

Data and Programs for the Paper
‘Tempo-Adjusted Period Parity Progression
Measures, Fertility Postponement and
Completed Cohort Fertility’, *Demographic*
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Hans-Peter Kohler* José Antonio Ortega†

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Abstract

In Kohler and Ortega (2002a) [*online available at Demographic Research, Volume 6, Article 6*], we introduce a new set of tempo-adjusted period parity progression measures in order to account for two distinct implications caused by delays in childbearing: *tempo distortions* imply an underestimation of the quantum of fertility in observed period data, and the *fertility aging effect* reduces higher parity births because the respective exposure is shifted to older ages where the probability of having another child is quite low. Our measures remove the former distortion and provide means to assess the latter aging effect. The measures therefore provide an unified toolkit of fertility measures that (a) facilitate the description and analysis of past period fertility trends in terms of synthetic cohort measures, and (b) allow the projection of the timing, level and distribution of cohort fertility conditional on a postponement scenario. Due to their explicit relation to cohort behavior, these measures extend and improve the existing adjustment of the total fertility rate. We apply these methods to Sweden during 1970–1999. On this web page we provide the data and Splus programs to replicate the analyses in Kohler and Ortega (2002a). Moreover, we provide additional details about the estimation of the fitted intensity schedules and the calculation of the parity progression ratios with continued fertility postponement.

*Head of Research Group on Social Dynamics and Fertility, Max Planck Institute for Demographic Research, Doberaner Str. 114, 18057 Rostock, Germany. *Tel:* +49-381-2081-123, *Fax:* +49-381-2081-423, *email:* <mailto:kohler@demogr.mpg.de>, *www:* <http://user.demogr.mpg.de/kohler>.

†Departamento de Análisis Económico: Economía Cuantitativa, Universidad Autónoma de Madrid, 28049-Madrid, Spain, *Email:* joseantonio.ortega@uam.es, *www:* <http://www.adi.uam.es/~jaortega>.

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1 Introduction

Despite the widespread agreement about relevance of tempo distortions, there remains a substantial controversy about the appropriate way of measuring and adjusting for these effects. Bongaarts and Feeney (1998, henceforth BF) have recently proposed an adjusted total fertility rate that measures total fertility rate that would have been observed in a calendar year if there had been no change in the tempo of fertility during this year. Kohler and Philipov (2001) have extended this adjustment to include also variance effects, i.e., changes in the shape of the fertility schedule in addition to shifts in the mean age. Alternative models for adjusting the *TFR* have also been proposed (e.g., Brass 1990; Le Bras 1997), and virtually all of the above approaches are strongly influenced by the seminal work of Norman Ryder (Ryder 1964, 1983; see also the review of the literature on period fertility measures in Ortega and Kohler 2002a). In addition to the debate about the appropriate adjustment for tempo distortions, there is also a controversy about the relation between tempo-adjusted fertility measures and cohort fertility. For instance, while the

adjusted *TFR* in the Bongaarts-Feeney and Kohler-Philipov analyses provides an improved indicator of period fertility (or quantum), it cannot be used to infer the completed fertility of the current cohorts of women. This limitation is due to the fact that the completed cohort fertility depends on the future paths of fertility quantum *and* tempo. It can only be properly projected with appropriate assumptions about *both* aspects of future fertility trends. Moreover, investigating these implications of future quantum and tempo changes on cohort fertility requires an explicit consideration of the fact that only women who are currently at parity zero, one, two, etc., are exposed to giving birth to their first, second, third, etc., child. This sequencing of births is not explicitly considered in the calculation of the adjusted *TFR* or the standard *TFR*. In many circumstances, therefore, these measures provide little or even erroneous information about the parity distribution and the completed fertility of cohorts (Kim and Schoen 2000; Ortega and Kohler 2002b; van Imhoff 2001).

