

*(Appendix to Realized Beta: Persistence and Predictability,
by Andersen, Bollerslev, Diebold and Wu, 2004)*

Analysis of Monthly Realized Beta
Based on High-Frequency Intraday Data*

by

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Here we further explore the dynamic nature of realized betas and their components by studying monthly realized betas constructed from high-frequency intraday returns. The time span covered is shorter, because availability of intraday data is limited, but the improvement in the quality of the measurement still allows a precise characterization of the dynamics.

Data

Our analysis is based on data from the TAQ (Trade And Quotation) database. The TAQ data files contain continuously recorded information on the trades and quotations for the securities listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the National Association of Security Dealers Automated Quotation system (NASDAQ). The database is published monthly, and has been available on CD-ROM from the NYSE since January 1993; we refer the reader to the corresponding data manual for a more complete description of the actual data and the method of data-capture. Our sample extends from January 4, 1993 until September 30, 1999, for a total of 1,366 trading days. A complete analysis based on all trades for all stocks, although straightforward conceptually, is infeasible in practice. We therefore restrict our analysis to the same set of DJIA firms that we analyzed using daily data above. This also helps ensure a reasonable degree of liquidity. We display the specific stocks in Table 1H (“H” indicates “high-frequency,” in reference to the underlying 15-minute sampling). Finally, the corresponding market return is the DJIA cash index.

Although the DJIA stocks are among the most actively traded U.S. equities, the median inter-trade duration for all stocks across the full sample is 23.1 seconds, ranging from a low of 7 seconds for Merck & Co. Inc. (MRK) to a high of 54 seconds for United Technologies Corp. (UTX). As such, it is not practically feasible to push beyond this level the continuous record asymptotics and the length of the observation interval Δ in equation (3) of the main text. Moreover, because of the organizational structure of the market, the available quotes and transaction prices are subject to discrete clustering and bid-ask bounce effects. Such market microstructure features are generally not important when analyzing longer horizon interdaily returns but can seriously distort the distributional properties of high-frequency intraday returns; see, e.g., the textbook treatment by Campbell, Lo and MacKinlay (1997). For simplicity, we rely on artificially constructed equally-spaced intraday returns, as discussed for example by Andersen and Bollerslev (1997).¹ It turns out that a 15-minute sampling frequency is sufficient to alleviate the non-synchronous trading problem so that the market return is nearly uncorrelated and the individual stock return series are approximately serially uncorrelated and orthogonal to all but the first lagged and leading 15-minute market

¹ An alternative but much more complicated approach would be to utilize all of the observations by explicitly modeling the high-frequency frictions; see, also Andersen, Bollerslev, Diebold and Labys (2000), Andreou and Ghysels (2002), Hansen and Lunde (2003), Bandi and Russell (2003), and Zhang, Mykland and Ait-Sahalia (2003), among others, for further discussion or alternative procedures in this context.

returns. Following the logic of Scholes and Williams (1977),² our high-frequency based estimate of beta then consists of the contemporaneously measured beta plus the first-order lag and lead beta obtained from the covariance of the individual 15-minute stock return with the lagged (respectively lead) 15-minute market return divided by the market return variance estimate adjusted for the (almost negligible) first-order serial correlation in the market index. With the daily transaction record extending from 9:30 EST until 16:00 EST, there are a total of 26 fifteen-minute returns for each day, corresponding to $\Delta = 1/26 \approx 0.0385$ in the notation above, and thus about 570 underlying intraday observations for the monthly beta calculations. This frequency should be sufficiently high to reduce the measurement errors substantially while also avoiding the confounding influences from market microstructure frictions. The main advantage of using the intraday returns is that we obtain less noisy measures of the relevant variance and covariances – conditional on the chosen 15-minute frequency controlling appropriately for market microstructure features – because we are pushing closer to continuous sampling, while the drawback is the much shorter time span available for analysis. Because the analysis in the main text points toward the potential presence of short-run dependence in betas, we focus on the monthly interval to explore this possibility in some depth.

Point Estimates of Monthly Betas

Figures 1H and 2H reveal a strong shift upward in both market volatility and the stocks covariances with the market in the latter half of the sample, where the values are both higher and much more volatile. In contrast, there is no visual evidence of any shift in the realized beta series in Figure 3H. The latter figures occasionally display distinct outliers but these are not concentrated toward the second half of the sample, and the enhanced volatility in the market variance and the individual covariances in the second part of the 1990s is certainly not visible, implying that the dominant dynamic components in the variances and covariances tend to annihilate each other. As with the quarterly realized betas based on daily data, there is no sign of integer integration in any of the realized variance, covariance or beta measures and the median Ljung-Box statistic for the beta series of 48 in Table 2H is below that of the covariances, 59 and the market variance, 79, supporting the visual impression of relatively less serial dependence in the realized monthly betas.

Further confirmation of the relative strength of the serial dependence across the series is evident in Figures 4H-6H. The autocorrelations generally appear significant, or nearly so, for the realized market variance and the individual covariances out to an order of fifteen months. This is consistent with the significance of the autocorrelations out to lags four to six in the quarterly measures in Figures 4-6. Moreover, the median correlogram for the realized covariances again appears to be a dampened version of that for the market variance. In contrast, the individual realized betas rarely are significant beyond a lag

² See also the more recent discussion in Bollerslev and Zhang (2003).

length of five or six months, consistent with only the first few autocorrelation coefficients being significant in Figure 6. From Figure 6H it is evident that there is a large degree of heterogeneity in the results across the individual stocks, but the dominant pattern is clearly one with significantly positive coefficients at lower lags but also one with a sharply declining correlogram. This is again consistent with a cancellation occurring between the variance and covariance measures in the construction of the betas. Because none of the individual realized beta series display significant negative correlation at lower lags, the results point strongly toward the presence of a relatively short-lived but highly significant degree of positive serial dependence in the betas, as also evidenced by the median beta autocorrelation pattern in Figure 4H.

Continuing along the lines above, Figures 7H-9H present sample autocorrelations of the monthly realized measures, each prefiltered by $(1 - L)^{.42}$. Because the degree of fractional integration is invariant to the sampling frequency and temporal aggregation, it is natural to compare these figures to those from the quarterly realized volatility measures. The message is in important respects similar to that for quarterly realized betas computed from daily data: the market variance and the covariance measures seem consistent with a pure fractionally integrated process with $d = .42$. In contrast, however, the monthly realized beta series behave differently from their quarterly counterparts, as there is no direct indication of over-differencing this time. Of course, at the monthly frequency the serial dependence, as manifest in the correlogram, should appear stronger than for the quarterly series and thus generally blur our ability to distinguish the short run dependent case from the fractionally integrated one, in particular because the calendar period is now shorter. The discrepancy in behavior for the monthly and quarterly series for the realized betas is nevertheless an indication that the invariance property with respect to sampling frequency and time aggregation, as should occur if the series is truly $I(d)$, for $d > 0$, may be violated. This lends additional credibility to our earlier assertion in the main text that, all told, $I(0)$ dynamics for realized betas seemed reasonable. Consequently, we again also model the realized betas using a standard autoregression. Table 3H presents results for low-order $AR(p)$ models for each individual realized monthly beta series. The modulus of the dominant inverted root of the AR polynomial is now generally higher compared to Table 3, as is to be expected given the shorter monthly horizon for the betas. Figures 10H and 11H document that prefiltering using the estimated $AR(p)$ polynomial produces residuals with white noise characteristics and that the resulting predictability is lower than for the constituent components, the market variance and the covariances with the market. Overall, there are no signs that a null hypothesis of short memory dependence for the realized betas can be rejected.

