

On Cointegration and Exchange Rate Dynamics

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ABSTRACT

Baillie and Bollerslev (1989) have recently argued that nominal dollar spot exchange rates are cointegrated. Here we examine an immediate implication of their finding, namely, that cointegration implies an error-correction representation yielding forecasts superior to those from a martingale benchmark, in light of a large earlier literature highlighting the predictive superiority of the martingale. In an out-of-sample forecasting exercise, we find the martingale model to be superior. We then perform a battery of improved cointegration tests and find that the evidence for cointegration is much less strong than previously thought, a result consistent with the outcome of the forecasting exercise.

SINCE THE WORK OF Meese and Singleton (1982), a consensus has emerged that the dynamics of nominal dollar spot exchange rates during the post-1973 float are well approximated by time-series models with one autoregressive unit root. Moreover, since the work of Meese and Rogoff (1983), a consensus has also emerged that the simplest of all unit-root processes, the martingale, provides the best approximation to exchange rate dynamics.¹

In short, financial economists have yet to develop an exchange rate model, structural or nonstructural, with *ex ante* predictive performance statistically significantly better than a naive martingale.

In recent work, Baillie and Bollerslev (1989) make several important contributions with implications for the unit-root and martingale hypotheses. First, using sophisticated statistical procedures, they provide evidence supporting the unit-root hypothesis, thereby strengthening the consensus. Second, using similarly sophisticated procedures, they provide evidence sup-

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¹ Here we use “approximation” in the sense of linear projection. It may be that the martingale is also the best approximation in the sense of conditional expectation; see Diebold and Nason (1990).

porting a hypothesis of cointegration among exchange rates. They do not, however, address the relationship between cointegration and martingale behavior. It is our intention to do so.

Both the martingale and cointegration hypotheses strike chords of intuition. The martingale hypothesis appears to be a reasonable baseline from an "efficient markets" perspective, as does the cointegration hypothesis from a "common trends" perspective. But the two are incompatible. Cointegration implies that there exist one or more long-run relationships among exchange rate levels, deviations from which tend to be eliminated over time and are therefore useful in predicting future exchange rate changes, whereas *nothing* is useful for predicting future exchange rate changes if exchange rates evolve as a vector martingale.

We perform the obvious experiment. Using Baillie and Bollerslev's data on seven nominal dollar spot exchange rates, we estimate vector autoregressive (VAR) models for the system, allowing for varying amounts of cointegration, and we compare their out-of-sample forecasting performance to that of a martingale. The experiment is discussed in detail in Section I, and the results appear in Section II. We also present the results of an improved battery of cointegration tests, the results of which are linked to the outcome of the forecasting experiment. Section III offers a summary and conclusions.

I. Experimental Design

We study seven nominal daily spot exchange rates: the Canadian Dollar (CD), French Franc (FF), Deutsche Mark (DM), Italian Lira (LIR), Japanese Yen (YEN), Swiss Franc (SF), and British Pound (BP), all relative to the U.S. Dollar. In accordance with the literature, we work with the natural logarithms of all exchange rates, which are New York opening bid prices. The data, originally from the Data Resources Incorporated database, were kindly provided by Tim Bollerslev and are precisely those used in Baillie and Bollerslev (1989). They run from March 1, 1980 through January 28, 1985, for a total of 1,245 observations.

We estimate a p th order VAR for our system of seven exchange rates.² As is well known, we can write the system with no loss of generality as

$$\Delta S_t = \mu + \sum_{i=1}^{p-1} B_i \Delta S_{t-i} - \Pi S_{t-1} + \epsilon_t, \quad (1)$$

where S_t is the (7×1) vector of log spot exchange rates, μ is a (7×1) vector of constants, the B_i are (7×7) coefficient matrices, Π is a (7×7) matrix the rank of which equals the number of cointegrating vectors, and ϵ_t is a (7×1) vector of white noise disturbances.

