The Cambridge Companion to

KANT AND MODERN PHILOSOPHY

Edited by

Paul Guyer
University of Pennsylvania

CAMBRIDGE
UNIVERSITY PRESS

2006

GARY HATFIELD

2 Kant on the perception of space (and time)

Although the "Transcendental Aesthetic" is the briefest part of the first *Critique*, it has garnered a lion's share of discussion.[1] This fact reflects the important implications that Kant drew from his arguments there. He used the arguments concerning space and time to display examples of synthetic *a priori* cognition, to secure his division between intuitions and concepts, and to support transcendental idealism. Earlier, in the years around 1770, Kant's investigations into space and time had facilitated his turn toward "critical" philosophy. Prior to that time, Kant's main interests in space and time pertained to physics and metaphysics. As he entered the critical period, he delved into the cognitive basis of our experience of space (and time), and drew his conclusions about their ideality.

Kant's doctrines of space and time provoked extensive response in his own time and throughout the nineteenth century. These responses variously concerned the metaphysics, physics, epistemology, psychology, and geometry of space. Throughout the nineteenth century, philosophers, physiologists, and psychologists sought to extend or to refute Kant's theories of space. By the last decades of the nineteenth century, many had rightly concluded that the existence of non-Euclidean geometry as a candidate description of physical space refuted Kant's full doctrine of space – though some have hoped that his position might be saved by restricting it to "visual space."

This chapter first examines the background to Kant's work on space (and time) in the writings (primarily) of Descartes, Leibniz, Wolff, and Crusius. It then follows the development of Kant's own views, from his first writings through the second edition of the *Critique of Pure Reason*. Finally, it surveys the reception of his mature views in the nineteenth and twentieth centuries.

I. METAPHYSICS AND EPISTEMOLOGY OF SPACE AND TIME PRIOR TO KANT

Space and time, or the related concepts of extension and duration, attained special prominence in early modern philosophy because of their importance in the new science. Prior to Newton, the mechanical philosophies of Descartes, Galileo, Gassendi, Boyle, and others designated a subset of the Lockean "primary qualities" as the fundamental properties of matter: size, shape, position, and motion. Size and shape are, in early modern parlance, "modes" or "modifications" of extension or space; position is a spatial relation; and motion requires space and time. Subsequently, the absolute space and time of Newtonian mechanics sparked further debate, most notably the *Leibniz–Clarke Correspondence* (published in 1717).

Metaphysical questions surrounding the new science pertained to the nature of space and time and their relation to matter. Epistemological questions pertained to the cognition of space itself or extension in general (including geometry, understood to be the science of extension), and also to the operation of the senses in perceiving the actual spatial order of things. Various positions emerged in both domains, and debate continued to the time of Kant.

In the *Principles of Philosophy*, Descartes staked out a bold new position, which equated space with matter.[2] Famously, he held that the essence of matter is *extension*: spatial extension in three dimensions. There is no distinction between matter and space. Matter is not *in* space; rather, its own extension is required for there to be any spatiality at all. Consequently, Descartes ruled out, on metaphysical grounds, the existence of a true vacuum. He maintained that the world is a plenum, that is, it is completely filled with matter, which is infinitely divisible. Some of this matter is a fine dust called the aether, which fills in between larger particles (with no gaps). Descartes held a relational view of position, according to which the positions of bodies are defined in relation to other bodies. There is no coordinate system of spatial positions independent of material things.

The extension that constitutes the essence of matter was, in Descartes' view, the object of geometry. He held that the truths of geometry are known innately by the human mind, through the "pure intellect" operating independently of the senses. In perceiving these

geometrical truths, the intellect perceives also the essence of matter, and therefore comes to know what properties matter can and cannot have (independently of matter's existence). For instance, any matter that takes the shape of a right triangle must exhibit the Pythagorean relations between sides and hypotenuse. In this way, the pure intellect knows the real possibilities of bodies as they are in themselves.[3]

Descartes distinguished the purely intellectual apprehension of extension from the perception of the shape, size, and distances of extended things by means of the senses. Sense-perception depends on bodily organs. The mind must be affected so as to experience sensations and perceptions of spatial properties. In vision, the primary sensations are produced by a two-dimensional pattern in the brain that echoes the retinal image. The sensation itself is of a two-dimensional array of light and color. These sensations are altered into perceptions by a variety of psychological mechanisms, including unnoticed judgments that infer the distant sizes and shapes of things. Sense-perceptions are adequate for everyday life, but not for metaphysics. Indeed, Descartes described sensations of qualities such as color as "obscure and confused," because we are unable to tell from them whether there is a property in bodies resembling the color we experience. The pure intellect must tell us the essential properties of bodies as they are in themselves, which are modes of extension (primary qualities) only.[4]

Descartes' metaphysics of space was set against the doctrines of ancient atomism, as revived and promoted by Pierre Gassendi and others. According to atomism, matter comes as small, indivisible particles or "atoms." These are distributed through space itself, conceived as an empty container. Where there are no atoms, there is a vacuum (empty space).

Newton was an atomist who posited an absolute space and time as a (potentially empty) container.[5] He held that the containing space provides an absolute framework for motion. According to Newton's laws of motion, any change in motion (defined as an acceleration) requires a cause (an acting force). However, the changes involved pertain to absolute motion in relation to space, not merely to the motion of one body in relation to another. To see this, consider two bodies that are accelerating away from one another. There are three possibilities for the true story about their absolute motions (and hence about the true forces): (1) one body is at rest (or in inertial motion) and the second is accelerating (which requires that it is acted upon by force); (2) they are both accelerating (both are acted upon by forces); or (3) the first body is accelerating and the second is at rest (or in inertial motion). A gap between two bodies (considered to be alone in the universe, if you like) that widens at a given rate is consistent with three different causal scenarios.

Newton's postulation of an invisible, homogeneous, and potentially empty space (and time) was subject to criticism. Leibniz, in correspondence with Samuel Clarke (who acted as Newton's mouthpiece), advanced both metaphysical and epistemological objections. Epistemologically, he objected that absolute motion in relation to a containing space could not be discerned. (Suppose first that the universe is at rest in relation to absolute space, and then instead that it is in inertial motion; the difference is undetectable according to Newton's theory.) Metaphysically, he appealed to the principle of sufficient reason (among other arguments). He argued that God would have no reason to place the universe (holding its internal spatial relations constant) in one position in absolute space rather than another (or to create it at one instant rather than another in time). But, in his view, God always acts for a (nonarbitrary) reason. Further, he asked what the (potentially empty) containing space is supposed to be. If it is a substance, would it be coeternal with God? If it is a property, what is it a property of? Clarke wrote as if it were a property of God, or of God's sensorium, to which Leibniz responded scornfully that this would make God an extended thing, or at least give him extended parts or organs.[6]

Leibniz's own position, which is only partially revealed in the Leibniz–Clark correspondence, was that space is relational, phenomenal, and ideal. Leibniz argued that the essence of matter could not be extension, as Descartes had maintained, but must also include force. He also held that anything composite (as bodies are) must be constituted from, or at least based upon, simples. In positing his (infinity of) simple substances, or "monads," he conceived of them by analogy with minds, as immaterial (or "metaphysical") points, which have internal states but no external relations – causal, spatial, or otherwise (no windows or doors). The internal states are perceptions, which mirror the whole universe from a point of view. That is, they portray a spatially extended universe of bodies that can be described, in accordance with the mechanical philosophy, as

matter in motion (in a plenum). But those bodies are phenomenal and ideal: they are "well-founded phenomena," founded in their agreement with the perceptions of other individual substances (and with the divine vision of the universe).[7]

In the Leibniz–Clarke correspondence, without revealing his position that bodies are phenomena, Leibniz argued that space is constituted by relations among bodies. Space is the perception of the order of coexistences – or rather, of possible relations of coexistence. Bodies at an instant have a set of actual relations among themselves; the idea of space comes from recognizing that they could be otherwise ordered (switching two small bits of matter, or reordering it all). The mind thus recognizes space as the set of possible relations among bodies. Space is ideal just in the sense that it abstracts away from the actual relations among really existing bodies (in the language of the *Correspondence*) to represent possible relations.