All told, the results across the two sampling frequencies and time horizons are generally consistent. There is strong evidence of a much lower degree of dependence in the realized betas compared to the realized market variance and the realized covariances with the market return. Although the realized betas can be

approximated by a pure fractionally integrated process with a d of around .20 in the quarterly data – and perhaps an even higher degree of fractional integration in the shorter monthly sample – this may well be an artifact of the short sample span. There is clearly some heterogeneity across the stock betas but a standard short memory autoregressive process with significantly positive serial correlation for each of the individual realized betas appears robust across both estimation horizons and sample lengths.

Interval Estimates of Monthly Betas

Once we pass to the monthly beta estimation frequency, even the intraday sampling provides somewhat imprecise inference, as seen in Figure 12H. Nonetheless, the standard error bands remain tighter than for the quarterly beta estimates based on daily data, and it is feasible to establish significant time-variation in the monthly betas from Figure 12H.

Recall from the main text that there was not much difference in the assessment of the estimation uncertainty inherent in the quarterly beta measures obtained from the alternative Barndorff-Nielsen-Shephard and Newey-West procedures. This conclusion is largely confirmed from a comparison of the Newey-West standard error bands for the monthly betas based on the intraday data in Figure 13H with Figure 12H discussed above. The two sets of figures again appear qualitatively similar, although there is now evidence of significant differences in the width of the bands associated with some of the outliers in the series.

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Table1H
The Dow Jones Thirty

<u>Company Name</u>	<u>Ticker</u>	<u>Data Range</u>
Alcoa Inc.	AA	04/01/1993-09/30/1999
Allied Capital Corporation	ALD	04/01/1993-09/30/1999
American Express Co.	AXP*	04/01/1993-09/30/1999
Boeing Co.	BA	04/01/1993-09/30/1999
Caterpillar Inc.	CAT	04/01/1993-09/30/1999
Chevron Corp.	CHV	04/01/1993-09/30/1999
DuPont Co.	DD	04/01/1993-09/30/1999
Walt Disney Co.	DIS	04/01/1993-09/30/1999
Eastman Kodak Co.	EK	04/01/1993-09/30/1999
General Electric Co.	GE	04/01/1993-09/30/1999
General Motors Corp.	GM	04/01/1993-09/30/1999
Goodyear Tire & Rubber Co.	GT	04/01/1993-09/30/1999
Hewlett- Packard Co.	HWP	04/01/1993-09/30/1999
International Business Machines Corp.	IBM	04/01/1993-09/30/1999
International Paper Co.	IP	04/01/1993-09/30/1999
Johnson & Johnson	JNJ	04/01/1993-09/30/1999
JP Morgan Chase & Co.	JPM*	04/01/1993-09/30/1999
Coca-Cola Co.	KO	04/01/1993-09/30/1999
McDonald's Corp.	MCD*	04/01/1993-09/30/1999
Minnesota Mining & Manufacturing Co.	MMM	04/01/1993-09/30/1999
Philip Morris Co.	MO	04/01/1993-09/30/1999
Merck & Co.	MRK	04/01/1993-09/30/1999
Procter & Gamble Co.	PG	04/01/1993-09/30/1999
Sears, Roebuck and Co.	S	04/01/1993-09/30/1999
AT&T Corp.	T	04/01/1993-09/30/1999
Travelers Group Inc.	TRV*	04/01/1993-09/30/1999
Union Carbide Corp.	UK	04/01/1993-09/30/1999
United Technologies Corp.	UTX	04/01/1993-09/30/1999
Wal-Mart Stores Inc.	WMT*	04/01/1993-09/30/1999
Exxon Corp.	XON	04/01/1993-09/30/1999

Notes: The table summarizes company names and tickers, and the range of the data examined. We use the Dow Jones Thirty as of March 1997. Tickers with asterisks denote stocks with incomplete data, which we exclude from the analysis.

Table 2H
The Dynamics of Monthly Realized Market Variance, Covariances and Betas

v_{mt}^2	<u>Q</u>	<u>ADF¹</u>	<u>ADF²</u>	<u>ADF³</u>	<u>ADF⁴</u>					
	78.99	-3.989	-3.317	-2.814	-2.619					
		$cov(r_{mt}, r_{it})$				β_{it}				
	<u>Q</u>	<u>ADF¹</u>	<u>ADF²</u>	<u>ADF³</u>	<u>ADF⁴</u>	<u>Q</u>	<u>ADF¹</u>	<u>ADF²</u>	<u>ADF³</u>	<u>ADF⁴</u>
Min.	35.841	-4.658	-3.941	-3.531	-3.357	10.818	-5.474	-4.763	-5.732	-5.217
0.10	38.289	-4.543	-3.860	-3.451	-3.029	17.559	-4.753	-4.419	-4.158	-3.776
0.25	40.752	-4.358	-3.644	-3.253	-2.908	23.250	-4.475	-3.746	-3.332	-3.144
0.50	58.915	-4.099	-3.460	-3.094	-2.769	47.954	-3.822	-2.994	-2.778	-2.385
0.75	69.659	-3.848	-3.313	-2.815	-2.537	70.443	-3.392	-2.737	-2.576	-2.204
0.90	84.754	-3.685	-3.023	-2.486	-2.335	134.90	-3.173	-2.486	-2.194	-1.908
Max.	89.932	-3.599	-2.878	-2.374	-2.132	209.29	-2.445	-2.164	-1.802	-1.718
Mean.	57.340	-4.117	-3.464	-3.049	-2.735	58.182	-3.918	-3.224	-3.068	-2.736
St.Dev.	16.966	0.319	0.292	0.332	0.275	46.307	0.691	0.697	0.939	0.824

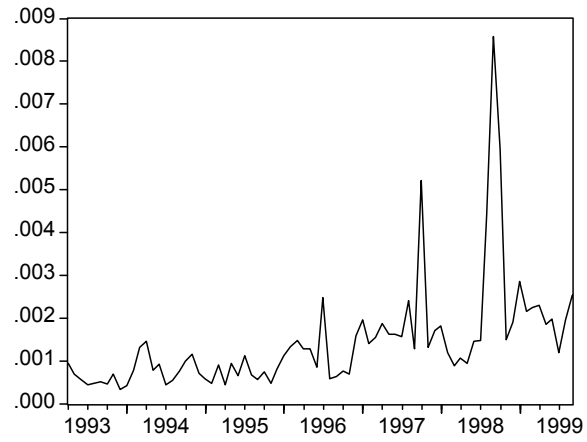
Notes: The table summarizes aspects of the time-series dependence structure of monthly realized market variance, covariances and realized betas. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation, and ADF^i denotes the augmented Dickey-Fuller unit root test with i augmentation lags. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minutes returns.

