"Rational Expectations, Random Walks, and Monetary Models of the Exchange Rate," *Proceedings of the American Statistical Association, Business and Economic Statistics Section, 1986*, 101-106. Washington, DC: American Statistical Association, 1987.

RATIONAL EXPECTATIONS, RANDOM WALKS, AND MONETARY MODELS OF THE EXCHANGE RATE

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1) Introduction

The descriptive accuracy and predictive superiority of simple random-walk spot exchange rate models have received substantial attention in the literature, as in Meese and Rogoff (1983a, 1983b), and Backus (1984) among many others. Diebold and Nerlove (1985, 1986) use time series methods, including formal unit root tests, to study the temporal structure of seven major weekly dollar spot rates during the post 1973 float, and find strong support for the presence of one unit root in each series. For most rates, some slight serial correlation remains after the application of a first difference, but in all cases, the identified model is very close to a random walk.

In light of these results, an interesting question is whether, and under what conditions, lower-frequency structural exchange rate models will generate random walk behavior. In this paper, a flexible-price monetary model is studied under rational expectations. The restrictions under which random walk behavior arises are characterized and shown to be testable. The results are then modified to characterize the conditions leading to more general nonstationary, as opposed to simple random walk, behavior. Finally, the restrictions imposed by the joint monetary-model/random walk hypothesis are tested empirically for the DM/$ rate, in two ways. The first involves the application of formal tests for a unit root in the driving process, conditional on plausible values for the income elasticity of money demand. The second involves testing for co-integration between driving variables, which avoids conditioning on structural parameter values.

2) Random Walk Behavior in A Monetary Model under Rational Expectations

The quasi-reduced form of the standard flexible-price monetary model (Frenkel (1976), Bilson (1979)), is given by:

$$(1) \quad S_t = m_t - a\, y_t + b\, i_t .$$

where

$$m = \log \left(\frac{m^L}{m^F}\right)$$

$$y = \log \left(\frac{y^L}{y^F}\right)$$

$$i = (i^L - i^F)$$

and m, y, and i respectively denote money stock, real income, and nominal interest rate, and "F" denotes "foreign" and and "L" denotes "local." S is the log spot rate in units of local currency per foreign currency unit. The structural assumptions behind this reduced form, including purchasing power parity and identical local and foreign money demand parameters, are well known and quite strong. The Frenkel-Bilson monetary model is, nevertheless, the most commonly estimated exchange rate model and provides a convenient vehicle for tractable analysis.

Elimination of i_t via the uncovered interest parity condition:

$$(2) \quad i_t = E_t S_{t+1} - S_t$$

(apart from second-order terms) yields the reduced form:

$$(3) \quad S_t = \frac{b}{1+b} E_t S_{t+1} + \frac{1}{1+b} (m_t - a y_t).$$

Solving for the forward solution (Whiteman (1983), <u>inter alia</u>), we have:

$$(4) \quad S_t = \frac{1}{1+b} \sum_{j=0}^{\infty} ((1+b)/b)^{-j} \; E_t Z_{t+j}$$

where the driving variable Z is defined by

$$(5) \quad Z_t = m_t - a\, y_t .$$

Under the assumption that Z follows an AR(1) process:

