_m(\omega_j;\theta)|-\tfrac{1}{2}\,\text{tr}\sum_j F_m^{-1}(\omega_j;\theta)\hat{F}(\omega_j)\right).$$

It is worth emphasizing how all parts of the spectrum contribute to loss in the multivariate case. Consider, for example, a bivariate model (variables x and y) under quadratic loss. Then

$$D(\omega_j;\theta)=\begin{pmatrix} d_{xx}(\omega_j;\theta) & d_{xy}(\omega_j;\theta)\\ d_{yx}(\omega_j;\theta) & d_{yy}(\omega_j;\theta)\end{pmatrix},$$

where

$$\begin{aligned} d_{xx}(\omega_j;\theta)&=f_{xx,m}(\omega_j;\theta)-f^*_{xx}(\omega_j),\\ d_{yy}(\omega_j;\theta)&=f_{yy,m}(\omega_j;\theta)-f^*_{yy}(\omega_j),\\ d_{xy}(\omega_j;\theta)&=f_{xy,m}(\omega_j;\theta)-f^*_{xy}(\omega_j),\\ d_{yx}(\omega_j;\theta)=f_{yx,m}(\omega_j;\theta)-f^*_{yx}(\omega_j)&=\overline{f_{xy,m}(\omega_j;\theta)}-\overline{f^*_{xy}(\omega_j)}=\overline{d_{xy}}(\omega_j;\theta).\end{aligned}$$

Thus,

$$\begin{aligned}\text{tr}\,(D'(\omega_j;\theta)D(\omega_j;\theta))&=[d^2_{xx}(\omega_j;\theta)+d_{xy}(\omega_j;\theta)d_{yx}(\omega_j;\theta)]\\ &\quad+[d^2_{yy}(\omega_j;\theta)+d_{xy}(\omega_j;\theta)d_{yx}(\omega_j;\theta)]\\ &=d^2_{xx}(\omega_j;\theta)+2|d_{xy}(\omega_j;\theta)|^2+d^2_{yy}(\omega_j;\theta)\\ &=[f_{xx,m}(\omega_j;\theta)-f^*_{xx}(\omega_j)]^2\\ &\quad+2[\text{re}\,(f_{xy,m}(\omega_j;\theta))-\text{re}\,(f^*_{xy}(\omega_j))]^2\\ &\quad+2[\text{im}\,(f_{xy,m}(\omega_j;\theta))-\text{im}\,(f^*_{xy}(\omega_j))]^2\\ &\quad+[f_{yy,m}(\omega_j;\theta)-f^*_{yy}(\omega_j)]^2.\end{aligned}$$

This expression shows clearly how the goodness of fit of both univariate spectra, as well as both the real and imaginary parts of the cross spectrum, contribute to loss.

4. APPLICATION: THE U.S. CATTLE CYCLE

In this section, we provide a detailed illustration of the use of our assessment and estimation techniques by applying them to an important model of the dynamics of beef cattle consumption and stock developed by Rosen, Murphy and Scheinkman (RMS, 1994). The so-called "cattle cycle," documented for example in Mundlak and Huang (1996), is a well-known phenomenon in agricultural economics. This simple yet rich model allows us to illustrate clearly the application of all the tools in our framework, and moreover, our findings provide new insight into the RMS model and its agreement with the data.

The data

We use annual data on U.S. cattle consumption and stock, 1900–1989, kindly supplied by Sherwin Rosen and originally obtained from *Historical Statistics: Colonial Times to 1970* and *Agricultural Statistics*, published by the U.S. Department of Agriculture. We plot the series in Figures 1 and 2; the cycle is visually apparent. Moreover, the series are clearly trending. Following RMS, we remove a linear trend from each series prior to additional analysis, allowing for a break in the slope of the trend in 1930; we also show the fitted trends in Figures 1 and 2.

