

THE DYNAMICS OF EXCHANGE RATE VOLATILITY: A MULTIVARIATE LATENT FACTOR ARCH MODEL

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SUMMARY

We study temporal volatility patterns in seven nominal dollar spot exchange rates, all of which display strong evidence of autoregressive conditional heteroskedasticity (ARCH). We first formulate and estimate univariate models, the results of which are subsequently used to guide specification of a multivariate model. The key element of our multivariate approach is exploitation of factor structure, which facilitates tractable estimation via a substantial reduction in the number of parameters to be estimated. Such a latent-variable model is shown to provide a good description of multivariate exchange rate movements: the ARCH effects capture volatility clustering, and the factor structure captures commonality in volatility movements across exchange rates.

1. INTRODUCTION

In this paper we specify and estimate a multivariate time-series model with an underlying latent variable whose innovations display autoregressive conditional heteroskedasticity (ARCH). Various aspects of this factor-analytic approach are sketched in Diebold (1986) and Diebold and Nerlove (1987); here we provide a more complete exposition, propose a new estimation procedure, and present a detailed substantive application to movements in seven major dollar spot exchange rates. To guide multivariate specification, we begin with a univariate analysis and relate the results to apparent random walk behaviour, leptokurtic unconditional distributions and convergence to normality under temporal aggregation.

The univariate results point to the need for a multivariate specification, but multivariate ARCH modelling is difficult, due to the large number of parameters which must be estimated. We therefore propose a multivariate latent-variable model in which the common factor displays ARCH. The conditional variance–covariance structure of the observed variables arises from joint dependence on a common factor; this leads to common volatility movements across rates, which are in fact observed.

The plan of the paper is as follows. In section 2 we discuss economic issues related to non-constant exchange rate volatility, and we underscore the importance of subsequent ARCH findings. In section 3 we briefly report the results of univariate analyses, focusing on unit roots and ARCH effects. The multivariate model with factor structure is specified, estimated, and compared to the univariate models in section 4. Section 5 concludes.

2. NON-CONSTANT EXCHANGE RATE VOLATILITY

The difficulty of explaining exchange rate movements during the post-1973 float with purchasing power parity, monetary, or portfolio balance models has become increasingly apparent. Meese and Rogoff (1983a,b) systematically document the out-of-sample empirical failure of these models, and they find that a simple random-walk model predicts the major rates during the floating period as well as (or better than) any of the alternatives, including a flexible price monetary model (Frenkel, 1976; Bilson, 1979), a sticky price monetary model (Dornbusch, 1976; Frankel, 1979), a sticky price monetary model with current account effects (Hooper and Morton, 1982), six univariate time-series models, a vector autoregressive model, and the forward rate.

In this paper we study seven major dollar spot rates: the Canadian dollar (CD), French franc (FF), Deutschemark (DM), Italian lira (LIR), Japanese yen (YEN), Swiss franc (SF), and British pound (BP). We find that, in the class of linear time-series models, the random walk is a very good approximation to the underlying conditional mean process; therefore we would not expect other linear models to dominate in terms of predictive performance. However, when the class of models under consideration is broadened to allow for possible nonlinearities, we find strong evidence of ARCH (Engle, 1982) in the one step ahead prediction errors, so that the disturbances in the 'random walk' are uncorrelated but not independent.

The finding of ARCH in exchange rates is important. First, ARCH provides a way of formalizing the observation that large changes tend to be followed by large changes (of either sign), and small by small, leading to contiguous periods of volatility and stability. Such temporal clustering of prediction error variances has been well documented in pioneering work on stochastic processes for financial assets, such as Mandelbrot (1963) and Fama (1965), and is visually apparent in exchange rate movements. Second, ARCH effects are consistent with the unconditional leptokurtosis in exchange rate changes documented by Westerfield (1977) and Boothe and Glassman (1987).¹ Finally, under mild regularity conditions, ARCH effects vanish under temporal aggregation as convergence to unconditional normality occurs.² This phenomenon has been observed by Fama (1976) and Boothe and Glassman (1987), and cannot be explained by the commonly used return-generating models in the stable family.

We also show that ARCH models may be used to generate statistically and economically meaningful measures of exchange rate volatility. The nature, time pattern, and economic effects of exchange rate volatility are recurrent topics in the literature. Volatility of exchange rates is of importance because of the uncertainty it creates for prices of exports and imports, for the value of international reserves and for open positions in foreign currency, as well as for the domestic currency value of debt payments and workers' remittances, which in turn affect domestic wages, prices, output, and employment. Under risk-aversion, risk premia form a wedge in arbitrage conditions (such as uncovered interest parity) and may therefore influence the determination of spot exchange rates. Such risk premia depend on the dispersion of the distribution of future spot rates, which varies over time. ARCH effects (if present), provide a parsimonious description of such an evolving conditional variance. By estimating an appropriate ARCH model we can solve for the implied time-series of conditional variances, and thus obtain a meaningful measure of volatility for that rate.

Finally, a finding of random walks with ARCH disturbances means that, although a

¹ Detailed characterizations of the conditional and unconditional moment structures of ARCH processes are provided by Engle (1982) and Milhoj (1986).

² This is shown by Diebold (1988), who obtains the result by applying a central limit theorem for dependent, identically distributed, random variables due to White (1984).

particular exchange rate change cannot be forecast, its changing variance can be forecast. Thus, ARCH may be exploited to obtain time-varying confidence intervals for point forecasts of exchange rate changes (zero for a random-walk model), and hence is naturally suited to the modelling of time-varying risk premia. In periods of high volatility these intervals are large, and in less volatile periods they are smaller. This stands in marked contrast to the standard constant variance random-walk model, which ignores the changing environment in which forecasts are produced and the associated temporal movements in forecast error variances.

3. UNIVARIATE ANALYSIS

We study weekly spot rates from the first week of July 1973 to the second week of August 1985.³ All data are seasonally unadjusted interbank closing spot prices (bid side), Wednesdays, taken from the *International Monetary Markets Yearbook*. Wednesdays were chosen because very few holidays occur on that day, and there is no problem of weekend effects.

In our sample 632 observations, fewer than eight holidays occurred on a Wednesday; when they did, the observation for the following Thursday was used. The use of point-in-time data avoids the introduction of spurious serial correlation via the Working (1960) effect. Following standard convention, all exchange rates except the pound are measured in units of local currency per dollar. We work with log spot rates; the log specification avoids prediction problems arising from Jensen's inequality (Meese and Rogoff, 1983a) and $(1 - L)\ln S_t$ has the convenient interpretation of approximate percentage change.⁴

We now proceed to consider conditional mean specification. A visual inspection indicated nonstationarity in each of the series. (The DM/\$ rate, which together with the BP/\$ rate will



Figure 1. Log DM/\$ rate

³Detailed univariate results are contained in Diebold (1988).

⁴Throughout this paper, the generic notation S_t denotes an exchange rate at time t .

be used for illustration, is shown in Figure 1.) The sample autocorrelation functions were calculated for each series up to lag 40, and clearly indicated homogeneous nonstationarity, as evidenced by the fact that all were positive, failed to damp, and had very smooth, persistent movements. Even the YEN, whose autocorrelation function declined the most quickly, had a sample autocorrelation of 0.848 at lag 20.