² Selection of p will be discussed subsequently.

The existence of cointegration places restrictions on the parameters of this representation. There are three cases of interest for our purposes:

- (i) When the number of cointegrating vectors is zero and there are no dynamics in the differenced system (that is, $p = 1$), the system is fully nonstationary and equation-by-equation least squares regression on a constant is well specified and fully efficient. This martingale model is denoted MART.
- (ii) When the number of cointegrating vectors is zero but there are dynamics in the differenced system (that is, $p > 1$), the system is fully nonstationary and a VAR in first differences is well specified. Least squares estimates of this VAR are fully efficient. The VAR in differences is denoted VARD.
- (iii) When the number of cointegrating vectors is greater than zero but less than seven, the system is only partially nonstationary. Efficient estimation requires the use of Johansen's (1991) procedure, or relatives such as Engle and Yoo (1992) or Stock and Watson (1993). The models are estimated imposing varying degrees of cointegration are denoted ECM1, ECM2, ..., ECM6, depending on the number of cointegrating vectors.³

All parameters are estimated using observations 1 through 489 and then recursively reestimated every 21 periods. Johansen's (1991) maximum likelihood procedure is used throughout.⁴ At each time subsequent to period 489, 1- through 126-step-ahead forecasts are constructed in real time, using Wold's chain rule of forecasting, treating parameters as fixed at their estimated values.⁵ The forecasts of ECM1, ..., ECM6 are then compared to MART and VARD. We evaluate forecasts of exchange rate log levels in terms of root mean squared prediction error (RMSPE) and mean absolute prediction error (MAPE), at horizons from 1 through 126 days.

II. Empirical Results

VAR lag length is selected using the Akaike and Schwarz information criteria (AIC and SIC), as reported in Table I. Never is a lag length greater than two selected in levels, so we allow one lag in our estimated models, all of which use differenced data. That is, we set $p = 2$ in equation (1), yielding

$$\Delta S_t = \mu + B\Delta S_{t-1} - \Pi S_{t-1} + \epsilon_t. \quad (2)$$

³ "ECM" stands for "error-correction model."

⁴ It should be pointed out that the Johansen procedure does not explicitly allow for ARCH effects and/or unconditionally leptokurtic innovations, both of which are known to be present in high frequency exchange rate dynamics (e.g., Diebold and Nerlove, 1989). But Gonzalo (1989) finds the good performance of Johansen's procedure to be remarkably robust to these and other deviations from classical assumptions.

⁵ On Wold's chain rule see, for example, Sargent (1987).

Table I
Order Selection Using the AIC and SIC Criteria

The table gives the Akaike information criterion (AIC) and Schwartz information criterion (SIC) for a variety of vector-autoregression lag lengths. The information criteria are defined by $AIC = \log|\hat{\Sigma}| + \left(\frac{2}{T}\right)d$, and $SIC = \log|\hat{\Sigma}| + \left(\frac{\log T}{T}\right)d$, where $\hat{\Sigma}$ is the estimated innovation covariance matrix, T is sample size, and d is the number of parameters estimated. The *AIC* favors the lag length producing the smallest *AIC* value, and the *SIC* favors the lag length with the smallest *SIC* value.

	Number of Lags in Levels System (p)				
	1	2	3	4	5
AIC	-78.27	-78.36	-78.34	-78.31	-78.27
SIC	-78.01	-77.90	-77.67	-77.44	-77.20

The RMSPE and MAPE results for the real-time forecasting exercise, reported at horizons of 1, 21, 42, 63, 84, 105, and 126 days, appear in Tables II and III. The message is clear: the forecasting performance of either MART or VARD (neither of which imposes cointegrating restrictions) is almost always best. In fact, of the 294 times that an ECM forecast could have been best in Table II (which records RMSPE), an ECM forecast actually *was* best only once.⁶ Similarly, of the 294 times that an ECM forecast could have been best in Table III (which records MAPE), an ECM forecast actually *was* best only twice.⁷ It is very likely that those rare occasions when ECM models are best are spurious artifacts of sampling variation.

