



Taylor & Francis  
Taylor & Francis Group



---

[Realized Variance and Market Microstructure Noise]: Comment

Author(s): Francis X. Diebold

Source: *Journal of Business & Economic Statistics*, Apr., 2006, Vol. 24, No. 2 (Apr., 2006), pp. 181-183

Published by: Taylor & Francis, Ltd. on behalf of American Statistical Association

Stable URL: <https://www.jstor.org/stable/27638865>

---

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <https://about.jstor.org/terms>



JSTOR

Taylor & Francis, Ltd. and American Statistical Association are collaborating with JSTOR to digitize, preserve and extend access to *Journal of Business & Economic Statistics*

- Barndorff-Nielsen, O. E., and Shephard, N. (2004a), "Econometric Analysis of Realised Covariation: High-Frequency Covariance, Regression and Correlation in Financial Economics," *Econometrica*, 72, 885–925.
- (2004b), "Power and Bipower Variation With Stochastic Volatility and Jumps" (with discussion), *Journal of Financial Econometrics*, 2, 1–48.
- Barucci, E., and Reno, R. (2002a), "On Measuring Volatility and the GARCH Forecasting Performance," *Journal of International Financial Markets, Institutions and Money*, 12, 182–200.
- (2002b), "On Measuring Volatility of Diffusion Processes With High-Frequency Data," *Economic Letters*, 74, 371–378.
- Carr, P., and Wu, L. (2004), "Time-Changed Lévy Processes and Option Pricing," *Journal of Financial Economics*, 71, 113–141.
- Epps, T. W. (1979), "Comovements in Stock Prices in the Very Short Run," *Journal of the American Statistical Association*, 74, 291–296.
- Hayashi, T., and Yoshida, N. (2005), "On Covariance Estimation of Non-Synchronously Observed Diffusion Processes," *Bernoulli*, 11, 359–379.
- Kanatani, T. (2004a), "High-Frequency Data and Realized Volatility," unpublished doctoral thesis, Kyoto University, Graduate School of Economics.
- (2004b), "Integrated Volatility Measuring From Unevenly Sampled Observations," *Economics Bulletin*, 3, 1–8.
- Lo, A., and MacKinlay, C. (1990), "The Econometric Analysis of Nonsynchronous Trading," *Journal of Econometrics*, 45, 181–211.
- Malliavin, P., and Mancino, M. E. (2002), "Fourier Series Method for Measurement of Multivariate Volatilities," *Finance and Stochastics*, 6, 49–61.
- Mancino, M., and Reno, R. (2005), "Dynamic Principal Component Analysis of Multivariate Volatilities via Fourier Analysis," *Applied Mathematical Finance*, 12, 187–199.
- Martens, M. (2003), "Estimating Unbiased and Precise Realized Covariances," unpublished manuscript (EFA 2004 Maastricht Meetings Paper No. 4299).
- Nielsen, M. O., and Frederiksen, P. H. (2005), "Finite-Sample Accuracy of Integrated Volatility Estimators," unpublished manuscript, Cornell University, Dept. of Economics.
- Precup, O. V., and Iori, G. (2005), "Cross-Correlation Measures in High-Frequency Domain," unpublished manuscript, City University, Dept. of Economics.
- Reno, R. (2003), "A Closer Look at the Epps Effect," *International Journal of Theoretical and Applied Finance*, 6, 87–102.
- Scholes, M., and Williams, J. (1977), "Estimating Betas From Nonsynchronous Trading," *Journal of Financial Economics*, 5, 309–327.
- Sheppard, K. (2005), "Measuring Realized Covariance," unpublished manuscript, University of Oxford, Dept. of Economics.