The sample partial autocorrelation functions were also calculated for each of the seven exchange rates, and the results were qualitatively the same for each series: each had a very large and highly significant value (extremely close to one) at lag 1, while the values at all other lags were insignificantly different from zero. Specifically, the lag 1 sample partial autocorrelations for the CD, FF, DM, LIR, YEN, SF, BP were, respectively, 0.99, 1.00, 1.00, 1.00, 1.00, 1.00, and 0.99. Thus, the distinct cutoff in the sample partial autocorrelation functions after lag 1, the smooth and slowly declining behaviour of the sample autocorrelation functions, and the values of the highly significant first sample partial autocorrelation strongly suggested first-order homogeneous nonstationarity in general, and the random walk in particular, for each series.

This preliminary evidence was supported by a battery of formal unit root tests. Solo's (1984) test is a Lagrange multiplier (LM) test for unit roots in general ARMA models; since it is an LM test, it requires estimates only under the null of a unit root. We therefore began by differencing the series and formulating appropriate models. However, use of model selection procedures such as Akaike's (1974) and Schwarz's (1978) information criteria (AIC and SIC, respectively), as well as visual inspection of the sample autocorrelation functions, revealed no evidence of a moving average component in any of the seven $(1-L)\ln S_t$ series. The simpler Dickey-Fuller test for unit roots in autoregressive series (Dickey, 1976; Fuller, 1976) was therefore employed, allowing for high-order autoregressive lag operator polynomials, as well as trend under the alternative.⁵ The test amounts to regressing $(1-L)\ln S_t$ on an intercept, trend term, $\ln S_{t-1}$, and lags of $(1-L)\ln S_t$, and testing the 't-statistic' on $\ln S_{t-1}$ against the appropriate null distribution, tabulated in Fuller (1976) as τ_τ , which is *not* Student's t .

The results of τ_τ tests for unit roots in AR(7) representations are given in Table I. The basic message is clear: we consistently fail to reject the unit root null. In addition, the small magnitude and general statistical insignificance of the coefficients on lagged $\Delta \ln S_t$ values (not shown) indicated very little serial correlation in any of the first-differenced series. The Hasza-Fuller (1979) joint test of the null hypothesis of two unit roots (with trend allowed under the alternative) is also shown in Table I. The results are given in the column labelled 'HF $_\tau$ '. We reject the null conclusively for each rate, confirming the result of one, and only one, unit root in each series.⁶

Visual inspection of the $\Delta \ln S$ series revealed no evidence of serial correlation, although there did seem to be persistence in the conditional variances, as we discuss in detail below. Such effects are evident in the plot of $\Delta \ln S_{DM}$ given in Figure 2. The sample autocorrelations were calculated for each $\Delta \ln S$ series up to lag 40, and in each case they strongly indicated white noise, as did the sample partial autocorrelation functions. (Allowance for the possible presence of ARCH only strengthens the conclusions, as shown in Diebold, 1988.) As a conservative safeguard against specification error, however, the models estimated subsequently make use of AR(3) conditional mean representations, in order to account for any weak serial correlation that might be present.

⁵ This class of tests has recently been shown by Schwert (1987a,b) to perform quite well, even under misspecification of the ARMA representation.

⁶ In addition, Pantula (1985) has shown that the asymptotic distribution of the Dickey-Fuller test statistic is invariant to ARCH. This convenient property does not hold for standard tests of stationary serial correlation; see Domowitz and Hakkio (1983) and Diebold (1988).

Table I. Test for unit root in $\ln S$, trend allowed under the alternative

Currency	τ_τ	HF_τ
CD	-2.74	31.40***
FF	-1.16	30.90***
DM	-1.12	28.34***
LIR	-1.47	26.71***
YEN	-1.86	22.84***
SF	-1.26	24.49***
BP	-1.33	23.70***

The symbols *, **, and *** respectively denote significance at the 10, 5, and 2 per cent levels. Tests are for unit root(s) in an AR(7) representation, allowing for trend under the alternative. τ_τ is the Dickey-Fuller test for a single unit root, and HF_τ is the Hasza-Fuller joint test for two unit roots.

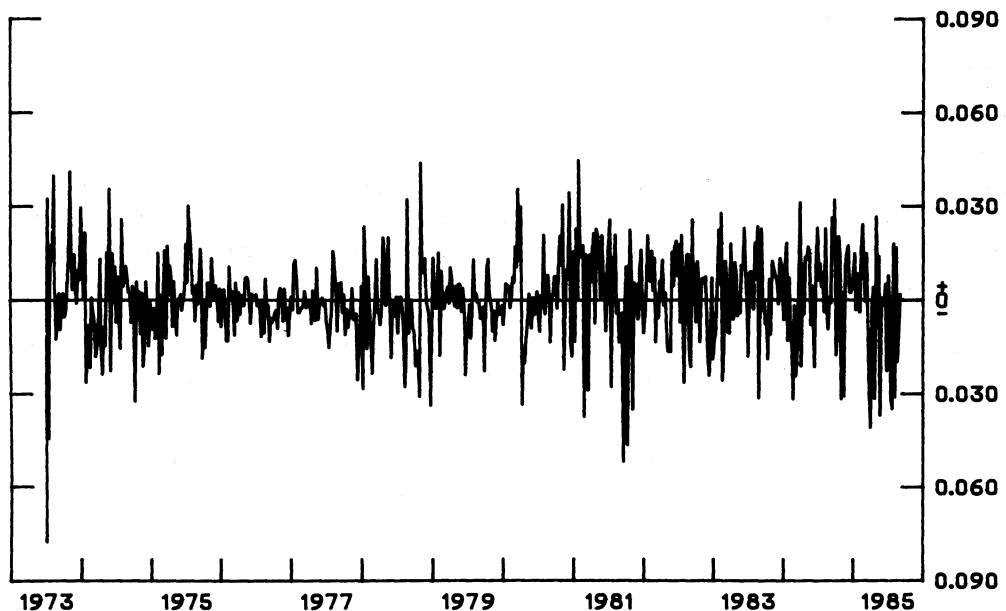


Figure 2. Change in log DM/\$ rate

Finally, in order to assess the distributional properties of the $\Delta \ln S$ series, various descriptive statistics are reported in Table II, including mean, variance, standard deviation, skewness, kurtosis, the Kiefer-Salmon (1983) Lagrange multiplier normality test, and a variety of order statistics. Included among the order statistics is the studentized range, which may also be used to test normality. In particular, the hypothesis of normality is rejected for each exchange rate, whether the studentized range test or the Kiefer-Salmon test is used. Further evidence on the nature of deviations from normality may be gleaned from the sample skewness and kurtosis measures. While skewness of each series is always very close to zero, the kurtosis is very large,

Table II. Weekly nominal dollar spot rates, test statistics, $\Delta \ln S$

Statistic	CD	FF	DM	LIR	YEN	SF	BP
Mean (μ)	0.00049	0.00126	0.00034	0.00189	-0.00017	-0.00024	-0.00104
t ($\mu = 0$)	2.35**	2.28**	0.61	3.77***	-0.33	-0.37	-1.92*
Variance	0.00003	0.00019	0.00019	0.00016	0.00016	0.00027	0.00018
Std. Dev.	0.00526	0.01390	0.01381	0.01260	0.01276	0.01640	0.01360
Skewness	0.56098	0.26069	-0.08594	0.44196	-0.21592	-0.1072	0.34407
Kurtosis	4.70565	2.53659	1.23452	8.08811	3.26364	1.495	3.2979
KS	633.61***	178.99***	38.97***	1667.12***	280.55***	58.74***	268.34***
KS1	31.90***	6.86***	0.77	18.54***	4.88**	1.20	12.24***
KS2	601.71***	172.13***	38.20***	1648.58***	275.67***	57.54***	256.1***
Maximum	0.03754	0.07478	0.05776	0.09679	0.06980	0.06616	0.07246
Q3	0.00309	0.00788	0.00826	0.00725	0.00641	0.00892	0.00543
Median	0.00067	0.00070	0.00050	0.00061	0.00030	0.00039	-0.00058
Q1	-0.00240	-0.00545	-0.00732	-0.00373	-0.00520	-0.00872	-0.00839
Minimum	-0.01762	-0.04583	-0.04839	-0.07490	-0.05671	-0.05421	-0.05322
Mode	0	0	0	0	0	0	0
SR	10.49***	8.68***	7.69***	13.63***	9.91***	7.34***	9.24***

KS = Kiefer-Salmon normality test, decomposed into KS1 (skewness test) and KS2 (kurtosis test).