$$(6) \quad Z_t = \rho\, Z_{t-1} + v_t$$

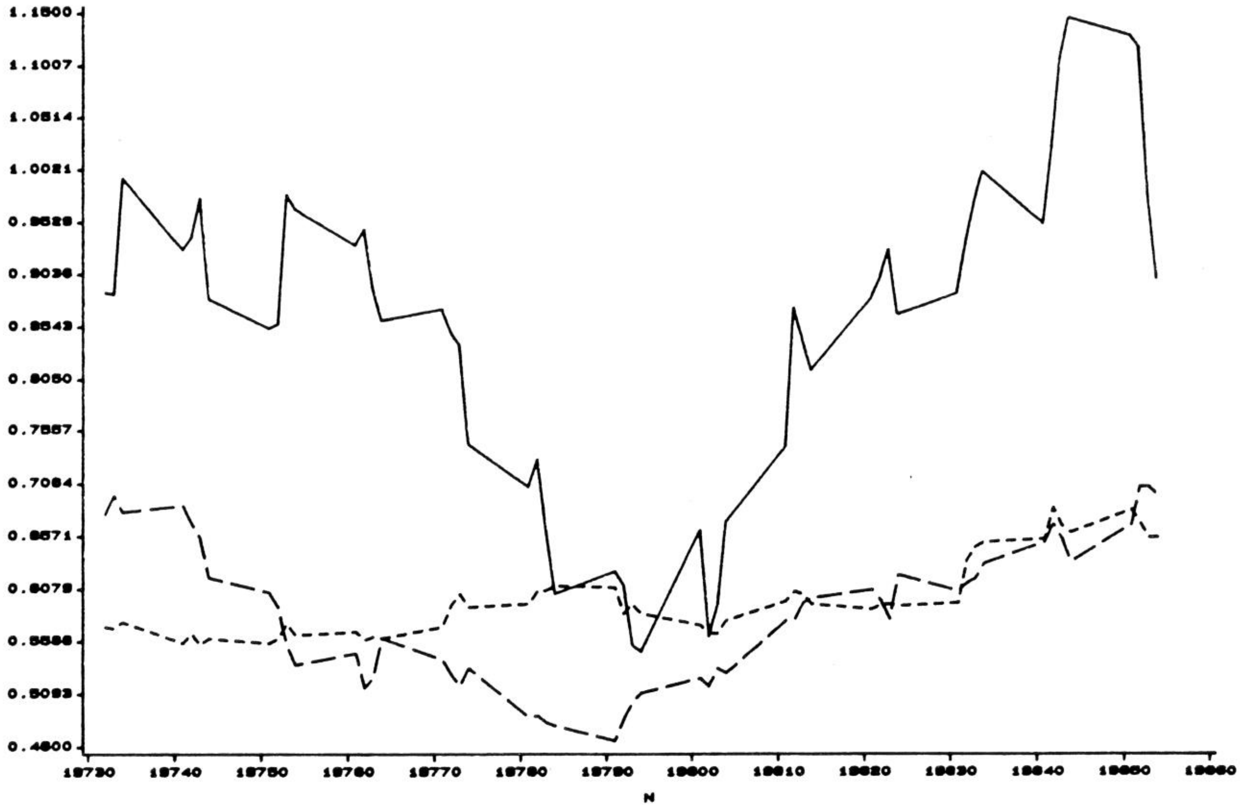
we know that $E_t Z_{t+j} = \rho^j Z_t$.

Thus (4) reduces to:

$$(7) \quad S_t = \frac{1}{1 + b(1 - \rho)} \; Z_t .$$

Clearly, if Z_t follows a random walk (i.e. $\rho = 1$), then $S_t = Z_t$ and is therefore a random walk as well. It is <u>not</u> true, however, that interest inelastic money demand schedules (i.e. $b = 0$) imply a random walk for e. (Compare Backus (1984).) In that situation, we have $S_t = Z_t$ as before, but <u>Z is not a random walk</u> (unless $\rho = 1$), which means that <u>S cannot follow a random walk</u>. Thus, $\rho = 1$ is both necessary and sufficient for random walk exchange rates in the simple rational expectations monetary model with AR(1) driving process. The restriction is not immediately testable, however, because the parameter "a" is unknown, making Z unobservable. We will return to this in the empirical analysis of Section 4, but we first pause to consider driving processes more general than (6).

Figure 1



S (SOLID), Y (DOT), M (DASH)

3) Random Walk Behavior with a General Autoregressive Driving Process

Suppose now that Z follows the general p^{th} order autoregression:

(8) $\Phi(L)\ Z_t = \varepsilon_t$

where

(9) $\Phi(L) = (1 - \phi_1 L - \ldots - \phi_p L^p)\ ,\ p < \infty$.

Then, by the well known Wiener-Kolmogorov formula (Nerlove, et al. (1979), inter alia) we obtain:

(10) $E_t Z_{t+j} = \gamma(L)\ Z_t$

where

(11) $\gamma(L) = \sum_{i=0}^{\infty} \gamma_j\ L^j = \Phi(L)\ [\Phi^{-1}(L) / L^j]_+$,

L is viewed as a complex variable with modulus less than unity, and the $[\ ]_+$ operator eliminates all negative powers of L. Thus,

(12) $S_t = (1 / (1 + b)) \sum_{j=0}^{\infty} \{((1 + b) / b)^{-j}$

$\cdot\ \Phi(L)\ [\Phi^{-1}(L) / L^j]_+\ Z_t\}$.

By the law of iterated projections (Sargent (1979)), $E_t Z_{t+j}$ will be a linear combination of only the p most recent Z values (for all j), so that the sum in (12), if convergent, will also be a linear combination of the p most recent Z values. We can therefore write:

(13) $S_t = (1 / (1 + b))\ \{c_0 Z_t + c_1 Z_{t-1} + \ldots$

$\ldots + c_{p-1} Z\ t\text{-}p\text{-}1\}$.

