The Intergenerational Conflict over the Provision of Public Education*

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Abstract

We study the intergenerational conflict over the provision of public education. This conflict arises because older households without children have weaker incentives to support the provision of high quality educational services in a community than younger households with school-age children. We develop an overlapping generations model for households in a system of multiple jurisdictions. This model captures the differences in preferred policies over the life-cycle. We show that the observed inequality in educational policies across communities is not only the outcome of stratification by income, but is also determined by the stratification by age and a political process that is dominated by older voters in many urban communities with low quality of educational services. The mobility of older households creates a positive fiscal externality since it creates a larger tax base per student. This positive tax externality can dominate the negative effects that arise because older households tend to vote for lower educational expenditures. As a consequence sorting by age can reduce the inequality in educational outcomes that is driven by income sorting.

JEL classification: D72, D91, H31, R12

1 Introduction

A fundamental premise of modeling competition among local jurisdictions is that households make their location decisions taking account of the quality and tax costs of local public goods and services.[1] Most recent research has focused on the provision of local public education. Since the demand for public education and the willingness to support high quality education at the ballot box is at least partially determined by income, households with higher income tend to locate in communities with higher expenditures and housing prices. This stratification of households by income is one important causal mechanism that supports large differences in the quality of public education provided by communities and school districts within many metropolitan areas in the U.S.[2] However, it is not the only mechanism.

The same logic that suggests households sort based on income implies that households will sort based on age since preferences for local public education are largely determined by the presence of school-age children. A household's consumption of local public education begins when its first child enters kindergarten and ends when its last child leaves high school. Younger households with school-age children have strong incentives to vote for high levels of educational expenditures. In contrast, older households with children that have graduated from high school have preferences for lower tax and spending policies. These differences in preferences over preferred education policies give rise to a generational conflict in local public good provision that is played out at the ballot box within many communities in the U.S. This generational conflict may be more important in understanding persistent differences in access to educational opportunities than sorting by income.

Mobility of households not only affects the identity of the pivotal voter, but also the

[1] This hypothesis, first proposed by Tiebout (1956), has been the subject of extensive formal modeling and empirical analysis.

[2] Much recent empirical work has focused directly on the extent to which households stratify based on differences in the quality of local public goods. See, for example, Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Bayer, McMillan, and Reuben (2004), Bajari and Kahn (2004), Sieg, Smith, Banzhaf, and Walsh (2004), Urquiola (2005), Ferreyra (2007), and Ferreira (2009).

magnitude of the tax bases in each community. Older households not only vote at the ballot box, but also vote with their feet by relocating to communities that better fit their needs. These incentives to relocate must be balanced against potential moving costs. The mobility of young and old households then implies that the age composition of the voting population, the identity of the decisive voters, and the tax base are endogenous in each community. Communities with low educational spending will tend to attract a disproportionate share of older households that prefer low property taxes and relatively inexpensive housing. As a consequence, older households tend to be in the majority in poorer, urban communities with low educational expenditures. Younger households with children dominate at the ballot box in suburban communities with high levels of expenditures. The inequality in educational policies is, therefore, not only the outcome of stratification by income, but is influenced by stratification by age and a political process that is dominated by older voters in many urban communities with low quality of educational services.

But, in contrast to sorting by income, the effects of sorting by age on inequality in educational outcomes is not obvious. While older households tend to vote for lower expenditures, they also provide a positive fiscal externality since they increase the tax base in most school districts while not adding to the cost of providing education. If this positive tax externality dominates the negative effects that arise because older households tend to vote for lower educational expenditures, then sorting by age can partially off-set the inequality in educational outcomes that is driven by income sorting.

The main contribution of this paper is then that we combine an overlapping generations model with a multiple jurisdictions model in a tractable way to study the generational conflict in local public good provision.[3] Our model captures four important

[3]Benabou (1996,2002) and Fernandez and Rogerson (1998) innovate by introducing an overlapping generation approach to study fiscal competition. In their models, young individuals do not make any decisions. Hence, there is no generational conflict in voting over expenditures. An alternative interpretation of their models is that households live one period, but have altruistic preferences. Households care about local educational expenditure since they affect the human capital accumulation of the next generation. In our model, there are two generations that vote and make locational decisions at the same time. This is necessary to capture the conflict over public good provision that arises in this structure.

dimensions by which households differ: income, moving cost, age, and family structure. Income is clearly a key factor influencing a household's ability and willingness to pay the housing price premium to live in a community with high quality public services. Moving costs, both financial and psychic, are important factors in the decision process. In addition to transactions costs, relocation often entails costs associated with moving away from friends, neighbors, and familiar surroundings and the associated costs of becoming established in a new neighborhood. While financial costs will typically be roughly proportional to house value, psychic costs are likely to exhibit greater variation across households. Finally, our model also captures the fact that relocation incentives vary over the life cycle. These incentives are largely driven by the presence or absence of children at home at various points during the life cycle.

In our model, adults live for two periods and thus can live in at most two different locations. We define the stationary equilibrium of our model. Our modeling approach allows us to characterize important properties of an equilibrium without relying on functional form assumptions or numerical analysis. One important property of residential sorting is that many community pairs that could be chosen over the life cycle are strictly dominated by other pairs in equilibrium. Restricting our attention to community pairs in the relevant choice set, we can order lifetime community-choice plans by a composite local public good measure and can provide conditions that guarantee that households stratify across these plans by wealth conditional on moving costs in equilibrium. Old households have weak incentives to move to a community that has higher levels of public good provision than the community chosen when young. We show that this conjecture is correct if the relative weight placed on the local public good is higher when young than when old. Older households that relocate do not "move up." This equilibrium property then generates a household sorting pattern in which older households tend to be in the majority in communities with lower expenditures and younger households are in the majority in communities with higher levels of expenditures. Households will thus not only sort by wealth, but also by age. Based on the characterization of household sorting by wealth and age, we can then characterize the intergenerational conflict that arises at the ballot box. We show that the the median-income voter will almost never

be the decisive voter in any community in our model as communities adopt different tax and expenditure policies in equilibrium.

Based on our theoretical characterization of equilibrium we can also develop an algorithm that can be used to show that equilibria exist for reasonable parameterizations. Moreover, we can gain additional insights into the quantitative properties of our model. We compute equilibria in which a reasonable fraction of households relocates to a different community when old in equilibrium. This property of equilibrium is consistent with evidence on turnover in local housing markets. This finding has important implications for the political decisions made in the communities. We find that older households that move in equilibrium tend to have higher levels of lifetime wealth. As a consequence, the mobility of older households creates a positive fiscal externality since it creates a larger tax base per student in lower quality communities. This positive fiscal externality can dominate the negative effects that arise because older households tend to vote for lower educational expenditures. Lower mobility costs tend to increase the importance of the fiscal externality increasing expenditures in poor communities and lowering expenditure in richer communities. In contrast to sorting by income, sorting by age then leads to a decrease in inequality in educational outcomes.

Recent empirical evidence supports the elements of our model. Harris, Evans, and Schwab (2001) use district level educational expenditure data, while controlling for Tiebout bias, and provide evidence that the elderly have relatively weak support for expenditure. Farnham and Sevak (2006) provide evidence of Tiebout sorting driven by empty-nest status, and also find that the extent of such sorting is reduced by school finance equalization policies and other factors. Fletcher and Kenny (2008) find support for median-voter choice of local schooling expenditure, with the best fit having the elderly opposing increased expenditures. Brunner and Ross (2010) use voter behavior data from two referenda in California that would change the super-majority needed to pass local school bond measures, with the second referendum passing, and find evidence of elderly opposition to schooling expenditures. Reback (2010) provides additional evidence of weak preferences for expenditure among the elderly, including those 55 to 64 (in contrast to earlier research), while taking account of "circuit breaker policies" that

reduce tax costs to those 65 and up.

The rest of the paper is organized as follows. Section 2 develops our theoretical model and defines equilibrium. Section 3 establishes key properties of our equilibrium. Section 4 introduce a parametrized version of our model and examines the quantitative properties of our model. Section 5 offers conclusions.

2 An OLG Model with Multiple Jurisdictions

We develop an overlapping generations model with multiple jurisdictions to study the generational conflict over the provision of education.[4]

Consider a closed economy in which activity occurs at discrete points of time, t = 1,2, ... The economy consists of J communities. At each point of time, each community provides a local public good g, which is financed by a property tax with rate denoted τ.[5] Each community has a fixed supply of land, and thus a supply of housing services that is not perfectly elastic.[6] Let p^h_{jt} denote the net rental price of a unit of housing services in community j in period t. We assume competitive housing markets and:

Assumption 1 *Housing is owned by absentee landlords. Housing supply in community j is stationary and given by $H^j_s(p^h_{jt})$, a non-decreasing continuous function.*

Let p = (1+τ)p^h denote the gross-of-tax rental price of a unit of housing services.

[4]Our theoretical model builds on previous work by Ellickson (1973), Westhoff (1977), Epple, Filimon, and Romer (1984), Goodspeed (1989), Epple and Romer (1991), Nechyba (1997), Fernandez and Rogerson (1996), Benabou (1996a, 1996b), Durlauf (1996), Fernandez and Rogerson (1998), Glomm and Lagunoff (1999), Henderson and Thisse (2001), Benabou (2002), Rothstein (2006) and Ortalo-Magne and Rady (2006).

[5]We suppress time and community subscripts when obvious. Subscripts have the obvious ranges unless we state otherwise.

[6]For example, suppose that housing units are produced by combining land with an elastically supplied factor according to a Cobb-Douglas production function. Then a constant elasticity housing supply function is implied that shifts right with the community's land endowment. See Epple and Romer (1991).

There is a continuum of individuals each of whom lives for three periods, one period as a child and two periods as an adult. Thus at each point of time the economy consists of three overlapping generations, denoted child (c), young adult (y), and old adult (o). Each young household has one child who lives at home and attends public school, with per child expenditure g.[7] Children become young adults and move away from home at the same time that young adults transition to old age. Hence, there are no children in old households. Each young adult is characterized by a lifetime wealth denoted by w and achievement of the household's child, denoted a. The achievement of the child, a(g), is determined by g in the household's community.