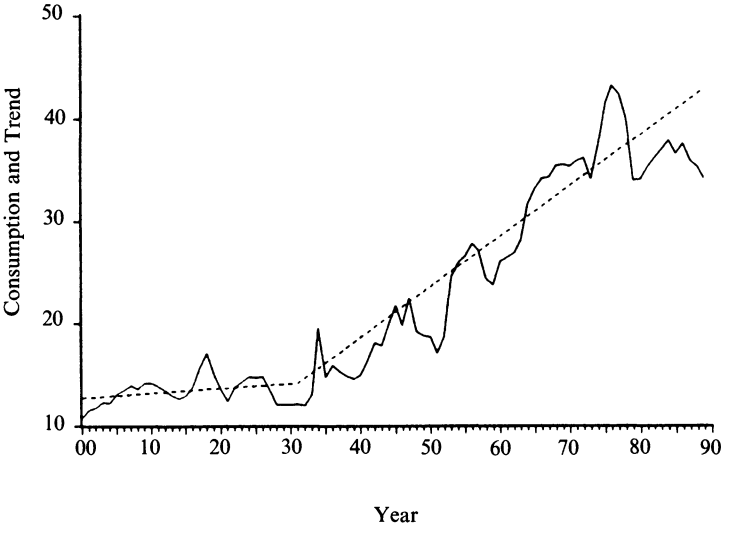

FIGURE 1

U.S. cattle consumption, 1900–1990, actual and estimated trend. (We show cattle consumption (solid line) and the estimated kinked-linear trend (dashed line))

We obtain the estimated data spectrum, presented in Figure 3, by smoothing the sample autocovariance function using a Bartlett window with truncation lag 24. We make use, here and throughout, of a matrix graphic with univariate spectra plotted on the main diagonal, coherence in the upper-right corner, and phase in the lower-left corner. In addition, following standard practice, we graph log spectra rather than spectra throughout and refer to the log spectra simply as "spectra." Not all frequencies are of equal interest, however. The frequencies most relevant to an investigation of the cattle cycle, typically thought to have a period of roughly ten years, are not those in the entire $[0, \pi]$ range,

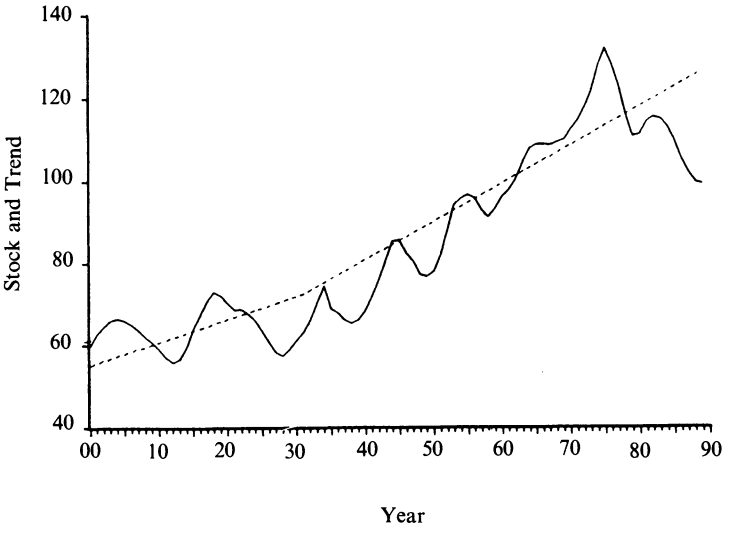

FIGURE 2

U.S. cattle stock, 1900–1990, actual and estimated trend. (We show cattle stock (solid line) and the estimated kinked-linear trend (dashed line))

but rather those in a *subset* that excludes very low and very high frequencies. This presents no problem for our procedures and in fact provides a good opportunity to illustrate the ease with which they can be tailored to study specific applications. Thus, for much of our analysis, we concentrate on the frequency band corresponding to periods of 30 years to 4 years, indicated by the shaded region in Figure 3 and subsequent figures.

Four features of the point estimates of the data spectrum stand out. First, the consumption spectrum (and to a lesser extent, the stock spectrum) displays a power concentration at roughly a ten-year cycle. Second, both the consumption and stock spectra otherwise have Granger's (1966) typical spectral shape, with high power at low frequencies, and declining power throughout the frequency range. Third, the coherence between consumption and stock is generally high and varies across frequencies, with a maximum (about 0·85) at roughly a ten-year cycle. Finally, the phase shift, measured in years by which consumption leads stock, varies with frequency; within the band of interest, the maximum phase shift of roughly one year again occurs at a ten year cycle.

In Figure 4 we present the data spectrum along with 90% confidence tunnels computed using the conservative Bonferroni technique in conjunction with the Cholesky-factor bootstrap. All of the point estimates display substantial uncertainty, as manifest in the wide 90% confidence tunnels. Such uncertainty associated with estimated spectra is typical of economic time series, although it often goes unacknowledged.