This makes clear the fact that unless Z follows a random walk (i.e. $\phi_1 = 1, \phi_2 = \ldots = \phi_p = 0$), S_t cannot follow a random walk. Furthermore, if Z follows a stationary autoregressive process, then S will as well, and if Z follows an integrated autoregressive process then so too will S. Thus, even in the case of a general AR(p) driving process, the interest elasticity of money demand contains no information which impinges on the random walk hypothesis. Further generalization to the case of ARMA and ARIMA processes is immediate.

4) Testing the Random Walk Restrictions

The log DM/$ rate is plotted over 1973II - 1985IV in Figure 1, as are m and y. The movements in the log spot rate are well known and need no further description. The sample path of S is typical of a random walk realization, and the sample autocorrelations in Table 1, which decay very slowly, support the random walk hypothesis as well. Similarly, the sample partial autocorrelations in Table 1 indicate a random walk via the large, significant value at lag one, and insignificant, small values thereafter. Formal tests for a unit root in the autoregressive lag-operator polynomial of an AR(1) representation of S, given in Table 3, fail to reject the null at any level. These tests include the $\hat{\tau}$ test (zero mean and absence of trend assumed under the alternative), the $\hat{\tau}_\mu$ test (non-zero mean allowed under the alternative) and the $\hat{\tau}_\tau$ test (trend allowed under the alternative) as described in Dickey, Bell and Miller (1986) and tabulated in Dickey (1976). Given the relatively small sample (51 observations), power considerations are of key importance, particularly in light of the fact that realistic alternatives are close to the null. Because of this, the uniformly most powerful test of Bhargava (1986) was also used, but it too failed to reject the null.
A wide range of diagnostics verified that the distributional and temporal properties of ΔS are indicative of uncorrelated Gaussian noise; for reference, the sample autocorrelations and partial autocorrelations of ΔS are given in Table 2.

The results on temporal dependence for m and y are similar, in terms of the presence of one unit root in each, with no serial correlation present in first differenced form. Again, sample autocorrelations and partial autocorrelations for both levels and first differences are contained in Tables 1 and 2, while the unit root test results are

Table 1
Autocorrelations (with Bartlett Standard Errors) and Partial Autocorrelations

Autocorrelations	S	y	m	$Z_{.1}$	$Z_{.3}$	$Z_{.6}$
1	.92(.14)	.92(.14)	.91(.14)	.91(.14)	.91(.14)	.90(.14)
2	.83(.23)	.83(.23)	.82(.23)	.81(.23)	.81(.23)	.79(.23)
3	.74(.28)	.74(.28)	.72(.28)	.71(.28)	.71(.28)	.69(.28)
4	.64(.32)	.62(.32)	.62(.31)	.62(.31)	.60(.31)	.58(.31)
5	.54(.34)	.51(.34)	.53(.34)	.52(.34)	.50(.33)	.47(.33)
6	.42(.36)	.41(.35)	.43(.35)	.42(.35)	.40(.35)	.36(.34)
7	.32(.37)	.31(.36)	.34(.36)	.33(.36)	.31(.36)	.27(.35)
8	.22(.37)	.23(.37)	.24(.37)	.23(.37)	.20(.36)	.16(.35)
9	.12(.38)	.16(.37)	.18(.37)	.16(.37)	.14(.36)	.09(.35)
10	.03(.38)	.10(.37)	.12(.38)	.11(.37)	.09(.37)	.05(.36)

Partial Autocorrelations	S	y	m	$Z_{.1}$	$Z_{.3}$	$Z_{.6}$
1	.92	.92	.91	.91	.91	.90
2	-.13	-.07	-.11	-.11	-.11	-.11
3	-.01	-.07	-.06	-.05	-.04	-.01
4	-.16	-.24	-.04	-.04	-.07	-.10
5	-.04	.02	-.05	-.07	-.08	-.10
6	-.16	-.02	-.07	-.07	-.06	-.05
7	.07	-.05	-.04	-.03	-.01	-.01
8	-.17	.03	-.12	-.13	-.15	-.17
9	-.01	.01	.13	.14	.15	.16
10	-.14	.00	-.04	-.02	-.03	-.03

Table 2
Sample Autocorrelations (with Bartlett Standard Errors) and Partial Autocorrelations