Johansen's trace statistics for cointegration, reported in Table IV (in the column labeled "1980–1985," in reference to the Baillie-Bollerslev data), shed light on the superiority of the forecasts from noncointegrated models. Although the fitted model contains a constant term, as in equation (1), the appropriate critical values, reported in Table V, differ depending upon whether or not the true data-generating process does or does not contain a constant term. In reality, of course, we don't *know* the true data-generating process, so we compare the test statistics to *both* sets of critical values.⁸ As is apparent from the table, there is no evidence of cointegration in either case.

III. Concluding Remarks

Our results lend empirical support to Granger's (1986) claim that one should not expect to find cointegration in asset markets. Using real-time,

⁶ The ECM2 model beat MART and VARD by a very slight margin at 105-steps-ahead for the British Pound (RMSPE = 0.0468 for ECM2 vs. RMSPE = 0.0470 for MART and VARD).

⁷ The ECM1 beat MART and VARD by very slight margins at 42- and 84-steps-ahead for the Canadian Dollar (0.0105 for ECM1 vs. 0.0106 for MART and VARD; 0.0152 for ECM1 vs. 0.0154 for MART and VARD).

⁸ The highly accurate tables of Osterwald-Lenum (1992) are used.

Table II
Root Mean Squared Prediction Error

The error-correction model is $\Delta S_t = \mu + B\Delta S_{t-1} - \Pi S_{t-1} + \epsilon_t$, where S denotes a log exchange rate, which may be any of CD (Canadian Dollar), FF (French Franc), DM (Deutsche Mark), LIR (Italian Lira), YEN (Japanese Yen), SF (Swiss Franc), or BP (British Pound). In the error-correction model, μ is a (7×1) parameter vector, B and Π are (7×7) parameter matrices, and ϵ is white noise. STEPS is the number of steps ahead forecasted. The forecasting models are denoted ECM1–ECM6 (error-correction models with varying numbers of cointegrating relationships), VARD (vector autoregression in first differences of logs), and MART (martingale).

