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Better to Give than to Receive: Predictive Directional Measurement of Volatility Spillovers

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Abstract: Using a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, we propose measures of both total and directional volatility spillovers. We use our methods to characterize daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets, from January 1999 through January 2010. We show that despite significant volatility fluctuations in all four markets during the sample, cross-market volatility spillovers were quite limited until the global financial crisis that began in 2007. As the crisis intensified so too did the volatility spillovers, with particularly important spillovers from the stock market to other markets taking place after the collapse of Lehman Brothers in September 2008.

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1. Introduction

Financial crises occur with notable regularity, and moreover, they display notable similarities (e.g., Reinhart and Rogoff, 2008). During crises, for example, financial market volatility generally increases sharply and spills over across markets. One would naturally like to be able to measure and monitor such spillovers, both to provide "early warning systems" for emergent crises, and to track the progress of extant crises.

Motivated by such considerations, Diebold and Yilmaz (DY, 2009) introduce a volatility spillover measure based on forecast error variance decompositions from vector autoregressions (VARs).[1] It can be used to measure spillovers in returns or return volatilities (or, for that matter, any return characteristic of interest) across individual assets, asset portfolios, asset markets, etc., both within and across countries, revealing spillover trends, cycles, bursts, etc. In addition, although it conveys useful information, it nevertheless sidesteps the contentious issues associated with definition and existence of episodes of "contagion" or "herd behavior".[2]

However, the DY framework as presently developed and implemented has several limitations, both methodological and substantive. Consider the methodological side. First, DY relies on Cholesky-factor identification of VARs, so the resulting variance decompositions can be dependent on variable ordering. One would prefer a spillover measure invariant to ordering. Second, and crucially, DY addresses only *total* spillovers

[1] VAR variance decompositions, introduced by Sims (1980), record how much of the H-step-ahead forecast error variance of some variable, i, is due to innovations in *another* variable, j.

[2] On contagion (or lack thereof) see, for example, Forbes and Rigobon (2002).

(from/to each market i, to/from all other markets, added across i). One would also like to examine *directional* spillovers (from/to a particular market).

Now consider the substantive side. DY considers only the measurement of spillovers across identical assets (equities) in different countries. But various other possibilities are also of interest, including individual-asset spillovers within countries (e.g., among the thirty Dow Jones Industrials in the U.S.), across asset classes (e.g., between stock and bond markets in the U.S.), and of course various blends. Spillovers across asset classes, in particular, are of key interest given the recent global financial crisis (which appears to have started in credit markets but spilled over into equities), but they have not yet been investigated in the DY framework.

In this paper we fill these methodological and substantive gaps. We use a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, and we explicitly include directional volatility spillovers. We then use our methods in a substantive empirical analysis of daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets over a ten year period, including the recent global financial crisis.

We proceed as follows. In section 2 we discuss our methodological approach, emphasizing in particular our new use of generalized variance decompositions and directional spillovers. In section 3 we describe our data and present our substantive results. We conclude in section 4.

2. Methods: Generalized Spillover Definition and Measurement

Here we extend the DY spillover index, which follows directly from the familiar notion of a variance decomposition associated with an *N*-variable vector autoregression. Whereas DY focuses on *total* spillovers in a *simple* VAR framework (i.e., with potentially order-dependent results driven by Cholesky factor orthogonalization), we progress by measuring *directional* spillovers in a *generalized* VAR framework that eliminates the possible dependence of results on ordering.

Consider a covariance stationary *N*-variable VAR(*p*), $x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t$, where $\varepsilon \sim (0, \Sigma)$ is a vector of independently and identically distributed disturbances. The moving average representation is $x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}$, where the *N*x*N* coefficient matrices A_i obey the recursion $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + ... + \Phi_p A_{i-p}$, with A_0 an *N*x*N* identity matrix and $A_i = 0$ for $i<0$. The moving average coefficients (or transformations such as impulse-response functions or variance decompositions) are the key to understanding the dynamics of the system. We rely on variance decompositions, which allow us to parse the forecast error variances of each variable into parts attributable to the various system shocks. Variance decompositions allow us to assess the fraction of the *H*-step-ahead error variance in forecasting x_i that is due to shocks to x_j, $\forall\ j \neq i$, for each *i*.

Calculation of variance decompositions requires orthogonal innovations, whereas our VAR innovations are generally contemporaneously correlated. Identification schemes such as that based on Cholesky factorization achieve orthogonality, but the variance decompositions then depend on ordering of the variables. We circumvent this problem by exploiting the generalized VAR framework of Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998), hereafter KPPS, which produces variance decompositions invariant to ordering. Instead of attempting to orthogonalize shocks, the generalized approach allows correlated shocks but accounts for them appropriately using the historically observed distribution of the errors. As the shocks to each variable are not orthogonalized, the sum of contributions to the variance of forecast error (that is, the row sum of the elements of the variance decomposition table) is not necessarily equal to one.

Variance Shares

Let us define *own variance shares* to be the fractions of the *H*-step-ahead error variances in forecasting x_i due to shocks to x_i, for i=1, 2,..,N, and *cross variance shares*, or *spillovers*, to be the fractions of the *H*-step-ahead error variances in forecasting x_i due to shocks to x_j, for i, j = 1, 2,.., N, such that $i \neq j$.

Denoting the KPPS *H*-step-ahead forecast error variance decompositions by $\theta_{ij}^{g}(H)$, for $H = 1, 2, ...$, we have

$$\theta_{ij}^{g}(H) = \frac{\sigma_{ii}^{-1}\sum_{h=0}^{H-1}(e_i^{'}A_h\Sigma e_j)^2}{\sum_{h=0}^{H-1}(e_i^{'}A_h\Sigma A_h^{'}e_i)} \qquad (1)$$

where Σ is the variance matrix for the error vector ε, σ_{ii} is the standard deviation of the error term for the *i*th equation and e_i is the selection vector with one as the *i*th element and zeros otherwise. As explained above, the sum of the elements of each row of the variance decomposition table is not equal to 1: $\sum_{j=1}^{N}\theta_{ij}^{g}(H) \neq 1$. In order to use the information available in the variance decomposition matrix in the calculation of the spillover index, we normalize each entry of the variance decomposition matrix by the row sum as[3]:

$$\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N} \theta_{ij}^{g}(H)} \tag{2}$$

Note that, by construction, $\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = 1$ and $\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = N$.