Autocorrelations	ΔS	Δy	Δm	ΔZ .1	ΔZ .3	ΔZ .6
1	.06(.14)	.01(.14)	.01(.14)	.00(.14)	-.01(.14)	-.01(.14)
2	-.09(.14)	-.08(.14)	.09(.14)	.08(.14)	.08(.14)	.05(.14)
3	.11(.14)	.09(.14)	.12(.14)	.12(.14)	.12(.14)	.12(.14)
4	-.01(.14)	.04(.14)	.26(.14)	.26(.14)	.25(.14)	.24(.14)
5	.08(.15)	-.18(.15)	-.04(.15)	-.05(.15)	-.05(.15)	-.08(.15)
6	-.08(.15)	-.13(.15)	.04(.15)	.04(.15)	.02(.15)	.01(.15)
7	.07(.15)	-.00(.15)	.23(.15)	.23(.15)	.21(.15)	.19(.15)
8	-.03(.15)	-.11(.15)	-.21(.16)	-.22(.16)	-.22(.16)	-.22(.16)
9	-.11(.15)	-.03(.15)	.14(.17)	.12(.17)	.08(.16)	.01(.16)
10	.08(.15)	.05(.15)	.09(.17)	.10(.17)	.13(.17)	.17(.16)

Partial Autocorrelations	ΔS	Δy	Δm	ΔZ .1	ΔZ .3	ΔZ .6
1	.06	.01	.01	.00	-.01	-.01
2	-.09	-.08	.09	.08	.08	.05
3	.12	.09	.12	.13	.12	.12
4	-.03	.03	.26	.26	.25	.25
5	.11	-.17	-.06	-.06	-.06	-.08
6	-.12	-.13	-.02	-.02	-.04	-.03
7	.12	-.04	.19	.19	.18	.15
8	-.11	-.10	-.30	-.30	-.29	-.28
9	-.04	.00	.17	.15	.09	.03
10	.03	.02	.08	.10	.15	.19

Table 3
Unit Root Tests

Variable	$\hat{\tau}$	$\hat{\tau}_\mu$	$\hat{\tau}_\tau$
S	-.21	-1.38	-1.46
y	.03	- .86	-1.41
m	.93	- .73	-2.24
$Z_{.1}$	-.05	-.97	-1.40
$Z_{.2}$	-.14	-1.08	-1.39
$Z_{.3}$	-.23	-1.19	-1.38
$Z_{.4}$	-.34	-1.30	-1.38
$Z_{.5}$	-.46	-1.41	-1.38
$Z_{.6}$	-.61	-1.51	-1.38
$Z_{.7}$	-.81	-1.61	-1.39
$Z_{.8}$	-1.13	-1.70	-1.40

* Significant at the 10% level

given in Table 3.