Assumption 2 *We assume that achievement $a(g)$ is increasing in g.*

When young, households have additive time-separable utility. The period utility when young is defined over the quantity of housing services h, child achievement, and young-aged numeraire consumption b. The period utility when old is defined over housing services consumed, the local public good, and old-aged numeraire consumption. Period utility when young is denoted U^y (b,h,a(g)), and U^o(b,h,g) for an old household. For notational convenience, we write $U^y(b,h,g) \equiv U^y(b,h,a(g))$.

Assumption 3 *The current period utility of a young household $U^y(b,h,g)$ and the utility function of an old household $U^o(b,h,g)$ are increasing, twice differentiable, and concave in (b,h,g). Lifetime utility of a young household is given by: $U^y(b,h,g) + \beta U^o(b,h,g)$, for common discount factor β.*

The period utilities of young and old households differ due to the presence or absence of a child, and we impose additional structure on these later. A key feature of our model is that old households have weaker preferences for public expenditures than younger households. In the context of education spending, one can justify weak preferences of old households by appealing to altruism. Moreover, older households care about the

[7] Since we assume each generation has the same mass, we implicitly assume single parents. We could also assume two parent households that have identical twins. Variation in ability and age of children in a household would add considerable complications to the model.

spill-over effects that arise from educational expenditures. These include lower crime levels and more attractive neighbors in the community

Households choose a community when young and may relocate to another as they enter old age. Moving between communities requires expenditure from wealth on mobility costs, m. We assume that households have heterogeneous mobility costs.

Assumption 4 *The joint distribution of lifetime income and mobility costs at time t, denoted by $F_t(w, m)$, is continuous with support $S = R^2_+$ and joint density $f_t(w, m)$ with $f_t(\cdot)$ everywhere positive on its support.*

Each period of adult life, a household will establish a community of residence, rent housing, vote on the community property tax, and consume. The precise timing of choices and household beliefs are specified below. The level of g in a community each period must satisfy community government budget balance, for majority choice of the property tax rate. Letting n^y_{jt} denote the mass of young households that live in community j in period t, we assume:

Assumption 5 *Local government balance prevails each period in community j.*

$$\tau_{jt} p^h_{jt} H^s_j(p^h_{jt}) \;=\; g_{jt} n^y_{jt}. \tag{1}$$

Assumption 6 *The timing of household choices and household beliefs, as well as the implications for determination of equilibrium community variables is given in Figure 1.*

Each period t begins with the type distribution of young adults F_t, and equilibrium unfolds in four stages. Households have rational expectations and anticipate all equilibrium values, while acting as price takers in housing markets. As households enter adulthood, they first commit to their young and old-aged community. The commitment to their old-aged community simplifies the voting problem in Stage 3, as discussed further below. In Stage 2, both young and old households rent units of housing, and the housing market clears in each community. In Stage 3, households vote on the tax rate, with the equilibrium tax rate the Condorcet winner. In Stage 4, current period

consumption is completed, and young households save for the future. The achievement levels of children in the community are established.

When households vote in Stage 3, they take as given current period housing consumption and the net housing price in their community, both already established. Old households anticipate all the effects on the implied subgames as they contemplate different taxes, specifically the level of g implied by (1), the gross housing price, and their numeraire consumption (as their level of housing consumption is already fixed). Young households are assumed to take as given the (p, g) pair on the equilibrium path in their committed future community, while otherwise anticipating equilibrium effects in the current and future period.[8] Thus, our equilibrium is not subgame perfect in that young households do not anticipate changes in (p, g) in their future community off the equilibrium path. This myopia assumption along with the assumption that their future community is committed permits us to establish the existence of voting equilibrium and characterize it. It is, of course, of interest to relax such myopia assumptions in future research.[9] Households do correctly anticipate all variables on the equilibrium path, and a young household's committed future community is the optimal community choice as they enter old age.

Consider the problem of choosing communities of a young household. Let $d^y_{jt} \in \{0, 1\}$ denote an indicator that is equal to one if a young household lives in community j at time t and zero otherwise. Similarly define $d^o_{jt} \in \{0, 1\}$ for old households. Households also determine consumption choices for housing and the composite private good numeraire. Anticipating the equilibrium values of gross housing prices and the $g's$, a

[8]While they take as given the (p, g) pair in their future community, they anticipate the effects on their future period housing and numeriare consumption when they contemplate how their lifetime utility would be impacted by changes in current-period tax rates.

[9]There are only a few studies that have analyzed voting in a dynamic model. Coate (2010) models forward looking behavior in local elections that determine zoning policies. His is able to a adopt more general approach to voting by adopting an otherwise simpler structure in which there is limited housing choice and heterogeneity, and housing prices are determined by construction costs. Krussell and Rios-Rull (1999) provide a dynamic model of taxation with forward looking voters that relies on numerical solution methods.

young household at date t with characteristics (w_t, m_t) maximizes lifetime utility:

$$\max_{d_t^y, h_t^y, b_t^y, d_{t+1}^o, h_{t+1}^o, b_{t+1}^o} \sum_{k=1}^{J} d_{kt}^y U^y(b_{kt}^y, h_{kt}^y, g_{kt}) \; + \; \beta \sum_{l=1}^{J} d_{lt+1}^o U^o(b_{lt+1}^o, h_{lt+1}^o, g_{lt+1}) \tag{2}$$

subject to the lifetime budget constraint

$$\sum_{k=1}^{J} d_{kt}^y \left(p_{kt}\, h_{kt}^y + b_{kt}^y\right) + \sum_{l=1}^{J} d_{lt+1}^o (p_{lt+1}\, h_{lt+1}^o + b_{kt+1}^o) = w_t - \sum_{k=1}^{J} \sum_{l \neq k} 1\{d_{kt}^y = d_{lt+1}^o = 1\}\; m_t \tag{3}$$

and residential constraints:

$$\begin{aligned} \sum_{k=1}^{J} d_{kt}^y &= 1 \\ \sum_{l=1}^{J} d_{lt+1}^o &= 1 \end{aligned} \tag{4}$$

where $1\{\cdot\}$ is an indicator function.[10] The last two constraints in (4) impose the requirement that the household lives in one and only one community at each point of time. Also, w_t is the present value of lifetime income, thus assuming perfect capital markets.[11] Finally, we have abstracted from discounting of future prices just for simplicity of exposition.

It is often convenient to express this decision problem using an indirect utility (or value) function. Given a household with wealth, w, moving cost, m, and community choice k when young and l when old, we can solve for the optimal demand for housing and other goods in both periods. Substituting these demand functions into the lifetime utility function yields the indirect utility function, which can be written:

$$V_{kl}^y = V(w - \delta_{kl} m, g_k, p_k, g_l, p_l) \tag{5}$$

where $\delta_{kl} = 1$ if $k \neq l$ and zero otherwise. Similarly, the indirect utility function of an old household that occupied community k when young and is occupying community l when old is:

$$V^o(w_n^o, g_l, p_l) = \max_{h_l} \; U^o(w_n^o - p_l h_l, h_l, g_l) \tag{6}$$

[10] Though all the choice variables in problem (2) are not actually chosen simultaneously, the solution variables do conform to the timing and beliefs of Assumption 6. For example, the planned value of old-aged housing consumption $h_{d,t+1}^o$ that soves (2) conforms to the equilibrium choice.

[11] Abstracting from uncertainties and liquidity constraints are obviously strong assumptions that should be relaxed in future research.

where $w_n^o = w - \delta_{kl} m - p_k h_k^y - b_k^y$.

Define the set of young households living in community j at time t as follows:[12]

$$C_{jt}^y = \left\{ (w_t, m_t) \middle| \, d_{jt}^y = 1 \right\} \tag{7}$$

The number of young households living in community j at time t is given by:[13]

$$n_{jt}^y = \int \int_{C_{jt}^y} f_t(w_t, m_t) \, dw_t \, dm_t \tag{8}$$

Similarly define the set of old households living in community j at time t as follows:

$$C_{jt}^o = \left\{ (w_{t-1}, m_{t-1}) \middle| \, d_{jt}^o = 1 \right\} \tag{9}$$

The number of old households living in community j at time t is given by:

$$n_{jt}^o = \int \int_{C_{jt}^o} f_{t-1}(w_{t-1}, m_{t-1}) \, dw_{t-1} \, dm_{t-1} \tag{10}$$

In this model all households are renters. Housing demand functions $h_j^y(\cdot)$ and $h_j^o(\cdot)$ can be derived by solving problem (2). Below we introduce subscripts t to indicate the dependence of housing demands on prices young and old households confront during their life.[14] Aggregate housing demand in community j at time t is then defined as the sum of the demand of young and old households:

$$H_{jt}^d = H_{jt}^y \; + \; H_{jt}^o \tag{11}$$

where

$$\begin{aligned} H_{jt}^y &= \int \int_{C_{jt}^y} h_{jt}^y(w_t, m_t) \; f_t(w_t, m_t) \, dw_t \, dm_t \\ H_{jt}^o &= \int \int_{C_{jt}^o} h_{jt}^o(w_{t-1}, m_{t-1}) \; f_{t-1}(w_{t-1}^y, m_{t-1}) \, dw_{t-1} \, dm_{t-1} \end{aligned}$$

[12] We can express C_{it}^y as the finite intersection of measurable sets that are defined by boundary indifference conditions. Hence C_{it}^y is measurable.

[13] Expressions (8) and (10) make use of the fact *almost every* household of a given type makes the same community choice.

[14] Housing demand when young and old solve (2) and thus depend on the vector $(p_{kt}^y, g_{kt}^y, p_{lt+1}^o, g_{lt+1}^o)$. Since these variables are predictable, we use the subscript t to indicate this dependence, thus greatly simplifying notation.