Table 3H
Inverted Roots of $AR(p)$ Models for Monthly Realized Betas

Stock	Inverted Roots (and Modulus of Dominant Inverted Root)				
AA	0.71	-0.28			(0.71)
ALD	0.78	-0.26 -0.34i	-0.26+0.34i		(0.78)
BA	0.68	-0.53			(0.68)
CAT	0.90	0.05+0.65i	0.05 -0.65i	-0.68	(0.90)
CHV	0.28				(0.28)
DD	0.54 -0.51i	0.54+.51i	-0.50 -0.49i	-0.50+0.49i	(0.74)
DIS	0.49				(0.49)
EK	0.82	0.14 -0.20i	0.14+0.20i	-0.42+0.41i	-0.42 -0.41i (0.82)
GE	0.54				(0.54)
GM	0.85	-0.49			(0.85)
GT	0.85	-0.29 -0.43i	-0.29+0.43i		(0.85)
HWP	0.55+0.34i	0.55 -0.34i	-0.15	-0.35+0.31i	-0.35 -0.31i (0.65)
IBM	0.25				(0.25)
IP	0.83	-0.29 -0.47i	-0.29+0.47i		(0.83)
JNJ	0.54				(0.54)
KO	0.77	-0.25 -0.46i	-0.25+0.46i		(0.77)
MMM	0.81	-0.20 -0.59i	-0.20+0.59i		(0.81)
MO	0.22				(0.22)
MRK	0.70+0.46i	0.70 -0.46i	-0.33+0.59i	-0.33 -0.59i	-0.39 (0.84)
PG	0.29				(0.29)
S	0.71	-0.43			(0.71)
T	0.62 -0.19i	0.62+0.19i	-0.46 -0.51i	-0.46+0.51i	(0.69)
UTX	0.69	-0.59			(0.69)
UK	0.77	-0.15+0.53i	-0.15 -0.53i		(0.77)
XON	0.70+0.27i	0.70 -0.27i	-0.23+0.74i	-0.23 -0.74i	-0.79 (0.79)

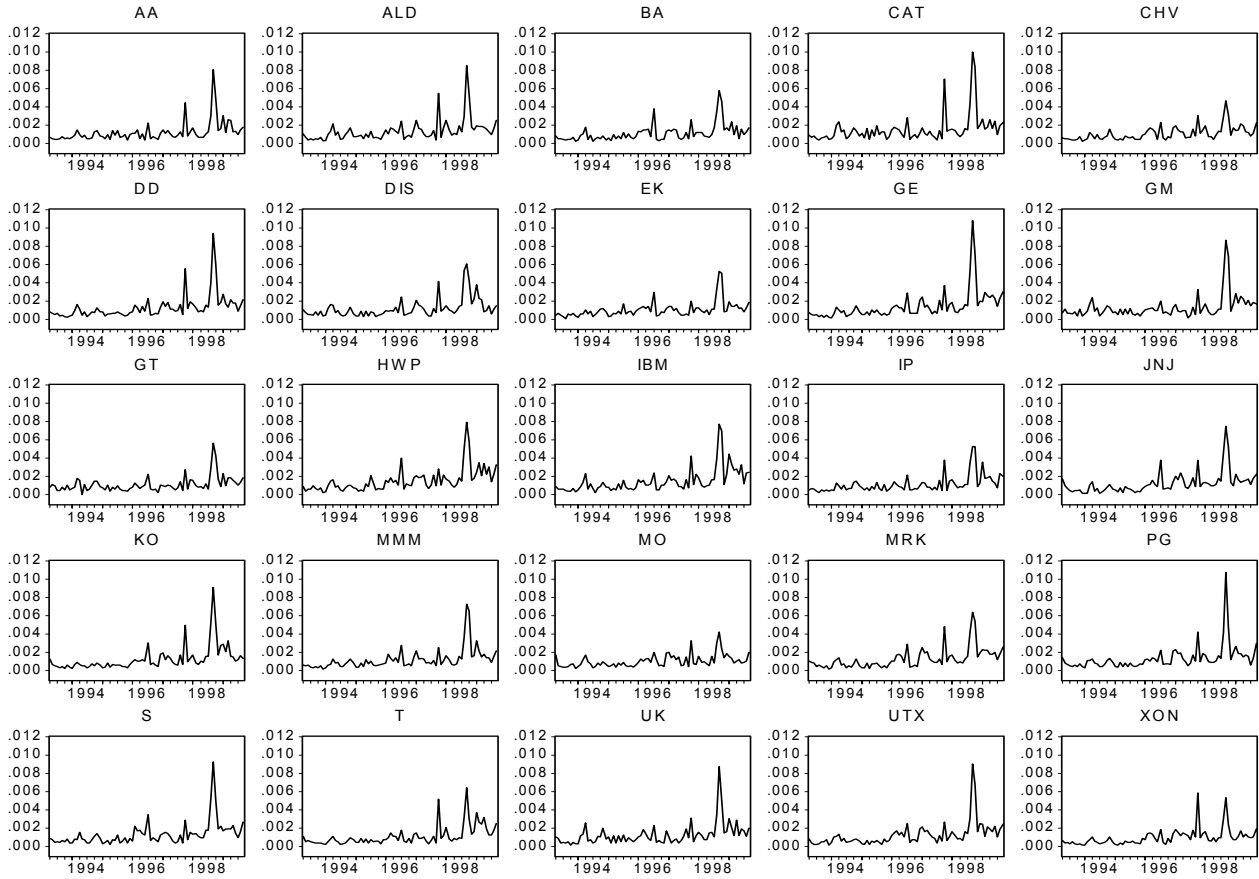
Notes: The table shows the inverted roots and modulus of the dominant root of the autoregressive lag operator polynomials $(1 - \hat{\phi}_1 L - \hat{\phi}_2 L^2 - \dots - \hat{\phi}_p L^p)$, where $\hat{\phi}_1, \hat{\phi}_2, \dots, \hat{\phi}_p$ are the least squares estimates of the parameters of $AR(p)$ models fit to the realized betas, with p selected by the AIC. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 1H
Time Series Plot of Monthly Realized Market Variance