The model

We begin with some accounting identities. The head count of all animals (y_t) is the sum of the adult breeding stock (x_t), the stock of calves (assumed equal to gx_{t-1}), and the stock of yearlings (assumed equal to gx_{t-2}), where g is a fertility parameter. That is,

$$y_t = x_t + gx_{t-1} + gx_{t-2}.$$

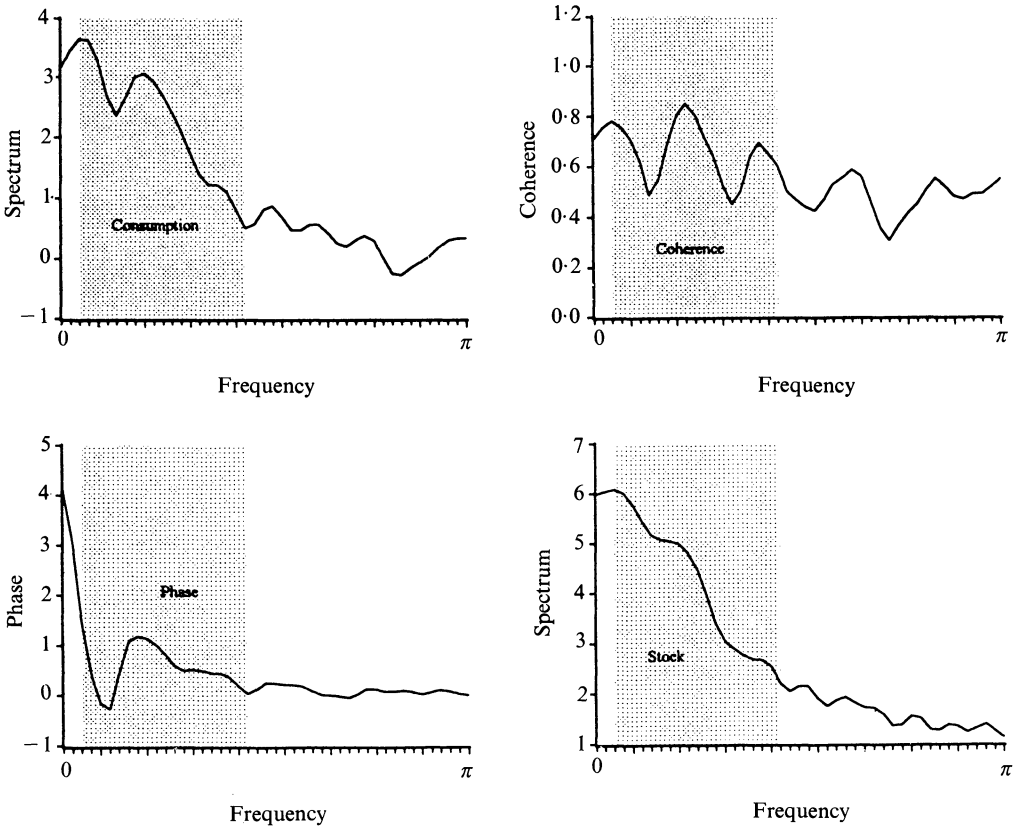

FIGURE 3

Estimated spectral density matrix, U.S. cattle consumption and stock. (We show the point estimate of each element of the spectral density matrix. The shaded frequency band corresponds to cycles with periods of 30 to 4 years)

The adult breeding stock consists of surviving stock from the previous period (assumed equal to $(1-\delta)x_{t-1}$) and the yearlings from $t-1$ entering the adult herd (gx_{t-3}) less the number that are marketed (c_t)

$$x_t = (1-\delta)x_{t-1} + gx_{t-3} - c_t.$$

We are concerned with the equilibrium determination of c_t and y_t. The risk-neutral rancher maximizes the present discounted value of expected profits, which involves equating the expected marginal benefit of marketing an animal for consumption to the expected marginal benefit of holding the animal for breeding. First, suppose that the rancher markets the animal for consumption. He receives net revenue $q_t = p_t = m_t$, where p_t is price and m_t is finishing cost. Alternatively, suppose the rancher holds an animal for breeding. Expected discounted net revenue is the sum of expected discounted revenue from selling tomorrow plus expected discounted revenue from marketing its offspring, less expected total holding costs (z_t), $E_t[\beta(1-\delta)q_{t+1} + \beta^3 g q_{t+3} - z_t]$. Total holding cost equals the sum of time t holding costs (h_t), discounted holding costs of the resultant time $t+1$ calves, and discounted holding costs of the resultant time $t+2$ yearlings. That is, $z_t = h_t + \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}$ (assuming proportional costs for calves and yearlings, γ_0 and γ_1).