In order to overcome this limitation, we develop in Kohler and Ortega (2002a) [*online available at Demographic Research, Volume 6, Article 6*] tempo-adjusted period parity progression measures. These tempo-adjusted parity progression measures provide a new and unified ‘tool-kit’ that can be used for two related purposes. *First*, they remove tempo distortions and parity composition effects from the observed period fertility pattern and therefore provide an improved indicator of the period quantum of fertility. In particular, our tempo-adjusted period parity progression measures suggest a natural synthetic-cohort measure of period fertility and quantum, denoted *period fertility index*, that is equal to the total fertility of women who experience the tempo-adjusted period childbearing intensities (or adjusted age-specific parity progression probabilities) during their life-course. *Second*, our measures allow a demographically correct and consistent projection of the level, timing and distribution of the completed fertility of cohorts, who have not finished childbearing, conditional on the future paths of quantum and tempo. We derive these assumptions about future quantum and tempo from observed period fertility patterns. Our methods therefore provide an important input for population projections, and future analyses can combine these methods with stochastic forecasting models that explicitly consider the uncertainty about the future development of quantum and tempo. Moreover, the close relation between period fertility measurement and cohort projection reduce the traditional distinction between ‘cohort’ and ‘period’ approaches to fertility (e.g., Ní Bhrolcháin 1992): our methods provide proper period measures of fertility that can be combined with projection scenarios in order to assess the cohort fertility that results from period fertility patterns.

In addition, the investigation of fertility aging in our analyses is based on the comparison of two benchmark scenarios for the pace of fertility postponement during the life-course of a cohort: (a) a *postponement stops scenario* in which we calculate the parity progression measures assuming that the postponement comes to a halt after the reference year, i.e., assuming that there is no further delay of childbearing after the year for which the period PPRs are calculated (of specific importance in this context is the index of completed fertility in the postponement stops scenario, which is also denoted as *period fertility index*); (b) we contrast these calculations with a *postponement continues scenario* in which we assume that the tempo change (and also any changes in the variance of the fertility schedule) observed in a reference year

prevails over the life-course of a cohort. This postponement continues scenario thus allows us to calculate cohort fertility under the assumption that a cohort experiences a level of fertility *and* pace of fertility postponement that equals the level of fertility and change in the tempo of fertility observed in a reference year. In this approach we therefore include the postponement of fertility in the notion of synthetic cohorts: we derive fertility measures that reflect the timing, level and distribution of fertility in hypothetical cohorts that experience the fertility pattern observed in a given calendar year, where the term ‘fertility pattern’ encompasses the level, tempo *and* tempo change in a calendar year.

On this web page we provide the data and Splus programs to replicate the analyses in Kohler and Ortega (2002a, henceforth KO). Moreover, we provide additional details about the calculation of tempo-adjusted parity progression ratios and their application to the measurement of period fertility and the projection of cohort fertility.

All data and program files mentioned below (including the present html document ‘ko-ppr-programs.html’) are also available in a single *zip file*.

The data and Splus programs provided below are subject to our *disclaimer*.

This page is also available as a *ko-ppr-programs.pdf*, which is preferable if you intend to print archive the file.

2 Updates of Data and Programs

- March 1, 2001: preliminary version online
- March 1, 2002: publication of Kohler and Ortega (2002a) in *Demographic Research* and major revision of programs for the calculation of tempo-adjusted parity progression measures

3 Data for Sweden 1970–1999

The data required for the analysis include the births by age and parity in a calendar year and a measure of the person years lived by women who are ‘at risk’ of giving birth to a first, second, third, etc., child. The former information is identical to the data requirements for the adjustment of the total fertility rate. The latter information is more specific. For instance, the exposure can be estimated by the mid-year female population by age and parity. In countries with population registers it can be obtained from the exact counts of the person-years lived by age and parity during a calendar year. The primary advantage of these additional data is that they allow us to relate births of a given parity to the women who are at risk of giving birth to this parity, i.e., we can base our analyses on occurrence-exposure rates or *fertility intensities*. These fertility intensities reflect the ‘hazard’ of experiencing a next birth at a given age for women who are parity j in a calendar period.

The data the analyses in KO are obtained and updated from Andersson (1999) and include births by parity and age during 1970–1999 and estimates of the corresponding

person-years of exposure by age and parity in all calendar years from 1970–1999 (see also Andersson and Guiping 2001). These data are calculated from files with individual life stories of all Swedish women. Some observations have been censored, for instance at first emigrations, twin births, and some further events so the data do not comprise exactly all births and exposures during this period. Immigrant women are not included in the data (For a further description of the data see Andersson 1999). Due to the exclusion of the foreign-born population, the data yield a total fertility rate and related fertility measures that are slightly below published statistics for the Swedish resident population.