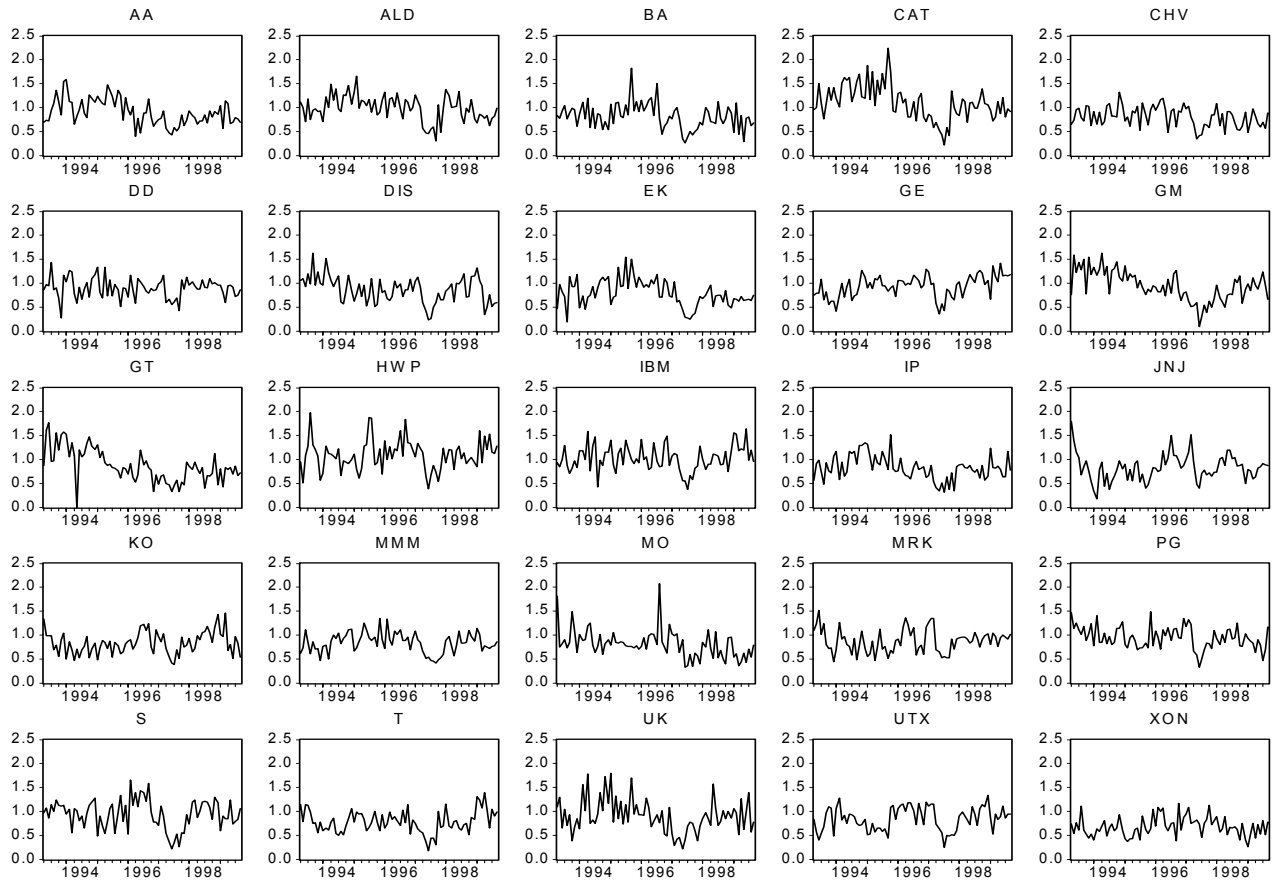
Notes: The figure shows the time series of monthly realized market variance. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly market variances from fifteen-minute returns.

Figure 2H
Time Series Plots of Monthly Realized Covariances



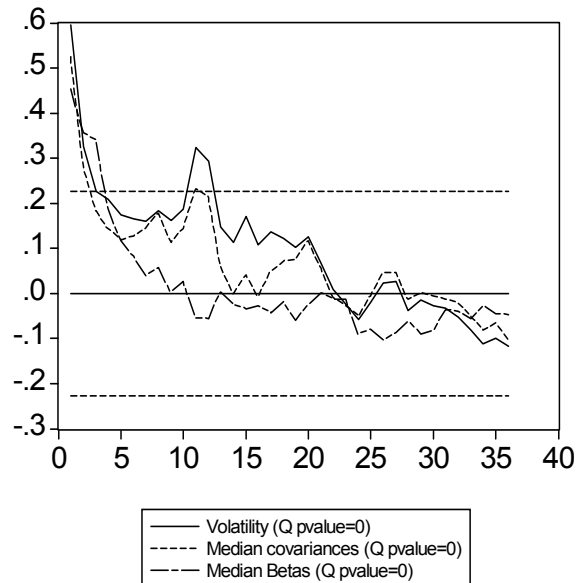
Notes: The Figure shows the time series of monthly realized covariances. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly covariances from fifteen-minute returns.

Figure 3H
Time Series Plots of Monthly Realized Betas



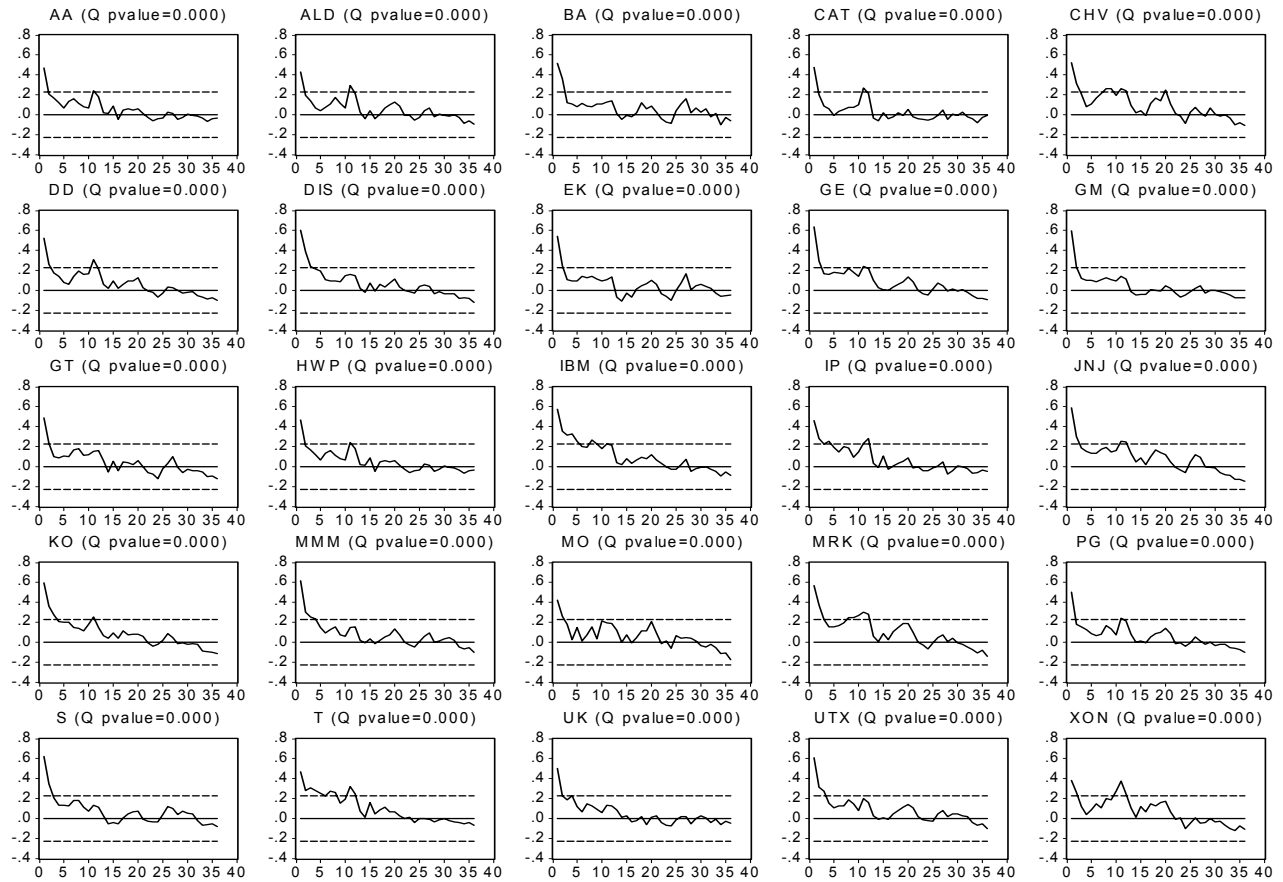
Notes: The Figure shows the time series of monthly realized betas. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly betas from fifteen-minute returns.