This point about the ideality of space is consistent with but independent of the claim that bodies are phenomenal (i.e., are well-founded phenomena). It is also independent of Leibniz's claim that there are neither direct causal interactions nor actual external relations between the monads themselves. Among this group of doctrines, some were better known in the eighteenth century than others. Leibniz discussed the ideality of space at length in the *Correspondence*. His system of preestablished harmony, as an alternative to mind–body interaction, had been published in 1695, and he referred to it in the *Correspondence*. However, the *Correspondence* is written as if the relations among material bodies are real, and as if material bodies causally interact with one another.[8] The *Correspondence* maintains the ideality of space without the *Monadology*'s noninteracting immaterial substances lacking external relations. (As should be apparent, Leibniz's "ideality" of space is not equivalent to Kant's transcendental idealism; but Kant's position has similarity with Leibniz's phenomenalism.)

Leibniz's account of sensory representations of space suggests that they are confused representations of the underlying reality (the monads), though this aspect of his phenomenalism about body was not well represented in his eighteenth-century published works. In published works, he affirmed the Cartesian point that the senses present confused images of things, at least as regards secondary qualities. He did not hold that these images should themselves be clarified by analyzing them; rather, clear ideas should be used to draw conclusions about their causes. Sensory images, even involving secondary qualities such as color, may have a regular relation to the cause of color in objects (a structure that reflects light in a certain way), but we cannot discern this fact by attending carefully to the sensory ideas. Rather, we must come to understand how light works and how it affects our nerves.[9]

If space is ideal and phenomenal, what implications does this have for geometry as the science of extension? It might seem as if Leibniz would adopt an abstractionist view of extension and geometry. Aristotelian philosophy had held that the object of geometry, "intelligible extension," is abstracted from sensory experience. This abstractionist position was challenged by Descartes and others (and later by Kant) to show how the cognitive basis of mathematics could be empirical, since mathematics achieves demonstrative certainty about perfect shapes, which are not found in sensory images.[10] Leibniz offered a version of this challenge, but he did not affirm Descartes' contention that the pure intellect can contemplate a purely intelligible extension. He agreed with Descartes that the intellect possesses innate ideas, or "seeds of eternity," that can serve to establish the universal, necessary truths of geometry.[11] By contrast with Descartes, who held that these ideas can be found by turning away from the senses, Leibniz held that they are awakened by sensory images. And yet Leibniz distinguished such ideas from sensory images, thereby affirming that there are thoughts without images – without suggesting that these thoughts directly present an intelligible extension. He gave a hint in the *New Essays* that geometry might be based on number and logic. In unpublished papers, he offered the hope that geometry could be reduced to logical identities without a need for spatial images.[12]

Christian Wolff was the dominant philosopher in Germany at mid-century. In some ways a follower of Leibniz, he did not adopt Leibniz's positions wholesale. He accepted that composite things are constituted from simples (though finite in number). He agreed that the simple substances are indivisible and unextended. He also adopted the relational view of space. However, unlike Leibniz's actual position (but more like the position Leibniz took in the *Correspondence*), Wolff held that (a finite number of) simple substances are aggregated to form continuously extended bodies. He affirmed real relations among simples, both spatial and causal. Contra Leibniz, he held that bodies causally influence one another directly. He merely tentatively endorsed Leibniz's "preestablished harmony" for mind–body relations.[13] By contrast, Alexander Baumgarten, a Wolffian philosopher of sorts, adopted a pre-established harmony for both body–body and mind–body interaction.[14]

As regards the ontology and epistemology of space, Wolff held that space is ideal but not merely phenomenal (in Leibniz's sense). Bodily extension is composed of unextended simple substances. Our perceptions of those substances represent their coexistent order as a spatial order, which it is (that is, it is a set of actual external relations among simples). Perceived space represents the real relations among things. Yet it does so only confusedly. We are in fact unable to understand how unextended simples can be composed to form continuously extended bodies. Accordingly, our spatial perceptions must be considered as confused. If they represented the simple substances clearly, we would be able to "see" or understand how unextended simples can yield extension.[15]

Christian August Crusius (whose work Kant admired in his early years) pounced on this implication of the Wolffian position, complaining that the relational view of space rendered it into a "*Götze*" (false idol) of the imagination.[16] Crusius aimed to establish a metaphysical basis for absolute space and to show how extended things could be composed of simple substances. He considered space to be neither a substance nor a property. As he put it, "substances must be in space"; space is not in the substances. Space is not an inhering property of anything; rather, properties inhere in the things that are in space. Neither is space a set of relations, for there are many relations – even of "next-to-ness" (as in a melody) – that are not spatial. Rather, spatial relations arise because things are *somewhere* in space.[17]

Crusius considered space to be an aspect of the reality of things as they really are. He held that the finite world of matter is composed of indivisible parts (substances) that fill space. From this merely finite world, we would not derive an infinite, absolute spatial framework. But Crusius sided with Newton and Clarke in holding that there is an infinite absolute space into which the finite world could be placed in one location or another. This infinite space is an abstraction from the existence of God, or from his omnipresence. Crusius held that God "fills" space, but is not extended (and that unextended souls "fill space" too). He sustained his distinction between being extended and being space-filling by defining an extended thing as something that has "actual parts." But God is indivisible and has no actual parts, hence is (in this technical sense) not extended, even though he fills space. Crusius also held that the elemental parts of matter (simple substances) fill space but are not extended. According to Crusius, there is no truly empty space (God is everywhere), but the infinite space abstracted from God's being can be empty of matter, or not.[18]

Through his unextended but space-filling material elements, Crusius claimed to show how extension can arise from unextended parts (thereby countering Wolff's claim that the spatial representations are inherently confused). The unextended (indivisible but space-filling) elements of matter, when put alongside one another, form a composite thing, which is divisible and so extended. Because the elements are already spatial, they can be composed to yield a continuous space, actually divisible into simples, and infinitely divisible in thought. He also claimed to provide a basis for cognizing infinite, absolute space, by abstraction from an unextended but space-filling God. He further held that magnitude, as the object of mathematics, is an abstraction from existing things: God and bodies.[19] This makes geometry rely for its object on content abstracted from the reality of things. However, Crusius (by contrast with Kant's subsequent critical attitude) held that mathematics could nonetheless achieve perfectly general definitions that would apply to all instances (and support demonstrative reasoning). It could do so because, in abstracting its object, it pays attention solely to magnitude itself. By contrast, philosophy treats of things together with their accidents. Crusius therefore reasoned that in mathematics alone, a single instance could provide the exemplar for mathematical definitions that would apply to all other instances.[20]

The theory of the senses, and especially vision, attracted philosophical discussion throughout the eighteenth century, stimulated by Descartes' *Dioptrics* and Berkeley's *New Theory of Vision*, among others. The psychological process of spatial perception was widely discussed in Germany. While Kant was working on his first *Critique*, J. N. Tetens published an extensive discussion of the perception of size in individual objects by means of the senses.[21]

A central problem in these discussions was to explain why we experience objects at close range to remain a constant size, even at different distances. If an object is seen at four and then at eight paces from the viewer, the size of its image on the retina is reduced by half (along any axis). Most eighteenth-century theorists held that we nonetheless experience the object as of a constant size. Tetens agreed, and suggested that in the second case the sensation is altered to produce an image of the full-sized object. He observed that some theorists ascribed this alteration to associative processes in which true sizes are associated with various projected sizes and cues for distance, while others ascribed it to a process of reasoning, in which projected size and distance are combined according to geometrical relations. Tetens rejected both types of theory. Instead, he described the process as a kind of abstraction from variations in projected size. In the normal course of things, we come to recognize objects when they are near to us and fill the visual field. When the object is further away, we recognize it as the same object, but do not notice its small projected size. We abstract from the small size, and experience the object as we did under the circumstances in which we first came to recognize it, with its "normal" size. As in the other accounts, the result of this psychological operation is phenomenally immediate, and we do not even notice that the sensation has been altered.