Using Assumption 5, the housing market in community j is in equilibrium at time t if:

$$H_{jt}^d = H_j^s(p_{jt}^h) \tag{12}$$

Our absentee housing ownership assumption is imposed primarily for simplicity. The alternative would be to assign property rights over land. Households would then obtain revenues from rental income. The income effects from this would be very minor. This alternative would, however, significantly complicate the public choice problem for households who happen to live where they own land.[15] We avoid the additional complexity by assuming absentee owners of land.

The property tax and thus local public good is chosen by majority vote with the described voter beliefs, subject to (1). A majority voting equilibrium is a public good level weakly preferred by at least half the community population in pairwise comparisons to all other feasible levels.[16]

We are now in a position to define formally an equilibrium for our model.

Definition 1

An equilibrium for this economy is defined as an allocation that consists of a sequence of joint distributions of wealth and moving costs, $\{F_t(w,m)\}_{t=1}^{\infty}$*, a vector of prices, taxes, and public goods denoted by* $\{p_{1t}, \tau_{1t}, g_{1t}, ..., p_{Jt}, \tau_{Jt}, g_{Jt}\}_{t=1}^{\infty}$*, consumption plans for each household type, and a distribution of households among communities,* $\{C_{1t}^y, ..., C_{Jt}^y, C_{1t}^o, ..., C_{Jt}^o\}_{t=1}^{\infty}$*, such that:*

1. *Households maximize lifetime utility and live in their preferred communities.*
2. *Housing markets clear in every community at each point of time.*
3. *Community budgets are balanced at each point of time.*
4. *There is a majority voting equilibrium in each community at each point of time.*

[15] Since households are atomistic and thus no one household affects voting equilibrium, residential choices would not be affected by land ownership.

[16] Given (1), it is equivalent to describe voting as over the local public good or property tax.

The last component of our model deals with the intergenerational income transmission process. We make the following assumption.

Assumption 7 *A child with achievement a_t, starts as a young adult with lifetime wealth w_{t+1}:*

$$\ln w_{t+1} \quad = \quad q(a_t, w_t, \epsilon_{t+1}) \tag{13}$$

where ϵ_{t+1} denotes an idiosyncratic shock. The dependence of children's income on parental income, w_t, captures intergenerational income persistence. Moreover, we assume that $q(\cdot)$ is increasing in all three elements for $w_t > 0$, but that $q(a_t, 0, \epsilon_{t+1}) = -\infty$.

A stationary equilibrium for our economy is then defined as follows.

Definition 2
A stationary equilibrium is an equilibrium that satisfies the following additional conditions:

1. *Constant prices, tax rates and levels of public good provision, i.e. for each community j, $p_{jt} = p_j$, $\tau_{jt} = \tau_j$, and $g_{jt} = g_j$ $\forall t$.*
2. *A stationary distribution of households among communities, i.e. for each community j, we have $C^o_{jt} = C^o_j$ and $C^y_{jt} = C^y_j$ $\forall t$.*
3. *A stationary distribution of household wealth and moving costs, i.e. $F_t(w, m) = F(w, m)$ $\forall t$.*

3 Properties of Equilibrium

3.1 Existence and Uniqueness of Equilibrium

An element of existence of equilibrium of the model is existence of voting equilibrium in Stage 3. We have:

Proposition 1 *Given residential commitments, a voting equilibrium exists in all communities.*

Proof of Proposition 1:

Consider a community j which is characterized by a pair of housing price and public good provision (p_{jt}, g_{jt}). Combining the equation relating net and gross housing prices, $p_{jt} = p^h_{jt}(1 + \tau_{jt})$, and the community budget constraint (1), we obtain:

$$p_{jt} = p^h_{jt} \; + \; \frac{g_{jt} n^y_{jt}}{H^s_{jt}} \tag{14}$$

Given our timing assumptions, all variables in this expression except (p_{jt}, g_{jt}) have been determined prior to voting. Thus the set of feasible alternatives yields a linear relationship between the choice of g_{jt} and the resulting gross-of-tax housing price p_{jt}.

In each community j, there are two types of voters, young and old. Given the correct beliefs of each voter about feasible alternatives in equation (14), we can characterize each voter's decision problem and thus characterize the voter's behavior.

First consider an old household that has chosen to live in community j after living in community i when young. The household's old age income is given by $w^o_{nt} = w_{t-1} - p_{it-1} h^y_{it-1} - b^y_{it-1} - \delta_{ij} m_{t-1}$. The household's budget constraint when old is given by: $w^o_{nt} = p_{jt} h^o_{jt} + b^o_{jt}$. Let h^o_{jt} be the amount of housing the household has chosen. The quantity h^o_{jt} is then fixed at the time that voting occurs. Substituting the community budget constraint that prevails at the time of voting into the voter's budget constraint, we obtain:

$$w^o_{nt} = p^h_{jt} h^o_{jt} \; + \frac{g_{jt} n^y_{jt}}{H^s_{jt}} \; h^o_{jt} \; + \; b^o_{jt} \tag{15}$$

The voter's utility function is $U^o(g_{jt}, h^o_{jt}, b^o_{jt})$. At the time of voting, all elements of the preceding budget constraint and utility function have been determined except (g_{jt}, b^o_{jt}). Quasi-concavity of the utility function (Assumption 3) and convexity of the budget constraint imply that the voter's induced preference over g_{jt} is single-peaked (Denzau and Mackay, 1976).

Next consider a young voter that lives in community j at t and plans to live in community k in $t + 1$. The development is analogous to that for old voters, and we

thus summarize briefly. At the time of voting in community j, this household will have purchased housing h^y_{jt}. The budget constraint of the young voter is then:

$$w_t = p^h_{jt} h^y_{jt} \; + \; \frac{g_{jt}\, n^y_{jt}}{H^s_{jt}}\, h^y_{jt} \; + b^y_{jt} \; + p_{kt+1} h^o_{kt+1} + b^o_{kt+1} + \delta_{jk}\, m_t \tag{16}$$

The young voters utility function is: $U^y(b^y_{jt}, h^y_{jt}, g_{jt}) \; + \; \beta U^o(b^o_{kt+1}, h^o_{kt+1}, g_{kt+1})$. At the time of voting, the community tax base, H^s_{jt}/n^y_{jt}, and the voter's housing consumption, h^y_{jt}, have been determined. The voter takes current and future prices (p^h_{jt}, p_{kt+1}) and future government provision, g_{kt+1}, as given. Quasi-concavity of the voter's utility function, $U^y + \beta U^o$, and convexity of the budget constraint then imply that induced preferences over g_{jt} are single-peaked (Slutsky, 1975).

The existence of a voting equilibrium follows from single-peakedness of preferences of all voters. Q.E.D.

In general, the pivotal voter will not be the voter with median income. Indeed, there will often be more than one household type that is pivotal. For example, a wealthy old household and a poor young household may both be pivotal, both having the same most-preferred tax rate and expenditure level. Voting equilibrium will be unique if the density of the preferred level of the public good is positive in the vicinity of its median, which we consistently find in our computations.

We do not have a general proof of existence of stationary equilibrium in the model. However, we compute stationary equilibrium "exactly" in realistically calibrated versions of the model. Computation of equilibrium entails performing numerical integration and setting tolerance levels for convergence. Thus, equilibrium is "exact," conditional on a degree of numerical accuracy. We have implemented the algorithm in GAUSS using quadrature techniques and in C using Monte Carlo integration. Both programs yield almost identical results for the set of equilibria reported in the paper.[17] Based on our computational and sensitivity analysis, we can conclude that equilibria exist and can be computed up to an arbitrary degrees of numerical accuracy.

[17] A version of the GAUSS program is available on our web site for other researchers who wish to replicate our results.

With respect to uniqueness of stationary equilibrium, there are three issues. First, as is common in multi-community models, equilibrium typically exists with communities that are ex post identical. These "non-Tiebout" equilibria are uninteresting and easily rejected empirically (see, e.g., Epple and Sieg, 1999). We analyze sorting equilibria here. Second, the non-convexities in the model associated with community choice preclude use of standard techniques to establish uniqueness of sorting equilibria. Last, the endogeneity of the income distribution in stationary equilibrium may not be unique.

While there are several sources of potential multiplicity, we find in our computational analysis that stationary (sorting) equilibria are robust. When we perturb an equilibrium that we have computed, the algorithm converges back to the original equilibrium. These computational experiments suggest that equilibrium is at least locally unique. We do not, however, have a formal proof of uniqueness of sorting equilibrium.

3.2 Equilibrium with Household Sorting

We need to show that the equilibrium of this model captures the generational conflict in voting over local public good provision. We will show that older households tend to be in the majority in communities with low quality of educational services. Young households tend to dominate in communities with higher quality of education. We derive this result in a sequence of propositions. First, we characterize residential sorting patterns. We then provide sufficient conditions to establish the result that older households only move down in the quality hierarchy if they move at all. We then consider the implication of the residential sorting patterns for the age composition of communities and the resulting voting majorities.

Upon entering adulthood, young households choose an initial and an old-age community of residence, correctly anticipating housing prices and local public good provision. Let k and l denote, respectively, the initial and old-age communities, $k, l \in \{1, 2, ..., J\}$. If $k \neq l$, then the household bears moving cost with present value of m. We adopt the convention of numbering the communities so that $g_{j+1} > g_j$. Since households correctly anticipate g's and p's, gross housing prices will also ascend with the community number.

We now place some restrictions on the form of the household utility function that greatly facilitate the analysis.

Assumption 8 *The utility function*

$$U^a(b,h,g) \;=\; u^a_g(g) \;+\; u^a(b,h),\;\; a \;\in\; \{y,o\}, \tag{17}$$

is separable and $u^a(b,h)$ *is homogeneous of degree* ψ.

Define $V^y_{kl} = V^y(g_k, g_l, p_k, p_l, \tilde{w})$ as the indirect lifetime utility of a young household choosing residential plan kl, where $\tilde{w} \;=\; w - \delta_{kl} m$ is lifetime wealth adjusted for any moving cost. Given Assumption 8, we show in Appendix A that the indirect utility of a young household can be written as:

$$V^y_{kl} = G(g_k, g_l) \;+\; \tilde{w}^{-\psi} W(p_k, p_l); \tag{18}$$

with G an increasing function of (g_k, g_l) and W a decreasing function of (p_k, p_l).