Figure 4H
Sample Autocorrelations of Monthly Realized Market Variance, Median Sample Autocorrelations of Monthly Realized Covariances and Median Sample Autocorrelations of Monthly Realized Betas



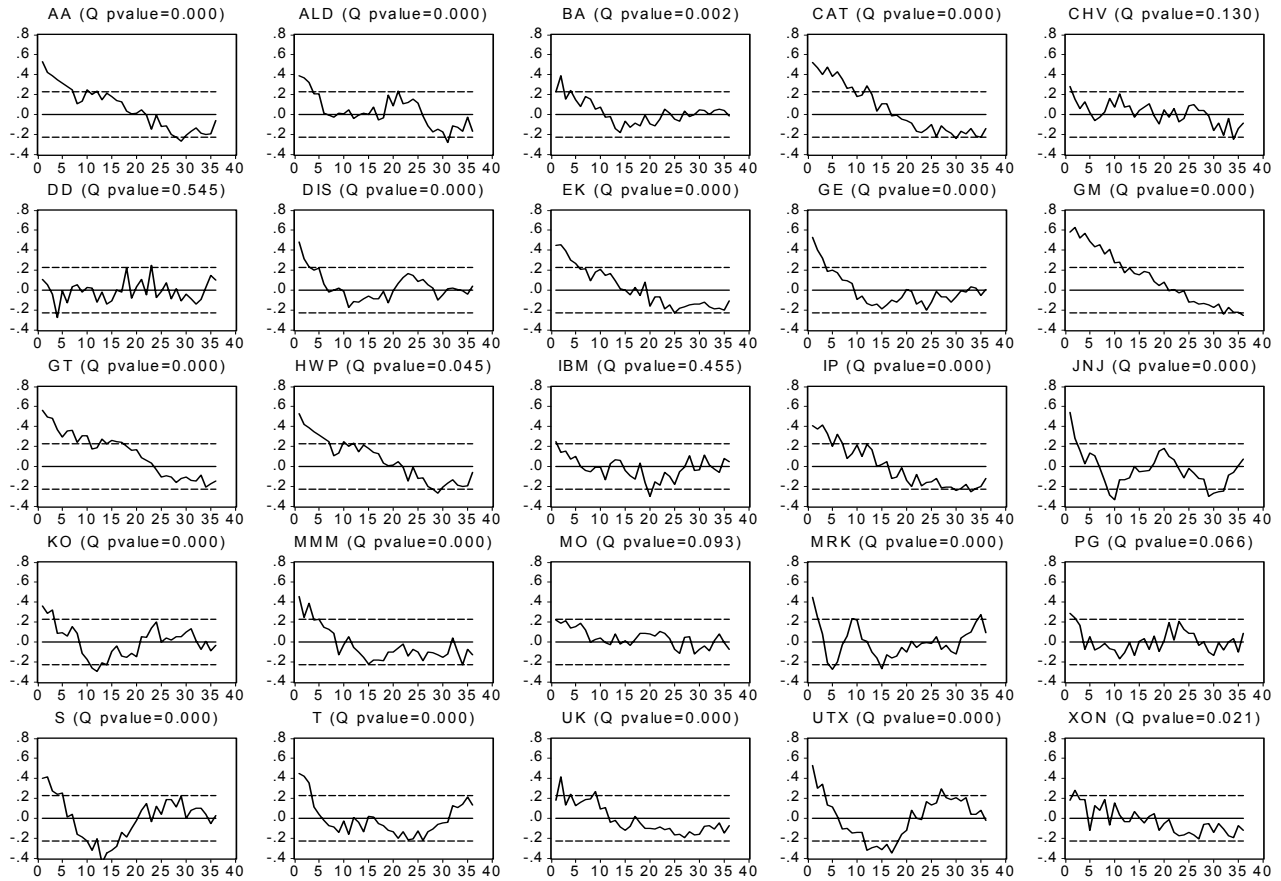
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized market variance, the medians across individual stocks of the first 36 sample autocorrelations of monthly realized covariances and the medians across individual stocks of the first 36 sample autocorrelations of monthly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 5H
Sample Autocorrelations of Monthly Realized Covariances



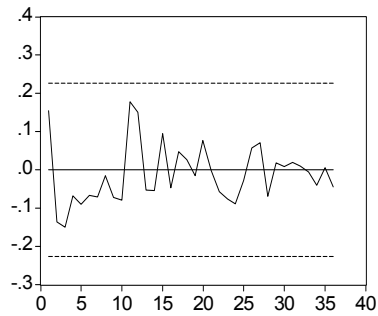
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized covariances. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized covariances from fifteen-minute returns.

Figure 6H
Sample Autocorrelations of Monthly Realized Betas



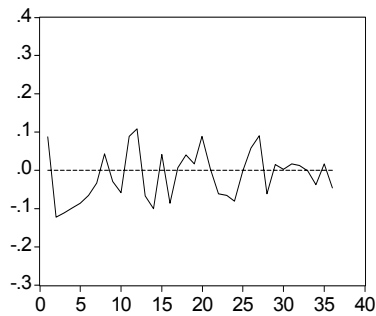
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized betas from fifteen-minute returns.

Figure 7H
Sample Autocorrelations of Monthly Realized Market Variance Prefiltered by $(1-L)^{42}$



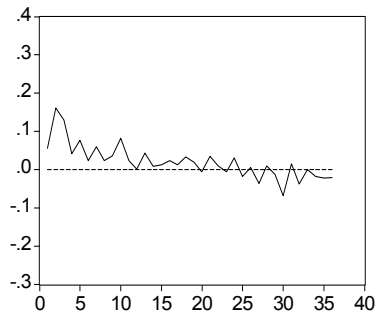
Q pvalue=0.337

Median Sample Autocorrelations of Monthly Realized Covariances Prefiltered by $(1-L)^{42}$