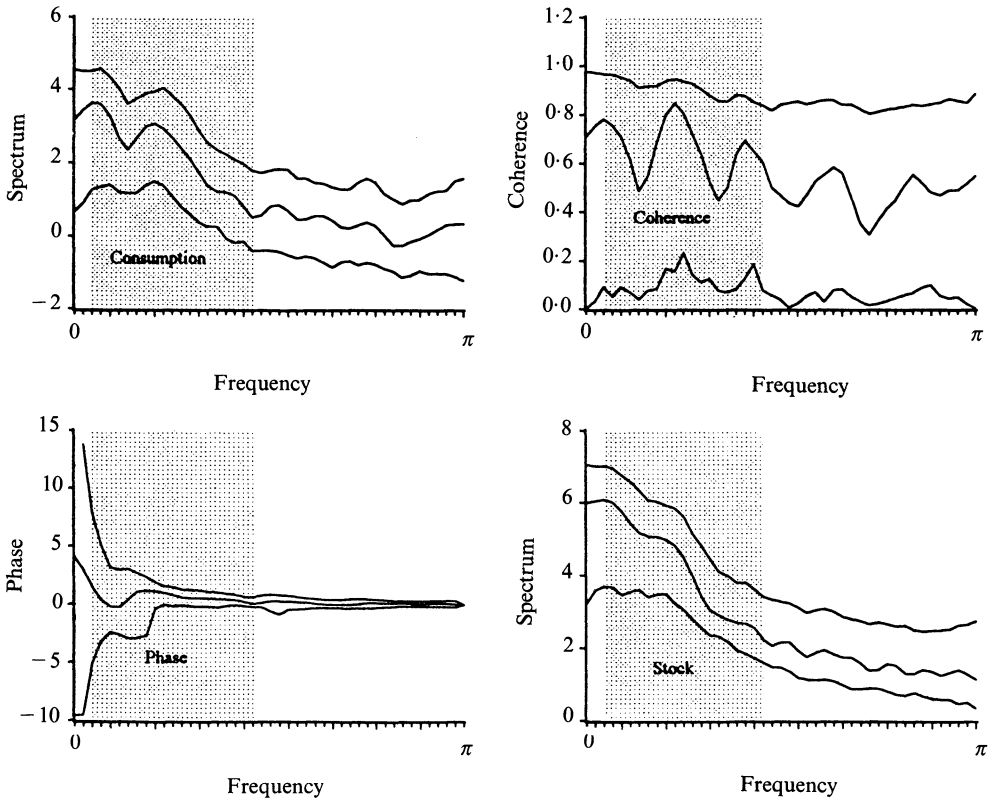

FIGURE 4

Estimated spectral density matrix and confidence tunnels U.S. cattle consumption and stock. (We show the point estimate together with a 90% confidence tunnel for each element of the spectral density matrix. The shaded frequency band corresponds to cycles with periods of 30 to 4 years)

In equilibrium, the expected marginal net revenue from marketing for consumption equals the expected marginal discounted net revenue from holding for breeding; that is,

$$E_t[q_t] = E_t[\beta(1-\delta)q_{t+1} + \beta^3 g q_{t+3} - z_t].$$

We close the model by specifying the exogenous processes $\{m_t, h_t, d_t\}$ as first-order autoregressions.[7] Following RMS, we assume that each of the three shocks has common serial-correlation parameter ρ.

The model structure implies that the reduced-form equations for c_t and y_t can be expressed in terms of a single disturbance, ω_t, which is a linear combination of the independent innovations from the three AR(1) driving processes. In particular, $c_t \sim$ ARMA(2, 1) and $y_t \sim$ ARMA(4, 2)

$$(1-\lambda_1 L)(1-\rho L)c_t = -(1-\phi_1 L)\omega_t.$$

$$(1-\lambda_1 L)(1-\phi_2 L)(1-\phi_3 L)(1-\rho L)y_t = (1+gL+gL^2)\omega_t,$$

7. d_t is a preference shock. We have not discussed the demand side of the model, because we do not use it in estimation.