The optimal residential choice plan of young adults maximizes V^y_{kl} over (k,l) taking anticipated p's and g's as given. It is also convenient to adopt a notation in which locational choices can be characterized by a single index subscript i. Let $i \in I_{kl}$, $I_{kl} = \{kl|\, k,l \;=\; 1,2,...,J\}$, indicate a residential plan. Let $P_i \equiv -W(p_k, p_l)$ for $i = kl$, which we refer to as the composite price of residential plan i. Note that P_i is increasing in (p_k, p_l). Using this definition, we have that indirect utility from residential plan i is given by:

$$V^y_i \;=\; G_i \;-\; (w - \delta_i m)^{-\psi} P_i, \tag{19}$$

where $G_i \;\equiv\; G(g_k, g_l)$ for $i \;=\; kl$. As a final step, let $T \;\equiv\; m/w$ denote proportional moving costs and again rewrite indirect utility using type-dependent price P^T_i.

$$V^y_i \;=\; G_i \;-\; w^{-\psi} P^T_i; \tag{20}$$

where

$$P^T_i \;\equiv\; \begin{cases} P_i & \text{if } i \text{ does not move } (k = l) \\ P_i(1-T)^{-\psi} & \text{if } i \text{ moves } (k \neq l) \end{cases}. \tag{21}$$

Household type (w, T) then chooses a residential plan i to maximize V_i^y in (20) taking (G_i, P_i^T), $i \in I_{kl}$, as given.

Household choices then satisfy the following three properties:

- (P1) Indifference curves V_i^y = const. in the (G_i, P_i^T) plane are linear with slope w^ψ.
- (P2) Indifference curve satisfy single crossing, with "slope increasing in wealth (SIW)."
- (P3) $dP_i^T/dT > 0 \; for \; k \neq l$; choices with moving are effectively more expensive as m rises.

Properties (P1) – (P3) are intuitive and simply confirmed. (P1) will greatly simplify the analysis that follows. The single crossing property in (P2) means that the indifference curves defined in (P1) cross at most once, and with slopes increasing in wealth. (P2) and (P3) are keys to the character of sorting over communities over the life cycle.

With J communities, there are J^2 residential plans that could feasibly be chosen. Using properties of the choice problem, we can develop restrictions on the set of plans that are actually chosen and then develop an algorithm for mapping household types into their equilibrium residential plans. Let $B^0 \equiv \{G_i, P_i^T \big| i \in I_{kl}\}$ denote the set of bundles, corresponding to residential plans, that are feasible for households with $T = m/w$. Let H^T denote the convex hull of B^0 and let $\underline{B}^0(T)$denote the set of residential plans (G_i, P_i^T) on the lower boundary of H^T. Formally, $\underline{B}^0(T)$is defined:

$$\underline{B}^0(T) \equiv \{ (G_i, P_i^T) \in B^0 \big| \text{no distinct } (\tilde{G}_i, \tilde{P}_i^T) \in H^T \text{exist with } \tilde{G}_i \geq G_i \text{ and } \tilde{P}_i^T \leq P_i^T \} \tag{22}$$

Figure 2 shows two examples from some of our computational analysis of these concepts for a case with J = 4.

We make the following assumption:

Assumption 9 *A T exists that prohibits moving in equilibrium for all wealth types.*

We then have the following main result that characterizes the relevant choice set:

Proposition 2

Households with relative moving cost T choose in equilibrium all residential plans in $\underline{B}^0(T)$. As a consequence, we have:

(i) Non-moving residential plans chosen by households with a prohibitive T comprise the set of all non-moving residential plans chosen in equilibrium by any households.

(ii) Moving plans chosen by households with the minimum T comprise the set of all moving residential plans chosen in equilibrium by any households.

The proof of Proposition 2 and the remaining propositions are collected in an appendix.

To see informally what underlies these results refer to Figure 2. For given T, one can use (P1)-(P3) and draw indifference curves for a wealth type in the relevant panel. The household chooses the plan where the southeastern most indifference curve touches the choice set. Wealth can vary from 0 to ∞, and an indifference curve has slope w^ρ. Hence, for every residential plan in that T-types choice set, there will be a wealth type w choosing that residential plan Parts (i) and (ii) are confirmed by examining the effects of varying T on the convex hull of a T type's residential plans. In particular, bundles (G_i, P_i^T) do not vary with T for non-moving plans, but P_i^T increases with T for moving plans.

We can also show that equilibrium satisfies an "ascending bundles" property and is characterized by a conditional wealth stratification property. Let $J_e \leq J^2$ denote the number of residential plans chosen by any household.[18] Number these plans $1, 2, \ldots, J_e$ such that $G_1 < G_2 < \ldots < G_{J_e}$.

Proposition 3

(i) Ascending Bundles: Given two residential plans chosen in equilibrium by household with T satisfying $G_i > G_j$, then $P_i^T > P_j^T$.

(ii) Conditional Wealth Stratification: For given T, if $w_2 > w_1$ and household with wealth w_2 chooses plan with G_i and household with wealth w_1 chooses plan with G_j $(j \neq i)$, then $i > j$.

[18] Later we show that $J_e < J^2$ under reasonable restrictions.

These results can be confirmed using the properties of the optimal residential choice set and the indifference curves of young households.

Note that the subset of the J_e plans chosen by different T types varies. Figure 3 shows an example with $J = 4$ from our computational analysis of the equilibrium partition of young households by type (w, T) across residential plans kl. In this example, only five of the residential plans entailing moving arise in equilibrium. There are four no-moving plans and thus $J_e = 9$.

In our computational analysis below, we adopt the following lifetime utility function:

$$U \;=\; a \;+ \frac{1}{\rho}[\,\alpha_h h_k^\rho \;+\; \alpha_b b_k^\rho \;+\; \beta_g g_l^\rho \;+\; \beta_h h_l^\rho \;+\; \beta_b b_l^\rho],\;\; \rho \;<\; 0; \tag{23}$$

and the following achievement function:

$$a \;=\; \frac{\alpha_g}{\rho_a} g_k^{\rho_a}; \tag{24}$$

The specification in (23) and (24) is a variant of a CES specification that satisfies our general assumptions and is tractable while retaining substantial flexibility. If $\rho = \rho_a$, the standard CES case arises. Note that discounting of future values is impounded in the β's.

Substituting the achievement function into the utility function, we obtain:

$$U \;=\; [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} \;+\; \frac{\beta_g}{\rho} g_l^\rho] + \frac{1}{\rho}[\,\alpha_h h_k^\rho \;+\; \alpha_b b_k^\rho \;+\; \beta_h h_l^\rho \;+\; \beta_b b_l^\rho],\;\; \rho \;<\; 0; \tag{25}$$

After some manipulation one obtains indirect utility:

$$V_i^y \;=\; G_i \;-\; (w - \delta_i m)^\rho P_i; \tag{26}$$

where:

$$\begin{aligned} P_i &= -\frac{1}{\rho} z_{kl}^{-\rho} \left[\alpha_h (\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{\rho}{1-\rho}} + \alpha_b + \beta_h (\frac{\alpha_b}{\beta_h} p_l)^{-\frac{\rho}{1-\rho}} + \beta_b (\frac{\alpha_b}{\beta_b})^{-\frac{\rho}{1-\rho}} \right]; \\ z_{kl} &= [p_k (\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{1}{1-\rho}} + 1 + p_l (\frac{\alpha_b}{\beta_h} p_l)^{-\frac{1}{1-\rho}} + (\frac{\alpha_b}{\beta_b})^{-\frac{1}{1-\rho}}]; \\ G_i &= [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^\rho]. \end{aligned} \tag{27}$$

and where we have again used the notation that residential plan $i = kl$.

Keeping in mind that $\rho < 0$, one can see that all the properties of the preceding more general case are satisfied. In particular the composite public good G_i is increasing in the g's and the composite price P_i is increasing in the p's.

Restricting the relative values of the parameters of the utility function, we can provide conditions such that no household will move when old to a community with higher g.

Assumption 10 *The utility function satisfies the following parameter restrictions:*

$$\alpha_g(\alpha_h/\alpha_b)^{1/(\rho-1)} \;>\; \beta_g(\beta_h/\beta_b)^{1/(\rho-1)} \text{ and } \rho_a \geq \rho. \tag{28}$$

Hence we have the following important result that impacts the age distribution within communities.

Proposition 4
No household will choose a community with higher (p, g) pair when old than when young in a stationary equilibrium.

The willingness to pay a higher housing price to live in a community with higher g increases with the coefficient on g in the period utility function and decreases with the coefficient on housing. While the presence of children when young indicates that both $\alpha_g > \beta_g$ and $\alpha_h > \beta_h$ are to be expected, the condition of Proposition 4 implies that the relatively stronger preference for g when young outweighs the relatively stronger preference for housing so that moving to a higher (p, g) community when old would not result.

We have characterized the residential sorting patterns by age that are generated by our model. We can explore the implication of these sorting patterns for collective choices. As we have shown, one key implication of our model is that older households do not want to move up the community hierarchy after the children graduate from high school. These sorting implications suggest that older households will be in a majority in poor communities with low educational services and younger households will dominate in communities with high levels of education services. We formalize this intuition in this section.

There will be at least some mobility in equilibrium:

Proposition 5 *Consider the set of communities $j > 1$ that are chosen by young households with zero moving costs. Some households will move down from these communities in equilibrium.*

Proposition 6 then characterizes potential voting majorities in equilibrium.

Proposition 6 *The young will be in the majority in the highest g community, and the old will be in the majority in at least one lower g community, necessarily so in the lowest g community.*

Note that the first results follows since some households will move out of community J as the enter old age and no such households will move in (by Proposition 4).[19] The second result follows from the first result and that Proposition 5 implies some households will necessarily move into the lowest g community as they enter old age while none will move out (again by Proposition 4).