Q3 = Third quartile.

Q1 = First quartile.

SR = Studentized range.

Significance levels: * = 10 per cent, ** = 5 per cent, *** = 1 per cent.

ranging from 1.23 for the DM to 8.09 for the LIR.⁷ In addition, the Kiefer-Salmon Lagrange multiplier statistic, distributed as χ^2_2 under the null of normality, may be decomposed into two asymptotically independent χ^2_1 variates, the first being an LM test for normal skewness and the second an LM test for normal kurtosis. These show that most of the non-normality in each series is due to leptokurtosis.

We have seen that the conditional mean of each exchange rate is, to a close approximation, linearly unpredictable; however, this need not be true for the conditional variances. The ARCH model of Engle (1982) is particularly relevant in the time-series context.⁸ Suppose that:

$$\varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_{t-p} \sim N(0, \sigma_t^2), \quad (1)$$

$$\sigma_t^2 = f(\varepsilon_{t-1}, \dots, \varepsilon_{t-p}), \quad (2)$$

where

$$\varepsilon_t = \Delta \ln S_t - \sum_{k=1}^3 \rho_k \Delta \ln S_{t-k}.$$

Throughout this paper we adopt the following natural parameterization:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2, \quad (3)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \dots, p$. The conditional variance of ε_t is allowed to vary over time as a linear function of past squared realizations. In the expected value sense, then, today's

⁷ Throughout this paper, 'kurtosis' refers to excess kurtosis, so that a value of zero corresponds to normality.

⁸ See also Hsieh (1988), Bollerslev (1987a) and Milhoj (1987) for variations on this theme, with particular attention to the modelling of asset price movements.

variability depends linearly on yesterday's variability, so that large changes tend to be followed by large changes, and small by small, or either sign. The ARCH model formalizes this phenomenon and enables us to test for it rigorously.

Estimation and hypothesis testing for the ARCH model have been treated by Engle (1982) and Weiss (1985). The log-likelihood function is:

$$\ln L(\rho, \alpha; \Delta \ln S) = \text{const} - \sum_{t=1}^T \ln \sigma_t - \frac{1}{2} \sum_{t=1}^T \frac{\varepsilon_t^2}{\sigma_t^2}. \quad (4)$$

The simplicity of $\ln L$ under the null of no ARCH makes the Lagrange multiplier test extremely attractive. Because of the large sample size we used the TR^2 version given in Engle (1982), which indicated strong ARCH effects. Information criteria were used to determine appropriate order; in no case did we identify an order greater than twelve. We therefore estimated ARCH(12) models for each series.

In addition, it may be argued on *a priori* grounds that the α_i , $i = 1, \dots, p$ should be monotonically decreasing. This follows from the basic intuition of the ARCH model, which is that high volatility 'today' tends to be followed by similar volatility 'tomorrow' and vice-versa. In this spirit it is unreasonable to let a squared innovation from the distant past have a greater effect on current conditional variance than a squared innovation from the recent past. This intuition may be enforced by restricting the α_i , $i = 1, \dots, p$ to be linearly decreasing:

$$\sigma_t^2 | \Psi_{t-1} = \alpha_0 + \theta [p\varepsilon_{t-1}^2 + (p-1)\varepsilon_{t-2}^2 + \dots + \varepsilon_{t-p}^2]. \quad (5)$$

$$\Psi_{t-1} = \{\varepsilon_{t-1}, \dots, \varepsilon_{t-p}\}. \quad (6)$$

The estimates of the linearly constrained ARCH models are given in Table III, along with the maximized log likelihoods, iterations to convergence, the sum of the $\hat{\alpha}_i$, and the unconditional variance.⁹ The estimated models are third-order AR representations (with allowance for a non-zero mean) with twelfth-order linearly constrained ARCH disturbances:

$$\begin{aligned} (1 - \rho_1 L - \rho_2 L^2 - \rho_3 L^3) \Delta \ln S_t &= \mu + \varepsilon_t, \\ \varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_{t-12} &\sim N(0, \sigma_t^2), \\ \sigma_t^2 &= \alpha_0 + \theta \sum_{i=1}^{12} (13-i)\varepsilon_{t-i}^2. \end{aligned} \quad (7)$$

As expected, the intercept and AR parameters are often insignificant and always very small, while the ARCH parameters are highly significant and of substantial magnitude. The conditional mean intercept term is insignificant for all exchange rates. All but two of the 21 autoregressive lag coefficients for the seven currencies are positive, all are very small, and most are insignificant, as expected. Convergence was obtained for each exchange rate in no more than 13 iterations of Davidon-Fletcher-Powell, and the log likelihoods were noticeably single-peaked, leading to the same parameter estimates for a variety of startup values.¹⁰

The estimated conditional variances are easily obtained. We begin with the estimated disturbances:

$$\hat{\varepsilon}_{jt} = \Delta \ln S_{jt} - \text{const}_j - \hat{\rho}_{1j} \Delta \ln S_{j,t-1} - \hat{\rho}_{2j} \Delta \ln S_{j,t-2} - \hat{\rho}_{3j} \Delta \ln S_{j,t-3}, \quad (8)$$

⁹For conformity with subsequent results, the reported maximized log likelihoods are for $1000 \Delta \ln S$.

¹⁰The first 12 observations are used as initial conditions for the conditional variance. The point likelihoods are therefore summed over $t = 13, \dots, T$ to construct the sample likelihood.

Table III. Weekly nominal dollar spot rates, ARCH model estimates

	CD	FF	DM	LIR	YEN	SF	BP
μ	0.00029 (1.48)	0.00077 (1.61)	-0.00016 (-0.33)	0.00065 (2.10)*	-0.00021 (-0.46)	-0.00023 (-0.42)	-0.00088 (-1.81)*
ρ_1	0.12436 (2.81)***	0.06323 (1.48)	0.09167 (2.20)**	0.06318 (1.49)	0.05542 (1.22)	0.06323 (1.49)	0.05452 (1.24)
ρ_2	0.07845 (1.81)*	0.09044 (2.11)**	0.07200 (1.71)*	0.06785 (1.52)	0.07959 (1.77)*	0.03115 (0.72)	0.03981 (0.90)
ρ_3	-0.02651 (-0.60)	0.05090 (1.21)	-0.00239 (-0.06)	0.06138 (1.38)	0.08140 (1.78)*	0.02060 (0.48)	0.04679 (1.06)
$\sqrt{\alpha_0}$	0.00364 (11.90)***	0.00797 (10.12)***	0.00731 (8.69)***	0.00367 (6.27)***	0.00803 (13.72)***	0.00761 (7.20)***	0.00800 (13.65)***
θ	0.08372 (10.00)***	0.09664 (12.97)***	0.09912 (13.72)***	0.12287 (20.37)***	0.09184 (13.89)***	0.10505 (14.96)***	0.09430 (15.74)***
iterations	12	12	11	12	11	11	11
$\ln L$	-1310.10	-1887.00	-1880.24	-1765.71	-1845.78	-1976.72	-1871.15
$\Sigma \alpha_i$	0.547	0.728	0.766	1.178	0.658	0.861	0.694
$\alpha_0/(1 - \Sigma \alpha_i)$	0.000029	0.000234	0.000228	NA	0.000189	0.000417	0.000209

Significance levels: * 10 per cent, ** 5 per cent, *** 1 per cent.