Q pvalue=0.478

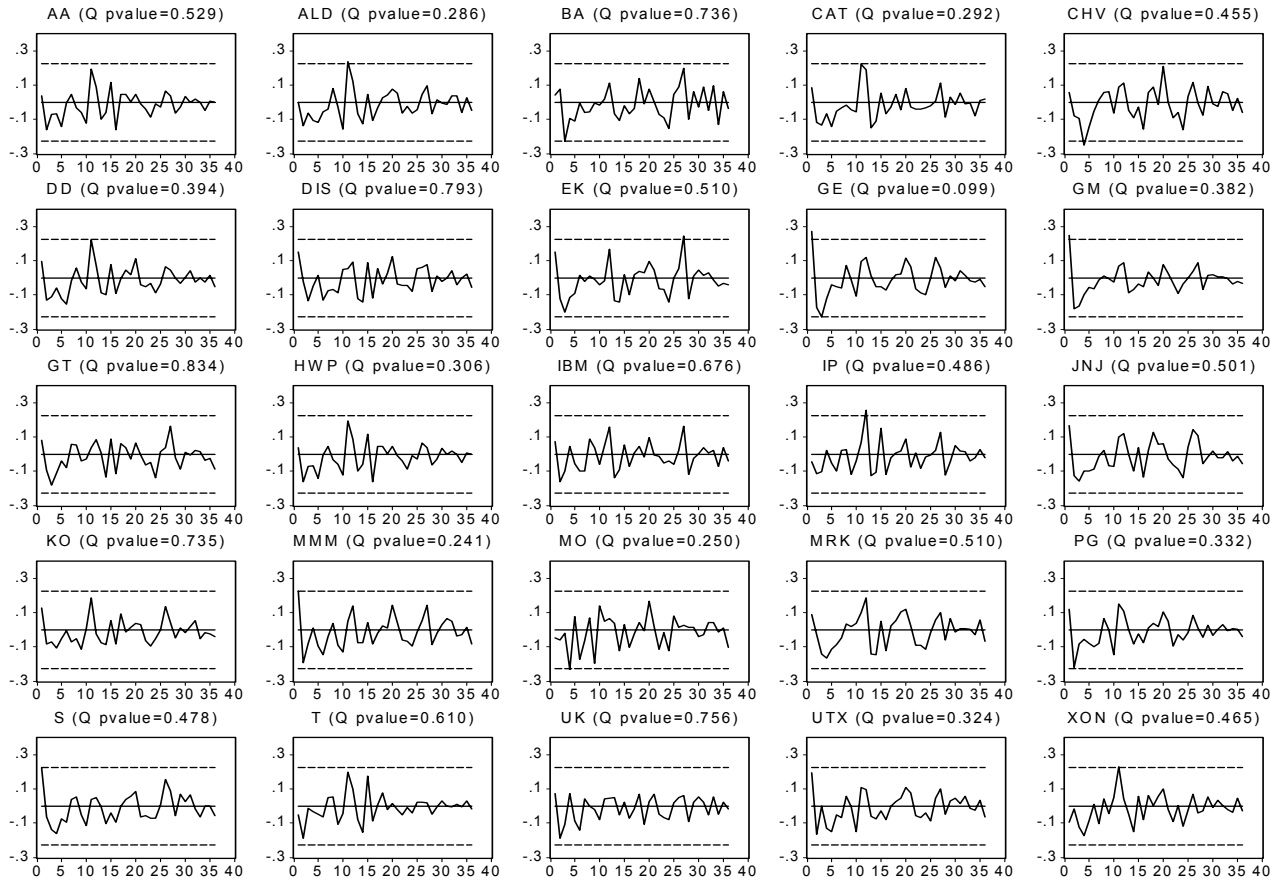
Median Sample Autocorrelations of Monthly Realized Betas Prefiltered by $(1-L)^{42}$



Q pvalue=0.252

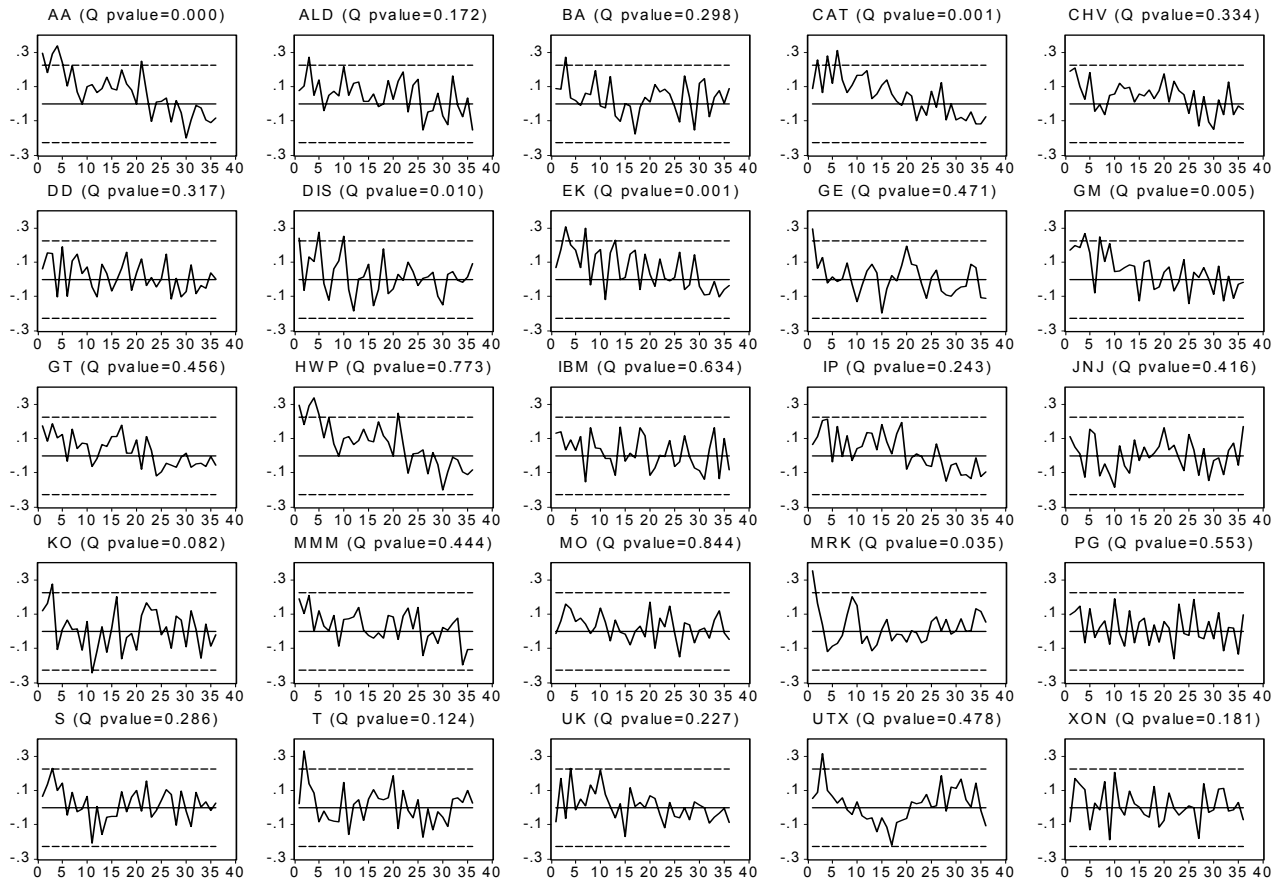
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized market variance, the medians across individual stocks of the first 36 sample autocorrelations of monthly realized covariances and the medians across individual stocks of the first 36 sample autocorrelations of monthly realized betas all prefiltered by $(1-L)^{42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the median of Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance from fifteen-minute returns.