4 Quantitative Properties of Equilibrium

The quantitative analysis has two objectives. First, we show that equilibria of the model exist and can be computed for reasonable specifications of the model.[20] Second, we show that the model can generate equilibria that are broadly consistent with many quantitative facts that we observe in the data. In particular, the model can generate household sorting patterns by income and age among the set of communities that are broadly consistent with our empirical characterization of household sorting observed in the Boston metropolitan area. The computed equilibria are also consistent with the observed mobility patterns if we use relatively low mobility costs. These are meaningful exercises despite the fact that equilibrium may not be unique.

[19] Some zero moving cost households will choose community J while young since some of them have arbitrarily high wealth.

[20] Appendix B presents an algorithm that can be used to compute equilibria.

4.1 Parameterization and Calibration

In our quantitative analysis we restrict attention to models with four communities. To implement the algorithm, we must fully specify the model, choosing functional forms and assigning parameter values. First, we assume that the community housing supply has constant elasticity θ and is given by

$$H^s_{jt} = [p^h_{jt}]^\theta \tag{29}$$

Thus we assume that the four communities have the same housing supply function and in this sense are of "equal size." We set the supply elasticity, θ, equal to 3.[21]

We then calibrate the eight parameters of the utility function as follows. We set $\rho_a = \rho$ as explained below, leaving 7 parameters. The strategy is then to set parameters to match predictions of the baseline model to empirical estimates of expenditure shares and demand elasticities. The α's and β's are set so that equilibrium predictions approximately conform to empirical values of: (i) relative expenditure of lifetime wealth while young versus old; (ii) the housing expenditure shares while young and old; (iii) proportional expenditure on local public goods; and (iv) a constant share of expenditure on the numeraire good while young and old. Since the ordinality of utility makes one parameter free, calibrating to the latter five conditions pin down the α's and β's. We employ data from the Consumer Expenditure Survey to obtain the shares in (i) and (ii), treating the data as if it pertains to a single cohort moving through the life cycle. We take households aged 35-44 as typical of young households in our model, and households aged 65-74 as typical of old households who have relocated. Households spend 60% of lifetime wealth when young and 40% when old. Approximately 26 percent of expenditures at each life stage are for housing services.

While we have emphasized education as a key factor influencing household location choices, we include in local government expenditure the other components that potentially influence location choices in estimating (iii); specifically expenditures for public safety (police and corrections), fire, sanitation, health, transportation, debt expense, and government administration. These totaled $901.8 billion in 2004. Personal income in

[21] This is consistent with empirical evidence as discussed in detail in Epple, Gordon, and Sieg (2010).

2004 was \$9,731 billion, implying local government expenditure equal to 8.7% of income. Of this total, \$474 billion (52.5%) was for education.[22] Using this strategy, we obtain: $\alpha_h = 0.096$, $\beta_h = 0.053$, $\alpha_g = 0.075$, $\beta_g = 0.028$, $\alpha_b = 1.00$, and $\beta_b = 0.57$. We then choose $\rho = -.4$ as this yields price elasticities between -.7 and -.8 for all goods.

Our algorithm requires that we specify an initial distribution of household income. We approximate the initial income distribution using a log-normal. In 2005, U.S. mean and median incomes were \$63,344 and \$46,326. These imply that $\mu_{\ln y} = 10.743$ and $\sigma^2_{\ln y} = .626$. We treat each of the two periods of adult life in our model as "representative years." This implies that wealth equals twice annual income, $w = 2y$, and hence $\ln(w) = \ln(2)+\ln(y)$. This and the distribution of $\ln(y)$ imply $\ln(w) \sim N(11.436, .626)$. The mean and standard deviation of w are then \$112,638 and \$78,018. Calibrating wealth as twice annual income is convenient in then permitting us to interpret the equilibrium values of variables as typical annualized values for a young and an old household respectively.

Our achievement function is given by equation (24). We assume that the logarithm of wealth when an adult for a child with achievement a is given by

$$\ln w_{t+1} \;=\; \gamma_p\, a_t \;+ \gamma_w \ln w_t \;+\; \epsilon_{t+1} \tag{30}$$

where ϵ_{t+1} is normally distributed with mean μ_ϵ and variance σ^2_ϵ. To calibrate the intergenerational income transmission function, we consider the stationary equilibrium in the one community-case. In stationary equilibrium, the distribution of wealth is invariant across generations. Moreover, we require that the transmission function generates an income distribution with mean and variance reported above. This provides two moment conditions for the four parameters to be calibrated. The other two moments are obtained using the correlation of parent and child earnings and the elasticity of spending on educational outcomes. The literature suggests that the correlation of parent and child earnings is approximately .4 (Solon, 1992).

The effect of spending on educational outcomes is more difficult to establish since there is a lack of agreement in the empirical literature about the magnitude of this

[22] Sources: Statistical Abstract, 2008, Table 442. Local Governments Expenditure and Debt by State: 2004.

effect. Fernandez and Rogerson (2003) adopt a utility function that also has education spending entering the utility function is the same way as our function above. Fernandez and Rogerson (1998) review evidence regarding the elasticity of earnings with respect to education spending, concluding that the evidence suggests a range of 0 to .2. We choose an elasticity, .1, in the middle of this range.[23]

We choose parameters of the income transmission function which, in equilibrium, satisfy the four moment conditions discussed above. It is then straight-forward to show that there is closed-form solution that maps the moment conditions into the parameter estimates. We obtain the following estimates $\mu_\epsilon = 7.11$, $\sigma_\epsilon = .573$, $\gamma_p = 49.32$, and $\gamma_w = .4$.

Next, to calibrate the moving cost distribution, we consider the empirical age distributions in metropolitan areas. Figure 4 plots the ratio of old to young households as a function of median community income for the 92 municipalities in the Boston SMSA in 1980.[24] We define cohorts representative of our young and old households. For the former, we choose age 35 to 49 and, for the latter, age 55 to 69.[25] We find that the proportion of old to young households is inversely related to community income.

The plots in Figure 4 suggest a calibration of the distribution of moving cost so that our model can replicate the observed age ratios. We aggregated all communities by income into four groups with population proportions approximately equal to those in our four-community equilibrium. Next, we calculated the ratio of old to young households in each of these groups. The results are in column 2 of Table 1. One might argue that households will typically be in the age range 30 to 44 when their first child enters school. Hence, as a second calculation, we treated the young as cohort 30 to 44. The results

[23] They also review the evidence, concluding that the exponent on expenditure is in the range from 0 to -3. The value $\rho_a = -.4$ that we have chosen for the other component (ρ) of utility falls within this range.

[24] Our plot for 1980 is chosen to precede the pronounced effects of non-stationary changes arising from maturing of the baby boom generation.

[25] The metropolitan population in the former cohort is 7% larger than the metropolitan population in the latter. Since our model presumes equal cohort sizes, we increase all community populations in the 55 to 69 cohort by 7%.

are in column 3 of Table 1. It is important to note that the 30 to 44 cohort in 1980 is substantially larger than the 35 to 49 cohort, the former being heavily influenced by the baby boom generation. Thus, while we present it for completeness, the 3rd column is of questionable value for calibration of our stationary equilibrium. One might also argue that households do not contemplate relocating until their children have completed college. Hence, as a third calculation we defined ages 60 to 74 as the old cohort, with results in column 4 of Table 1.

Table 1: Cohort Ratios

1980				
1	2	3	4	5
Community	(55-69)/(35-49)	(55-69)/(30-44)	(60-74)/(35-49)	Model Prediction
1	1.128	1.025	1.155	1.178
2	1.184	1.181	1.231	1.123
3	0.926	0.956	0.910	0.989
4	0.742	0.804	0.683	0.777

To calibrate moving costs, we take moving costs as a share of income, T, to be log-normally distributed. We chose parameters of our moving cost distribution to generate an equilibrium with cohort ratios roughly in accord with those summarized in columns 2 through 4 of Table 1. With some experimentation, we settled on $\ln(T) \sim N(.00925, .0026)$ with a correlation of $\ln(T)$ and $\ln(w)$ equal to zero. This yields the cohort ratios in column 5 of Table 1.

The age ratios implied by our model are also broadly consistent with observed migration patterns in the Boston Metropolitan area. Migration data are publicly available for municipalities in New England from the year 2000 U.S. Census. We plot in Figure 5 the net inflow of individuals over the age of 50 among residents in the Boston MA as a percentage of the population for each municipality in the Boston metropolitan area. We find a strong negative correlation with median income. Poorer communities have a positive net inflows while richer communities have positive net outflows. These findings are in line with the prediction of our model and provide empirical support for our modeling approach.

4.2 Quantitative Analysis of the Intergenerational Conflict

We now provide a quantitative analysis of the intergenerational conflict over provision of local educational services. Table 2 illustrates two different equilibria with four communities. The lower panel conforms to the model that we have presented. In the upper panel we also consider an equilibrium with a foundation grant for local public good spending that is financed by a given proportional income tax. In the equilibrium with a foundation grant, the proceeds from the income tax revenues finance a constant economy-wide expenditure on g, and jurisdictional property tax revenues provide a local supplement. Since income is exogenous in this model, an income tax is equal to a lump sum tax. For simplicity we assume that an income tax is paid by young adult households and the tax base is total lifetime income. In 2006, state and local government revenues for primary and secondary education were approximately equal. Thus, we choose the foundation grant to equalize state and local expenditures on education. As we noted above, education expenditures are 52.5% of local expenditures. With state funding equal to half this amount in our calibrated equilibrium, we obtain a foundation grant of $2,600 per young household. All the properties of the model with no foundation grant carry over with trivial adjustments.[26]

The potential lack of uniqueness implies that comparative static exercises have to be taken with caution. When we compute the equilibrium with the $2600 foundation grant, we start with the baseline equilibrium with no foundation. We then compute a sequence of equilibria with increasing foundation grants until we are up to $2600. We find that the mapping that characterizes equilibrium allocations is continuous in that model parameter. Moreover, we cannot find any other equilibria nearby. These computations suggest that equilibrium is at least locally unique. We cannot, however, rule out that there may exist other equilibria that are not close to the original, baseline equilibrium.