Without entering further into the details of Teten's position, we may note that Kant was aware of the fact that typical accounts of perception posited association or judgment – or, in Teten's case, a sensory act of abstraction – to underlie visual appearances. This is apparent from his discussion of the moon illusion, which he ascribes to the influence of imagination (presumably, through association) in making the moon appear larger at the horizon than overhead (A 295–7/B 351–4).

2. KANT'S EARLY WRITINGS ON SPACE AND SPATIAL COGNITION

Kant discussed space and spatiality in his early works on physics and metaphysics, adopting a quasi-Leibnizian, relational view of space. He considered himself to have solved certain problems that plagued the positions of Leibniz, Wolff, and the Wolffians. In essence, he came to see space as an appearance of real relations, but one that was neither merely ideal nor confused. He arrived at these improvements by applying Crusius's ideas to the question that Wolff had found insoluble: how simple substances might fill space without being rendered divisible.

The *New Elucidation* of 1755 examined the "first principles of metaphysical cognition."[22] It first covered the principle of contradiction and the principle of a "determining ground" or sufficient reason, before turning to the principles of succession and coexistence. These latter two principles concern the causal sequence of changes of states in substances over time, and the basis of the coexistent relations of states and so the basis of space. In the section on the principle of succession, Kant argued that changes depend on the "reciprocal dependency" of substances on one another. If individual substances were in causal isolation (as Baumgarten in fact argued), they would have no impetus to change states and would therefore remain always in the same condition (1:410–11). (Once an isolated substance was in one state, it would have no basis for determining itself to change states.) Change arises through the interaction of substances, or their mutual causal dependence. These interactions establish the relations that exist among coexistent substances, as well. In this way, Kant considered himself to have overturned the Wolffians (in fact, his position on causal influence goes against Baumgarten, but not Wolff himself), and also Leibniz's preestablished harmony between soul and body (1:411–12).

Kant held that substances would not, merely in virtue of their existence, stand in any relation to one another. Rather, it is only through real reciprocal causal relations (grounded in a divine conception of their mutual relations) that things obtain real relations among themselves.[23] These relations then constitute the space of the substances, as in the Leibnizian relational view of space. As Kant put it, "place, position, and space are relations of substances, in virtue of which substances, by means of their reciprocal determinations, relate to other substances which are really distinct from themselves and are in this way connected together in an external connection" (1:414). In the work of 1755, Kant said little about the cognition of space, merely observing that the "concept of space" is constituted by cognition of "the interconnected actions of substances," that is, their reciprocal actions (1:415).

In the *Physical Monadology* of 1756, Kant advanced a conception of space as an appearance. The work was devoted to the problem of reconciling the infinite divisibility of space as posited by geometry with the simple substances advanced by the metaphysicians (including Leibniz, Wolff, Baumgarten, and, in his own way, Crusius). Kant accepted that space is infinitely divisible. He also accepted that bodies are constituted from simples, though he specified that their number should be finite. He departed from Leibniz and Wolff in saying that "each monad is not only in space; it also fills a space" (1:480). Talk of simple substances "filling space" was of course applied by Crusius to God, souls, and the simple substances constituting matter.

Kant developed and altered Crusius's position, combining the relational view of space found in Leibniz and Wolff with talk of "monads," now conceived in Crusian fashion. He treated monads as "filling space" without being (physically or metaphysically) divisible. Crusius had said that the space of simple, indivisible substances is technically not extended (because indivisible), but Kant dropped this definition of extension. However, again echoing Crusius,[24] he held that the monads stand in real relations through causal interaction. But departing from Crusius, he held that space arises solely from these interactions (rather than the monads being in a space provided by God's omnipresence). As Kant saw things, in the light of his relational view of space, each monad determines "the little space of its presence" (1:480) through forces that it exerts on the substances next to it. As in the *New Elucidation*, an order of relations among coexistent things arises from causal relations of mutual dependence among substances, though now these relations are explained as interactive forces.

Space is "the appearance of the external relations of unitary monads" (1:479). With this doctrine, Kant in effect claimed to solve Wolff's problem of how to derive a continuous space from indivisible simples, thereby avoiding the bane of previous supporters of monads, who had "regarded it as their duty to maintain that the properties of geometrical space were imaginary" (1:480). Although he did not mention names, his position departed from both the phenomenal space of Leibniz and the confused representations of Wolff. Contrary to Leibniz, Kant's space is not merely phenomenal; it is the appearance of real relations (that form a real external space). But contrary to Wolff, this appearance need not be regarded as confused, because Kant has claimed to solve the problem of how a finite composition of simples could yield a continuous space.

In 1756, when Kant described space as an appearance, it may seem as if he was asserting its phenomenality (in Leibniz's sense) or ideality (in his later sense). He says that the continuously divisible space of mathematics arises as "an appearance of the external relations" among indivisible simples (1:479). But he also says that these simple substances "fill space" through their "sphere of activity" (1:480–1). The divisibility found in appearance is grounded in the continuous space-filling actions of simple substances. The space of appearance echoes the space created by the monads.

3. THE SHIFT TO ABSOLUTE SPACE, AND THE CRITICAL TURN

Having started with a relational view of space, Kant changed his mind by 1768, when his *Concerning the Ultimate Ground of the Differentiation of Directions in Space* appeared. He now advocated an "absolute and original space" in which physical things are located (2:383). His arguments for this space hinged largely on the consideration of incongruent counterparts. The arguments did not directly establish a mind-independent absolute space, but they revealed that the Leibnizian or relational view, as Kant had understood it, could not capture certain distinctions that exist in our descriptions of space. By contrast, a view of space as an empty, absolute container could account for these distinctions.

Incongruent counterparts are spatial structures (shapes) in which all internal sizes and relations are identical, but which cannot be made to coincide spatially. Typical examples include objects that are (precise) mirror images of one another, such as left and right hands, or left and right ears. For true counterparts, if one measured all the relations among the fingers of left and right hands and wrote them down, the listed measurements would be identical: the thumb would be *x* units long and its joint would be *y* units from the knuckle of the index finger. Although the listed measurements for each hand would not differ, a right hand will not fit into the space of a left hand (or into its glove).

Kant's argument that a relational theory is precluded by the existence of incongruent counterparts depends partly on his conception of the relational view. He has in mind a view in which space is constituted from relations among simples. His argument also depends on what should be asked of a theory. Kant expects the theory to account for our "intuitive judgments about extension" (2:378). That is, he expects a metaphysical theory of space to account for the descriptive judgments we make about space. Now Kant asks: from describing relations among simples, without orienting them to directions in space, can we distinguish the description of two actually distinct (according to our perception), incongruent counterparts? Kant correctly answered "no."[25]

To see this, continue the set of measurements on your hands, seeking a description that someone else (or you on another occasion) could use to construct a congruent hand. Holding the position of one hand fixed, measure the length of each finger, the distances between each pair of adjacent joints, the distance from the wrist knob to each knuckle, and so on, and then go on to the other. Considered purely as internal relations defined by the structure of each hand, these two sets of measurements cannot be distinguished. To distinguish them, one must orient the hands in space, and note that on the left hand (as viewed from above with palm down) the index finger is to the left of the thumb, whereas on the right hand it is to the right. Such descriptions use the orienting directions of *up*, *down*, *right*, and *left*. To convey these directions in a description, they must be related either to the directionality in our individual perceptual spaces to a common external frame (absolute space). This can be verified by trying to use one of the descriptions to construct a specific hand. Failing an appeal to an external frame (to right and left, up and down), one will be unable to provide distinct instructions for constructing either a left or a right hand.

In 1768, Kant's conclusion was that "the ground of the determination of a corporeal form does not depend simply on the relation and position of its parts to each other; it also depends on the reference of that physical form to universal absolute space, as it is conceived by the geometers" (2:381). This is the universal and absolute space of Newton. Bodies possess a distinctive structure in the relations among their parts that can be described only by appeal to directions defined within this encompassing space. If, as in Kant's conclusion just quoted, the "ground of the determination" pertains to our descriptions or cognitions of space, then his argument is unassailable. But if he means to say that an existing thing could not exhibit handedness independently of our ability to distinguish that handedness descriptively, then the argument fails. A set of oppositely handed relations might exist, whether we are able to find a framework for describing them or not. Kant's metaphysical argument depends on an epistemic argument about what can be conveyed cognitively in descriptions of parts and their relations.