Total Spillovers

Using the volatility contributions from the KPPS variance decomposition, we can construct a total volatility spillover index:

$$S^{g}(H) = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \bullet 100 = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{N} \bullet 100. \tag{3}$$

This is the KPPS analog of the Cholesky factor based measure used in Diebold and Yilmaz (2009). The total spillover index measures the contribution of spillovers of volatility shocks across four asset classes to the total forecast error variance.

Directional Spillovers

Although it is sufficient to study the total volatility spillover index to understand how much of shocks to volatility spill over across major asset classes, the generalized VAR approach enables us to learn about the direction of volatility spillovers across major asset

[3] Alternatively, we can normalize the elements of the variance decomposition matrix with the column sum of these elements and compare the resulting total spillover index with the one obtained from the normalization with the row sum.

classes. As the generalized impulse responses and variance decompositions are invariant to the ordering of variables, we calculate the directional spillovers using the normalized elements of the generalized variance decomposition matrix. We measure directional volatility spillovers received by market *i* from all other markets *j* as:

$$S_{i\bullet}^{g}(H) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \bullet 100 \tag{4}$$

In similar fashion we measure directional volatility spillovers transmitted by market *i* to all other markets *j* as:

$$S_{\bullet i}^{g}(H) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{N} \tilde{\theta}_{ji}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ji}^{g}(H)} \bullet 100 \tag{5}$$

One can think of the set of directional spillovers as providing a decomposition of total spillovers into those coming from (or to) a particular source.

Net Spillovers

We obtain the net volatility spillover from market *i* to all other markets *j* as

$$S_{i}^{g}(H) = S_{\bullet i}^{g}(H) - S_{i\bullet}^{g}(H) \tag{6}$$

The net volatility spillover is simply the difference between gross volatility shocks transmitted to and gross volatility shocks received from all other markets.

Net Pairwise Spillovers

The net volatility spillover (6) provides summary information about how much in net terms each market contributes to volatility in other markets. It is also of interest to examine net pairwise volatility spillovers, which we define as:

$$S_{ij}^{g}(H) = \left(\frac{\tilde{\theta}_{ij}^{g}(H)}{\sum_{k=1}^{N} \tilde{\theta}_{ik}^{g}(H)} - \frac{\tilde{\theta}_{ji}^{g}(H)}{\sum_{k=1}^{N} \tilde{\theta}_{jk}^{g}(H)} \right) \bullet 100 \qquad (7)$$

The net pairwise volatility spillover between markets *i* and *j* is simply the difference between gross volatility shocks transmitted from market *i* to *j* and gross volatility shocks transmitted from *j* to *i*.

3. Empirics: Estimates of Volatility Spillovers across U.S. Asset Markets

Here we use our framework to measure volatility spillovers among four key U.S. asset classes: stocks, bonds, foreign exchange and commodities. This is of particular interest because spillovers across asset classes may be an important aspect of the global financial crisis that began in 2007.

In the remainder of this section we proceed as follows. We begin by describing our data in section 3a. Then we calculate average (i.e., total) spillovers in section 3b. We then quantify spillover dynamics, examining rolling-sample total spillovers, rolling-sample directional spillovers, rolling-sample net directional spillovers and rolling-sample net pairwise spillovers below.

<u>Stock, Bond, Exchange Rate, and Commodity Market Volatility Data</u>

We examine daily volatilities of returns on U.S. stock, bond, foreign exchange, and commodity markets. In particular, we examine the S&P 500 index, the 10-year Treasury bond yield, the New York Board of Trade U.S. dollar index futures, and the Dow-Jones/UBS commodity index.[4] The data span January 25, 1999 through January 29, 2010, for a total of 2771 daily observations.

In the tradition of a large literature dating at least to Parkinson (1980), we estimate daily variance using daily high and low prices.[5] For market *i* on day *t* we have

$$\tilde{\sigma}_{it}^2 = 0.361\left[\ln(P_{it}^{\max}) - \ln(P_{it}^{\min})\right]^2,$$

where $P_{it}^{\max}$ is the maximum (high) price in market *i* on day *t*, and $P_{it}^{\min}$ is the daily minimum (low) price. Because $\tilde{\sigma}_{it}^2$ is an estimator of the daily variance, the corresponding estimate of the annualized daily percent standard deviation (volatility) is $\hat{\sigma}_{it} = 100\sqrt{365 \bullet \tilde{\sigma}_{it}^2}$. We plot the four markets' volatilities in Figure 1 and we provide summary statistics of log volatility in Table 1. Several interesting facts emerge, including: (1) The bond and stock markets have been the most volatile (roughly equally so), with commodity and FX markets comparatively less volatile, (2) volatility dynamics appear highly persistent, in keeping with a large literature summarized for example in Andersen, Bollerslev, Christoffersen and Diebold (2006), and (3) all volatilities are high during the recent crisis, with stock and bond market volatility, in particular, displaying huge jumps.

Throughout the sample, stock market went through two periods of major volatility. In 1999, daily stock market volatility was close to 25 percent, but it increased significantly to above 25 percent and fluctuated around that level until mid-2003, moving occasionally above 50 percent. After mid-2003, it declined to less than 25 percent and stayed there until August

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[4] The DJ/AIG commodity index was re-branded as the DJ/UBS commodity index following the acquisition of AIG Financial Products Corp. by UBS Securities LLC on May 6, 2009.
[5] For background, see Alizadeh, Brandt and Diebold (2002) and the references therein.

2007. Since August 2007, stock market volatility reflects the dynamics of the sub-prime crisis quite well.

In the first half of our sample, the interest rate volatility measured by the annualized standard deviation was comparable to the stock market volatility. While it was lower than 25 percent mark for most of 2000, in the first and last few months of 2001, it increased and fluctuated between 25-50 percent. Bond market volatility remained high until mid-2005, and fell below 25 percent from late 2005 through the first half of 2007. Since August 2007, volatility in bond markets has also increased significantly.