Table 2 reports expenditures, tax rates, and housing prices in the stationary equilibria. First consider the equilibrium with a foundation grant. Expenditures range from

[26]Lifetime income is lowered by the required taxes and educational expenditure is adjusted up to reflect these revenues.

$4,012 in the low income community to $17,996 in the high-income community. Property tax rates range from 0.124 to 0.366.[27] This finding is consistent with the observation that households in the higher income communities prefer much higher levels of expenditures than the threshold level guaranteed by the foundation grant.

We are primarily interested in characterizing the age distribution of households within and among communities. Table 2 also reports the fraction of young and old households in each community. There are more old households in communities 1 and 2 than young households. This ratio reverses for the higher income communities 3 and 4. Households tend to downsize when old and move from communities with high expenditures to communities with low expenditures. These mobility patterns are also reflected in the average lifetime wealth of young and old households in each community. We find that old households that live in a given community have a higher wealth than young households. Old households that move to a lower community typically have a higher lifetime wealth than the household that always live in this community. As a consequence, the mobility of older households creates a positive fiscal externality since it creates a larger tax base per student in poor communities. This positive fiscal externality can at least partially off-set the negative effects that arise because older households tend to vote for lower educational expenditures.

Next we turn to a more complete characterization of the voting equilibrium. Table 2 reports the fraction of young and old households that prefer lower expenditures than the equilibrium level for each community. We find the expected strong generational divide in voting patterns. Young households typically prefer higher expenditure levels while the vast majority of older household prefer lower expenditure levels. This finding holds for all four communities and is especially pronounced for the high income communities in which almost all old households prefer lower expenditure levels.

[27] Note that these tax rates are on rents and not housing values.

Table 2: Quantitative Properties of Equilibrium

Decentralized Equilibrium with $2600 Foundation Grant: 4 Communities									
	Housing	Government		Populations		Wealth		Voting	
community	p	t	g	fraction young	fraction old	young	old	lower g young	lower g old
1	9.117	0.120	4,012	0.280	0.330	43,626	55,601	0.058	0.798
2	10.935	0.236	6,959	0.249	0.275	78,771	89,795	0.001	0.952
3	12.391	0.313	10,406	0.238	0.230	121,499	125,054	0.017	0.999
4	14.600	0.366	17,996	0.233	0.164	224,382	249,981	0.152	0.982
Decentralized Equilibrium without Foundation Grant : 4 Communities									
	Housing	Government		Populations		Wealth		Voting	
community	p	t	g	fraction young	fraction old	income young	income old	lower g young	lower g old
1	9.979	0.429	3668	0.209	0.246	38,713	49,569	0.059	0.874
2	11.828	0.422	6259	0.242	0.272	68,958	80,022	0.001	0.945
3	13.334	0.440	9297	0.261	0.258	107,833	110,407	0.006	0.999
4	15.901	0.428	17099	0.288	0.224	209,096	226,496	0.119	0.989

Comparing the equilibrium with a foundation grant with the one obtained without the foundation grant, we find that the same qualitative properties of the equilibrium carry over. However, there are also some pronounced quantitative differences. One obvious consequence of the lack of the foundation grant is that property taxes are uniformly higher since property tax revenues are being substituted for income tax revenues in all communities. On average expenditures in all communities are somewhat lower without the foundation grant. Less obvious is the finding that the high income community is larger and thus appears less "selective" under a pure property tax system (see Table 3). There is a stronger incentive to choose a richer community than under a foundation grant system since the latter system provides somewhat higher expenditures in relatively less wealthy communities.

Table 3 and Figure 3 provide some additional insights into the nature of the fiscal externalities that arise due to the mobility of older households. As we have discussed in the previous section, not all possible residential plans are used in equilibrium. For the equilibria in Table 2, there are nine residential plans that are optimal for households. These include four plans that involve no relocations and five plans with relocations. Figure 3 shows the partition of households among the equilibrium residential plans for the case with a foundation grant. No household with moving costs that exceed about 1 percent of lifetime income move as they enter old age. Figure 2 shows the associated convex hull of optimal plans for $T = 0$ and $T = .02$. Households that initially choose communities 2 and 3 either stay in these communities or move to community 1. In contrast, households from community 4 move to all three other communities. We also find that 7 % of all households find it optimal to relocate in the no foundation grant equilibrium. About 22 percent of the households that choose community 4 when young move to one of the three other communities as they enter old age. Figure 3 illustrates how the choice of residential plans varies with wealth and moving costs.

Comparing the equilibrium with a foundation grant to the equilibrium without a foundation grant, we find similar mobility patterns. We find that mobility increases slightly as we move to a foundation grant equilibrium. This is mainly because there are more households moving from communities 3 and 4 to community 1.

Table 3: Quantitative Properties of Residential Plans

Community	Community	No Foundation Grant		Foundation Grant	
Young	Old	Population	Wealth	Population	Wealth
1	1	0.208	38,713	0.280	43,626
2	1	0.002	48,353	0.001	56,096
2	2	0.240	68,975	0.248	78,801
3	1	0.006	75,596	0.011	85,502
3	3	0.255	108,606	0.228	123,195
4	1	0.031	117,577	0.039	133,258
4	2	0.030	169,183	0.026	193,621
4	3	0.003	246,919	0.003	281,539
4	4	0.224	226,496	0.165	249,981

To assess the effects of moving costs on equilibrium, we calculate equilibrium with very low moving costs (having distributional mean equal to .16 of the mean in the benchmark equilibrium). Table 4 presents the results, where we include the benchmark values for ease of comparison. We find that these changes in the distribution of moving costs have massive effects. When moving costs are very low, 37% move over the life cycle as compared to 7% in the benchmark. Examining the fractions of old and young that make up communities, we see that the old population dominate in the lowest-g community and the young dominate in the two higher-g communities. Associated with this increased relocation, the lowest-g community would increase markedly in household population relative to the benchmark from 23 % to 40 %, while the highest-g community would shrink from having 26 % to 15 % of the household population.

Most interesting is that the increased relocation would substantially lower the variability in g levels across communities, increasing public provision in the lowest-g community and lowering public provision in the highest-g community. Note that there are two effects that go in opposite directions. First, older households that move down to cheaper communities have typically weaker preferences for public goods. In Table 4 we also report the maximum income of a young and old household in each community who

Table 4: The Impact of Moving Costs: Sensitivity Analysis

Benchmark Equilibrium (no foundation grant)								
Community	g	t	p	Fraction of young	Fraction of old	voting for lower g max inc young	voting for lower g max inc old	tax base
1	3,668	.43	9.98	.208	.246	21026	87993	8549
2	6,259	.42	11.83	.242	.272	38143	165641	14818
3	9,297	.44	13.33	.260	.258	70057	288480	21085
4	17,099	.43	15.90	.289	.224	141451	574949	39898
Low Moving Cost Equilibrium								
1	6,615	.31	11.91	.241	.564	23909	98009	21404
2	8,262	.41	12.69	.245	.264	52182	215439	20280
3	9,963	.49	13.45	.244	.135	91872	376888	20173
4	14,968	.54	15.40	.270	.035	180106	732382	27528

would vote for lower expenditures than the status quo. Note that the maximum income for older households is typically four times as large as the maximum income for younger households reflecting differences in the valuations of education. The set of pivotal voters in each community thus typically consist of young households that are much poorer than the median income young household and older households that have much more wealth than the typical median old household. While the median voter theorem applies, as demonstrated in Proposition 1, the median voter is clearly not the voter with median income when there is sorting by age.

However, mobility has a second effect since it increases the tax base per student in communities that experience a net-inflow of older households and vice versa. Table 4 also reports the property tax base (average net housing expenditures) normalized by the number of students in the community. While the older households that move to lower-g

communities place less weight on g in their utility functions, they are relatively wealthy and increase the tax base per student in poorer communities. In our calibration we find that the effects of increased moving on tax bases outweigh the political economy effects. Facilitating moving of older households has an equalizing effect. Lower mobility costs tend to increase the importance of the fiscal externality yielding higher expenditures in the poor communities. This effect further draws into poorer communities somewhat more wealthy young households, reinforcing higher educational expenditures in those communities. Note that lowering mobility costs increases the young population in the two poorer communities and the reverse in the two richer communities. Our model reveals the importance of considering the general equilibrium effects of life cycle choices in assessing the generational divide in support for public educational expenditure.

5 Conclusions

Understanding the intergenerational conflict over public good provision is an important research area, and there is ample scope for future research. One interesting avenue for future research is to analyze the differences between households with and without children. The presence of households that never have children can be expected to affect the age composition of communities as well as the outcomes that arise from voting over public good levels.

Households without children present do not have strong preferences for public education, but care for a variety of other local expenditures such as police and fire expenditures or welfare and recreational expenditures. Education has the largest expenditure share of all local expenditures and typically accounts for at least 50 percent of all local expenditures on our sample of 119 Boston communities. A simple correlation analysis available upon request from the authors suggests that communities with older individuals tend to spend a larger share of resources on police, fire and other safety expenditures. Moreover, these communities also tend to spend a larger share on recreational expenditures. Future research should provide compelling models that explain the composition of expenditure types, and not just the level of expenditures within a system of jurisdictions. Allowing

for multi-dimensional voting is, however, a challenging problem.

Another important generalization is introduction of home ownership effects. Home owners with grown children may have an incentive to support high provision of education to maintain property values (Brueckner and Joo, 1991). These incentives depend on the household's beliefs about the way in which quality of public services impacts rental prices or the value of the home. Property owners have different preferences over public good provision than renters since owners are affected by capital gains or losses that may arise from changes in public policies. The key complication in such a generalization is in characterizing voting equilibrium.[28] Introducing ownership into our dynamic framework is a challenging but important task for future research.