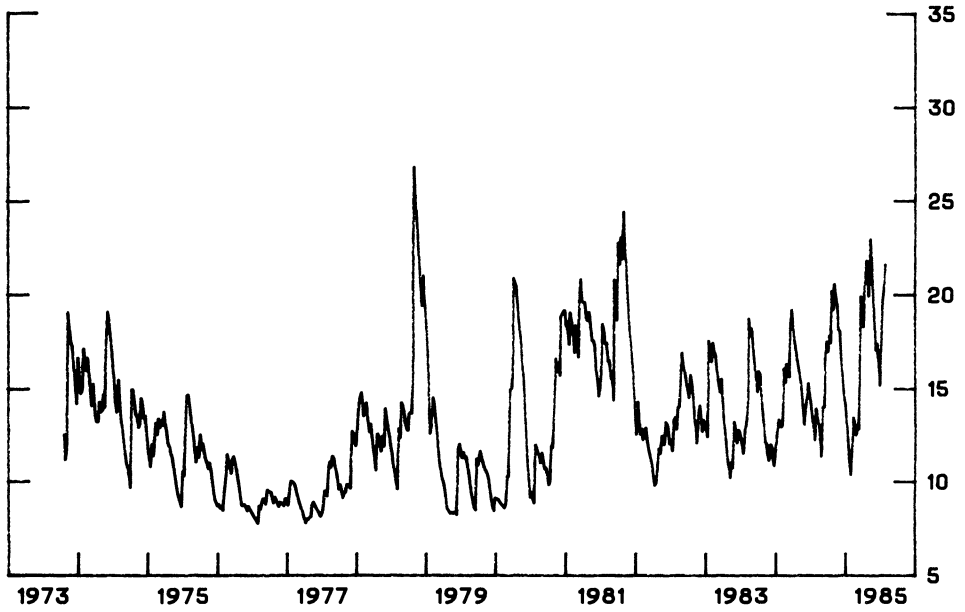


Figure 3. Conditional standard deviation, DM, univariate model

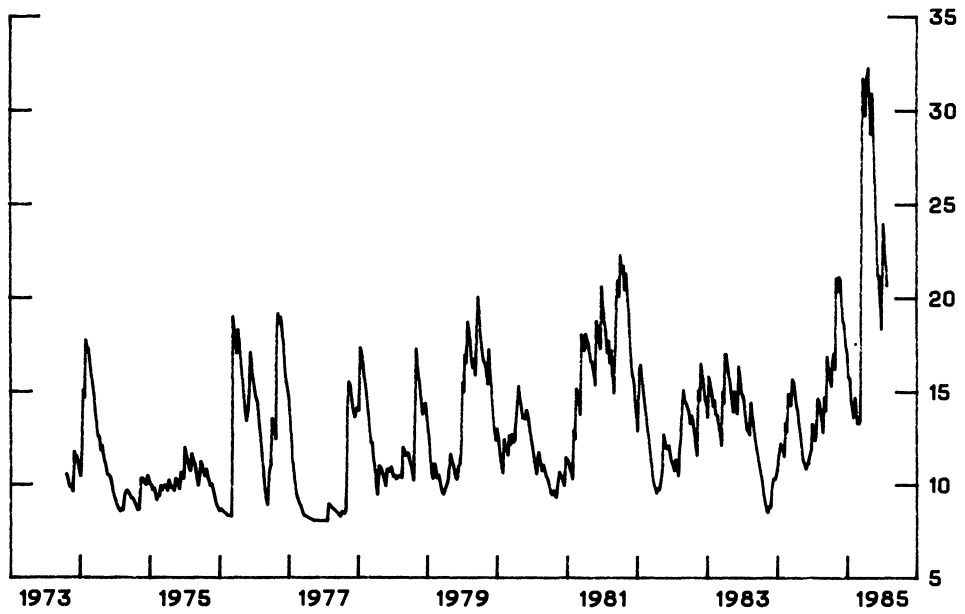


Figure 4. Conditional standard deviation, BP, univariate model

for $j = \text{CD, FF, DM, LIR, YEN, SF, BP}$. The estimated conditional variance is then given by:

$$\hat{\sigma}_{jt}^2 = \hat{\alpha}_{0j} + \hat{\theta} \sum_{i=1}^{12} (13 - i) \hat{\epsilon}_{j,t-i}^2, \quad (9)$$

$j = \text{CD, FF, DM, LIR, YEN, SF, BP}$. The estimated DM/\$ and \$/BP conditional standard deviations for linear ARCH(12) models are shown in Figures 3 and 4, respectively.¹¹

While there are substantial ‘own-country’ effects in the movements of the conditional variances of each of the seven rates, similarities in the qualitative conditional variance movements are apparent. There is a tendency towards high conditional variance in the very early part of the float, perhaps due to the uncertainty created by the 70 per cent increase in the posted price of Arabian crude oil of October 1973, and the additional 100 per cent increase in December 1973. Towards the middle of the 1970s we see generally smaller conditional variances as gloomy economic news translates into relatively smooth dollar depreciation, culminating in the historic lows achieved against the DM, YEN and other major currencies on 29 December 1977. The year 1978, particularly the latter part, brings a return of higher volatility, as large intervention efforts by the Federal Reserve and the Treasury begin to turn the dollar around. Another period of very high conditional variances arises in mid-1981, as interest rates in the 20 per cent range brought the dollar to new highs against the major European currencies. The CD also reached a post-1981 low on 31 July, closing at 80.9 US cents. As inflation subsided, so too did exchange rate volatility, but it did begin to grow again towards the end of the sample.

4. A MULTIVARIATE MODEL WITH FACTOR STRUCTURE

While the univariate ARCH models estimated above offer good statistical descriptions of exchange rate movements, they are not satisfying relative to a full multivariate model. They do, however, provide useful guidance for multivariate specification. The move to a multivariate framework is important for a number of reasons. First, non-zero covariances among exchange rate innovations require simultaneous multivariate estimation if full efficiency is to be achieved. In the present case of vector exchange rate modelling, all rates are bilateral dollar rates, which makes zero innovation covariances very unlikely. The portfolio-balance approach to exchange rates implies such covariation, because new information coming to the market (regarding the state of the US economy, for example) affects *all* dollar rates, as agents’ asset demands shift and portfolios are rebalanced.

Second, the conditional covariances may not be constant. Specifically, they may display temporal persistence, exactly like the conditional variances. If this is found to be the case, examination of the time-paths of conditional covariances may provide useful information. As noted by Bollerslev (1987b), for example, risk premia in asset pricing models depend either on the covariation among asset prices or the covariation among marginal rates of substitution; in general, we do not expect such risk premia to be time-invariant.

Third, further insight into the nature of exchange rate interaction may be gained via multivariate parameterization motivated by latent variable considerations. The role of ‘news’ in the determination of exchange rate movements (Frenkel, 1981) suggests not only correlation among exchange rate shocks, but also commonality in the conditional variance movements of the shocks (regardless of correlation), as all exchange rates react to the arrival of new information. In such a model, movements in each exchange rate may be ascribed to an

¹¹ For conformity with subsequent multivariate results, the conditional standard deviations displayed are those of $1000 \Delta \ln S_t$.

underlying factor representing news (common across all rates) upon which country-specific shocks are superimposed. In short, a common factor approach has a strong substantive motivation that simultaneously produces a parsimonious variance–covariance structure.