Figure 8H
Sample Autocorrelations of Monthly Realized Covariances Prefiltered by $(1 - L)^{42}$



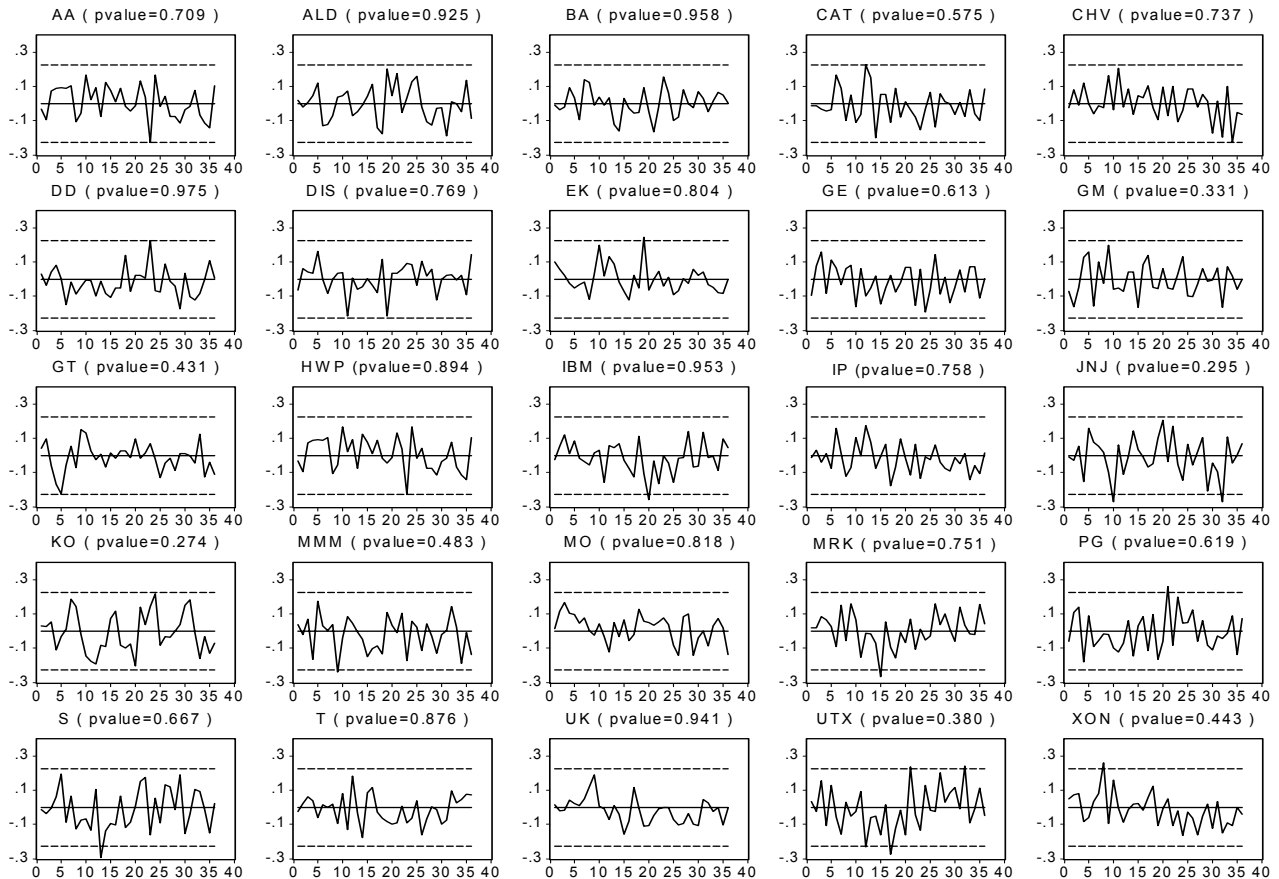
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized covariances prefiltered by $(1 - L)^{42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized covariances from fifteen-minute returns.

Figure 9H
Sample Autocorrelations of Monthly Realized Betas Prefiltered by $(1 - L)^{42}$



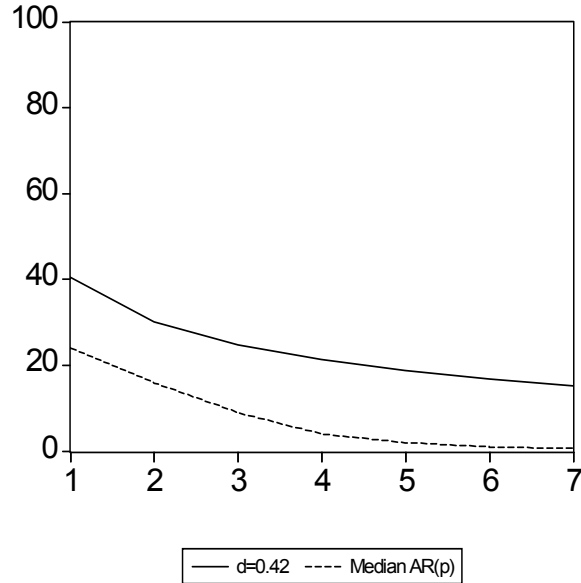
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas prefiltered by $(1 - L)^{42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized betas from fifteen-minute returns.

Figure 10H
Sample Autocorrelations of Monthly Realized Betas Prefiltered by Estimated $AR(p)$ Lag Operator Polynomial



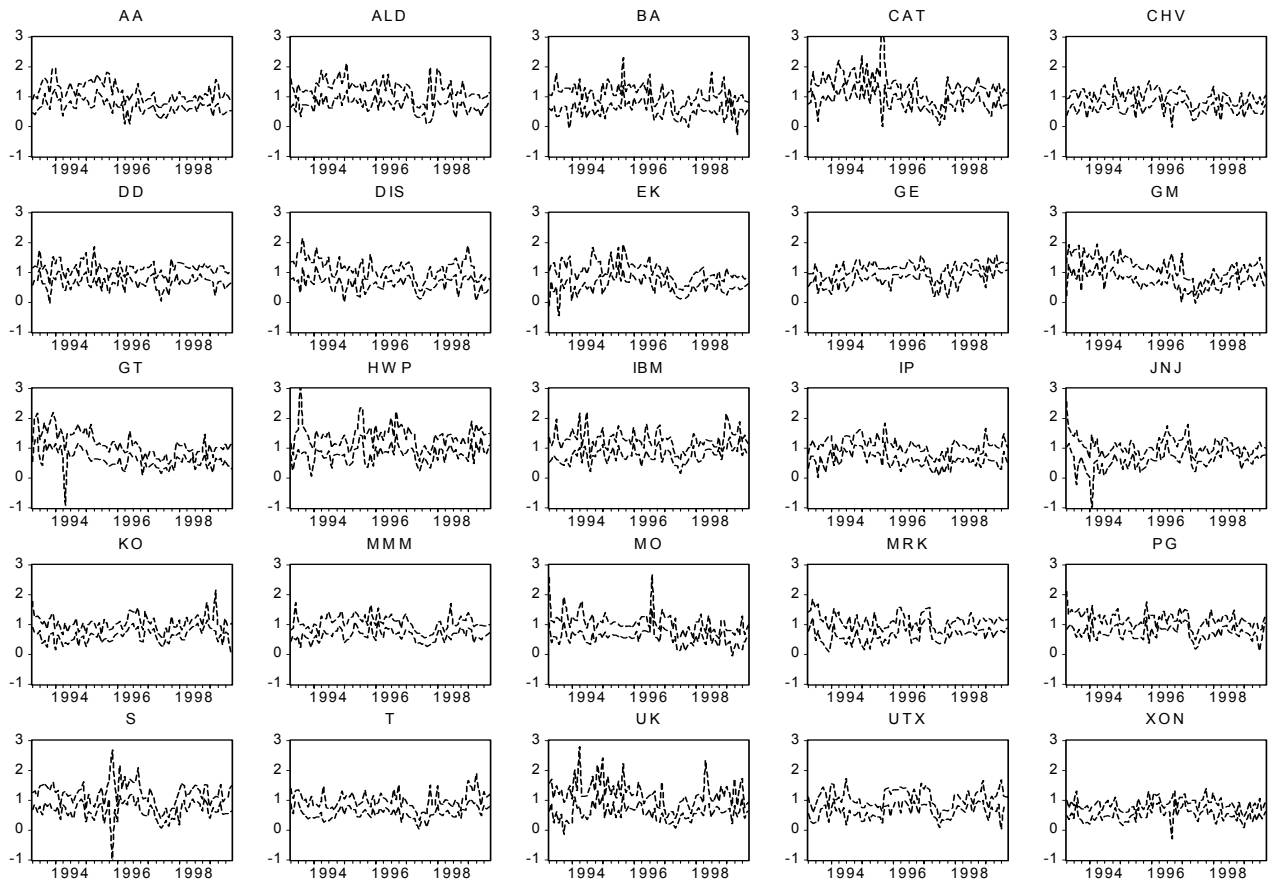
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas prefiltered by $(1 - \hat{\phi}_1 L - \hat{\phi}_2 L^2 - \dots - \hat{\phi}_p L^p)$, where $\hat{\phi}_1, \hat{\phi}_2, \dots, \hat{\phi}_p$ are the least squares estimates of the parameters of $AR(p)$ models fit to the realized betas, with p selected by the AIC. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 11H
Predictability of Market Volatility, Individual Equity Covariances with the Market, and Betas