Two years later, when he presented his inaugural *Dissertation*, Kant again supported an absolute spatial framework, but he now denied that such a space exists apart from our perceptions – a conception that he had, in 1768, already described as involving "difficulties" (2:383). He rejected both the relational view of Leibniz and the "English" view of space as an "*absolute* and boundless *receptacle* of possible things" (2:403). He now asserted that "Space is not something objective and real, nor is it a substance, nor an accident, nor a relation; it is, rather, subjective and ideal; it issues from the mind in accordance with a stable law as a scheme, so to speak, for co-ordinating everything which is sensed externally" (2:403). Kant now asserts the *ideality* of space in his critical sense of that term. He also holds that "the concept of space" is an "intuition" (2:402), which means that the representation of space is a concrete image (not a discursive concept).[26]

Kant's absolute spatial framework is no longer a Newtonian container, but is now a phenomenally given appearance. He sought to support this position by undermining attempts (which he associated with Leibnizian and Wolffian views) to abstract spatial representation from sensory experience. He contended that the concept of space is "presupposed" by the perceptions of the external senses; sensations can be located outside us, and next to one another, only if a space exists for so ordering them (2:402). This space is "a singular representation," embracing all spaces within itself, unlike an abstract concept, under which instances fall. It is "a pure intuition," which means it is not compounded from sensations. It provides the "form" or the structure in which all sensations are ordered. (We will return to similar arguments in the next section.)

To support the point that space is a pure intuition, Kant again employed the argument from incongruent counterparts, now restricted to the cognition and description of an ideal or phenomenal space (which suits the argument). Kant argued that our discursive description of an incongruent counterpart could be made determinate so as to distinguish left and right hands only if spatial intuition were already given with the directions up and down, right and left. That is, any merely conceptual description of the counterparts could not supply a directional framework; our spatial representations must arise as a concrete intuition or image. From this argument, he also concluded that geometry cannot content itself with mere discursive descriptions and universal concepts, but must appeal to concrete or "singular" intuitions (2:403). Discursive descriptions would be unable to capture directions in space without an ostensive basis (in an imagistic representation).

Kant's argument that "space is not something objective and real" (whether "substance," "accident," or "relation"), but "is, rather, subjective and ideal," depended on ruling out the two alternative theories named above: that space exists apart from perceptions as an absolute container or as relations among elements. He baldly described real absolute space (apart from perception) as "a fable" because "it invents an infinite number of true relations without there being any beings which are related to one another" (2:404). This is a version of the arguments found in Leibniz and Crusius, namely, that absolute space considered as an infinite receptacle cannot be a substance (otherwise it would compete with God as an infinite substance), nor an accident or relation, since by hypothesis there is no substance for it to be an accident or relation of. Further, Kant could not accept the Crusian abstraction of space from the omnipresence of God, for it spatializes God, whom he placed outside time and space (2:297, 414).

Kant offered an epistemological objection to relational space: it could not account for the necessity of geometry. In his view, if space were abstracted from the given relations of things, our knowledge of space would be empirical and so could not support apodictic certainty. However, geometry provides us with apodictic knowledge of spatial structure. This would be explained if our spatial representations were subjective and ideal (on the assumption that they must conform uniformly to Euclid's geometry). Hence, our subjective space is "the foundation of all truth in outer sensibility" (2:404). Because all spatial perceptions are constructed according to the same laws of sensibility, the properties of pure intuition (as described by geometry) must apply to all empirical intuition. Hence, the applicability of geometrical descriptions to the physical world (as experienced) is guaranteed.

With these arguments and conclusions, many of which will reappear in the "Transcendental Aesthetic" of the *Critique of Pure Reason*, Kant has begun his critical turn. Missing is the limitation of the intellect or understanding to possible experience. In the *Dissertation* (2:392, 402–5), Kant still held that the form of the intelligible world can be understood through causal relations (though no longer through spatial relations, which do not apply to intelligible beings). Once he had achieved the critical restriction of all cognition to actual or possible experience, the previously secured ideality of space and time entailed his mature position of transcendental idealism, thereby also entailing the impossibility of traditional metaphysical knowledge of things as they are in themselves.

4. SPACE (AND TIME) IN THE "TRANSCENDENTAL AESTHETIC"

Kant's arguments in the "Transcendental Aesthetic" of the *Critique of Pure Reason*, first published in 1781 and then revised in 1787, are intended to establish the ideality of an absolute (as opposed to relational) space and time. More precisely, he intended to establish the transcendental ideality of space, which meant that it was not only dependent on human perception, but was also an *a priori*, necessary, and universal representation. He employed several arguments to achieve this aim. Some of the arguments were conceptual: he contended that spatiality was presupposed by spatial representation, and that empty space was a more fundamental representation than space with objects. Other arguments were epistemological. Drawing on his conceptual arguments, he contended that the representation of space could not arise from experience (by abstraction from things as given in space). It is difficult to see how these arguments could establish Kantian ideality of space (that it pertains to perception only, not to things in themselves), at least not without other premises. Kant also maintained that the necessary and universal cognitions found in geometry could be explained only if space was an *a priori* form of representation that universally structures all intuition, and hence that

also accounts for the conformity of physical objects in experience to the geometry of that *a priori* spatial structure. This argument, if correct, might establish ideality.

These arguments appear in both the first and second editions of the Aesthetic, though in the second edition the argument from geometry is separated from the conceptual (and related epistemological) arguments. Here we follow the second version, in which Kant offers four numbered arguments in the "metaphysical exposition" of the concept of space. By establishing that our representations of space and time are fundamentally intuitions and not general concepts, these four arguments are intended to refute metaphysical and epistemological implications of the relational view of space – as well as, to a lesser extent, those of the (Newtonian) absolute view – and also to establish Kant's fundamental distinction between intuitions and concepts.

In preparation for his numbered arguments, Kant draws several key distinctions. He distinguishes *sensibility*, as a passive faculty of receiving representations, from the *understanding*, as the faculty of thought. He calls the "immediate" representations of sensibility *intuitions*, as distinct from *concepts* employed by the understanding. Within intuitions, he distinguishes the sensations proper (for vision, color, and intensity), which he calls the *matter* of sensory appearance, from the *form* of the appearance, which is "that which makes it that the manifold of appearance can be ordered in specific relations" (A 20/B 34). (Soon he will speak of the "form of intuition" in addition to the form of appearance.)

With this terminology in hand, Kant goes on to ask a set of questions regarding the status of space and time, laying out several alternative positions for consideration.

> What, now, are space and time? Are they actual beings? Are they mere determinations or else relations of things, but nonetheless of a sort that would in themselves belong to such things if they were not being intuited; or are they such that they inhere only in the form of intuition, and hence in the subjective constitution of our mind, in the absence of which these predicates could not be ascribed to anything whatsoever? [A 23/B 37–8]

The alternatives offered here are Newtonian absolute space and time (actual beings independent of objects in space); Crusian absolute spatial extension as a "determination" of God, or Cartesian spatial extension as a determination of matter; Leibnizian or Wolffian relational views; and Kant's own view. We should note that the first three are realist theories of space, according to which either space is a real being in itself or spatiality is a real property based in things. In Kant's own view, space is transcendentally ideal, that is, it inheres only in "the subjective constitution of our mind."

In the arguments of the Aesthetic, Kant purports to decide among these alternatives through arguments about the "concept of space" (the metaphysical exposition) and about the possibility of geometrical cognition. As he has done from 1768 onward, he seeks to draw conclusions about the ontological status of space (and time) from arguments about spatial (and temporal) cognition.