## Comment

### Francis X. DIEBOLD

Department of Economics, University of Pennsylvania, PA 19104, and National Bureau of Economic Research  
([fdiebold@sas.upenn.edu](mailto:fdiebold@sas.upenn.edu))

The research program of Hansen and Lunde (HL) is generally first rate, displaying a rare blend of theoretical prowess and applied sense. The present article is no exception. In a major theoretical advance, HL allow for correlation between microstructure (MS) noise and latent price. (I prefer "latent price" to such terms as "efficient price" or "true price," which carry a lot of excess baggage.) In a parallel major substantive advance, HL provide a pioneering empirical investigation of the *nature* of the correlation between MS noise and latent price, documenting a negative correlation at high frequencies. My admiration of the article hinges on the aforementioned contributions and is indeed most genuine. Nevertheless, much of what follows is rather critical of the extant literature, including certain key elements of HL's approach. My intention is for my criticism to be constructive, promoting and hastening additional progress.

#### 1. ON THE DYNAMICS OF LATENT PRICE

HL work in the framework

$$p(t) = p^*(t) + v(t) \quad (1)$$

and

$$dp^*(t) = \mu dt + \sigma(t) dW(t), \quad (2)$$

where  $p(t)$  is observed (log) price,  $p^*(t)$  is the latent (log) price,  $v(t)$  is MS noise,  $\mu$  is a fixed expected return (actually HL go even farther and restrict  $\mu = 0$ ), and  $dW_t$  is an increment of standard Brownian motion. Without additional assumptions, (1) is tautological, defining MS noise simply as  $v(t) = p(t) - p^*(t)$ . Hence everything hinges on the assumed specifications of  $p^*(t)$  and  $v(t)$ —neither of which is observable—and assumptions regarding their interaction.

Before HL's work, the literature effectively focused on specifications with latent price assumed to be uncorrelated with the MS noise,

$$\text{corr}(v(t), dW(t)) = 0. \quad (3)$$

HL progress by allowing instead

$$\text{corr}(v(t), dW(t)) = \rho. \quad (4)$$

Importantly, their allowance for  $\text{corr}(v(t), dW(t)) \neq 0$  is in accordance with both MS theory (more on this later) and with empirical fact (as HL emphasize). Nonetheless, HL's specifications (1), (2), and (4) remain quite limited relative to one allowing for *time-varying expected returns and jumps*, as in

$$dp^*(t) = \mu(t) dt + \sigma(t) dW(t) + \kappa(t) dq(t), \quad (5)$$

where  $\mu(t)$  is the time-varying expected return,  $\kappa(t) = p(t) - p(t-)$  is jump size, and  $q(t)$  is a counting process with possibly time-varying intensity  $\lambda(t)$  such that  $P[dq(t) = 1] = \lambda(t) dt$ .

First, consider the possibility of time-varying expected returns. Note that  $p^*(t)$  is a real-world price, not a risk-neutral price, so there is no reason for  $p^*(t)$  to follow a martingale. Hence allowance for time-varying expected returns is important in principle. In practice, one might argue that, at least in high-frequency environments [e.g., hourly returns used to construct daily realized volatility (RV)], time variation in expected returns is likely to be negligible and thus can be safely ignored. Fair enough, but at least three caveats are in order. First, and

obviously, interest sometimes centers *not* on high frequencies, but rather on lower frequencies, such as annual RV constructed from underlying monthly returns, particularly in historical asset market studies covering long calendar spans. Second, there is evidence of nonmartingale behavior not only at long horizons, but also at short horizons (e.g., Lo and MacKinlay 2001). Hence even if interest *does* center on high-frequency returns, it is not obvious that time-varying expected returns can be safely ignored. Finally, Elliott's (1998) well-known work establishes that cointegration methods are not robust to even *slight* deviations from  $I(1)$  behavior in the underlying variables. In the present case (and moving to discrete time to match the standard cointegration framework), the required  $I(1)$  behavior is for  $p_t^*$  and hence  $p_t$ , which is apparently not guaranteed when time variation in expected returns is allowed. This concern is particularly relevant to the present article, a large part of which is devoted to cointegration analysis.