Commodity market volatility used to be very low compared to stock and bond markets, but it increased slightly over time and especially in 2005-2006 and recently in 2008. FX market volatility has been the lowest among the four markets. It increased in 2008 and moved to a 25-50 percent band following the collapse of Lehman Brothers in September 2008. Since then, FX market volatility declined, but it is still above its average for the last decade.

Unconditional Patterns: The Full-Sample Volatility Spillover Table

We call Table 2 as volatility spillover table. Its ij^{th} entry is the estimated contribution *to* the forecast error variance of market i coming *from* innovations to market j.[6] Hence the off-diagonal column sums (labeled contributions to others) or row sums (labeled contributions from others), are the “to” and “from” directional spillovers, and the “from

[6] All results are based on vector autoregressions of order 4 and generalized variance decompositions of 10-day-ahead volatility forecast errors. To check for the sensitivity of the results to the choice of the order of VAR we calculate the spillover index for orders 2 through 6, and plot the minimum, the maximum and the median values obtained in Figure A1 of the Appendix. Similarly, we calculated the spillover index for forecast horizons varying from 4 days to 10 days. Both Figure A1 and Figure A2 of the Appendix show that the total spillover plot is not sensitive to the choice of the order of VAR or to the choice of the forecast horizon.

minus to" differences are the net volatility spillovers. In addition, the total volatility spillover index appears in the lower right corner of the spillover table. It is approximately the grand off-diagonal column sum (or row sum) relative to the grand column sum including diagonals (or row sum including diagonals), expressed as a percent.[7] The volatility spillover table provides an approximate "input-output" decomposition of the total volatility spillover index.

Consider first what we learn from the table about directional spillovers (gross and net). From the "directional to others" row, we see that gross directional volatility spillovers to others from each of the four markets are not very different. We also see from the "directional from others" column that gross directional volatility spillovers from others to the bond market is relatively large, at 18.5 percent, followed by the FX market with the spillovers from others explaining 14.2 percent of the forecast error variance. As for net directional volatility spillovers, the largest are from the stock market to others (16.29-11.24=5.05 percent) and from others to the FX market (11.41-14.24=-2.8 percent).

Now consider the total (non-directional) volatility spillover, which is effectively a distillation of the various directional volatility spillovers into a single index. The total volatility spillover appears in the lower right corner of Table 2, which indicates that on average, across our entire sample, 12.6 percent of volatility forecast error variance in all four markets comes from spillovers. The summary of Table 2 is simple: Both total and directional spillovers over the full sample period were quite low.

[7] As we have already discussed in Section 2 in detail, the approximate nature of the claim stems from the properties of the generalized variance decomposition. With Cholesky factor identification the claim is exact rather than approximate; see also Diebold and Yilmaz (2009).

Conditioning and Dynamics I: The Rolling-Sample Total Volatility Spillover Plot

Clearly, many changes took place during the years in our sample, January 1999-January 2010. Some are well-described as more-or-less continuous evolution, such as increased linkages among global financial markets and increased mobility of capital, due to globalization, the move to electronic trading, and the rise of hedge funds. Others, however, may be better described as bursts that subsequently subside.

Given this background of financial market evolution and turbulence, it seems unlikely that any single fixed-parameter model would apply over the entire sample. Hence the full-sample spillover table and spillover index constructed earlier, although providing a useful summary of "average" volatility spillover behavior, likely miss potentially important secular and cyclical movements in spillovers. To address this issue, we now estimate volatility spillovers using 200-day rolling samples, and we assess the extent and the nature of spillover variation over time via the corresponding time series of spillover indices, which we examine graphically in the so-called total spillover plot of Figure 2.

Starting at a value slightly lower than 15 percent in the first window, the total volatility spillover plot for most of the time fluctuates between ten and twenty percent. However, there are important exceptions: The spillovers exceed the twenty percent mark in mid-2006 and most importantly by far exceed the thirty percent level, during the global financial crisis of 2007-2009.

We can identify several cycles in the total spillover plot. The first cycle started with the burst of the tech bubble in 2000 and the index climbed from 13 percent to 20 percent. In the second half of 2001 the index increased to 20 percent again, before dropping back to 10 percent at the end of January 2002. After hitting the bottom in mid-2002, the index went through three relatively small cycles until the end of 2005. The first cycle started in mid-2002 and lasted until the last quarter of 2003. The second cycle was shorter, starting in the first quarter of 2004 and ending in the third quarter. The third cycle during this period starts in the middle of 2004 and lasts until the end of 2005. All three cycles involve movements of the index between 10 and 15 percent.

After the rather calm era from 2003 through 2005, the spillover index recorded a significant upward move in May through the end of 2006. On May 9th 2006 the Federal Open Market Committee of the Federal Reserve decided to increase the federal funds target rate from 4.75 percent to 5.00 percent and signaled the likelihood of another increase in its June meeting.[8] After this decision the total spillover index increased from 12 percent at the end of April 2006 to 24 percent by November 2006. The fact that FED was continuing to tighten the monetary policy led to an increase in volatility in the bond and FX markets which spilled over to other markets.

Finally, the most interesting part of the total spillover plot concerns the recent financial crisis. One can see four volatility waves during the recent crisis: July-August 2007 (credit crunch), January-March 2008 (panic in stock and foreign exchange markets followed by an unscheduled rate cut of three-quarters of a percentage points by Federal Reserve and Bear Stearns' takeover by JP Morgan in March), September-December 2008 (following the collapse of Lehman Brothers) and in the first half of 2009 as the financial crisis started to have its real effects around the world. During the January-March 2008 episode, and especially following the collapse of Lehman Brothers in mid-September and consistent with

[8] Indeed, the FOMC increased the federal funds target rate to 5.25 percent in its June meeting and kept it at that level for more than a year until its September 2007 meeting.

an unprecedented evaporation of liquidity world-wide, the spillover index surges above thirty percent.