The first of his numbered arguments holds that "space is no empirical concept, which has been abstracted from outer experiences" (A 23/B 38). One potential origin for the representation of space would be that it arises empirically from the experience of objects in space. An advocate of a relational view of space might hold that space is constituted out of relations among things in themselves that cause the representation of space to arise in us through experience. A Newtonian absolute view also permits an empirical origin for our spatial representations, through interaction with objects in absolute space. In either case, Kant argues that we could not in fact acquire the representation of space by means of experience because any representation of sensations as spatially related already presupposes a capacity for spatial representation. It is this capacity – for representing sensations as "in another position in space from that in which I am located," and also for representing them "as outside and alongside one another" (A 23/B 38) – that already requires "the representation of space" as a ground for presenting the sensations with spatial relations. Whatever may be the status of space itself, spatial representation cannot be acquired as a result of experience.

The second argument is also intended to support the view that the representation of space cannot be acquired from the experience of bodies because it is prior to or more fundamental than that experience. Kant expresses this point by asserting that "space is a necessary representation, *a priori*, which underlies all outer intuitions" (A 23/B 38). It *underlies* all outer intuitions because "one can never form a representation of the absence of space, though one can very well conceive that no objects are to be found in it." It is therefore "the condition for the possibility of appearances." That is, no objects can be represented except as in space. Since space is required for the very possibility of appearances (of objects), it is a (conditionally) *necessary* representation (relative to appearances). As a condition of appearances, it is not "a determination that is dependent on appearances"; hence, it is not empirical but *a priori*. Finally, Kant would seem to assert also that space is a necessary feature of any human consciousness, for he says that we can "never form a representation of the absence of space"; this phrase suggests that all our (sensory, or world-related?) representations contain a spatial element.

The third argument is intended to show that "space is no discursive, or, as one says, general concept of the relations of things in general, but rather is a pure intuition" (A 24–5/B 39). This argument opposes the conclusion that space is an intuition to the notion that space is a "concept of the relations of things in general." By contrast with the fourth argument, the third one does not seem to rely on a conception of concepts as applying to many independent instances (as the concept *dog* is related to many dogs). Rather, this argument opposes the notion that space is a representation that arises empirically from the consideration of numerous elements (or "parts") of space, as in the Wolffian theory that Kant himself had previously embraced. Kant contends that the parts of space presuppose a single, all-encompassing space. They are created by introducing limitations into (or carving up) the continuous, concrete, unitary space of intuition. Kant does not explain why this unitary space must itself be an intuition rather than a type of concept that is not relational. Presumably, it has to do with the fact that "intuitions," in contrast with Kantian concepts, are "immediately given" and concrete representations (A 19/B 33), as elaborated in the subsequent argument. Finally, because the representation of space cannot be derived from relations among previously given elements or parts, it must be *a priori*. As such, Kant observes, it is able to sustain apodictic geometrical propositions about spatial relations (A 25/B 39).

The fourth argument is intended to establish as a general point that space is an intuition (a pure, *a priori* one), and not a concept. It does so by asserting that "space is an infinite *given* magnitude" (B 39). As suggested by the wording of the first-edition version, the infinity of this magnitude is not to be understood as something given all at once; rather, it amounts to "boundlessness in the progress of intuitions" (A 25). Presumably, this boundlessness occurs in the fact that (1) the space of intuition is given without a boundary, and so we can continuously traverse into new space in thought (or imagination), and (2) we can divide all finite parts of space *ad infinitum* (i.e., without ever coming to an indivisible part). In this potential sense, "all parts of space, to infinity, exist simultaneously" (B 40). This notion that the parts of space are represented as being "in it" (as parts to be carved out of a single, continuous space) marks the contrast between intuition and concept. For, as Kant explains, a concept can represent an infinity of instances "under it," which means that it applies to an infinity of independent, discrete objects. The parts of space are not represented apart from the one embracing space, nor as independent constituent parts of it; rather, they are dependent parts of that space and are found (potentially) in it. Again, because the representation of space precedes its parts or elements, Kant claims that it is *a priori*.

The first-edition version of the Aesthetic contained another numbered argument concerning the basis for geometrical cognition. This topic was transferred into a newly titled section of the Aesthetic in the second edition, called the "Transcendental Exposition," immediately following the four numbered arguments just discussed. This section addresses (for sensory representation) what was, according to both the first- and second-edition versions of the Introduction, the central problem of the *Critique*: "to uncover the ground of the possibility of synthetic *a priori* judgments with appropriate generality, to gain insight into the conditions that make every kind of them possible, and not merely to designate this entire cognition (which comprises its own species) in a cursory outline, but to determine it completely and adequately for every use in a system in accordance with its primary sources, divisions, domain, and boundaries" (A 10; see also B 14–24). Hence, while the previous arguments lay the groundwork by explicating the cognitive origins and status of space (and time, in the subsequent section), the "Transcendental Exposition" uses these results to explain the possibility of a domain of synthetic *a priori* cognition, namely, that found in geometry, and thereby fulfills a central mission of the Aesthetic.

The way in which the *a priori* representation of space enters into geometrical demonstration is treated by Lisa Shabel in the

following chapter. Here we should note that Kant's explication of the possibility of geometrical knowledge helps us to understand his distinction between concepts and intuitions; to the extent that his explication is accepted, it serves to support that distinction. In his explication, Kant invokes the account of mathematical methodology found in the "Transcendental Doctrine of Method": the "construction of concepts" "in intuition" (A 713–17/B 741–5). Two points are especially important for our purposes. First, to sustain geometrical demonstrations, intuition must provide representations that are "*a priori*" and "immediate" (B 41); this latter condition, as articulated in the "Doctrine of Method," means that intuitions are "individual," "concrete," and "particular." Unlike the philosophical use of concepts, which "considers the particular only in the general," that is, in a general concept that applies to many discrete and independently given particulars, the mathematical construction of concepts in intuition "considers the general in the particular, nay, even in a single instance" (A 714/B 742). These instances are intuitions that "display" the objects of mathematical cognition "in concreto," though of course also *a priori* (A 715–16/B 743–4).

Second, the wording of the "Transcendental Exposition," with its talk of "the properties of space" and of "objects" that pertain to "outer intuition" (B 40–1), suggests that Kant takes geometrical knowledge to apply to physical space and objects in space. Indeed, elsewhere Kant makes clear that he considered geometry to apply to physical space and objects in space (A 27–8/B 43–4, A 40–1/B 57–8, A 157/B 196, A 165/B 206, A 224/B 271, A 240/B 299, B 147; 4:283–4, 287–8). This means that any explanation of the possibility of geometrical cognition not only would need to explain what kind of representation is needed for geometrical cognition, but also must answer the question: "how can outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of the latter can be determined *a priori*?" (B 41). The answer here is that the "concept" or representation (that is, space as a form of intuition) must have "its seat merely in the subject, as its formal constitution for being affected by objects" (B 41). In other words, the *a priori* applicability of geometrical judgments and principles to all objects of cognition is to be secured by the transcendental ideality of space as a form of intuition. (This also supports Kant's "empirical realism" regarding our knowledge of objects in space [A 28/B 44].)

The arguments of the metaphysical and transcendental expositions are successful to various degrees. The conceptual point that the capacity for spatial representation is presupposed for the spatial ordering of sensations held up well against onslaughts in the nineteenth century (discussed below). The claim that space is prior to its parts accorded with the geometrical knowledge of its time, but was challenged by subsequent models of geometrical extension as composed of points. The conception of the basis of geometrical proof also was challenged by the algebraization of geometry and the discovery of non-Euclidean geometries in the nineteenth century.

The most pressing question concerns whether the arguments really show that space itself is ideal and *a priori,* that is, whether they effectively support Kant's preferred option among those named above. Suppose that the numbered arguments are effective in showing that the *representation* of space is prior to the presentation of spatially ordered sensations. That, by itself, would not show that *space* is a mere representation (would not establish transcendental idealism). Neither, presumably, would the conclusion that Euclid's diagrammatic proofs require an *a priori* spatial medium to capture their apodictic certainty.