Now consider jumps. Jumps are an important feature of empirical reality, and frameworks that ignore them do so at their own peril. This insight arises repeatedly in many studies presenting estimates of parametric jump-diffusion models for asset returns and is reinforced and amplified by recent non-parametric volatility analyses (see, e.g., Andersen, Bollerslev, and Diebold 2005 and references therein) and examinations of the financial market reaction to macroeconomic news (e.g., Andersen, Bollerslev, Diebold, and Vega 2003). Indeed, given that incorporation of MS noise and jumps are widely acknowledged as two of the most pressing items on the RV research agenda, it is unfortunate that although the RV jumps literature acknowledges MS noise (e.g., Barndorff-Nielsen and Shephard 2004, 2006a,b; Andersen et al. 2005), the latest significant advance of the RV-MS literature (namely HL) does not acknowledge jumps. I look forward to additional work from HL to rectify that situation.

## 2. ON THE INADEQUACY OF LINEAR/GAUSSIAN METHODS

For simplicity of exposition, assume that both the observed and latent prices evolve at transaction times and that transaction times are equally spaced. The HL framework corresponds to a linear discrete-time state-space system,

$$p_t = p_t^* + \varepsilon_t, \tag{6}$$

$$p_t^* = p_{t-1}^* + \sigma_t u_t, \tag{7}$$

and

$$\begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix} \stackrel{\text{iid}}{\sim} N \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon u} \\ \sigma_{\varepsilon u} & 1 \end{pmatrix}. \tag{8}$$

The system may or may not be conditionally Gaussian, depending on the dynamics of  $\sigma_t$ . If, for example,  $\sigma_t$  has generalized autoregressive conditional heteroscedasticity (GARCH) structure such as

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{9}$$

then the system is conditionally Gaussian, but if  $\sigma_t$  has stochastic volatility structure such as

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \eta_t, \quad \eta_t \stackrel{\text{iid}}{\sim} (0, \sigma_\eta^2), \tag{10}$$

then the system is not conditionally Gaussian. Of course, the realized volatility framework makes only minimal assumptions

on  $\sigma_t$  and is compatible with both GARCH and stochastic volatility, among many others.

Quite apart from such details, however, the intrinsic mechanics of the leading MS noise candidates (and the ones explicitly used by HL to motivate their approach)—namely, bid/ask bounce and discrete price quotes—induce fundamental violations of the linear/Gaussian state-space framework that appear to have been ignored in the RV-MS literature thus far. To see this, dispense for the moment with volatility dynamics, because including them only complicates matters without changing the basic point.

Consider in particular an MS model in the tradition of Hasbrouck (1999a,b), incorporating both bid/ask bounce and discrete pricing,

$$p_t = \begin{cases} \text{floor}(q_t^b, \text{ticksize}) & \text{if } \pi_t = 0 \\ \text{ceiling}(q_t^a, \text{ticksize}) & \text{if } \pi_t = 1; \end{cases} \tag{11}$$

$$q_t^b = p_t^* - c_t, \quad q_t^a = p_t^* + c_t; \tag{12}$$

$$p_t^* = p_{t-1}^* + u_t; \tag{13}$$

$$\pi_t \stackrel{\text{iid}}{\sim} \text{Bernoulli}(.5); \tag{14}$$

$$c_t \stackrel{\text{iid}}{\sim} N(\mu_c, \sigma_c^2); \quad \text{and} \tag{15}$$

$$u_t \stackrel{\text{iid}}{\sim} N(0, \sigma_u^2), \tag{16}$$

where  $q_t^b$  is the bid price,  $q_t^a$  is the ask price,  $2c_t$  is the bid/ask spread that represents the positive stochastic cost of dealer quote exposure,  $\pi_t$  is a bid/ask indicator variable,  $\text{floor}(\cdot, d)$  rounds its argument down to the closest multiple of  $d$  and  $\text{ceiling}(\cdot, d)$  rounds up, and  $u_t$ ,  $c_t$ , and  $\pi_t$  are contemporaneously and serially independent.