Consider first the multivariate model of Kraft and Engle (1982), which is a direct generalization of the earlier univariate model, except that the temporal evolution of an entire variance–covariance matrix is now modelled. If Ψ_{t-1} is an information set containing:

$$\{\varepsilon_{j,t-1}, \dots, \varepsilon_{j,t-p}\}, \quad j = 1, \dots, N, \quad (10)$$

then the N -variate ARCH(p) exchange rate system is given by:

$$\varepsilon_t | \Psi_{t-1} \sim N(0, H_t(\Psi_{t-1})), \quad (11)$$

where $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{Nt})'$. H_t is an $(N \times N)$ symmetric positive definite conditional variance–covariance matrix, where:

$$H_{ij,t} = \alpha_{ij,0} + \varepsilon'_{t-1} C_{1,ij} \varepsilon_{t-1} + \dots + \varepsilon'_{t-p} C_{p,ij} \varepsilon_{t-p}. \quad (12)$$

Each element of H_t is therefore a sum of quadrature forms in $\varepsilon_{t-1}, \dots, \varepsilon_{t-p}$, and depends on the p th-order past histories of all innovation squares and cross-products. The full variance–covariance matrix may then be assembled as:

$$H_t = H_0 + (I \otimes \varepsilon_{t-1})' C_1 (I \otimes \varepsilon_{t-1}) + \dots + (I \otimes \varepsilon_{t-p})' C_p (I \otimes \varepsilon_{t-p}), \quad (13)$$

where C_k is an $(N^2 \times N^2)$ matrix with $(N \times N)$ blocks $C_{k,ij}$. It will be convenient for our purposes to vectorize the lower triangle of H_t and write:

$$h_t = a_0 + A_1 \eta_{t-1} + \dots + A_p \eta_{t-p}, \quad (14)$$

where:

$$h_t = \text{vec}(\text{LT}(H_t)),$$

$$\dim(h_t) = \dim(a_0) = \dim(\eta_{t-i}) = \frac{N^2 + N}{2}, \quad i = 1, \dots, p$$

$$\dim(A_i) = \left(\frac{N^2 + N}{2} \times \frac{N^2 + N}{2} \right), \quad i = 1, \dots, p.$$

The operators ‘vec’, ‘dim’, and ‘LT’ are the vectorization, dimension, and lower triangle operators, respectively, and the vector η_{t-i} contains all squared innovations and innovation cross-products at lag i .

Consistent, asymptotically efficient and normal parameter estimates are obtained by maximizing the log likelihood, which is a direct multivariate analogue of the univariate case. The point log likelihoods are given by

$$\ln L_t = -(N/2) \ln 2\pi + 1/2 \ln |H_t^{-1}| - \frac{1}{2} \varepsilon_t' H_t^{-1} \varepsilon_t, \quad (15)$$

and the likelihood for the sample is the sum of the point likelihoods.

The model developed thus far has

$$K = \frac{(N^2 + N)}{2} + p \frac{(N^2 + N)^2}{2}$$

parameters to be estimated. For a seven-variable ARCH(12) exchange rate system, $K = 9536$! Our task, then, is to impose various restrictions designed to reduce the number of free parameters, while simultaneously not imposing too much prior information on the data.

The system as written above allows each conditional variance and covariance to depend on p lags of the squared innovations and innovation cross-products of every exchange rate in the system. A more manageable and intuitively reasonable parameterization is obtained by allowing each conditional variance to depend only on own lagged squared innovations and each conditional covariance to depend only on own lagged innovation cross-products. This corresponds to diagonal A matrices, and it reduces the number of parameters by two orders of magnitude, since we now have only

$$K = \frac{N^2 + N}{2} + p \frac{(N^2 + N)}{2}$$

parameters to estimate. For $N = 7$ and $p = 12$, $K = 364$, which is still too large a number to handle.

Fortunately, however, we may use the results of section 3, in which we argued for a linearly decreasing ARCH lag weight structure, to provide further parametric economy. We retain $A_i = \text{diag}$, $i = 1, \dots, p$, but we now require that A_i be a scalar matrix with diagonal elements $(p - i + 1)$, $i = 1, \dots, p$, and we write:

$$h_t = a_0 + M(A_1\eta_{t-1} + \dots + A_p\eta_{t-p}), \quad (16)$$

where M is an

$$\left(\frac{N^2 + N}{2} \times \frac{N^2 + N}{2} \right)$$

diagonal matrix. Rewrite the system as:

$$h_t = a_0 + M \sum_{i=1}^p (p - i + 1)\eta_{t-i}. \quad (17)$$

This leads to $K = (N^2 + N)$ parameters to be estimated, which for our seven-variate ARCH(12) exchange rate system is 56. This is still a very large number of parameters; in addition, such a specification cannot account for the fact that substantial commonality appears to exist in exchange rate volatility movements.

A factor analytic approach enables us to simultaneously address both problems. Consider the seven-variate system:

$$\varepsilon_t = \lambda F_t + e_t, \quad (18)$$

$(7 \times 1) \quad (7 \times 1) \quad (1 \times 1) \quad (7 \times 1)$

where:

$$\begin{aligned} E(F_t) &= E(e_{jt}) = 0, \text{ for all } j \text{ and } t, \\ E(F_t F_{t'}) &= 0, \text{ } t \text{ not equal to } t', \\ E(F_t e_{jt'}) &= 0 \text{ for all } j, t, t', \\ E(e_{jt} e_{kt'}) &= \begin{cases} \gamma_j & \text{if } j = k, t = t' \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

(All expectations are understood to be conditional.) If in addition:

$$F_t | F_{t-1}, \dots, F_{t-12} \sim N(0, \sigma_t^2), \quad (19)$$

$$\sigma_t^2 = \alpha_0 + \theta \sum_{i=1}^{12} (13 - i) F_{t-i}^2, \quad (20)$$

then it follows that:

$$H_t = \sigma_t^2 \lambda \lambda' + \Gamma, \quad (21)$$

where $\Gamma = \text{cov}(e_t)$. Thus, the j th time- t conditional variance is given by:

$$H_{jji} = (\lambda_j^2 \alpha_0 + \gamma_j) + \lambda_j^2 \theta \sum_{i=1}^{12} (13 - i) F_{t-i}^2, \quad (22)$$

and the jk th time- t conditional covariance is:

$$H_{jkt} = (\lambda_j \lambda_k \alpha_0) + \lambda_j \lambda_k \theta \sum_{i=1}^{12} (13 - i) F_{t-i}^2. \quad (23)$$

The intuitive motivation of such a model is strong. The common factor F represents general influences which tend to affect all exchange rates. The impact of the common factor on exchange rate j is reflected in the value λ_j . The ‘unique factors’, represented by the e_j , reflect uncorrelated country-specific shocks. The conditional variance–covariance structure (H_t) of the observed variables arises from their joint dependence on the common factor F . All conditional variance and covariance functions depend on the common movements of F . The parameters of those functions are different, however, depending on the λ and γ values.

Before beginning the empirical analysis, all data are multiplied by 1000, to help avoid the formation of non-positive definite point conditional covariance matrices (H_t) while iterating. It is of interest to note that such rescaling changes only the intercept parameters in the ARCH variance and covariance equations. In particular, multiplying all data by a constant K multiplies all intercepts by K^2 . This result enables judicious choice of start-up values for multivariate ARCH estimation. In addition, we take:

$$\varepsilon_{jt} = \Delta \ln S_{jt} - \text{const}_j - \hat{\rho}_{1j} \Delta \ln S_{j,t-1} - \dots - \hat{\rho}_{3j} \Delta \ln S_{j,t-3}, \quad (24)$$

$j = \text{CD}, \dots, \text{BP}$, where the parameter estimates are as given in Table III. Such conditioning on the univariate ARCH conditional mean parameter estimates is necessary for numerical tractability.