Conditioning and Dynamics II: Rolling-Sample Gross Directional Volatility Spillover Plots

Thus far we have discussed the *total* spillover plot, which is of interest but discards directional information. That information is contained in the "Directional *TO* Others" row (the sum of which is given by $S_{i\cdot}^{g}(H)$ in equation 4) and the "Directional *FROM* Others" column (the sum of which is given by $S_{\cdot i}^{g}(H)$ in equation 5).

We now estimate the above-mentioned row and column of Table 2 dynamically, in a fashion precisely parallel to the earlier-discussed total spillover plot. We call these *directional* spillover plots. In Figure 3, we present the directional volatility spillovers from each of the four asset classes *to* others (corresponds to the "directional *to* others" row in Table 2). They vary greatly over time. During tranquil times, spillovers from each market are below five percent, but during volatile times, directional spillovers increase close to 10 percent. Among the four markets, gross volatility spillovers from the commodity markets to others are in generally smaller than the spillovers from the other three markets.

In Figure 4, we present directional volatility spillovers *from* others to each of the four asset classes (corresponds to the "directional *from* others" column in Table 2). As with the directional spillovers *to* others, the spillovers *from* others vary noticeably over time. The relative variation pattern, however, is reversed, with directional volatility spillovers *to* commodities and FX increasing relatively more in turbulent times.

Conditioning and Dynamics III: Rolling-Sample Net Directional Volatility Spillover Plots

Above we briefly discussed the gross spillover plots, because our main focus point is the net directional spillover plot presented in Figure 5. Each point in Figure 5a through 5d corresponds $S_i^g(H)$ (equation 6) and is the difference between the "Contribution from" column sum and the "Contribution to" row sum. In addition, as we described briefly at the end of section 2, we also calculate net pairwise spillovers between two markets (equation 7) and present these plots in Figure 6.

Until the recent global financial crisis net volatility spillovers from/to each of the four markets never exceeded the three percent mark (Figure 5). Furthermore, until 2007 all four markets were at both the giving and receiving ends of net volatility transmissions, with almost equal magnitudes. Things changed dramatically since January 2008. Net volatility spillovers from the stock market stayed positive throughout the several stages of the crisis, reaching as high as six percent after the collapse of the Lehman Brothers in September 2008.

As we have already introduced the net spillover and net pairwise spillover plots, we can now have a detailed analysis of the spillovers from each market to the others using Figures 5 and 6. From 1999 to 2009, there were three major episodes of net volatility spillovers taking place from the stock market to other markets (Figure 5a): during 2000, in 2002 thru 2003, and after January 2008. In our sample period, the first round of volatility spillovers took place from the stock market with the burst of the technology bubble in 2000. As the troubles of the technology stocks intensified after March 2000, the spillover index reached close to 20 percent in the second through the last quarter of 2000 (Figure 2). At the time, the bulk of the volatility spillovers from the stock market were transmitted first to the bond, and then, to the commodity markets (Figure 6a and 6b).

The second period when the stock market was a net transmitter of volatility to other markets spanned from the second half of 2002 to the third quarter of 2003. Technology stocks continued to be under pressure until October 2002 as the Nasdaq Composite Index hit its lowest level since 1997. In addition, the Iraqi crisis and the prospects of a war in the region increased volatility in the US stock markets.[9] During this episode, total spillover index increased from 7.5 percent in June 2002 to 15 percent in June 2003. Net volatility spillovers from the stock market reached close to 3 percent (Figure 5a), and affected mostly the bond market (Figure 6a). The fact that stock market was at the same time a net receiver of volatility from the commodity market (Figure 6b) shows the link between increased volatility in stock markets and the impending Iraqi War at the time.

While the first two episodes of net volatility spillovers from the stock market were important, the third took place during the worst financial crisis that hit the global financial markets. Since January 2008, the total spillovers jumped to above 30 percent twice, in the first quarter and the fourth quarters of 2008. During these two bouts of hefty volatility spillovers across financial markets, net spillovers from the stock market jumped to more than three and seven percents, respectively (Figure 5a). The volatility from the stock market was transmitted to all three markets, but especially to the FX market (close to five percent) following the collapse of the Lehman Brothers (Figure 6c). Actually, during the global financial crisis FX market also received sizeable net volatility spillovers from the bond market (Figure 6e) as well as the commodity market (Figure 6f).

Net volatility spillovers from the bond market tend to be smaller than net spillovers from other markets. We identify three episodes of net volatility spillovers from the bond

[9] Leigh et al. (2003) showed that a 10 percentage point rise in the probability of a war on Iraq lowered the S&P500 by about one and a half percentage points.

market: the second half of 2000 through 2001; the end of 2005 through 2006; and throughout 2007 (Figure 5b). In 2000, the spillovers went in the direction of the FX market (Figure 6e). In 2001, on the other hand, the bulk of volatility spillovers from the bond market were transmitted to the stock market (Figure 6a) and the commodity market (Figure 6d). In the second half of 2005 and the first half of 2006 spillovers from the bond market were transmitted mostly to the FX market. In 2007, on the other hand, the bond market spillovers affected the FX market mostly followed by the commodity market.

We identify four episodes of net volatility spillovers from the commodity markets: Throughout 2002, in the first five months of 2003 (before and immediately after the invasion of Iraq in March 2003), in late 2004 and through 2005, and in the second half of 2009 (Figure 5c). During 2002-2003, the commodity market was a net transmitter of volatility (Figure 5c). The oil prices started to increase from less than $20 at the end of 2001 to close to $40 by February 2003, before falling to almost $25 by the end of April 2003. Volatility spillovers from commodity markets increased in 2003 just before and during the invasion of Iraq by U.S. forces. Volatility spillovers from the commodity markets also increased, at the end of 2004 and early 2005, when the surge in Chinese demand for oil and metals surprised investors sending commodity prices higher (these shocks mostly transmitted to the bond and FX markets, See Figures 6d and 6e), and especially from March 2009 through September 2009 (shocks mostly transmitted to the FX market). The volatility shocks in the commodity market in 2002 and during the initial phases of the Iraqi invasion spilled over mostly to the stock market (Figure 6b), but also to the bond market (Figure 6d). During the late 2004-early 2005 and the first half of 2008, the volatility shocks in the commodity market mostly spilled over to the bond market (Figure 6d), but also to the FX market (Figure 6f). While commodity market was a net recipient of modest levels of volatility shocks from the stock and bond markets, it was a net transmitter to the FX market during 2009.