The argument from geometry, when extended to geometrical claims about space itself, might well do the trick. Suppose one took it to be certain that geometry must apply (with necessity) to physical space (as Kant thought). Then the argument would succeed if it could be shown that such applicability can be explained only if physical space is itself ideal and the product of a subjective form of intuition that underwrites geometrical demonstration. This would sustain the conclusion that space and the objects in it are ideal.[27]

Moreover, one can understand why Kant might have believed that the numbered arguments, which purport to show that the representation of space cannot be acquired from objects, would bear on the status of space. Kant's "critical question" as related to Marcus Herz in 1772 (10:130), which informs the first *Critique* at crucial junctures and underlies Kant's Copernican revolution (B xvi–xvii), inquired after the ground of the relation between objects and representations. In the *Critique,* Kant identified only two possibilities for this relation: either "the object alone makes the representation possible," or "the representation alone makes the object possible" (A 92/B 124). These alternatives do not allow that the representation and the object would independently establish the same possibilities (e.g., would independently establish the properties of space). Kant presumably ruled out that option because he was interested in the necessary applicability of representations to objects, and he saw his transcendental idealism as the only way to achieve that. Under these circumstances, in showing that the representation of space must be *a priori,* he might well have considered himself to have established that it is not made possible by objects (and does not pertain to objects). Hence, it must make objects possible (as in transcendental idealism). Even if one does not share Kant's goal (of necessary applicability) or his parsing of the alternatives, one can see how Kant could believe that with this added premise about the alternatives, his numbered arguments would support the transcendental ideality of space (and time) – though in any case he considered the explanation of the applicability of geometrical knowledge to physical objects as the real clincher (A 40–1/B 57–8; 4:287–8, 292).

5. SPACE AS AN OBJECT (SECOND-EDITION "TRANSCENDENTAL DEDUCTION")

Once we have been told that space and time are *a priori* forms of intuition, the question remains of what exactly we have been told. Does this mean that our sense perceptions present us with a world of objects in space and time, without any other cognitive activity? Clearly not. The Kantian notions of "cognition" and of "experience" require that our cognition and experience of a world of objects is mediated by concepts that synthesize intuitions to yield judgments (A 50–1/B 74–5, A 156/B 195). But what about our knowledge of space as an *a priori* form of intuition? Does that arise from intuitions that are given to us already presenting the properties of space, or must the understanding be involved in synthesizing the spatial structures we know in intuition?

Kant made clear in both the first- and second-edition versions of the "Transcendental Deduction of the Pure Concepts of the Understanding" (i.e., the categories) that in order to be cognized, spatial structures must be subject to synthesis by the understanding (A 99, 115–25; B 137–8, 147, 150–6, 160–2). In the second-edition Deduction he explained that in the Aesthetic he had not properly expressed this requirement, but that he now was in a position to qualify and correct what he said there. This passage, which occurs as a footnote to §26, is worth quoting in full:

> Space, represented as *object* (as is really required in geometry), contains more than the mere form of intuition, namely the *comprehension* of the manifold given in accordance with the form of sensibility in an *intuitive* representation, so that the *form of intuition* merely gives the manifold, but the *formal intuition* gives the unity of representation. In the Aesthetic I ascribed this unity merely to sensibility, only in order to note that it precedes all concepts, though to be sure it presupposes a synthesis, which does not belong to the sense but through which all concepts of space and time first become possible. For since through it (as the understanding determines the sensibility) space or time are first *given* as intuitions, the unity of this *a priori* intuition belongs to space and time, and not to the concept of the understanding (§24). [B 160–1]

The distinction here between the "form of intuition" and "formal intuition" implies that space, as an object, is cognized or known only through the synthetic activity of the understanding. In two earlier passages in the second-edition Deduction, Kant had explained that space can be known only through the synthetic activity of producing objects in intuition. The first passage says that

> the mere form of outer sensible intuition, space, is not yet cognition at all; it only gives the manifold of intuition *a priori* for a possible cognition. But in order to cognize something in space, e.g., a line, I must *draw* it, and thus synthetically bring about a determinate combination of the given manifold, so that the unity of this action is at the same time the unity of consciousness (in the concept of a line), and thereby is an object (a determinate space) first cognized. [B 137–8]

In §24, he says that "we cannot think a line without *drawing* it in thought, we cannot think a circle without *describing* it, we cannot represent the three dimensions of space at all without *placing* three lines perpendicular to each other at the same point" (B 154). He is not of course here talking about things that we are forced to do by habit; rather, the "drawing," "describing," and "placing" are *requirements* for "thinking" the objects in question.

Interpreters have long puzzled over Kant's picture of the interaction between the understanding and sensibility in synthesizing spatial objects such as lines or circles. Of particular interest here are questions concerning what is given in the "manifold" of spatial (or temporal) intuition from the senses, and what is provided by the understanding's synthesis. Three positions have been formulated about how the "forms" of intuition deliver material to the understanding. According to one position, called "forms as mechanisms," the forms of intuition are laws or rules for ordering the matter of sensations into spatial and temporal structures. According to a second, called "forms as representations," space and time are empty representations, independent of matter, into which the matter of sensation is arranged. According to a third, called "forms as orders of intuited matter," intuitions initially come with the matter ordered in a spatiotemporal manner. This third position requires no laws of or rules for this ordering. Further, because spatial and temporal structures are orders of matter, it does not allow the possibility of empty spatial and temporal forms into which matter would be placed in a spatial or temporal arrangement.[28]

Many questions arise in any attempt to decide among these views of what space and time as forms of intuition might be. One question concerns where the synthetic activity of the understanding fits into the various positions. In the forms as mechanisms view, would the understanding apply the laws or rules, or would sensibility do it? And if the understanding was responsible for the synthesis, would it supply the laws or rules, or follow laws or rules prescribed by sensibility? Further, if, as in the third view, sensibility simply provides ordered, intuited matter, does that mean it directly yields perceptions of a spatially ordered world? But why then does Kant suggest that empirical perception of objects depends on the activity of understanding in the "figurative synthesis" of the imagination (B 151–2)?

We will not be able to sort out these various positions here. However, there is greater commonality among (the more reasonable versions of) these positions than may at first be apparent. First, all three positions allow that form and matter are in some way distinct; matter by itself would not be spatially or temporally ordered. Second, they all agree that the synthetic activity of the understanding is required for cognizing objects, as objects, in space. Third, they all agree that the forms of intuition are responsible in some way for the constraint that intuitions can be ordered (or are ordered) with spatial and temporal relations. The constraint that human intuitions are spatial and temporal is provided by sensibility, whether in the way in which it passively creates ordered intuitions ("orders" view), in the rules by which it constrains the synthetic activity of the understanding (reasonable version of the "mechanisms" view), or through forms as containing representations into which matter is placed by sensibility or the understanding ("representations" view). Hence, on all views, sensibility yields (by itself or jointly with the understanding) intuitions as spatially continuous and infinitely divisible images (occurring in temporal succession). In cognizing these images in an *a priori* manner, through its own imaginative activity (drawing lines and the like), the understanding can explore the properties of space and time and achieve *a priori* knowledge of them (B 152, 155; A 157/B 196). Since, on all views, perceptual experience of objects is constrained by the forms of intuition, this exploration yields knowledge of the spatial and temporal properties of all possible objects of experience. That, in the end, was the conclusion Kant wished to highlight, and it must constrain any attempt to sort out his theory of the relation between the forms of intuition and the synthetic activity of the understanding, whether in pure *a priori* or empirical cognition.

Kant took up issues surrounding space and time again in two later sections of the *Critique*, the "Amphiboly of the Concepts of Reflection," where he criticized the Leibnizian theory, and in the "Antinomy of Pure Reason," where he sought to show the impossibility of ever decisively deciding the cosmological questions of whether the world is infinite in space and time or finite, and whether it consists of simple parts, or not. The Antinomies are treated in Chapter 8 of this volume.

6. RECEPTION OF KANT'S CRITICAL THEORY OF SPACE

Kant's theory of space has been continuously discussed from the time of its publication. These discussions have questioned all aspects of the theory: his transcendental idealism, his theory of space and time as *a priori* forms of intuition, and his conception of the epistemic basis of (Euclidean) geometry and its applicability to physical space and the objects in it.