where ϕ_1 is the one unstable root and $\{\phi_2, \phi_3\}$ are the two stable roots of

$$\phi^3 - (1-\delta)\phi^2 - g = 0,$$

and λ_1 is the one stable root of

$$g\beta^3\lambda^3 + (1-\delta)\beta\lambda - 1 = 0.$$

The associated univariate spectra are

$$f_c(\omega) = \sigma_\omega^2 \frac{|(1-\phi_1 e^{i\omega})|^2}{|(1-\lambda_1 e^{i\omega})(1-\rho e^{i\omega})|^2},$$

$$f_y(\omega) = \sigma_\omega^2 \frac{|(1+ge^{i\omega}+ge^{2i\omega})|^2}{|(1-\lambda_1 e^{i\omega})(1-\phi_2 e^{i\omega})(1-\phi_3 e^{i\omega})(1-\rho e^{i\omega})|^2},$$

and the cross spectrum is

$$f_{cy}(\omega) = \frac{-(1-\phi_1 e^{i\omega})(1-\phi_2 e^{i\omega})(1-\phi_3 e^{i\omega})}{(1+ge^{i\omega}+ge^{2i\omega})} f_y(\omega).$$

These equations provide a full description of the model in the frequency domain. σ_ω^2 is a complicated function of the structural parameters, including some from the demand side of the model. All of the parameters of present interest, however, may be identified from the other reduced-form parameters, with the exception of γ_0 and γ_1. We therefore treat σ_ω^2 as a free parameter and estimate it subject to no restrictions.

RMS do not estimate the cattle cycle model. Rather, they choose values for the behavioural parameters and report that the calibrated model fits the data well. In the following sections, we explicitly estimate the model and assess its performance.

Estimating the model

Estimation requires specification of an explicit loss function. We use a loss function that incorporates the focus in the cattle cycle literature on cycles of roughly 10 years. We exclude frequencies corresponding to periods of more than 30 years or less than 4 years, which from the standpoint of our earlier discussion of frequency downweighting corresponds to giving equal weight to frequencies in the band of interest, and giving zero weight to frequencies outside the band. Minimization of such a loss function, which measures divergence between model and data spectra only within a particular frequency band, leads us to an estimator that we call band-restricted maximum likelihood (Band-ML).[8]

We perform the minimization using the simplex algorithm, which is a derivative-free method, as implemented in the Matlab fmins.m procedure. Using penalty functions, we constrain the discount factor to be between 0·65 and 1·00, the fertility rate to be between 0·00 and 1·00, the death rate to be between 0·00 and 1·00, the persistence parameter to be between 0·00 and 1·00, and the scale parameter to be between 0·10 and 7·00. We start the iterations with the RMS parameter values for the discount rate, fertility rate, death rate, and persistence parameter; RMS did not report a value for the scale parameter, which we start at 1·7. In our experience, estimation is numerically straightforward and stable; the estimated parameter vector is always in the interior of the constraint set,

8. Gaussian Band-ML is the maximum-likelihood analogue of Engle's (1974) band-spectral linear regression. Band-ML may of course be undertaken for models more complicated than simple linear regression, such as the present one.

TABLE 1

Parameter estimates
Band-restricted maximum likelihood estimation

Parameter	β	g	δ	ρ	σ_ω^2
Estimation or Calibration Method					
Band-ML	0·86	0·67	0·08	0·21	2·10
	(0·03)	(0·09)	(0·03)	(0·10)	(0·37)
RMS	0·909	0·85	0·10	0·60	(NA)
	(NA)	(NA)	(NA)	(NA)	(NA)

Notes: β is the discount factor, g is the fertility rate, δ is the death rate, and ρ is the persistence parameter. Band-ML denotes band-restricted maximum likelihood estimation, with the frequency band used for estimation corresponding to periods from 30 to 4 years. Standard errors, based on 200 bootstrap replications, appear in parentheses. RMS denotes the Rosen–Murphy–Scheinkman calibrated parameters. (They have no standard errors, because they were not estimated.)

convergence is fast, and alternative starting values produce virtually identical estimates.[9]

We display the Band-ML estimates in Table 1; for comparison, we also display the RMS parameter values. Several of the Band-ML parameter estimates are similar to those chosen by RMS. In particular, the estimate of the death rate parameter (0·08) is nearly identical to the RMS value (0·10), the estimate of the producer's discount factor (0·86) is close to the RMS value (0·91), and the estimated fertility parameter (0·67) is lower than but nevertheless close to the RMS value (0·85). As stressed in Diebold (1998), however, even small differences in parameters can make important differences for activities such as forecasting.