In the case of FX markets, there were three major episodes of positive net spillovers. Net volatility spillovers from FX markets had little impact on volatility in other markets, perhaps with the exception of the modest spillovers at the end of 2001 and early 2002, from the end of 2002 through first half of 2003, and finally in the second half of 2006 (Figure 5d). Net volatility spillovers from the FX market increased at the end of 2001 and early 2002. It also increased in May 2006, following the FED's decision to tighten the monetary policy further (Figure 5d). In both episodes, the volatility shocks in the FX market spilled over to the stock market and the commodity market (Figures 6c and 6f).

4. Concluding Remarks

We have provided both gross and net *directional* spillover measures that are independent of the ordering used for volatility forecast error variance decompositions. When applied to U.S. financial markets, our measures shed new light on the nature of cross-market volatility transmission, pinpointing the importance during the recent crisis of volatility spillovers from the stock market to other markets.

We are of course not the first to consider issues related to volatility spillovers (e.g., Engle et al. 1990; King et al., 1994; Edwards and Susmel, 2001), but our approach is very different. It produces continuously-varying indexes (unlike, for example, the "high state / low state" indicator of Edwards and Susmel), and it is econometrically tractable even for very large numbers of assets. Although it is beyond the scope of this paper, it will be interesting in future work to understand better the relationship of our spillover measure to a variety of others based on measures ranging from traditional (albeit time-varying) correlations (e.g., Engle, 2002, 2009) to the recently-introduced CoVaR of Adrian and Brunnermeier (2008).

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Figure 1. Daily U.S. Financial Market Volatilities

(Annualized Standard Deviation, Percent)

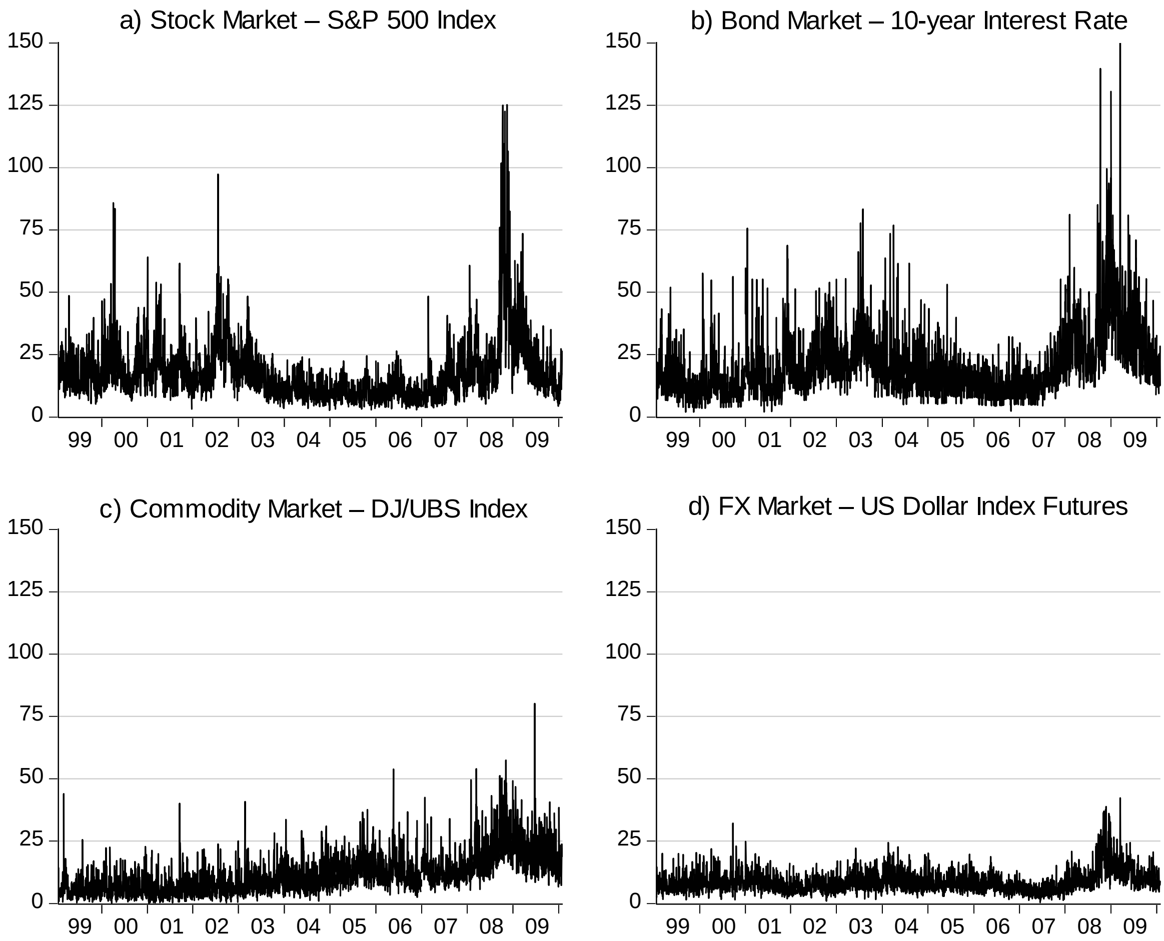


Table 1: Log Volatility Summary Statistics, Four Asset Classes

	Stocks	Bonds	Commodities	FX
Mean	-9.70	-9.44	-10.69	-11.00
Median	-9.74	-9.44	-10.50	-10.99
Maximum	-5.45	-4.23	-6.34	-7.62
Minimum	-13.09	-13.79	-18.33	-16.86
Std. Deviation	1.19	1.19	1.54	0.98
Skewness	0.21	0.019	-0.73	-0.21
Kurtosis	3.18	3.16	4.21	3.87