Among the many discussions, I will consider some representative instances and main trends. One trend in German metaphysics of the nineteenth century, represented by J. F. Herbart and R. H. Lotze, was to view the universe as constituted of simple substances called "reals." This was a return to a Leibnizian (and early Kantian) metaphysics. The positions of these two authors differed in their conformity to Leibniz's actual doctrines and offered opposing evaluations of Kant's theory of space. Herbart chastised Kant's argument for the ideality of space as based on explaining the possibility of geometrical cognition, and contended that metaphysics must come before geometry. In his metaphysics, he posited reals as simple substances in causal interaction. He considered these causal interactions to provide the basis for an intelligible space (graspable independently of the senses), and also as the basis for sensory space. The capacity for spatial representation arises from experience (though no one, including Herbart, ever successfully explained how), by contrast with what he took to be Kant's nativism about space and time as *a priori* forms. Geometry takes as its object the continuous spaces abstracted from experience and certified by metaphysics.[29] By contrast, Lotze posited reals as simple substances that do not interact, accepted Kant's conclusion of the ideality of space, but provided his own arguments for that conclusion, stemming from the unreality of external relations and the subsequent need to see space as arising from the mind's imposition of relations onto its representation of the reals. He accorded geometry its own authority, independent of metaphysics.[30]

Many German sensory physiologists and psychologists in the nineteenth century viewed Kant's doctrine of space as a psychological thesis about the innateness of spatial perception. They then lined up in support or opposition to Kant's (alleged) nativism. In the first part of the nineteenth century, Johann Georg Steinbuch developed a radically empiricist theory of sensory perception, according to which even the bare capacity for spatial representation is acquired through experience (involving ideas arising from muscular activity, a theory developed cleverly but in the end not convincingly).[31] A few years later, Caspar Theobald Tourtual argued in favor of a Kantian nativism as regards spatial representation itself and the localization of objects in space through sensory perception.[32] Both Steinbuch and Tourtual rejected Kant's transcendental idealism, and asserted that spatial perception reveals the real spatial properties of physical objects as they are in themselves. A third sensory physiologist, Johannes Müller, developed a nativistic position distinct from Tourtual's. Tourtual had considered himself to be true to the Kantian form–matter distinction in treating sensations as nonspatial and positing an ordering activity of the mind that innately places this matter into spatial order.[33] Müller considered the sensations of vision to be spatial from the beginning. In effect, he posited that the retina feels its own spatiality, so that the spatiality of sensations is based on the spatiality of the human body (considered as a thing in itself). Müller explicitly rejected Kant's theory that geometry requires an *a priori* basis, and contended, with Herbart, that geometry could achieve necessity even while working by abstraction from an empirically based representation of space.[34]

The single most important event for the evaluation of Kant's theory of space was the discovery of non-Euclidean geometries in the nineteenth century and the subsequent conclusion that physical space-time is non-Euclidean in the twentieth. Kant had contended not merely that the space of experience is Euclidean and grounds Euclidean demonstrations, but that (owing to transcendental idealism) we can therefore know *a priori* that physical space and physical objects are described by Euclid's geometry with apodictic certainty. In Kant's view, Euclid's description of spatial structure provides universal and necessary principles of the structure of physical space and physical objects. Generations of scientists and philosophers, including Hermann Helmholtz and Rudolf Carnap, challenged Kant's position. Helmholtz argued that the existence of non-Euclidean geometries, and the fact that we might make measurements that, given certain assumptions, would yield the conclusion that space is non-Euclidean, refuted Kant's claim that Euclid's geometry necessarily describes physical space.[35] The question of the structure of physical space then becomes a matter of empirical investigation. Even if each geometry were found to be a deductive system with its own internal necessity, the question of the fit between a given geometrical structure and the physical structure of the world would be empirical. Henri Poincaré later contended that the choice of geometry was conventional: one might choose always to posit an Euclidean space, and revise mechanics in the light of that choice (a position suggested by Lotze).[36] But as Carnap observed, even in that case the very possibility of choosing a non-Euclidean convention refutes the Kantian claim to necessity. Moreover, Einstein decided in favor of a realistic, not conventionalist, claim about the structure of physical space-time, according to which it is non-Euclidean. That conclusion directly contradicts Kant's claims about physical space and time.[37]

In the twentieth century there was some tendency among Anglo-American analytic commentators to seek to defend Kant's theory of

geometry from refutation by advances in mathematics and physics. One strategy is to suggest that Kant's position was not refuted, because the view that geometry intrinsically describes physical space differs in conception from the internally consistent but abstract mathematical geometries of the nineteenth century.[38] It is true that the understanding of geometry as a mathematical discipline changed in the nineteenth century, and that a new distinction arose between abstract mathematics and its application to nature. Further, it may be granted that Kant was a good expositor of the role of spatiality in the geometrical demonstrations of his day. Nonetheless, he did assert that Euclid's geometry necessarily describes physical space, and that is wrong.

Another strategy is to retreat to the claim that Kant could be right about Euclid's geometry applying to our own subjective spatial representations, including our "visual space." P. F. Strawson, in particular, has sought a notion of "phenomenal geometry" to which Kant's theory might apply. Strawson would abandon Kant's theory that Euclid's geometry necessarily describes physical space, but retain it for phenomenal space. Strawson, however, gives up on the idea that this "phenomenal geometry" describes the phenomenal space of visual perception – which in any case may not be a standard Euclidean space.[39] There remains very little for Kant to be right about, as regards the necessary relation of Euclid's geometry to any aspect of our experience. In the end, we are better off acknowledging the insightfulness of Kant's philosophical reconstruction of the actual Euclidean proof procedures, while allowing that Kant's theory that physical (or visual) space is necessarily Euclidean should be abandoned.

NOTES

1. Scaled to its relative size within the first *Critique*, the Aesthetic received exceptional attention in the older, monumental commentaries: H. Vaihinger, *Commentar zu Kants Kritik der reinen Vernunft*, 2 vols., incomplete (Stuttgart: Union deutsche Verlagsgesellschaft, 1881–1892), N. Kemp Smith, *A Commentary to Kant's "Critique of Pure Reason"*, 2d edn. (London: Macmillan, 1923), H. J. Paton, *Kant's Theory of Experience*, 2 vols. (New York: Macmillan, 1936). Most recently: Lorne Falkenstein, *Kant's Intuitionism: A Commentary on the Transcendental Aesthetic* (Toronto: University of Toronto Press, 1995); also Arthur Melnick, *Space, Time, and Thought in Kant* (Dordrecht: Kluwer Academic Publishers, 1989).

2. René Descartes, *Principles of Philosophy*, tr. V. R. Miller and R. P. Miller (Dordrecht: Kluwer, 1983), Pt. 2, arts. 1–25. On the mechanical philosophy more generally, see R. S. Westfall, *Construction of Modern Science: Mechanisms and Mechanics* (Cambridge: Cambridge University Press, 1971), chap. 2, and Gary Hatfield, "Metaphysics and the New Science," in D. Lindberg and R. Westman, eds., *Reappraisals of the Scientific Revolution* (Cambridge: Cambridge University Press, 1990), pp. 93–166.
3. Descartes, *Meditations on First Philosophy*, tr. J. Cottingham (Cambridge: Cambridge University Press, 1996), Fifth Meditation.
4. Descartes, *Principles*, Pt. 1, arts. 63–74. On Descartes' theory of the senses, see Hatfield, "Descartes' Physiology and Its Relation to His Psychology," in J. Cottingham, ed., *Cambridge Companion to Descartes* (Cambridge: Cambridge University Press, 1992), pp. 335–70; on his theory of intellect in relation to the senses, see Hatfield, "The Senses and the Fleshless Eye: The *Meditations* as Cognitive Exercises," in A. Rorty, ed., *Articles on Descartes' Meditations* (Berkeley: University of California Press, 1986), pp. 45–79.
5. On Newton's atomism, see Alan E. Shapiro, "Newton's optics and atomism," in I. B. Cohen and George E. Smith, eds., *Cambridge Companion to Newton* (Cambridge: Cambridge University Press, 2002), pp. 227–55, on pp. 245–51; on space and time, see Robert DiSalle, "Newton's philosophical analysis of space and time," in the same volume, pp. 33–56.
6. G. W. Leibniz and S. Clarke, *Leibniz–Clarke Correspondence*, tr. H. G. Alexander (Manchester: Manchester University Press, 1956), pp. 16–17, 25, 66, 68.
7. Among Leibniz's works discussing these positions and available in the eighteenth century, the best known is the *Monadology*, which, along with many other writings, is translated in G. W. Leibniz, *Philosophical Papers and Letters*, 2d edn., tr. L. E. Loemker (Dordrecht: Reidel, 1969). Phenomenalism about matter, a doctrine Leibniz held from the time of his correspondence with Arnauld (1686), is only implicit in the *Monadology*; the doctrine was published in 1698 and 1720 (*Philosophical Papers*, pp. 496, 623). Leibniz's more limited position positing a pre-established harmony between mind and body was published in 1695, in subsequent defenses, and in the *Leibniz–Clarke Correspondence*, pp. 18, 41, 84.
8. *Leibniz–Clarke Correspondence*, p. 12. If bodies are phenomena, the ideality of space arises among phenomenal material entities. On pre-established harmony, see the previous note.
9. Leibniz, *New Essays on Human Understanding*, tr. P. Remnant and J. Bennett (Cambridge: Cambridge University Press, 1981), pp. 132–3, 382–3, 403–4 (originally published in French in 1765). If bodies are well-founded phenomena, then these causal hypotheses will be descriptions of fine-grained relations among actual or possible bodily phenomena, in accordance with a mechanistic approach.