The Band-ML estimate of the persistence parameter, however, differs substantially from the RMS value. RMS choose a fairly persistent value of 0·6; in contrast, we find that optimizing the Band-ML loss function requires very little persistence in the driving process (0·2). This implies that the RMS model has a strong internal propagation mechanism: the model takes shocks with relatively little serial correlation and transforms them into series that display substantial persistence in equilibrium. This dimension of the RMS model differs fundamentally from standard dynamic equilibrium models used in macroeconomics, international economics, and public finance. As Watson (1993) and others have noted, models used in those fields typically have weak internal propagation mechanisms—they require highly persistent underlying shocks to generate a realistic amount of serial correlation in the variables determined in equilibrium.

In addition to finding the parameter estimates that maximize agreement between model and data, we can assess their sampling uncertainty within our framework. Standard errors are of some use in that regard, in spite of the fact that the sampling distributions need not be Gaussian. We compute them using 200 replications of the Cholesky factor bootstrap procedure, and we report them in parentheses below the estimated parameters in Table 1. More generally, our bootstrap procedures allow us to estimate the *entire* sampling distributions of the estimated parameters; we report on them in Figure 5. The

9. One wrinkle does arise: simple economic models such as this one, in which one shock drives the evolution of a higher-dimensional system, have a singular spectral density F_m. This presents a problem for implementation of the Gaussian ML and Band-ML estimators, which involve F_m^{-1}. We have found that the problem can be satisfactorily skirted in practice, because the model spectrum is typically obtained by simulating a long realization from the model, whose spectrum is then consistently estimated. This has the effect of introducing just enough "measurement error" to avoid a singular spectrum.

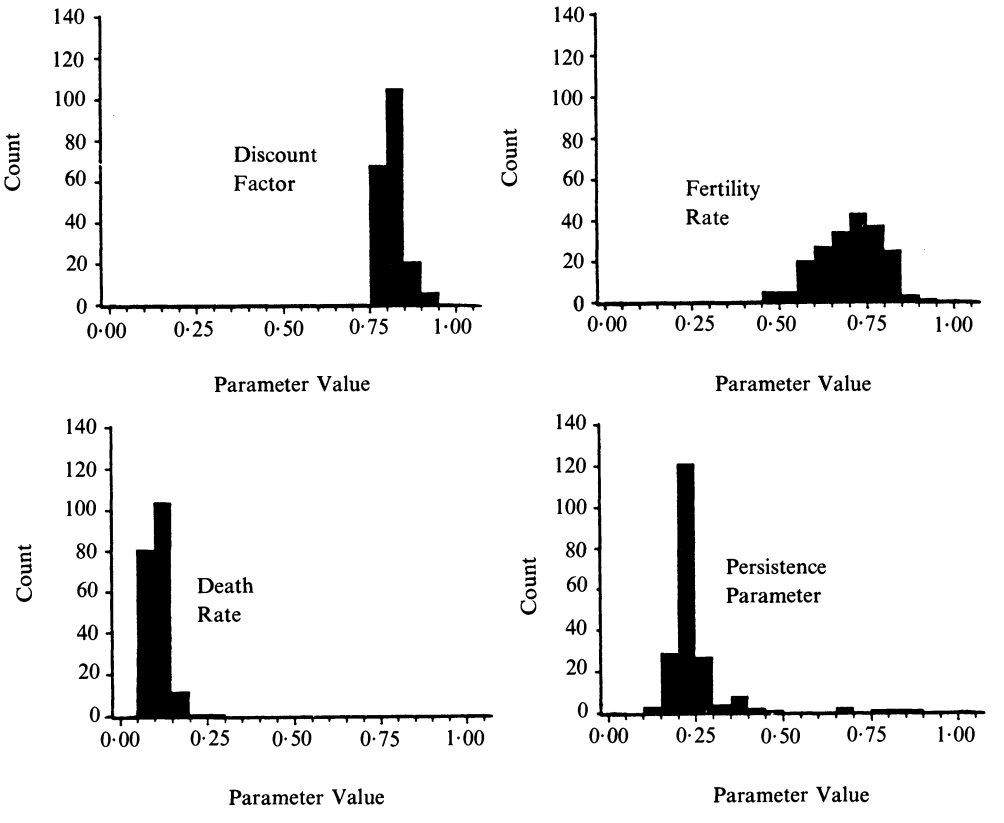

FIGURE 5

Bootstrap estimates of sampling distributions. (Estimated sampling distributions are based on 200 bootstrap replications)

estimated sampling distributions of the discount factor, the depreciation rate, and the persistence parameter are fairly concentrated, while the estimated sampling distribution of the fertility rate is more dispersed.