Table 2: Volatility Spillover Table, Four Asset Classes

	Stocks	Bonds	Commodities	FX	Directional *FROM* Others
Stocks	88.76	7.28	0.34	3.62	11.24
Bonds	10.17	81.49	2.69	5.65	18.51
Commodities	0.46	3.69	93.71	2.14	6.29
FX	5.66	6.99	1.59	85.76	14.24
Directional *TO* Others	16.29	17.95	4.63	11.41	
Directional Including Own	105.0	99.4	98.3	97.2	**Total Spillover Index (**50.3/400): **12.6%**

Figure 2. Total Volatility Spillovers, Four Asset Classes

Figure 3. Directional Volatility Spillovers, *FROM* Four Asset Classes

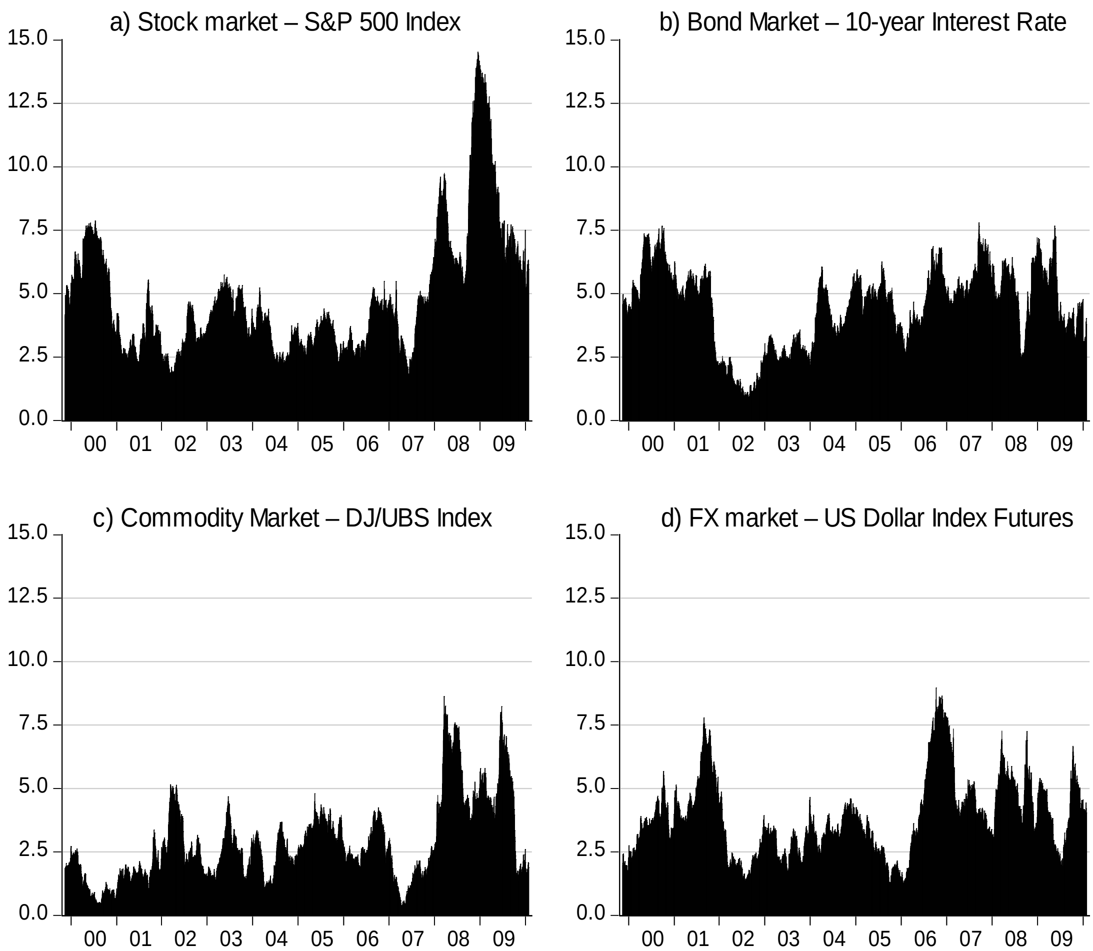


Figure 4. Directional Volatility Spillovers, *TO* Four Asset Classes

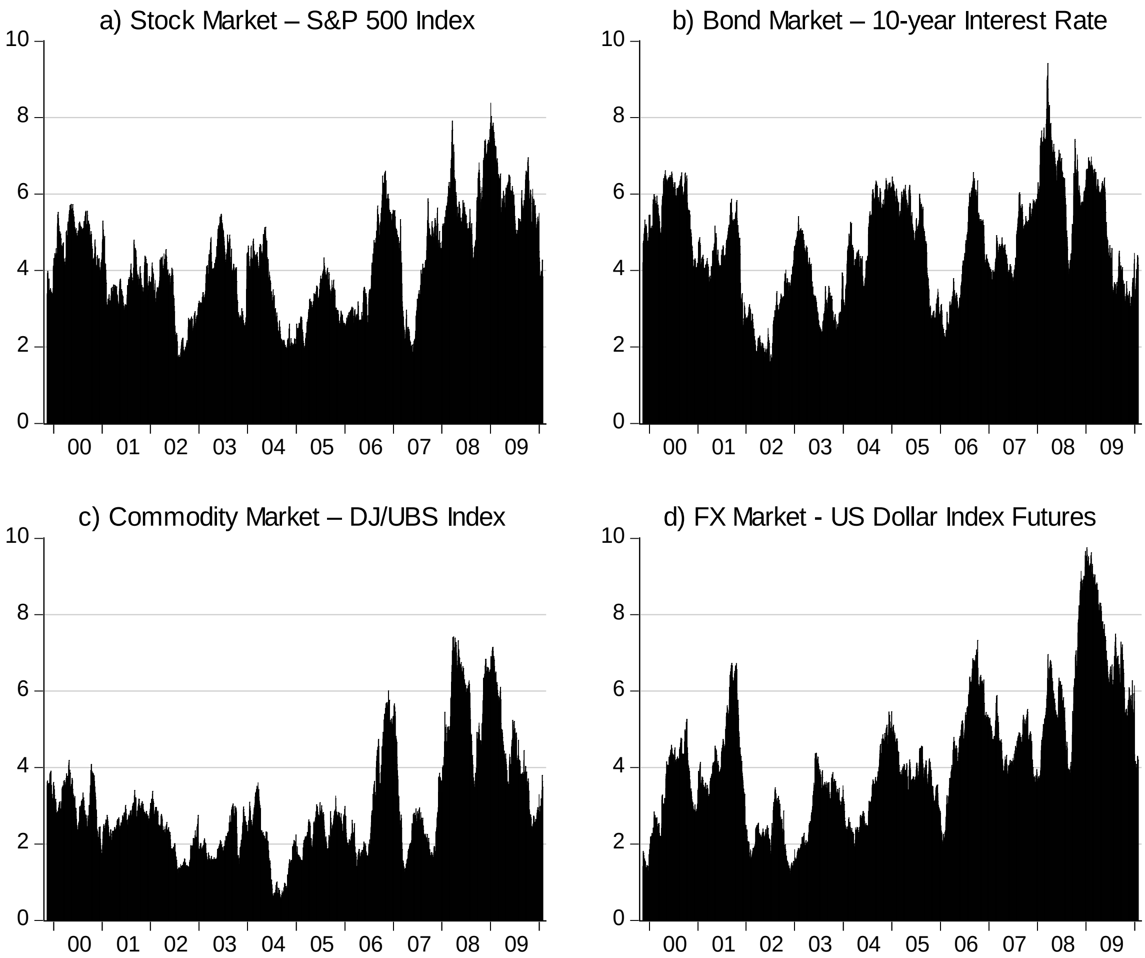


Figure 5. Net Volatility Spillovers, Four Asset Classes

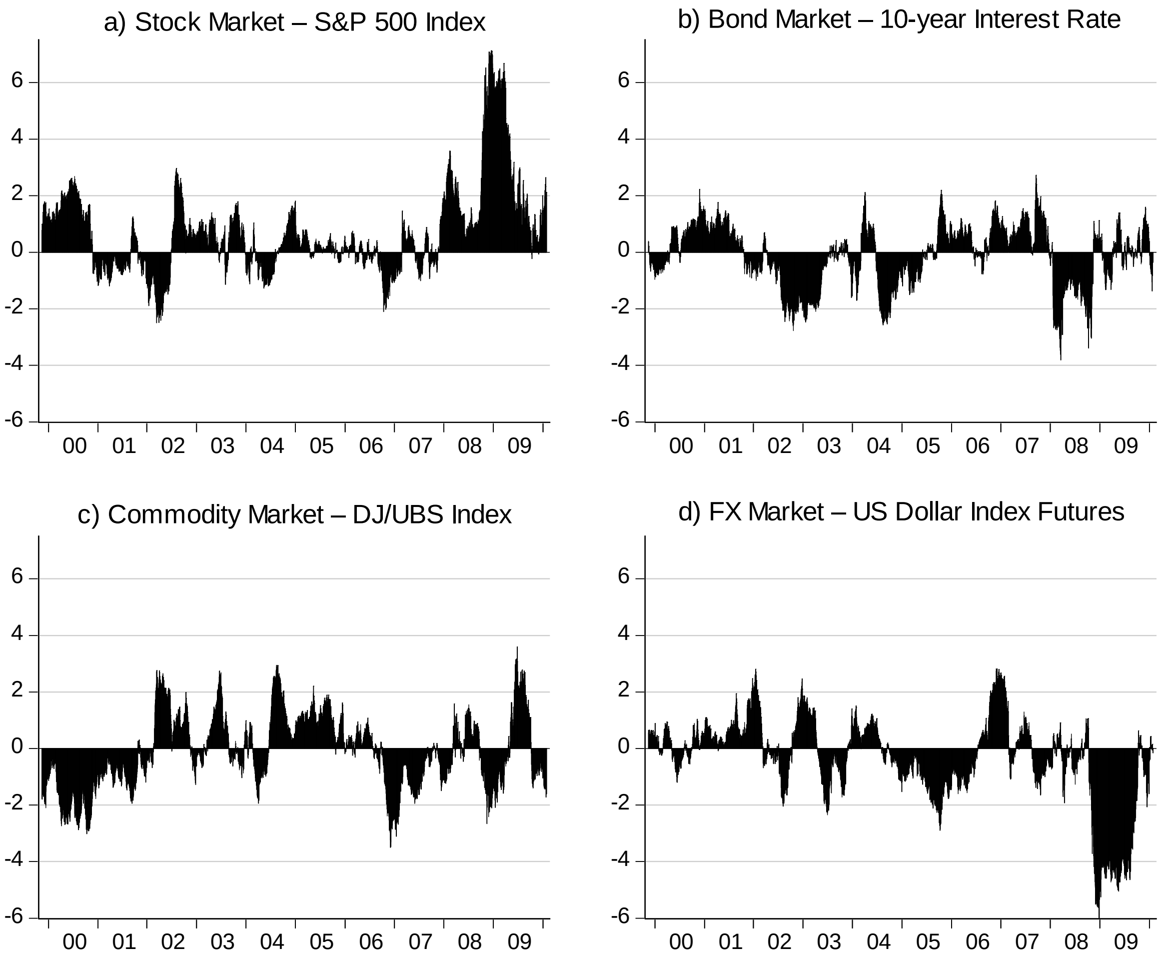
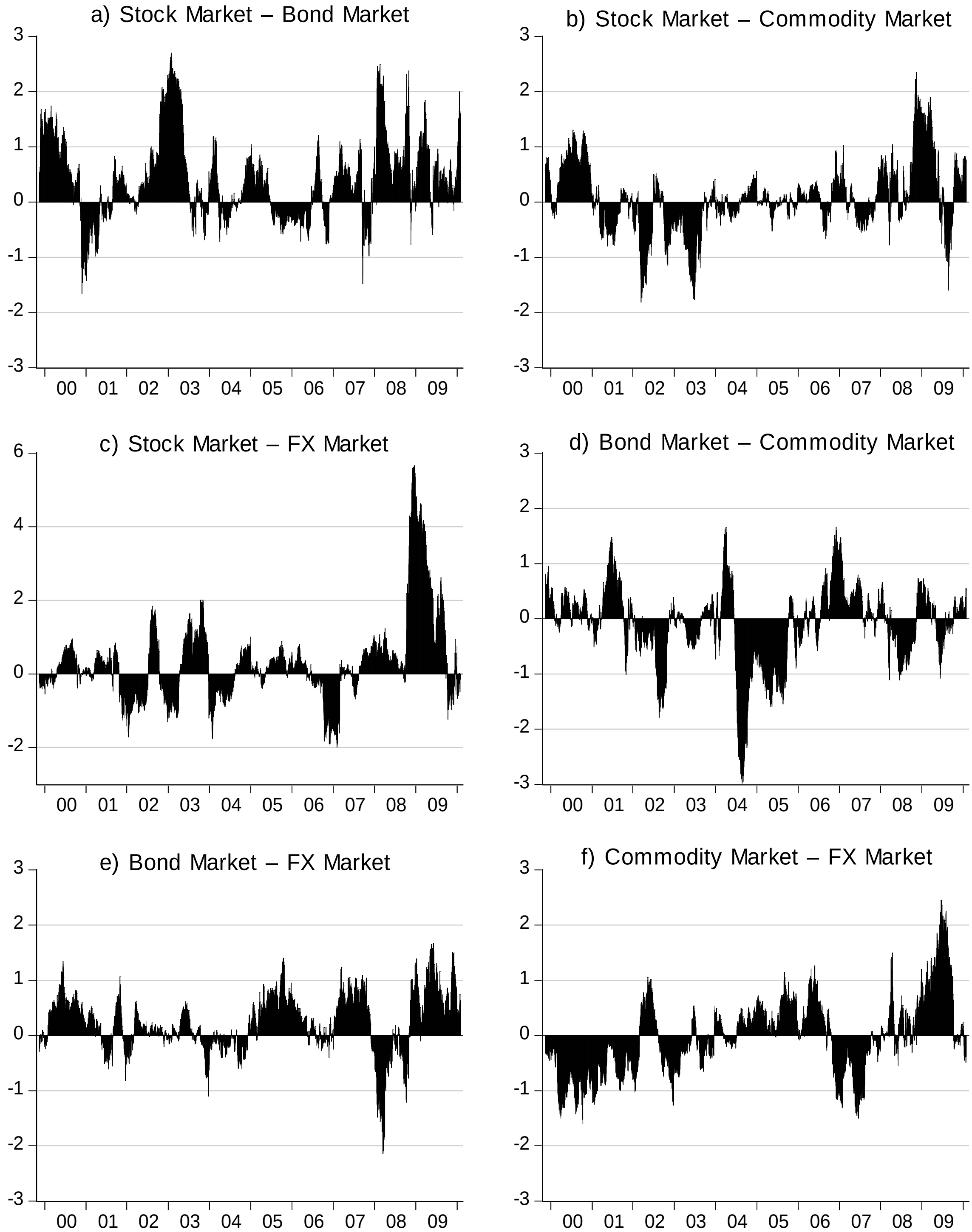