The data are available in tab-delimited ASCII files below. All data are in period-cohort format, that is, they trace real cohorts over time and pertain to parallelograms with vertical sides in the Lexis diagram. Age pertains to the age at the beginning of the period, and cohorts age by one year during the calendar year.

- standard age-specific fertility rates by birth order (also denoted as “incidence rates”): *birth order 1, birth order 2, birth order 3, birth order 4*.
- age-specific childbearing intensities (denoted $m_j(a, t)$ in KO): *birth order 1, birth order 2, birth order 3*; in our analyses, we combine births of order 4 and higher and treat the population of parity 3 or higher as the respective population “at risk”. The childbearing intensities for this open-ended category are calculated by our programs.
- parity distribution at mid-period: *parity 0, parity 1, parity 2, parity 3 and higher*.

4 Empirical Implementation

In the following we briefly discuss the empirical implementation and estimation of the tempo-adjusted period parity progression measures proposed in this paper. *S*-plus programs to perform these calculations are provided below.

Once the childbearing intensities, age-specific fertility rates and parity-distribution have been obtained (see previous section), the calculation of tempo-adjusted parity progression measures proceeds in two steps. First, we estimate mean- and variance changes in the schedules of period childbearing intensities for each calendar year and compute the adjusted childbearing intensities. Second, we use these tempo-adjusted childbearing intensities to calculate fertility measures for the synthetic cohort.

The first step requires that we calculate the mean and variances of the adjusted intensity schedules in all calendar years. In the presence of variance effects, the respective estimation is somewhat more complicated than in the standard BF adjustment of the total fertility rate because variance effects distort the shape of the intensity schedule (see Kohler and Philipov 2001 for a detailed discussion of the distortions in the observed fertility pattern that are caused by variance effects). The shape of the observed intensity schedule therefore differs from the adjusted intensity schedule at parity j whenever variance effects are present. In order to properly

estimate the adjusted intensity schedule along with its mean- and variance changes we therefore implement the following iterative procedure:

1. We estimate the mean age and variance of all period intensity schedules for parities $j = 1, \dots, J$, where J is the highest parity (the Splus functions below assume that $J = 3$ (i.e., births of order 4), and they can accommodate situations where J is an open-ended category that includes births of order 4 and higher. Since childbearing intensities for higher parities tend to be quite variable at young ages, it is advisable to restrict the intensities to ‘relevant’ ages (for instance, in our analyses for Sweden we have dropped the childbearing intensities for third and fourth births below the ages of 20 and 21, since third and fourth birth fertility below these ages does not contribute importantly to the overall period and cohort fertility).
2. We estimate the mean and variance changes in the observed childbearing intensities. In order to achieve a robust estimation, we use IRW methods based on state-space smoothing (García-Ferrer et al. 2001; Young 1994), where the ratio of the variance of the noise in the mean age series to the variance in the derivative (the so-called Noise Variance Ratio, NVR) is fixed.

This IRW model is specified as follows. The measurement equation states that the observed series (in our case the mean age and standard deviation of the period intensity schedule at parity j) is the sum of an unobserved trend component, T_t , and a noise component with variance σ_ε^2 :

$$Y_{qt} = T_t + \varepsilon_t \quad (1)$$

The transition equations state that the trend component changes in each period through the addition of a slope component (the derivative estimate) which changes overtime through the addition of a noise component η_t with variance σ_η^2 :

$$\begin{aligned} T_t &= T_{t-1} + S_{t-1} \\ S_t &= S_{t-1} + \eta_t \end{aligned}$$

The system depends on a single parameter, the noise variance ratio $NVR = \sigma_\varepsilon^2 / \sigma_\eta^2$, that reflects the variance of noise in the observed series Y_t relative to the noise in the underlying trend component T_t . The extraction of the trend and slope components implies the forward application of the Kalman filter algorithm and the backward application of Fixed Interval Smoothing. We implement this smoothing technique because of its desirable spectral properties: there is a relationship between the NVR and the cut-off frequency of the filter.