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[28]Owner-occupants who anticipate capital gains and losses when voting have been incorporated in static models(Epple and Romer, 1991), and those investigations reveal that ownership substantially affects voter incentives and equilibrium outcomes. Coate (2011) provides a dynamic analysis of voting over zoning policies when owners take capitalization effects into consideration.

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A Additional Proofs

The indirect utility is given by:[29]

$$\begin{aligned} V^y &= \underset{h_k,h_l}{Max}\,[u_g^y(g_k) + u_g^o(g_l) + u^y(b_k,h_k) + u^o(b_l,h_l)] \\ &\quad s.t.\ p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w} \\ &= G(g_k,g_l) + \underset{h_k,h_l}{Max}\,[u^y(b_k,h_k) + u^o(b_l,h_l)] \\ &\quad s.t.\ p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w}\,; \end{aligned} \tag{31}$$

where $G(g_k,g_l) \equiv u_g^y(g_k) + u_g^o(g_l)$ is an increasing function of (g_k,g_l). Since $u^a(b,h)$ is homogeneous of degree ψ, it follows from Theorem I in (Lau, 1970) (p. 376) that the maximand in the lower line of (31) equals $\tilde{W}\left(\frac{p_k}{\tilde{w}},\frac{p_l}{\tilde{w}}\right)$, a function homogeneous of degree $-\psi$ and decreasing in its arguments . Then: $V^y = G(p_k,p_l) + \tilde{w}^{-\psi}W(p_k,p_l)$.

Proof of Proposition 2:

Households with T maximize V_i^y as defined in (20) – (21). Since V_i^y is increasing in G_i and decreasing in P_i^T, households choose among the residential plans in $\underline{B}^0(T)$. Since w ranges from 0 to ∞, the slope of an indifference curve in the (G_i, P_i^T) plane ranges from 0 to ∞ (Assumption 4) as well, implying all plans in $\underline{B}^0(T)$ are chosen by some households with T.

(i) Obviously all non-moving residential plans chosen by households with the prohibitive T are in the set of chosen residential plans by all households. To confirm that only these non-moving plans are equilibrium ones, observe from (21) that, since P_i^T is increasing in T for moving plans and independent of T for non-moving plans, lowering T can eliminate but cannot add non-moving plans to $\underline{B}^0(T)$. From the result in the previous paragraph, it follows that no households with lower T than the prohibitive T will choose an alternative non-moving plan.

(ii) Let T_m denote the minimum T. (This equals zero under assumption 4, but the result does not require this.) Obviously all moving plans chosen by such households are in the equilibrium set of moving plans. To confirm only such moving plans are in the equilibrium set of all households, suppose household "2" with (w_2, T_2), $T_2 > T_m$, chooses

[29]The discount factor β is subsumed in the old age utility function with no loss of generality.

a moving plan lk in equilibrium that is not chosen by any households with T_m. Consider household "1" with $(w_1, T_1) = (w_2 \frac{1-T_2}{1-T_m}, T_m)$. Note that $w_1 < w_2$. Households 1 and 2 obtain the same level of utility from all moving plans (by (20) – (21)). Household 1 obtains lower utility from all non-moving plans than does household 2, since household 1 has lower wealth (and moving costs are irrelevant). But then household 1 would share household 2's preference for moving plan lk, a contradiction. Q.E.D.

Proof of Proposition 3: First we show that the plan with $G = G_1$ corresponds to $lk = 11$ and the plan with $G = G_{J_e}$ corresponds to $lk = JJ$.

The residential plans on the lower boundary of the convex hull of all feasible plans corresponds to just non-moving plans for any types with T that will never move in equilibrium. Plans $lk = 11$ and $lk = JJ$ are the endpoints of the lower boundary of the convex hull for all of these types. The result then follows from Assumption 9.

(i) If $P_j^T \geq P_i^T$, then choice of plan j would contradict maximization of V^y (recall (20)).

(ii) Using that households chose residential plans to maximize V^y, wealth stratification follows from the ascending bundles property and SIW. Q.E.D.

Proof of Proposition 4: The proof is by contradiction, so suppose a household makes such a choice. Then that choice solves the program:

$$\begin{aligned} \max_{h_k, b_k, h_l, b_l} \quad & U = [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] \\ & s.t.\ w - m = p_k h_k + b_k + p_l h_l + b_l \end{aligned} \tag{32}$$

with $(p_k, g_k) < (p_l, g_l)$. Let:

$$\begin{aligned} L^* \equiv \ & [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] + \\ & \lambda[w - m - p_k h_k - b_k - p_l h_l - b_l] \end{aligned} \tag{33}$$

denote the Lagrangian function at the household's optimum, where λ denotes the multiplier on the budget constraint. Thus, $V_{kl}^y(p_k, g_k, p_l, g_l) \equiv L^*(p_k, g_k, p_l, g_l)$. Using the latter and (33), compute, respectively, slopes of the indifference curves over (p, g) pairs

while young and (p, g) pairs while old:

$$\left.\frac{dp_k}{dg_k}\right|_{V^y_{kl}=const.} = -\frac{\partial V^y_{kl}/\partial g_k}{\partial V^y_{kl}/\partial p_k} = -\frac{\partial L^*/\partial g_k}{\partial L^*/\partial p_k} = \frac{\alpha_g g_k^{\rho_a - 1}}{\lambda h_k}; \tag{34}$$

and

$$\left.\frac{dp_l}{dg_l}\right|_{V^y_{kl}=const.} = -\frac{\partial V^y_{kl}/\partial g_l}{\partial V^y_{kl}/\partial p_l} = -\frac{\partial L^*/\partial g_l}{\partial L^*/\partial p_l} = \frac{\beta_g g_l^{\rho - 1}}{\lambda h_l}; \tag{35}$$

where the last equality in each of (34) and (35) uses the Envelope Theorem. Using the first-order conditions from (33), one obtains:

$$h_k = \frac{w-m}{z_{kl}} \left(\frac{\alpha_b}{\alpha_h} p_k\right)^{1/(\rho-1)} \tag{36}$$

$$h_l = \frac{w-m}{z_{kl}} \left(\frac{\beta_b}{\beta_h} p_l\right)^{1/(\rho-1)}. \tag{37}$$

Substituting (37) into (35) and (36) into (34) and evaluating slopes at a common (p, g) point, one finds that the indifference curve over (p, g) pairs while young are everywhere steeper than the indifference curve over (p, g) pairs while old if $\alpha_g(\alpha_h/\alpha_b)^{1/(\rho-1)} g^{\rho_a - \rho} > \beta_g(\beta_h/\beta_b)^{1/(\rho-1)}$. This condition holds under Assumption 10.[30] In a stationary equilibrium, the (p, g) pairs available in each period of life are the same.

One can then use the relative slopes of these indifference curves to show that the young households choice of plan kl implies the household would prefer to stay in community k when old, a contradiction. (Contact the authors for a more detailed proof.) Q.E.D.

Proof of Proposition 5: Suppose not. Find the lowest numbered community $j > 1$ for which no households with 0 moving cost move down. We know the poorest households will choose (1,1). Using that $g_1 < g_j$ and $p_1 < p_j$ for all communities $1 < j$, from (27) one can see by inspection that P(1,1) < P(1,j) < P(j,j) and G(1,1) < G(1,j) < G(j,j), where (i,j) indicates residence plan with j chosen while young and i when old. It follows that plan (1,j) is on the lower bound of the convex hull of the plans (1,1), (1,j), and (j,j) and plan (1,j) would be preferred by some households with 0 moving costs, a

[30] Here we are presuming $g \geq 1$. In economies with realistic wealth levels, the presumption that equilibrium spending in all communities is more than a dollar per student is innocuous.

contradiction. Now find the next higher numbered plan (k,k), k > j, chosen by some 0 moving cost households and suppose no households move down from community k. By analogous argument there exists some 0 moving cost households that prefer (k,j) to both (k,k) and (j,j), and there exist some households that prefer (k,1) to (k,k) and (1,1). It follows that some 0 moving cost households would move down from k.[31] Higher yet numbered communities chosen by 0 moving cost households while young must have some downward movers by the same argument. Further, by continuity of all the relevant functions, there will exist households with arbitrarily small moving costs that move down in each case as do the 0 moving cost households (i.e., a positive measure of households will so move).

B Computation of Stationary Equilibria

Given a stationary distribution of wealth and moving costs, a stationary equilibrium in this model is determined fully by a vector $\{p_j, g_j, \tau_j\}_{j=1}^{J}$. Computing an equilibrium is, then, equivalent to finding a root to a system of $3 * J$ nonlinear equations. For each community, the three equations of interest are the housing market equilibrium in (12), the balanced budget requirement in (1), and the majority rule equilibrium requirement.

The full algorithm, therefore consists of an outer loop that searches over admissible distributions of wealth and moving costs and an inner loop that computes a stationary equilibrium holding the joint distribution fixed. The algorithm in the inner loop finds a root of $3 \times J$ dimensional system of linear equations. More specifically, the algorithm can be describes as follows:

1. Fix the joint distribution of wealth and moving costs.

2. Compute equilibrium for that distribution:

   (a) Given a vector (p_j, τ_j, g_j) we can compute p_j^h from the identity $p_j = (1+\tau_j)p_j^h$.

[31] Note that this does not imply every downward moving plan will be followed, but that at least one will. For example, if k = 3, it could be that every household that prefers (3,2) to (3,3) and (2,2) also prefers (3,1) to (3,2). Thus there may not be any households that chose (3,2).

(b) For each young household type (w, mc), we can compute the optimal residential choices for both time periods. Hence we can characterize household sorting across the J communities.

(c) Given the residential decisions, we can characterize total housing demand, as well as total government revenues for each community.

(d) Given p_j^h, we can compute housing supply for each community, and check whether the housing market clears in each community.

(e) Given g_j, we can check whether the budget in each community is balanced.

(f) For each young household and each old household living in community j determine whether the household prefers lower expenditures than the status quo and check whether g_j is a majority rule equilibrium

(g) Iterate over (p_j, τ_j, g_j) until convergence obtains.