The MS model (11)–(16) also constitutes a state-space system, relating the observed  $p_t$  to the latent  $p_t^*$ , but the bid/ask bounce and rounding render it intrinsically *non-Gaussian*. Against this background, I worry about the adequacy of the linear/Gaussian tools on which HL rely, including HAC estimation based on sample autocorrelations, Gaussian-cointegrated vector autoregressions, and so on.

In closing this section, let me also note that the state-space framework raises the possibility of direct filtering or smoothing of MS noise from observed returns *before* proceeding to compute RV. This is of independent interest, because one might want to use MS-corrected returns for various purposes beyond construction of MS-corrected RV. For the reasons discussed earlier, however, the optimal filter will generally be nonlinear, and hence the Kalman filter will be suboptimal. Diebold and Vega (2002) explored optimal nonlinear MS noise filtering, and Owens and Steigerwald (2006) independently explored Kalman MS noise filtering. An interesting and open issue concerns the goodness of approximation of the Kalman filter to the fully optimal filter.

Alternatively, if interest centers exclusively on volatility, then one can first construct realized volatility and then filter to reduce MS noise (as well as estimation error due to incomplete convergence of RV to underlying integrated volatility). This is the approach that HL implicitly take, although they do not attempt to construct an optimal nonlinear filter tailored to the structure of the MS noise. It was also the approach taken

explicitly by Andersen, Bollerslev, Diebold, and Wu (2005, 2006), who analyzed explicit state-space systems but used only a Kalman filter.

### 3. ON THE THEORETICAL AND EMPIRICAL CORRELATION BETWEEN MICROSTRUCTURE NOISE AND LATENT PRICE

In a pioneering substantive contribution, HL discovered a negative relationship between latent returns and MS noise at high frequencies. Does the negative correlation match the predictions of MS theory? HL claim, rather casually, that the answer is “yes,” arguing that “the correlation between noise and efficient price arises naturally in some models of market microstructure effects, including (a generalized version of) the bid–ask model by Roll (1984)... and models where agents have asymmetric information, such as those by Glosten and Milgrom (1985) and Easley and O’Hara (1987, 1992).”

In fact, the situation is more nuanced, with some MS considerations suggesting a positive correlation and others suggesting a negative correlation, so that the net correlation could be empirically positive, negative, or even zero. The issues are well-illustrated by the generalized Roll model emphasized by HL (see Roll 1984; Hasbrouck 1999a,b, 2004), which can produce either positive or negative correlation, as follows.

We first generate a positive correlation. Focusing, for simplicity, only on the bid/ask bounce (i.e., ignoring the discreteness of price quotes), rewrite the system (11)–(16) in condensed notation as

$$p_t = p_t^* + cq_t, \quad p_t^* = p_{t-1}^* + u_t, \quad (17)$$

where  $q_t$  indicates direction of trade (1 for a buy and  $-1$  for a sell). Also, refine (13) by decomposing the latent price increment as

$$u_t = \lambda q_t + \omega_t, \quad (18)$$

where  $\lambda$  is the reaction of latent price to a trade (and hence captures some aspects of private information) and  $\omega_t$  denotes public information. Simple calculations then reveal that

$$\begin{aligned} \text{cov}(\Delta p_t^*, \Delta(p_t - p_t^*)) &= E((\lambda q_t + \omega_t)(c(q_t - q_{t-1}))) \\ &= c\lambda E(q_t^2) = c\lambda > 0, \end{aligned} \quad (19)$$

if  $q_t$  is independent of  $\omega_t$  and independent over time, as is commonly assumed.

Now let us generate a potentially negative correlation by allowing for sluggish adjustment of transaction prices, which could arise for a variety of reasons, such as learning. We replace (17) with

$$p_t = p_{t-1}^* + cq_t, \quad (20)$$

yielding

$$\text{cov}(\Delta p_t^*, \Delta(p_t - p_t^*)) = (c - \lambda)\lambda - E(\omega_t^2), \quad (21)$$

which can be negative if  $c$  is small relative to  $\lambda$  and the variance of  $\omega_t$  is large. It is interesting to note that MS models in the tradition of Glosten and Milgrom (1985), Kyle (1985), and Easley and O’Hara (1987, 1992) all assume similar adjustment lags and hence predict negative correlation.