Currency	STEPS	ECM6	ECM5	ECM4	ECM3	ECM2	ECM1	VARD	MART
CD	1	0.0025	0.0025	0.0025	0.0025	0.0025	0.0024	0.0024	0.0024*
	21	0.0160	0.0156	0.0155	0.0147	0.0142	0.0117	0.0114	0.0114*
	42	0.0223	0.0210	0.0210	0.0199	0.0185	0.0156	0.0155	0.0155*
	63	0.0271	0.0249	0.0243	0.0231	0.0213	0.0188	0.0185	0.0185*
	84	0.0309	0.0287	0.0277	0.0268	0.0245	0.0218	0.0214	0.0214*
	105	0.0322	0.0301	0.0290	0.0283	0.0255	0.0231	0.0228	0.0228*
	126	0.0344	0.0325	0.0313	0.0307	0.0274	0.0249	0.0245	0.0245*
FF	1	0.0074	0.0074	0.0073	0.0073	0.0073	0.0073	0.0071	0.0071*
	21	0.0424	0.0428	0.0419	0.0391	0.0396	0.0382	0.0350	0.0349*
	42	0.0584	0.0585	0.0584	0.0527	0.0544	0.0536	0.0456	0.0456*
	63	0.0704	0.0680	0.0671	0.0594	0.0634	0.0626	0.0519	0.0519*
	84	0.0790	0.0737	0.0740	0.0659	0.0708	0.0690	0.0580	0.0580*
	105	0.0873	0.0789	0.0798	0.0701	0.0765	0.0738	0.0627*	0.0628
	126	0.0915	0.0822	0.0832	0.0724	0.0797	0.0786	0.0673*	0.0674
DM	1	0.0066	0.0066	0.0065	0.0065	0.0065	0.0065	0.0064	0.0064*
	21	0.0378	0.0391	0.0378	0.0324	0.0323	0.0324	0.0309	0.0309*
	42	0.0514	0.0537	0.0528	0.0425	0.0430	0.0453	0.0413	0.0413*
	63	0.0608	0.0624	0.0607	0.0467	0.0480	0.0512	0.0462	0.0462*
	84	0.0693	0.0693	0.0679	0.0528	0.0537	0.0572	0.0511*	0.0512
	105	0.0791	0.0777	0.0769	0.0613	0.0622	0.0654	0.0572*	0.0573
	126	0.0839	0.0826	0.0829	0.0680	0.0674	0.0734	0.0638*	0.0639
LIR	1	0.0059	0.0059	0.0059	0.0058	0.0058	0.0058	0.0058	0.0058*
	21	0.0342	0.0347	0.0337	0.0299	0.0295	0.0290	0.0285	0.0285*
	42	0.0483	0.0490	0.0478	0.0398	0.0398	0.0407	0.0376	0.0376*
	63	0.0572	0.0568	0.0546	0.0429	0.0440	0.0461	0.0419	0.0419*
	84	0.0645	0.0623	0.0604	0.0472	0.0489	0.0505	0.0463*	0.0464
	105	0.0745	0.0705	0.0686	0.0543	0.0568	0.0572	0.0524	0.0524*
	126	0.0817	0.0779	0.0766	0.0621	0.0637	0.0653	0.0598*	0.0599
YEN	1	0.0068	0.0068	0.0067	0.0067	0.0067	0.0066	0.0066	0.0065*
	21	0.0436	0.0426	0.0396	0.0382	0.0378	0.0337	0.0324	0.0324*
	42	0.0662	0.0632	0.0586	0.0563	0.0563	0.0490	0.0467	0.0467*
	63	0.0840	0.0781	0.0714	0.0683	0.0681	0.0575	0.0551	0.0551*
	84	0.0996	0.0909	0.0843	0.0813	0.0798	0.0667	0.0643	0.0643*
	105	0.1150	0.1020	0.0948	0.0899	0.0895	0.0750	0.0714	0.0714*
	126	0.1264	0.1108	0.1026	0.0948	0.0968	0.0816	0.0773	0.0773*
SF	1	0.0068	0.0069	0.0068	0.0068	0.0068	0.0068	0.0067	0.0066*
	21	0.0403	0.0399	0.0395	0.0352	0.0348	0.0348	0.0313	0.0313*
	42	0.0586	0.0563	0.0560	0.0484	0.0479	0.0521	0.0425*	0.0426
	63	0.0688	0.0643	0.0633	0.0543	0.0539	0.0604	0.0487	0.0487*
	84	0.0752	0.0682	0.0681	0.0599	0.0591	0.0670	0.0561*	0.0562
	105	0.0838	0.0744	0.0744	0.0671	0.0657	0.0730	0.0614	0.0614*
	126	0.0932	0.0825	0.0827	0.0779	0.0714	0.0802	0.0685	0.0685*

Table II—Continued

Currency	STEPS	ECM6	ECM5	ECM4	ECM3	ECM2	ECM1	VARD	MART
BP	1	0.0063	0.0063	0.0063	0.0062	0.0062	0.0062	0.0061	0.0060*
	21	0.0369	0.0373	0.0382	0.0336	0.0319	0.0296	0.0262	0.0262*
	42	0.0520	0.0528	0.0558	0.0483	0.0437	0.0414	0.0343	0.0343*
	63	0.0552	0.0560	0.0612	0.0510	0.0448	0.0437	0.0371	0.0371*
	84	0.0557	0.0559	0.0628	0.0494	0.0440	0.0450	0.0425	0.0425*
	105	0.0598	0.0590	0.0643	0.0505	0.0468*	0.0479	0.0470	0.0470
	126	0.0672	0.0669	0.0715	0.0585	0.0551	0.0540	0.0513	0.0512*

* Indicates minimum value in each row.

out-of-sample forecasting exercises and improved hypothesis-testing procedures, we find that the null hypothesis of no cointegration cannot be rejected for a system of nominal dollar spot exchange rates, 1980 to 1985, and that no improvements in forecasting performance are obtained from the use of cointegrated VARs.