Our framework also enables us to examine the joint distribution of the estimated parameters. In Table 2 we present bootstrap estimates of the correlations between the estimated parameters. Perhaps the most interesting relationship is the strong negative correlation between the discount factor and the fertility rate, which occurs because the discount factor and the fertility rate enter multiplicatively in one of the cubic equations that define the ARMA polynomials. This implies that the loss function trades off high fertility rates for low discount factors, and suggests that fixing either one of the parameters at the higher RMS value would tend to result in an even *lower* estimate for the other.

Assessing the model

To assess divergence between model and data, we plot the model spectrum evaluated at the Band-ML parameter estimates in Figure 6, together with the earlier-discussed point estimates and 90% confidence tunnels for the data spectrum, produced with 200 replications

TABLE 2

Estimated parameter correlations
Band-restricted maximum likelihood estimation

	β	g	δ	ρ
β	1·00			
g	−0·73	1·00		
δ	0·49	−0·37	1·00	
ρ	−0·19	0·10	0·06	1·00

Notes: β is the discount factor, g is the fertility rate, δ is the death rate, and ρ is the persistence parameter. Estimated parameter correlations are based on 200 bootstrap replications. The frequency band used for estimation corresponds to periods from 30 to 4 years.

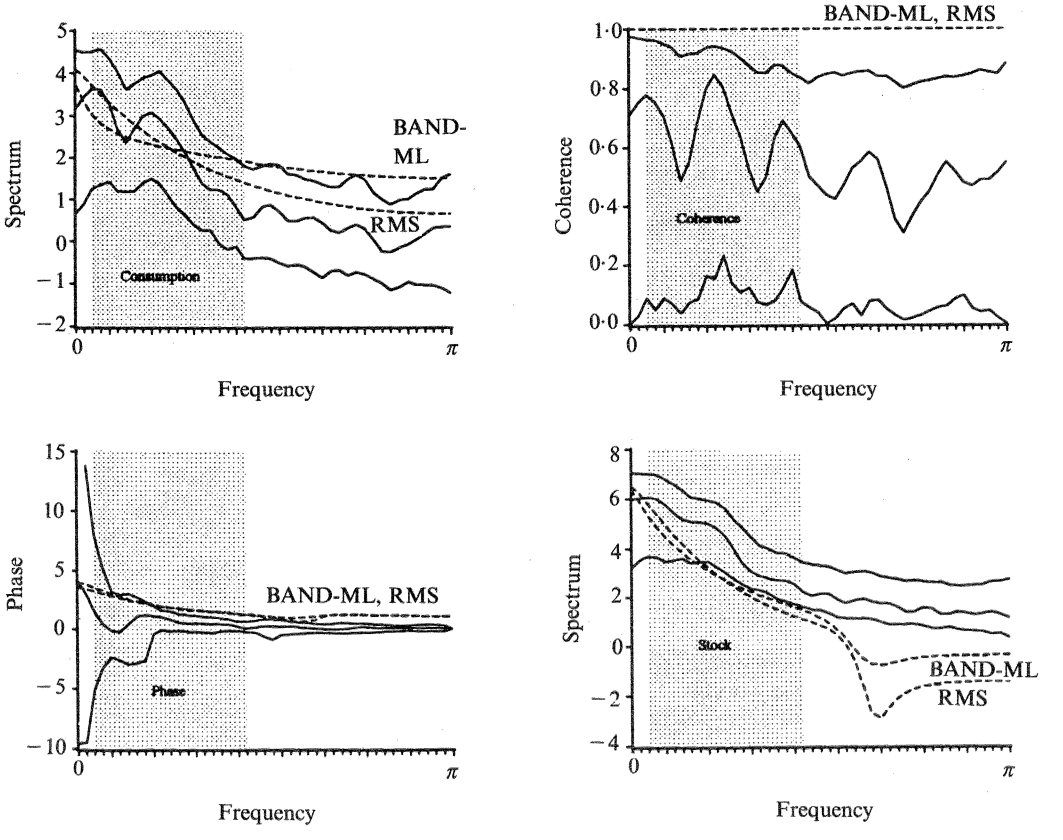

FIGURE 6

Model spectra, and data spectra confidence tunnels, U.S. cattle consumption and stock. (We show the 90% confidence tunnel for the data spectrum, together with the model spectrum evaluated at the band-restricted maximum likelihood parameter values, for each element of the spectral density matrix. The shaded frequency band corresponds to cycles with periods of 30 to 4 years)