Figure 6. Net Pairwise Volatility Spillovers



Note: The left axis scale ranges from -3 to 3 percent in all panels except for panel c), where it ranges from -3 to 6 percent.

APPENDIX

Figure A1. Sensitivity of the index to VAR lag structure (Max, Min and Median values of the index for VAR order of 2 through 6)

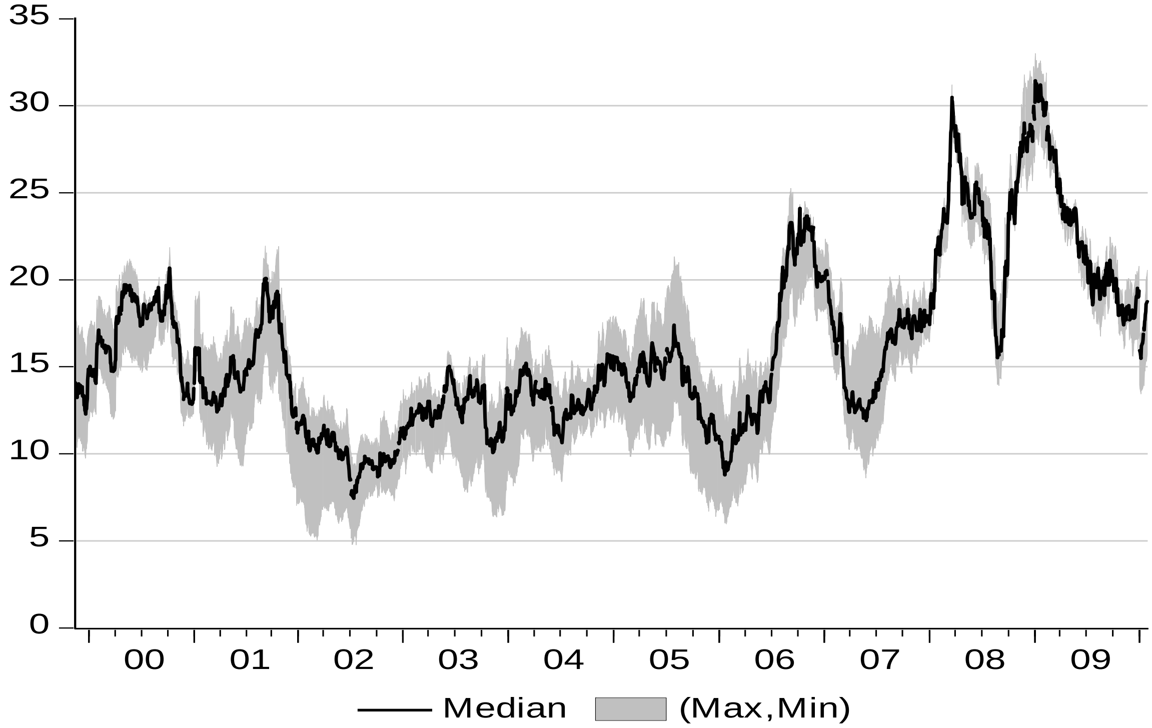


Figure A2. Sensitivity of the Index to Forecast Horizon (Min, Max and Median values over 5 to 10-day horizons)