The upshot is that it is trivial to write down a microstructure model with either a negative or a positive contemporaneous correlation between latent returns and MS noise. The basic insight is that certain factors promote *overreaction* of observed price to movements in latent price and hence a positive correlation, whereas other factors promote *underreaction* of observed price to movements in latent price and hence a negative correlation. Different models (or different variants of the same model) emphasize different factors.

Empirically, moreover, several such factors may be operating simultaneously, and whether they aggregate to something negative or positive is an empirical matter. Hence the negative estimates obtained by HL do not necessarily “support” or “refute” any particular MS model.

In any event, a rich vein remains to be mined. The discussion in HL (and thus far in this comment) focuses only on the contemporaneous correlation between latent price and MS noise. The contemporaneous correlation is just the tip of the iceberg, however. The entire cross-correlation structure associated with the transmission of latent price movements into eventual observed price movements is of great interest, and little is known about it, whether theoretically or empirically. Diebold and Strasser (2006) provided an initial exploration.

### ACKNOWLEDGMENTS

For useful discussion the author thanks Torben Andersen, Tim Bollerslev, Doug Steigerwald, Clara Vega, and especially Georg Strasser, who provided outstanding research assistance.

### ADDITIONAL REFERENCES

- Andersen, T. G., Bollerslev, T., and Diebold, F. X. (2005), “Roughing It Up: Including Jump Components in the Measurement, Modeling and Forecasting of Return Volatility,” unpublished manuscript, Northwestern University, Duke University, and University of Pennsylvania; available at [http://www.ssc.upenn.edu/~fdiebold/papers/paper56/abd\\_081605.pdf](http://www.ssc.upenn.edu/~fdiebold/papers/paper56/abd_081605.pdf).
- Andersen, T., Bollerslev, T., Diebold, F. X., and Vega, C. (2003), “Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange,” *American Economic Review*, 93, 38–62.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Wu, J. (2005), “A Framework for Exploring the Macroeconomic Determinants of Systematic Risk,” *American Economic Review*, 95, 398–404.
- (2006), “Realized Beta: Persistence and Predictability,” in *Advances in Econometrics 20: Econometric Analysis of Economic and Financial Time Series*, Part B, eds. T. Fomby and D. Terrell, Oxford: Elsevier Science, pp. 1–40.
- Diebold, F. X., and Strasser, G. (2005), “On the Correlation Structure of Microstructure Noise in Theory and Practice,” unpublished manuscript, University of Pennsylvania, Dept. of Economics.
- Diebold, F. X., and Vega, C. (2002), “Filtering Microstructure Noise,” unpublished manuscript, University of Pennsylvania, Dept. of Economics.
- Elliott, G. (1998), “On the Robustness of Cointegration Methods When Regressors Almost Have Unit Roots,” *Econometrica*, 66, 149–158.
- Hasbrouck, J. (1999a), “Security Bid/Ask Dynamics With Discreteness and Clustering: Simple Strategies for Modeling and Estimation,” *Journal of Financial Markets*, 2, 1–28.
- (1999b), “The Dynamics of Discrete Bid and Ask Quotes,” *Journal of Finance*, 6, 2109–2142.
- Kyle, A. (1985), “Continuous Auctions and Insider Trading,” *Econometrica*, 53, 1315–1335.
- Lo, A. W., and MacKinlay, A. C. (2001), *A Non-Random Walk Down Wall Street*, Princeton, NJ: Princeton University Press.
- Owens, J. P., and Steigerwald, D. G. (2006), “Noise Reduced Realized Volatility: A Kalman Filter Approach,” in *Advances in Econometrics 20: Econometric Analysis of Economic and Financial Time Series*, Part A, eds. T. Fomby and D. Terrell, Oxford: Elsevier Science, in press.