Why are our results, which show no cointegration and hence no forecast improvement from allowing for it, so markedly different from those of Baillie and Bollerslev (1989)? Our explanation is as follows. They used the best technology available when they wrote, namely, that of Johansen (1988). Unfortunately, however, the 1988 procedure does not allow for a drift in the estimated model, whereas it is now generally agreed that drift *should* be included, unless there is irrefutable prior information to the contrary. Once the possibility of drift is acknowledged, the evidence for cointegration vanishes. Evidence of cointegration arises only under the assumption that drift is *known* to be absent.⁹ Such an assumption is tenuous at best; economists are certainly not in agreement regarding the presence of drift.¹⁰

Our results are of course not the last word on the subject; there are many important directions for future research. The most obvious is extension of the analysis to the entire post-1973 float, as opposed to the five-year Baillie-Bollerslev subsample in which the dollar was continuously appreciating.¹¹ A longer span of data should facilitate more precise inference regarding the low frequency dynamics of interest. In work in progress, a daily dataset has been assembled for 1973 to 1991 and is being used to perform such an analysis. Preliminary results appear similar to those obtained using the Baillie-Bollerslev data. For example, the Johansen trace statistics reported in Table IV (in the column labeled "1973–1991") reinforce the inference obtained earlier: no cointegration.

⁹ In an appendix to this paper, available from the authors upon request, we perform a successful replication of the cointegration tests in Baillie and Bollerslev (1989).

¹⁰ Compare, for example, Diebold and Nason (1990) to Engel and Hamilton (1990).

¹¹ In this article, of course, we have intentionally worked with the Baillie-Bollerslev sample, in order to make our results fully comparable.

Table III
Mean Absolute Prediction Error

The error-correction model is $\Delta S_t = \mu + B\Delta S_{t-1} - \Pi S_{t-1} + \epsilon_t$, where S denotes a log exchange rate, which may be any of CD (Canadian Dollar), FF (French Franc), DM (Deutsche Mark), LIR (Italian Lira), YEN (Japanese Yen), SF (Swiss Franc), or BP (British Pound). In the error-correction model, μ is a (7×1) parameter vector, B and Π are (7×7) parameter matrices, and ϵ is white noise. STEPS is the number of steps ahead forecasted. The forecasting models are denoted ECM1–ECM6 (error-correction models with varying numbers of cointegrating relationships), VARD (vector autoregression in first differences of logs), and MART (martingale).