We begin by discussing a preliminary two-step estimation and testing procedure, and then we discuss simultaneous parameter estimation using a Kalman filter. In the first step of the two-step procedure we factor the unconditional covariance matrix and extract a time series of latent factor values $\{\hat{F}_t\}$. In the second step we test for and model ARCH effects in the extracted factor. In this way preliminary estimates of the model parameters λ_j , γ_j , ($j = \text{CD}, \dots, \text{BP}$), as well as α_0 and θ , are obtained, which are subsequently used as start-up values for simultaneous estimation.

First, we factor the unconditional covariance matrix, which we denote by H . Without loss of generality, we set the unconditional variance (denoted σ^2) of the common factor to unity, to establish the factor’s scale. Then:

$$H = \lambda \lambda' + \Gamma. \quad (25)$$

There are 14 parameters to be estimated (7λ and 7γ) from 28 independent covariance equations. This is necessary, but not sufficient, for identification. The Joreskog (1979) sufficient conditions for identification are also satisfied, however, since $\sigma^2 = 1$ and Γ is diagonal. We next obtain estimates of the λ and γ values, as well as an extracted time series of factor values $\{\hat{F}_t\}$.¹² The

¹² First-step estimates of factor loadings (λ) and country-specific shock variances (γ), as well as the extracted time-series of latent factor values, are obtained using SAS procedure FACTOR. Alternatively, one could estimate the parameters by maximizing a likelihood constructed with a Kalman filter, and extract the factor with a Kalman smoother, as in Watson and Engle (1983).

Table IV

	CD	FF	DM	LIR	YEN	SF	BP
CD	1·00000	—	—	—	—	—	—
FF	0·27662	1·00000	—	—	—	—	—
DM	0·25856	0·81059	1·00000	—	—	—	—
LIR	0·19834	0·65705	0·64533	1·00000	—	—	—
YEN	0·09878	0·49253	0·52593	0·41756	1·00000	—	—
SF	0·22164	0·70914	0·82814	0·55323	0·53976	1·00000	—
BP	-0·24631	-0·59277	-0·60461	-0·47431	-0·37155	-0·53196	1·00000

correlation matrix of the ε vector is as shown in table IV, which indicates strong cross-equation interaction. The eigenvalues of this matrix are 4·06, 0·94, 0·62, 0·53, 0·46, 0·26, 0·14, indicating the presence of one strong common factor, corresponding to the large maximum eigenvalue of 4·06. The estimated (λ_j, γ_{jj}) pairs are (1·47, 25·14), (11·70, 47·87), (12·84, 19·48), (8·66, 80·44), (7·27, 107·47), (13·94, 69·59), (-8·82, 106·96), for the CD, FF, DM, LIR, YEN, SF, and BP, respectively.¹³

Second, we test for the model ARCH effects in $\{\hat{F}_t\}$. The Lagrange multiplier test statistic for white noise against an ARCH(12) alternative has a value of 53·01, which is highly significant. We therefore replace σ^2 with:

$$\sigma_t^2 = \alpha_0 + \theta \sum_{i=1}^{12} (13-i) \hat{F}_{t-i}^2. \quad (26)$$

The unconditional variance is still normalized to unity, by imposing the restriction $\theta = (1 - \alpha_0)/78$. We then obtain the second-step estimates of α_0 and θ , as with the earlier univariate estimates, conditional upon the extracted factor $\{\hat{F}_t\}$. Convergence is obtained in four iterations of Davidon–Fletcher–Powell, yielding $\hat{\alpha}_0 = 0·027$, which implies a θ estimate of 0·012. Given that we now have preliminary estimates of all model parameters and the common factor, it is of interest to evaluate the multivariate log likelihood obtained by summing (15) and to compare it to the sum of the univariate log likelihoods, in order to get a rough feel for the improvement yielded by the multivariate factor approach. This multivariate log likelihood takes a value of -11,640·45, which is substantially greater than the sum of -12,536·70 for the seven independent univariate ARCH models.

Having established the likely presence of a common factor displaying ARCH effects, we now turn to simultaneous parameter estimation, which is done by casting the model in state-space form, using the Kalman filter to obtain the innovation vectors and their covariance matrices, and constructing the multivariate log likelihood via a prediction error decomposition. Such an approach is closely related to the DYMIMIC model of Watson and Engle (1983). The state-space form of the model is given by

$$F_t = v_t, \quad (\text{transition equation}) \quad (27)$$

$$\varepsilon_t = \lambda F_t + e_t, \quad (\text{observation equation}) \quad (28)$$

where:

$$v_t | v_{t-1}, \dots, v_{t-12} \sim N(0, \sigma_t^2), \quad (29)$$

$$\sigma_t^2 = \alpha_0 + \theta \sum_{i=1}^{12} (13-i) F_{t-i}^2, \quad (30)$$

¹³The negative BP correlations with other currencies are expected because of the reciprocal units in which the BP is measured. The negative λ value for BP is similarly expected.

and $e_t \sim N(0, \Gamma)$, $E(v_t e_s) = 0$, $E(e_t F_0) = 0$ for all s, t . To estimate the model we must take account of the fact that σ_t^2 is not measurable with respect to observable factor extractions. We therefore replace (30) with

$$\sigma_t^2 = \alpha_0 + \theta \sum_{i=1}^{13} (13-i) \bar{F}_{t-i}^*, \quad (31)$$

where the \bar{F}_{t-i}^* are Kalman-filter-based state vector estimates, to be defined shortly. The log likelihood which we construct is therefore best viewed as an approximation. To identify the model, we again normalize the unconditional variance of the latent factor to unity. The Kalman filter is used to obtain the time series of (7×1) innovation vectors ε_t and their conditional covariance matrices H_t . The log likelihood is then formed as:

$$\ln L = \text{const} - 1/2 \sum_{t=1}^T (\ln |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t). \quad (32)$$

The prediction equations for the state vector F (which in this case is a scalar) and its covariance matrix Ω (which in this case is just a scalar variance) are very simple:

$$\bar{F}_{t/t-1}^* = 0 \quad (33)$$

$$\bar{\Omega}_{t/t-1}^* = \sigma_t^2, \quad (34)$$

where

$$\sigma_t^2 = \alpha_0 + (1 - \alpha_0)/78 \sum_{i=1}^{12} (13-i) \bar{F}_{t-i}^*.$$

Similarly, the updating recursions have the simple form:

$$\bar{F}_t^* = \bar{\Omega}_{t/t-1}^* \lambda' H_t^{-1} \varepsilon_t, \quad (35)$$

$$\bar{\Omega}_t^* = \bar{\Omega}_{t/t-1}^* - \bar{\Omega}_{t/t-1}^* \lambda' H_t^{-1} \lambda \bar{\Omega}_{t/t-1}^*, \quad (36)$$

where the innovation ε_t has covariance matrix:

$$H_t = \lambda \bar{\Omega}_{t/t-1}^* \lambda' + \Gamma, \quad (37)$$

$$= \sigma_t^2 \lambda \lambda' + \Gamma. \quad (38)$$

In order to maintain conformity with the univariate analyses, the likelihood function is constructed over observations 13–632, using as $\bar{F}_1, \dots, \bar{F}_{12}$ the values of $\bar{F}_1, \dots, \bar{F}_{12}$ obtained from the first step of the two-step estimation procedure.