3. Update the joint distribution of wealth and moving costs using the law of motion in (13).

4. Check for convergence of wealth and moving cost distributions.

C Figures

Figure 1: Timing of Choices and Household Beliefs

Period t: $F_t(w,m)$ is given.

	Young Households	Old Households	Communities/Markets
Stage 1:	Commit to communities when young and old, anticipating all continuation equilibrium variables.	Establish old-aged community as committed when young, and bear moving cost if moved from young community.	Community type distributions determined with respect to age and current wealth.
Stage 2:	Rent housing in young community as price taker, anticipating all continuation equilibrium values.	Rent housing as price taker, anticipating all continuation equilibrium values.	Housing markets clear, establishing net housing prices in each community.
Stage 3:	Vote for local tax rate taking as fixed equilibrium (p,g) in old-aged community, otherwise anticipating all continuation equilibrium values.	Vote for local tax rate anticipating all continuation equilibrium values.	Tax and g determined through local budget balance in each community, as well as gross housing prices.
Stage 4:	Save optimally, anticipating all continuation equilibrium values, and consume (h,g,b).	Consume (h,g,b) exhausting wealth, and then die.	Achievement of young determined in each community.

Figure 2: The Relevant Choice Set

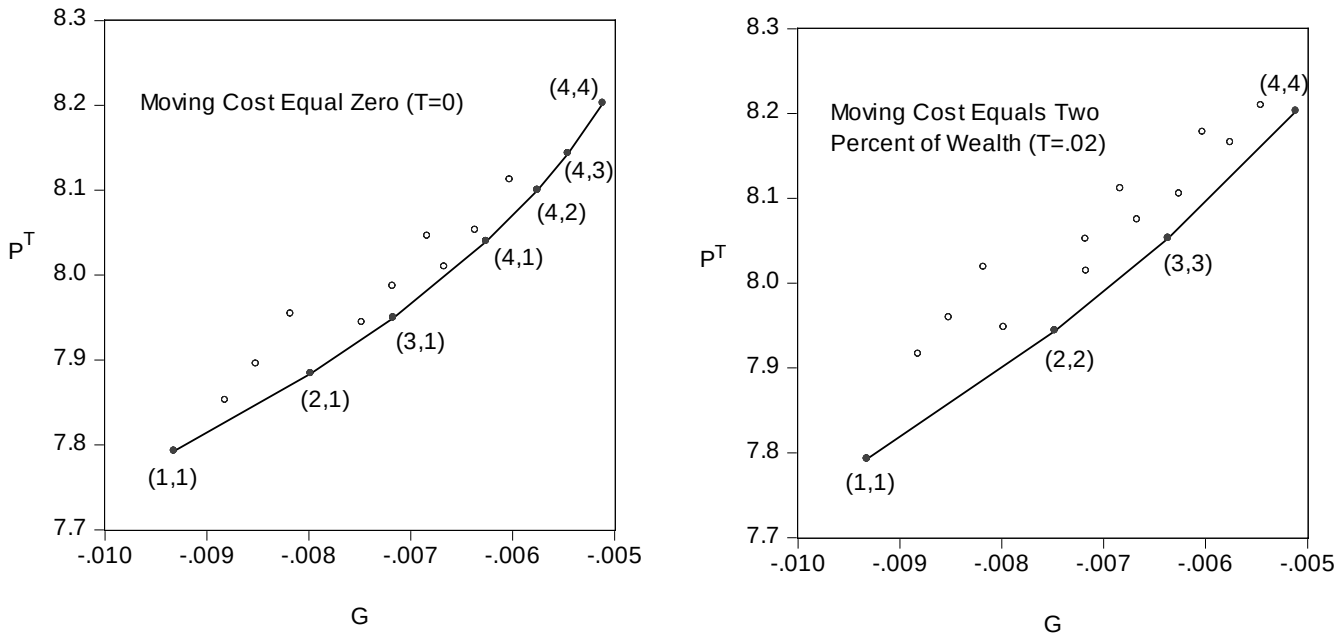

Figure 3: Residential Plans in Equilibrium

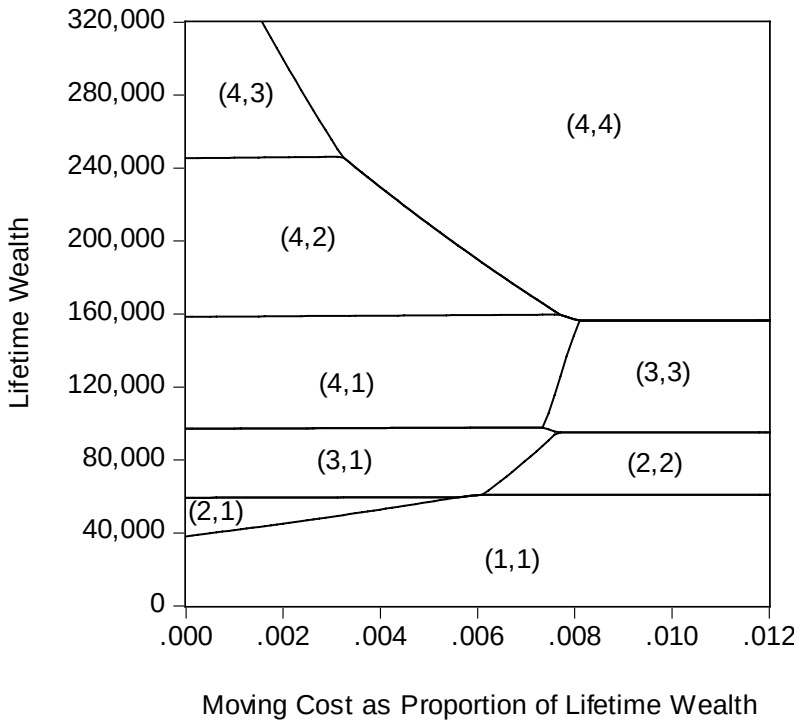

Figure 4: Ratio of Old to Young Households by Community in 1980

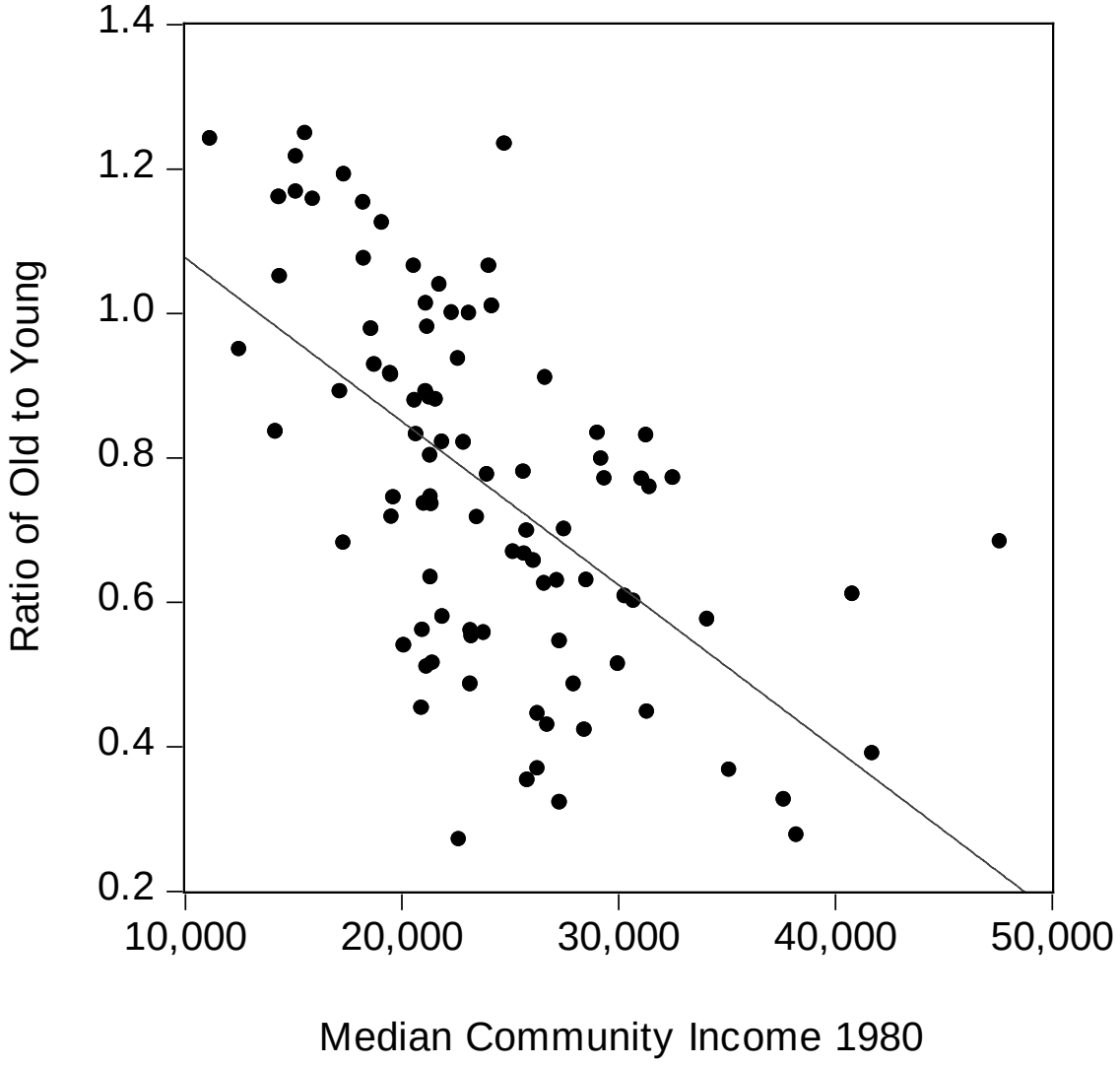

Figure 5
Domestic Inflow Less Outflow Of Individuals
Aged 50 and Over As A Proportion of Population
Municipalities in Boston SMSA in Year 2000