Currency	STEPS	ECM6	ECM5	ECM4	ECM3	ECM2	ECM1	VARD	MART
CD	1	0.0018	0.0018	0.0018	0.0017	0.0017	0.0017	0.0017	0.0017*
	21	0.0114	0.0112	0.0111	0.0102	0.0093	0.0080	0.0080	0.0080*
	42	0.0167	0.0157	0.0154	0.0141	0.0125	0.0105*	0.0106	0.0106
	63	0.0209	0.0195	0.0185	0.0175	0.0156	0.0132	0.0131	0.0131*
	84	0.0242	0.0225	0.0217	0.0201	0.0176	0.0152*	0.0154	0.0154
	105	0.0257	0.0243	0.0229	0.0217	0.0193	0.0170	0.0169	0.0169*
	126	0.0274	0.0263	0.0249	0.0239	0.0208	0.0191	0.0189	0.0189*
FF	1	0.0053	0.0053	0.0053	0.0052	0.0052	0.0052	0.0051	0.0050*
	21	0.0318	0.0320	0.0310	0.0293	0.0295	0.0286	0.0256	0.0256*
	42	0.0430	0.0433	0.0438	0.0407	0.0422	0.0423	0.0344	0.0344*
	63	0.0513	0.0498	0.0486	0.0456	0.0486	0.0482	0.0402	0.0402*
	84	0.0591	0.0551	0.0560	0.0523	0.0558	0.0534	0.0465	0.0465*
	105	0.0665	0.0603	0.0615	0.0568	0.0610	0.0583	0.0517	0.0517*
	126	0.0701	0.0632	0.0645	0.0596	0.0644	0.0639	0.0569	0.0569*
DM	1	0.0051	0.0051	0.0051	0.0050	0.0050	0.0050	0.0050	0.0049*
	21	0.0303	0.0317	0.0303	0.0258	0.0254	0.0249	0.0238	0.0238*
	42	0.0415	0.0447	0.0444	0.0354	0.0347	0.0363	0.0328	0.0328*
	63	0.0487	0.0523	0.0508	0.0390	0.0384	0.0409	0.0363*	0.0364
	84	0.0562	0.0596	0.0582	0.0451	0.0442	0.0480	0.0409	0.0409*
	105	0.0628	0.0659	0.0646	0.0515	0.0511	0.0556	0.0479	0.0479*
	126	0.0655	0.0688	0.0689	0.0560	0.0567	0.0620	0.0533*	0.0534
LIR	1	0.0045	0.0045	0.0045	0.0045	0.0045	0.0044	0.0045	0.0044*
	21	0.0267	0.0271	0.0262	0.0233	0.0228	0.0222	0.0213	0.0213*
	42	0.0355	0.0369	0.0367	0.0309	0.0303	0.0308	0.0282	0.0282*
	63	0.0431	0.0445	0.0431	0.0331	0.0330	0.0343	0.0308	0.0308*
	84	0.0513	0.0513	0.0498	0.0381	0.0383	0.0394	0.0348	0.0348*
	105	0.0604	0.0598	0.0579	0.0443	0.0455	0.0467	0.0416	0.0416*
	126	0.0656	0.0653	0.0640	0.0506	0.0521	0.0543	0.0483	0.0483*
YEN	1	0.0048	0.0048	0.0047	0.0047	0.0047	0.0047	0.0046	0.0046*
	21	0.0299	0.0294	0.0278	0.0267	0.0265	0.0247	0.0235*	0.0236
	42	0.0448	0.0422	0.0397	0.0376	0.0375	0.0357	0.0337	0.0337*
	63	0.0618	0.0564	0.0495	0.0463	0.0465	0.0428	0.0409	0.0408*
	84	0.0797	0.0709	0.0628	0.0583	0.0578	0.0530	0.0507	0.0507*
	105	0.0943	0.0828	0.0716	0.0640	0.0650	0.0600	0.0573	0.0573*
	126	0.1038	0.0921	0.0805	0.0706	0.0716	0.0669	0.0633	0.0633*
SF	1	0.0053	0.0053	0.0053	0.0052	0.0053	0.0053	0.0052	0.0051*
	21	0.0307	0.0304	0.0301	0.0279	0.0276	0.0272	0.0244	0.0244*
	42	0.0429	0.0405	0.0407	0.0364	0.0363	0.0397	0.0330	0.0330*
	63	0.0511	0.0471	0.0464	0.0424	0.0419	0.0476	0.0372	0.0372*
	84	0.0593	0.0514	0.0522	0.0491	0.0468	0.0541	0.0439*	0.0440
	105	0.0686	0.0589	0.0580	0.0558	0.0516	0.0590	0.0487	0.0487*
	126	0.0748	0.0637	0.0639	0.0637	0.0550	0.0649	0.0560	0.0560*