A few observations are in order. First, note that due to the degenerate Markov nature of the state vector $F_t = v_t$, the updating recursion for $\bar{\Omega}_t^*$ is never used. In the fully dynamic case in which:

$$F_t = \phi F_{t-1} + v_t, \quad (39)$$

we would have:

$$\bar{\Omega}_{t/t-1}^* = \phi^2 \bar{\Omega}_{t-1}^* + \sigma_t^2, \quad (40)$$

evaluation of which requires $\bar{\Omega}_{t-1}^*$ and hence the updating recursion. The lack of serial correlation in the exchange rate innovation vector ε_t makes this unnecessary. Second, the innovation covariance matrix (38) delivered by the Kalman filter is similar in structure to the one used in the two-step estimation procedure. The Kalman filter, however, produces real-time extractions of F_t , as opposed to the two-step procedure which conditions on first-step

Table V. Common factor loadings and unique factor standard deviations

	CD	FF	DM	LIR	YEN	SF	BP
$\hat{\lambda} =$	1·60 (0·57)	11·96 (4·11)	13·07 (4·51)	9·18 (3·14)	7·71 (2·69)	14·24 (4·89)	-9·33 (3·23)
$\hat{\gamma} =$	5·20 (0·14)	6·74 (0·23)	4·52 (0·24)	8·83 (0·27)	10·33 (0·31)	8·38 (0·28)	10·23 (0·30)

extractions. Finally, again due to the lack of serial correlation in the transition equation, the smoothed (at end of sample) estimates of the state vector are simply the real-time updated values (F_t^*); this is very convenient.

Parameter estimates are shown in Table V, together with their asymptotic standard errors. Convergence was obtained in 44 iterations of Davidon–Fletcher–Powell, using the two-step estimates as start-up values. All factor loadings are highly significant. The European currencies FF, DM, LIR, SF and BP load most heavily on the factor; the YEN loads less heavily, and the CD loads substantially less heavily.¹⁴ This categorization is reasonable in light of the highly integrated nature of the European economies, particularly France, Germany and Switzerland. In addition, Germany, France and Italy are members of the European Monetary System (EMS), founded in March 1979, before which they participated in the ‘snake’. The parity bands for the LIR established under the EMS are wider than those for other currencies, which may provide some insight into the slightly smaller common-factor loading and larger specific-factor variance for the LIR.¹⁵

Examination of the percentage of exchange rate innovation variance explained by the common factor enables us to assess the balance achieved between common-factor-induced commonality and specific-factor-induced divergence in volatility movements. The common factor explains 9, 77, 93, 54, 37, 77, and 47 per cent of variation in the CD, FF, DM, LIR, YEN, SF and BP, respectively. Thus, the currencies with small factor loadings do not, in general, have fully counterbalancing small specific factor variances. In fact, the LIR, YEN and BP, currencies with relatively lower loadings, have the largest specific shock variances. The rankings of percentage variation explained are therefore similar to those of the absolute factor loadings.

The estimated ARCH effects in the common factor are quite strong. We obtain an estimate of 0·16 for $\sqrt{\alpha_0}$, with an asymptotic standard error of 0·06, which (due to the unit unconditional variance constraint) implies a θ estimate of 0·012. The associated sum of the ARCH lag weights, given by $78\hat{\theta}$, is therefore 0·975, which is quite close to the unit circle, and somewhat greater than most of the univariate θ estimates. Presumably, the volatility persistence associated with the common factor reflects an effort to accommodate the high volatility persistence found earlier for the LIR. The time series of extracted common factor values is shown in Figure 5. The temporal behaviour of the factor is similar to that of the DM, which reflects the DM’s central role in the international monetary system and is consistent with the

¹⁴ The small factor loading for the CD, however, should not be interpreted as indicating ‘no ARCH’. The significant dependence of the CD on the common factor indicates that it should be conditionally heteroskedastic, although we expect the ARCH effects to be less pronounced than those of the other currencies, due to the small loading. This is consistent with the results of Table III, in which the estimated ARCH parameter θ is smallest for the CD.

¹⁵ While this discussion is meant to be suggestive, it is certainly not exhaustive. Member countries adhere to parity bands set for *intra*-EMS exchange rates (e.g. FF/DM and LIR/DM), while, the rates studied here are all *dollar* rates. The effect of the EMS on volatility of *intra*-EMS exchange rates is examined in greater detail in Diebold and Pauly (1988b) and Bollerslev (1987b).

large percentage of DM variation explained by the common factor. Finally, the maximized log likelihood is -11604.29 , which exceeds the sum of the maximized univariate log likelihoods by more than 900.

The estimated conditional standard deviations for the factor model are obtained by substituting all multivariate estimates into the square root of (22). The univariate and multivariate results for the DM are compared in Figure 6. The two estimates display high