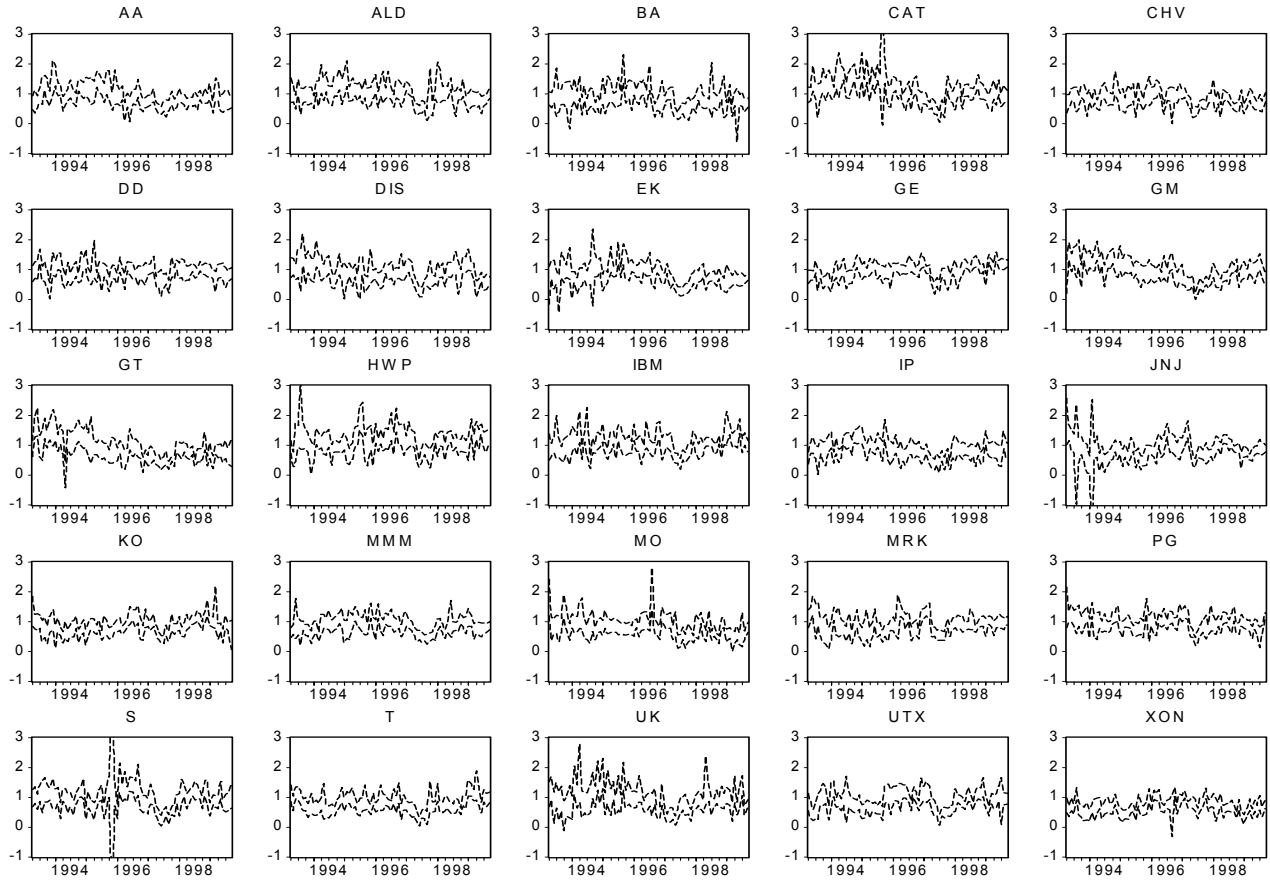
Notes: We define predictability as $P_j = 1 - \text{var}(e_{t+j,t}) / \text{var}(e_{t+40,t})$, where $\text{var}(e_{t+j,t}) = \sigma^2 \sum_{i=0}^{j-1} b_i^2$, σ^2 is the variance of the innovation \mathbf{e}_t , and the b_i 's are moving average coefficients; i.e., the Wold representation is $y_t = (1 + b_1L + b_2L^2 + b_3L^3 + \dots)\mathbf{e}_t$. We approximate the dynamics using a pure long-memory model, $(1 - L)^{d/2} y_t = \mathbf{e}_t$, in which case $b_0 = 1$ and $b_i = (-1)b_{i-1}(d - i + 2)/(i - 1)$ and plot P_j for $j = 1, \dots, 7$ (solid line). Moreover, because we take $d = 0.42$ for market volatility and for all covariances with the market, all of their predictabilities are the same at all horizons. We approximate the dynamics using an $AR(p)$ model, with the autoregressive lag order p determined by the AIC and plot the median of P_j for $j = 1, \dots, 7$ among all 25 stocks (dashed line). The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations.

Figure 12H
Ninety-Five Percent Confidence Intervals for Monthly Beta, Fifteen-Minute Sampling



Notes: The figure shows the time series of ninety-five percent confidence intervals for the underlying monthly integrated beta, calculated using the results of Barndorff-Nielsen and Shephard (2003). The sample covers the period from 1993.4 through 1999.9. We calculate the realized monthly betas from fifteen-minute returns.

Figure 13H
Ninety-Five Percent Confidence Intervals for Monthly Beta, Fifteen-Minute Sampling (Newey-West)



Notes: The figure shows the time series of Newey-West ninety-five percent confidence intervals for the underlying monthly integrated beta. The sample covers the period from 1993.4 through 1999.9. We calculate the realized monthly betas from fifteen-minute returns.