Table III—Continued

Currency	STEPS	ECM6	ECM5	ECM4	ECM3	ECM2	ECM1	VARD	MART
BP	1	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0047	0.0046*
	21	0.0279	0.0283	0.0287	0.0248	0.0241	0.0226	0.0205	0.0204*
	42	0.0375	0.0387	0.0403	0.0347	0.0331	0.0313	0.0271*	0.0272
	63	0.0390	0.0416	0.0448	0.0352	0.0341	0.0340	0.0304	0.0304*
	84	0.0414	0.0438	0.0489	0.0378	0.0355	0.0370	0.0343	0.0343*
	105	0.0427	0.0452	0.0500	0.0385	0.0382	0.0394	0.0371	0.0370*
	126	0.0491	0.0527	0.0563	0.0458	0.0448	0.0435	0.0400	0.0399*

* Indicates minimum value in each row.

Table IV

Trace Statistics for Cointegration

The error-correction model is $\Delta S_t = \mu + B\Delta S_{t-1} - \Pi S_{t-1} + \epsilon_t$, where S denotes a log exchange rate, which may be any of CD (Canadian Dollar), FF (French Franc), DM (Deutsche Mark), LIR (Italian Lira), YEN (Japanese Yen), SF (Swiss Franc), or BP (British Pound). In the error-correction model, μ is a (7×1) parameter vector, B and Π are (7×7) parameter matrices, and ϵ is white noise. We report Johansen's trace statistic for various null hypotheses and for two sample periods. The 1980 to 1985 dataset is precisely that of Baillie and Bollerslev, while the 1973 to 1991 data are daily from March of 1973 through 1991.

Null Hypothesis	1980–1985	1973–1991
At most 6 cointegrating vectors	0.869	1.656
At most 5 cointegrating vectors	5.447	5.398
At most 4 cointegrating vectors	11.647	14.495
At most 3 cointegrating vectors	21.697	27.167
At most 2 cointegrating vectors	45.581	41.776
At most 1 cointegrating vector	72.297	65.963
No cointegration	108.688	114.928

Table V

Critical Values for Trace Statistics

The estimated error-correction model is $\Delta S_t = \mu + B\Delta S_{t-1} - \Pi S_{t-1} + \epsilon_t$, where S denotes a log exchange rate, which may be any of CD (Canadian Dollar), FF (French Franc), DM (Deutsche Mark), LIR (Italian Lira), YEN (Japanese Yen), SF (Swiss Franc), or BP (British Pound). In the error-correction model, μ is a (7×1) parameter vector, B and Π are (7×7) parameter matrices, and ϵ is white noise. We report the critical values of Johansen's trace test statistic, which depend on the size of the test, the precise null hypothesis being tested, and whether we admit the possibility of a nonzero drift.

Null Hypothesis	$\mu \neq 0$		$\mu = 0$	
	10%	5%	10%	5%
At most 6 cointegrating vectors	2.69	3.76	6.50	8.18
At most 5 cointegrating vectors	13.33	15.41	15.67	17.95
At most 4 cointegrating vectors	26.79	29.68	28.71	31.53
At most 3 cointegrating vectors	43.95	47.21	45.23	48.28
At most 2 cointegrating vectors	64.84	68.52	66.49	70.60
At most 1 cointegrating vector	89.48	94.16	90.39	95.18
No cointegration	118.50	124.24	118.99	124.25

Even the 1973 to 1991 sample is very short, however, for answering the low frequency questions of interest. We would like, of course, a century or two of data, but alas, the float began only recently! Given that the first-best strategy of obtaining longer calendar spans of data is infeasible, an obvious second-best strategy is to use more sophisticated econometric techniques on the limited data available. In this regard, perhaps long-memory models will be useful.¹² But whatever the nature of the consensus that may eventually be reached, one thing is clear: at present, and in contrast to first impressions, there exists substantial uncertainty regarding the existence and nature of cointegrating relationships among nominal dollar exchange rates.

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¹² For a concise survey, see Diebold and Nerlove (1990, section 5).