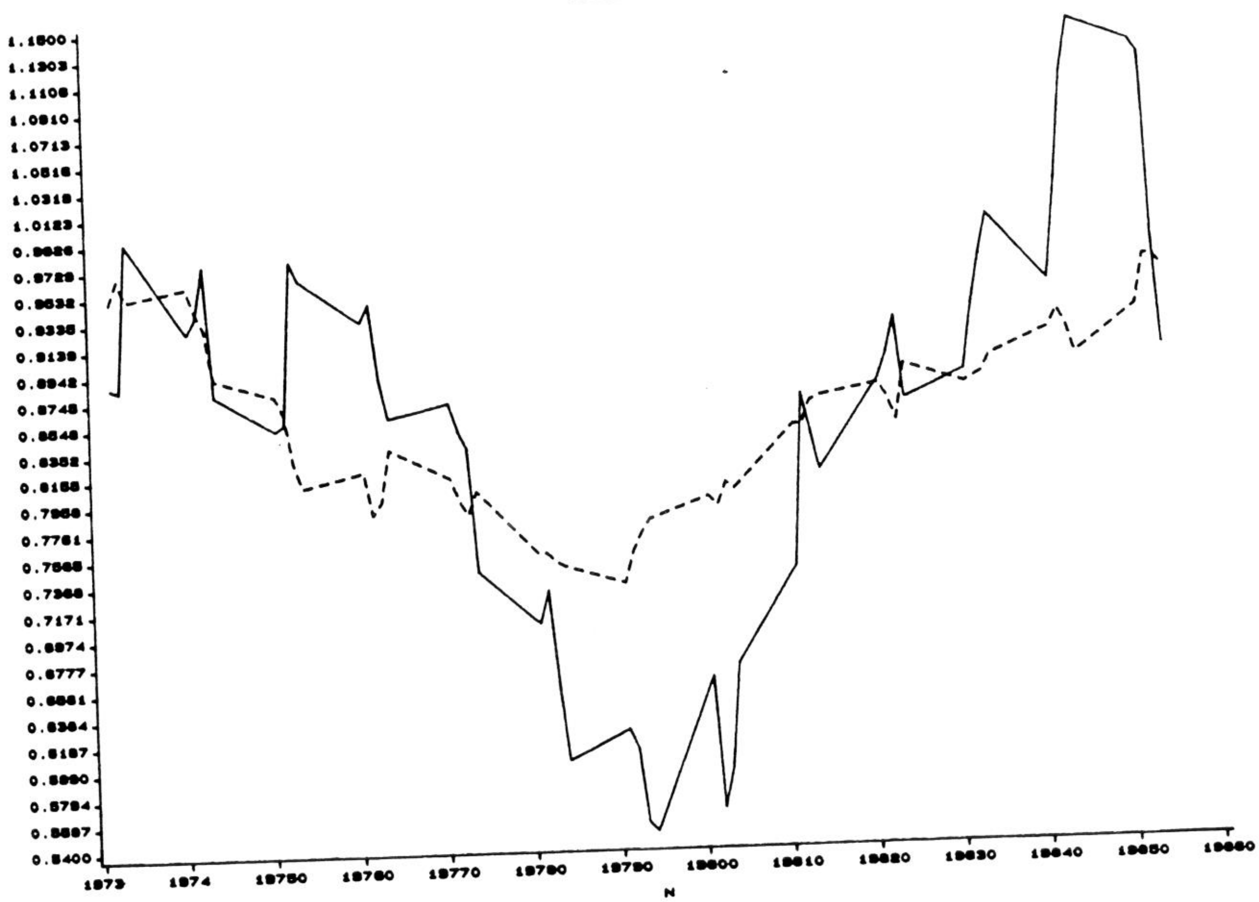
As stressed by Engle and Granger (1985), however, random walk behavior in m and y does not guarantee similar behavior in Z = m − ay, due to possible co-integration. Define Z_a = m − ay, for a = .1, .2, . . . , .8. This covers the entire plausible range of "a," the income elasticity of money demand. The unit root tests and other diagnostics of Tables 1, 2, and 3 strongly indicate random-walk behavior of Z_a, conditional on the various "a" values.

A test of random walk behavior in Z, which is not conditional on particular "a" values, may be obtained implicitly via a test for co-integration of m and y. Under the null of no co-integration, there does not exist an "a" such that Z_a = m − ay is stationary. Under the alternative of cointegration, on the other hand, there exists an "a" such that Z_a is stationary, even though both m and y contain a unit root. A simple test, proposed by Engle and Granger (1985) and closely related to the work of Bhargava (1986), is obtained by running the "equilibrium" regression m = ay + ε and testing the residuals for non-stationarity via the Durbin-Watson statistic, which tends to zero under the null of no co-integration (i.e. in the case of nonstationary residuals). Under the null, the respective 1%, 5%, and 10% critical values have been tabulated by Engle and Granger as .508, .372, and .312. The value of the Durbin-Watson test statistic obtained is .132, so we fail to reject at any level. Thus, m and y are not co-integrated, and Z displays nonstationary random-ralk behavior regardless of the value of "a."

5) Concluding Remarks

We have shown that the flexible-price monetary model passes one very basic test of adequacy: it generates realizations with stochastic properties similar to those of actual exchange rates. Unfortunately, however, the random walks produced by the model are related to the actual observed series only in the roughest qualitative essentials. Specifically, the Z series qualitatively captures the pre-1979 dollar

FIGURE 2



ACTUAL (SOLID) AND PREDICTED (DOT)

depreciation, as well as the rapid post-1979 dollar appreciation. The fit, in terms of levels of the generated realizations, is best for small "a" values, which is reasonable in light of what we know about the instantaneous income elasticity of money demand. For example, if we condition on a = .1 and allow for an intercept to pick up omitted, but constant, effects, then the parameterization which minimizes the residual sum of squares is given by:

$$S_t = .3285 + Z_t.$$

The actual and predicted series are graphed in Figure 2, in which it is readily apparent that the fluctuations of model-generated exchange rates, while qualitatively accurate, are quantitatively of substantially smaller amplitude. In addition, the model-based realizations exhibit less short-term (i.e. high frequency) variation. These anomalies may be due to overshooting, speculative bubbles, "news" effects, and/or model misspecification.

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