of the non-parametric Cholesky factor bootstrap. The diagonal elements of the figure provide comparative assessment of model and data univariate dynamics, and the off-diagonal elements provide comparative assessments of cross-variable dynamics.

Given the objective of constructing a simple model that is consistent with periodic behaviour in consumption and stock, a surprising finding is that neither the consumption

nor the stock model spectrum has a peak corresponding to a ten-year cycle, which suggests that the model does not easily produce cyclical behaviour at the Band-ML parameter values. Moreover, the rate of decay of the model consumption spectrum appears slower than that of the data spectrum; thus, although the model and data consumption spectra are insignificantly different over most of the relevant frequency range, they begin to deviate significantly for cycles with periods of 4 years or less. Second, and conversely, the rate of decay of the model stock spectrum appears significantly *faster* than that of the data spectrum. The two diverge not only at high frequencies, but also over much of the relevant frequency range. In particular, the model stock spectrum lies slightly outside the lower region of the 90% confidence tunnel for cycles of about 20 years and less.

Now let us examine phase and coherence. As with the model spectra, the model phase shift declines monotonically, which contrasts somewhat with the point estimate of the data phase shift, which has a local peak at roughly the ten-year cycle. In addition, the model generates significantly more phase shift than one sees in the data, particularly at the low and middle frequencies of interest. Finally, the model coherence reminds us of an inherent limitation of the model: because it is driven by a single shock, the model is singular, which forces unit coherence at all frequencies. Thus the model and data coherence diverge sharply; in spite of the fact that the confidence tunnel is very wide, the unit model coherence is always outside the data coherence confidence tunnel.

We also show in Figure 6 the model spectrum evaluated at the RMS parameter values. The major difference between the two parameterizations of the model, within the frequency band of interest, is revealed in the spectral density of consumption. The RMS parameters generate substantially more variation at low and middle frequencies than do the Band-ML parameters. The differences in the phase shift and the spectral density of stock across the two parameterizations of the model are modest.

All told, comparison of the Band-ML and RMS parameters, and model spectra evaluated at those parameters, reveals some similarities and some differences, with the similarities outnumbering the differences. The real insight afforded by our analysis is that the similar model behaviour across different parameterizations is due to several model deficiencies, and that the deficiencies cannot be remedied simply by exploring different estimators; rather, they need to be remedied at the source by improving the model. We hope that our results stimulate such work; an appropriately improved model might include richer dynamics for driving processes, incorporation of a demand side and prices, and explicit determination of trend.

5. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

We have described a framework for evaluating dynamic economic models that should be useful to applied economists in many fields. The framework is flexible—it can be used by researchers to evaluate purely calibrated models, and it can also be used by researchers interested in estimating parameters and conducting inference. Its frequency-domain foundations provide useful diagnostics that nicely complement alternative time-domain approaches. In particular, it is graphical and constructive, and it takes seriously several important issues in the quantitative analysis of simple, dynamic equilibrium models: model misspecification, the user's objectives, and small sample sizes.

Our analysis of the RMS model of cattle cycles illustrated the use of our tools for formally estimating models and performing statistical inference, as well as for assessing agreement between models and data at various parameter values, whether pre-set or estimated. In addition, it shed new light on the characteristics of the RMS model, and in particular, its strong internal propagation mechanism. Our analysis also revealed that the

model is deficient in several key respects, not the least of which is its inability to generate internal spectral peaks.

The ultimate goal of the research programme of which this paper is a part is to facilitate communication between researchers with potentially very different research objectives and strategies, thereby bringing modern dynamic economic theory into closer and more frequent contact with dynamic economic data. As economists use richer and more complicated models to understand a wider variety of data, we hope that our framework will find use in discerning the dimensions along which models are consistent—and inconsistent—with data. That information can in turn be used to help construct new and improved models.

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