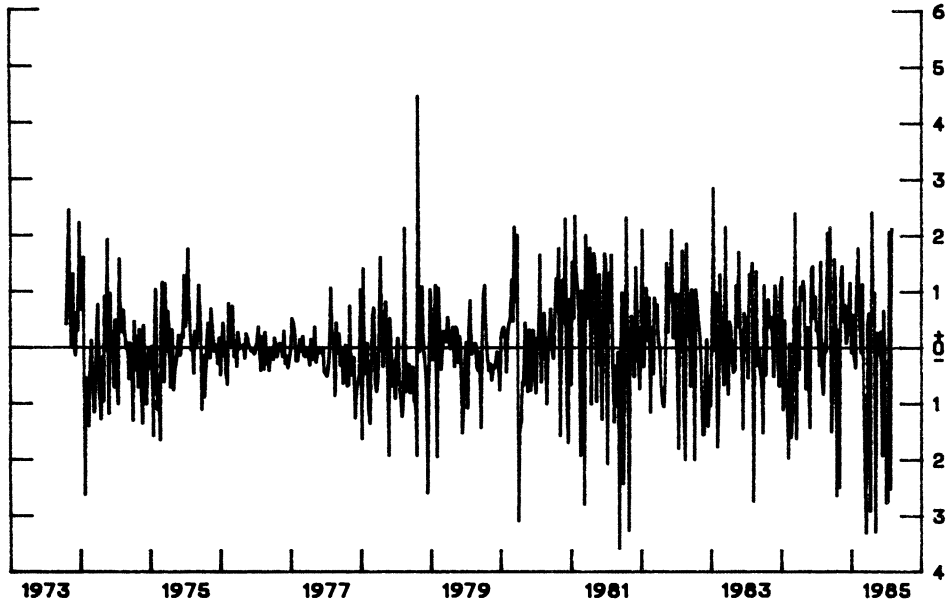


Figure 5. Estimated common factor

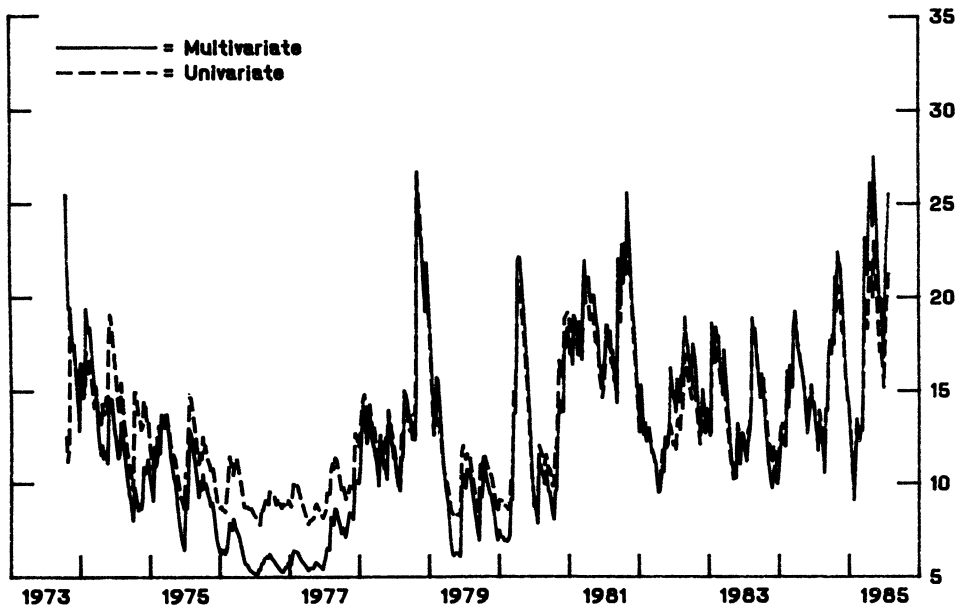


Figure 6. Conditional standard deviations, DM, univariate and multivariate models

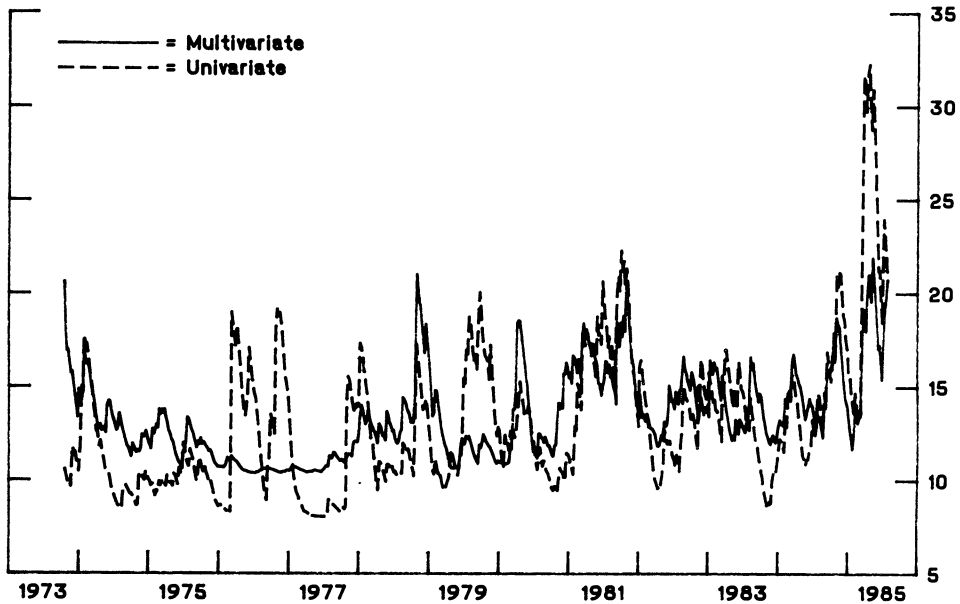


Figure 7. Conditional standard deviations, BP, univariate and multivariate models

coherence, particularly during the latter half of the sample, reflecting the high percentage of DM variation explained by the common factor. Some notable divergences do occur, however, such as that of 1976–1977, in which the multivariate estimate of conditional volatility is lower, and the corresponding multivariate conditional confidence interval becomes tighter. For contrast, we display in Figure 7 the univariate and multivariate conditional standard deviation estimates for the BP, a currency for which the common factor explains relatively *little* variation. The divergence between the two volatility estimates is greater than for the DM. As expected, the movements in univariate volatility estimates are generally more pronounced than those associated with the factor model, and some univariate conditional volatility movements, such as those of 1975–1976 and late 1979, have no counterpart in the multivariate volatility estimates.

5. CONCLUSIONS

We argue that nominal dollar spot rates during the post-1973 float are well described by an approximate vector random walk with ARCH innovations, and we show that the comovements have a factor structure that can be exploited for tractable parameter estimation. More precisely, we identify a particular type of martingale that appears to provide a useful description of exchange rate movements. We hope that our results will facilitate multivariate generalizations of existing univariate empirical studies such as Domowitz and Hakkio (1985), McCurdy and Morgan (1985), Engle, Lillien and Robbins (1987) and Diebold and Pauly (1988a), as well as extensions of existing multivariate studies such as Engle, Granger and Kraft (1984) and Bollerslev, Engle and Wooldridge (1988). In addition, we hope that our results will stimulate additional theoretical research on the conditional moment structure of equilibrium asset price fluctuations. While the conditional volatility movements which we document are *consistent* with recent general-equilibrium asset-pricing models of exchange rate determination, such as

Manuelli and Peck (1986), they do not arise as explicit *predictions* of such models. In this sense, perhaps, measurement is currently ahead of theory.

Several refinements would be desirable in future work. First, the statistical properties of the estimates produced by our two-step procedure merit further investigation, perhaps along the lines of Pagan (1984, 1986). In particular, while we can obtain a minimum-variance unbiased first-step extraction of $\{F_t\}$, we can never extract it with certainty, even with an infinitely large sample. Second, while we modify the model associated with our simultaneous estimation procedure to accommodate the fact that F_t is never observed, it would be preferable (but much harder) to preserve the original model and instead appropriately modify the Kalman filter recursions. Third, the high persistence which we find in the conditional variance of the common factor points to the potential usefulness of models with integrated common-factor conditional variance. Finally, it would be of interest to investigate the possible presence of ARCH (or GARCH, as in Bollerslev, 1986 and Engle and Bollerslev, 1986) in the country-specific factors.

The finding that movements in the major rates can be well approximated by a multivariate random walk with ARCH is certainly consistent with market efficiency: prices adjust rapidly and fully reflect all available information. But what precisely is the significance of ARCH? One explanation runs as follows. In an efficient exchange market, traders respond nearly instantaneously to incoming information; the supply of, and demand for, each currency is always in balance at each moment. But, of course, there must be differences of opinion about the significance of the information for any trades to take place. New information coming into the market affects supply and demand; old information is already incorporated in these schedules and thus in the previous equilibria. Most interpretations of market efficiency involve the idea that what is 'new' is not predictable from what is 'old', but predictability is usually interpreted in a linear sense; hence the conclusion that price changes should be serially uncorrelated. Linearity is quite restrictive, however. It is well known that a time-series may be predictable in terms of other variables or its own past in a nonlinear way, yet appear to be uncorrelated serially or with other variables if the relationship is nonlinear. This could be why we find evidence of ARCH, which is a form of nonlinear serial dependence. An alternative explanation is related to the nature of incoming information, which is of different kinds. For some kinds there is more disagreement and uncertainty about the significance or relevance of the information. When signals are relatively clear (i.e. easily and unambiguously interpretable) then, conditional upon those signals, exchange rate volatility is likely to be low. When there is disagreement about the meaning of incoming information, or when clearly relevant and significant information is scarce, we would expect greater market volatility. Our hypothesis, then, is that the quality of new information coming into the market is serially correlated.¹⁶ A (conditionally) volatile 'world' is associated with news of dubious relevance or significance. Conversely, when the 'world' is tranquil, signals are clear and easily interpreted. The 'state of the world' is a serially correlated thing; hence, we find ARCH.

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¹⁶The quantity of information coming to the market per unit of calendar time may also be serially correlated, which may lead to movements in conditional volatility in calendar time. See, for example, Stock (1987).

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