10. On Aristotelian abstraction and intelligible matter, see Hatfield, "Metaphysics and the New Science," pp. 98–9. On Descartes' challenge, see the Fifth Meditation.
11. Leibniz, *New Essays*, pp. 49–50; also, p. 74.
12. Leibniz, *New Essays*, pp. 137, 266; see also his *Philosophical Papers*, pp. 248–58. See also Shabel's chapter in this volume.
13. On extension, C. Wolff, *Cosmologia generalis*, new edn. (Frankfurt am Main: Renger, 1737), §§184, 221–30. On cause, Wolff, *Philosophia prima, sive ontologia*, new edn. (Frankfurt am Main: Renger, 1736), §§713–14, 880–2. On pre-established harmony as merely the best hypothesis, Wolff, *Psychologia rationalis*, new edn. (Frankfurt am Main: Renger, 1740), §§638–9. (All cited works are reprinted in C. Wolff, *Gesammelte Werke* [Hildesheim: Olms, 1962].)
14. A. G. Baumgarten, *Metaphysica*, 7th edn. (Halle: Hemmerde, 1779), §463, 762.
15. Wolff called space a "phenomenon," which he took to imply that it represents the properties and relations of substances in a confused manner (*Cosmologia*, §224–6). Wolff's term does not carry the implications of Leibniz's "well-founded phenomena." In Leibniz's phenomenalist position, spatial representations are not founded upon actual external relations (as in Wolff), but on other perceptions representing a common (intentional) spatial world.
16. C. A. Crusius, *Entwurf der nothwendigen Vernunft-Wahrheiten* (Leipzig: J. F. Gleditsch, 1745), §§48–50.
17. Crusius, *Vernunft-Wahrheiten*, §§48–53, 253, 351–6.
18. Crusius, *Vernunft-Wahrheiten*, §§52, 108–19, 252–3, 351–6, 440.
19. Crusius, *Vernunft-Wahrheiten*, §§1, 114–19.
20. Crusius, *Weg zur Gewissheit und Zuverlässigkeit der menschlichen Erkenntniss* (Leipzig: J. F. Gleditsch, 1747), §§5–10.
21. J. N. Tetens, *Philosophische Versuche über die menschliche Natur und ihre Entwicklung*, 2 vols. (Leipzig: Weidmann, 1777), pp. 431–59. On early modern theories of vision prior to Kant, see Hatfield, *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz* (Cambridge, MA: MIT Press, 1990), chap. 2.
22. Citations to Kant's works are explained in the frontmatter to this volume. I have used the translations of the various volumes of the Cambridge Edition, except for passages appearing in *Prolegomena to Any Future Metaphysics with Selections from the Critique of Pure Reason*, tr. G. Hatfield, rev. edn. (Cambridge: Cambridge University Press, 2004).

23. Kant distinguished his position from "physical influence" (*influxus physicus*), pre-established harmony, and occasionalism. Like the true Leibnizian doctrine of pre-established harmony (applied to all monads), he grounded mutually harmonious states of individual substances in the divine understanding. But unlike the Leibnizian position, he considered this harmony to arise "by means of efficient causes" and through a "universal interaction of substances" (1:415). He distinguished it from a physical influence arising from the individual substances themselves by arguing that his substances depend for their interaction on "the connection, by means of which they are linked together in the ideas entertained by the Infinite Being" (1:415). God decrees an order of efficient causal relations among the substances, establishing their relations of mutual dependency. Left to themselves, they could not interact with other things nor be in relation to them.
24. Crusius, *Vernunft-Wahrheiten*, §§350, 359.
25. For further discussion of this answer, see Martin Gardner, *Ambidextrous Universe* (New York: Basic Books, 1964).
26. As Falkenstein observes (*Kant's Intuitionism*, p. 394, n. 10), the fact that Kant contrasts intuitions with concepts (here and in the Aesthetic) does not mean that he denies we have concepts of space and time; rather, his point is that our spatial and temporal representations cannot be accounted for by general or abstract concepts; rather, our "original" and primary representations of space and time are intuitions (B 40).
27. Commentators have long suspected that Kant overlooked a "neglected alternative," that the space of intuition is *a priori* but that things in themselves are also spatial and in fact conform to the space of intuition. This alternative would be excluded by the present argument, unless one wanted to distinguish (in the neglected alternative) between physical objects and things in themselves while still ascribing spatiality to the latter. But then this alternative would rightly be neglected as arbitrary foot stomping. See also Paul Guyer, *Kant and the Claims of Knowledge* (Cambridge: Cambridge University Press, 1987), pp. 362–9.
28. These positions are reviewed (with citations) in Falkenstein, *Kant's Intuitionism*, chap. 2 (he favors the third one). On this topic, see also Melnick, *Space, Time, and Thought in Kant*.
29. J. F. Herbart, *Lehrbuch zur Einleitung in die Philosophie*, 4th edn. (Königsberg: Unzer, 1837), secs. 137, 157, 160; *Allgemeine Metaphysik* (Königsberg: Unzer, 1828–1829), secs. 142–3, 243–4, 258–60.
30. R. H. Lotze, *Metaphysic*, tr. B. Bosanquet, 2 vols. (Oxford: Clarendon, 1887), Bk. 2, chaps. 1–2.
31. J. G. Steinbuch, *Beytrag zur Physiologie der Sinne* (Nurnberg: Schragg, 1811).

32. C. T. Tourtual, *Die Sinne des Menschen* (Münster: Regensberg, 1827).
33. Tourtual, *Sinne des Menschen*, pp. 175–86.
34. J. Müller, *Elements of Physiology*, tr. W. Baly, 2 vols. (London: Taylor and Walton, 1838–1842); see Hatfield, *Natural and the Normative*, pp. 152–6.
35. H. Helmholtz, "Origin and Meaning of Geometrical Axioms," *Mind* 1 (1876), pp. 301–21.
36. H. Poincaré, *Science and Hypothesis*, tr. G. B. Halstel (New York: Dover, 1952), chaps. 3–5. Lotze, *Metaphysic*, 1:293.
37. R. Carnap, *Philosophical Foundations of Physics* (New York: Basic, 1966), chaps. 15–18.
38. D. P. Dryer, *Kant's Solution for Verification in Metaphysics* (London: Allen & Unwin, 1966), pp. 160–9.
39. P. F. Strawson, *Bounds of Sense* (London: Methuen, 1966), pt. 5. On the geometry of visual space as not standardly Euclidean, see Gary Hatfield, "Representation and Constraints: The Inverse Problem and the Structure of Visual Space," *Acta Psychologica* 114 (2003), pp. 355–78.