3. We estimate the adjusted intensity schedule at parity j on the basis of the observed childbearing intensities and the annual mean- and variance changes obtained in the previous step.
4. We re-estimate the mean- and variance change, $\gamma_j(t)$ and $\delta_j(t)$, for all calendar years based on the adjusted intensity schedules obtained in the previous step.

5. We return to step 3 and re-estimate the adjusted intensity schedule based on the new values of $\gamma_j(t)$ and $\delta_j(t)$. The iteration stops once the estimates for the mean- and variance changes converge.

Once the adjusted intensity schedules have been estimated for all calendar years, the tempo-adjusted parity progression measures proposed in this paper can be calculated by using discrete-time versions of the results in KP.

Once the data have been read into a specific KO data object (see Section 6.2 below), the analyses (including the graphics) of the Swedish fertility patterns in KO and Kohler and Ortega (2002b) is replicated by the following Splus commands that are discussed in more detail below:

```
## calculate adjusted TFR
se.data <- ko.add.tfr.adjustment(se.data)

## estimate mean and variance changes in KO
se.data <- ko.ppr.add.fitted(se.data)

## calculate tempo-adjusted period parity progression measures
se.ppr.comparison <- ko.ppr.comparison(se.data)

## cohort summary for nfx cohort data
se.cohort.summary <-
  ko.calc.cohort.summary(se.long.data,
                        first.cohort = 1930,
                        use.initial.par.dist = T)

## complete fertility of cohorts who have not completed childbearing in
## the years 1975,1999
se.complete.fert.1975 <-
  ko.ppr.complete.fert.3.scenarios(se.data,1975,
  par.dist =
    se.cohort.summary$par.dist[,se.cohort.summary$years==1975,],
  cum.nfx =
    se.cohort.summary$cum.nfx[,se.cohort.summary$years==1975,],
  cum.mab =
    se.cohort.summary$cum.mab[,se.cohort.summary$years==1975,]
  )
se.complete.fert.1999 <-
  ko.ppr.complete.fert.3.scenarios(se.data,1999,
  par.dist =
    se.cohort.summary$par.dist[,se.cohort.summary$years==1999,],
  cum.nfx =
    se.cohort.summary$cum.nfx[,se.cohort.summary$years==1999,],
  cum.mab =
    se.cohort.summary$cum.mab[,se.cohort.summary$years==1999,]
  )
```

```
## Create graphs with various fertility indicators
## (this step requires some additional definitions, see
## Section 7)
source("ko-ppr-tokyo-figures.ssc")
```

5 Splus Programs and Scripts

In the following, we discuss the above implementation of tempo-adjusted parity progression measures in some additional detail. In particular, the estimation of tempo- and variance changes in the parity-specific schedules of childbearing intensities and calculation of parity progression ratios in are implemented via several Splus functions that need to be defined by sourcing the following script files (i.e., by executing the Splus command `source('filename.ssc')` at the Splus command prompt). The functions defined by these script files are further explained below.

- *ko-smoother.ssc*: defines a Kalman filter based smoother that allows the estimation of the tempo change in a slightly different fashion as proposed in Bongaarts and Feeney (1998) with the advantage of removing random fluctuations in the mean age at birth; see discussion in KO as well as the discussions in Ortega and Kohler (2002b) and Garcia-Ferrer et al. (1993).
- *ko-ppr-functions.ssc*: defines all functions for the estimation of tempo- and variance changes, the calculation of parity progression measures, and the projection of cohort fertility.
- *ko-auxfunctions.ssc*: defines several auxiliary functions for reading and transforming the data.
- additionally, the following script file generate the Figures in KO and Kohler and Ortega (2002b): *ko-ppr-sweden-graphs-pprpaper.ssc* and *ko-ppr-tokyo-figures.ssc*.

6 A Quick Tour: Replicating the Analyses in KO

The following commands replicates the analyses in our paper. All these commands are also provided in the script file *ko-ppr-quicktour.ssc*.

6.1 Prepare the analyses

```
## if necessary, delete all existing objects in the Splus working
## directory

remove(objects(pattern = "*"))

## source the above script files to define Splus functions for the
## analyses
```



```
## load KO smoother
source("ko-smoother.ssc", synch = T)

## load auxiliary functions
source("ko-auxfunctions.ssc", synch = T)

## load functions for KO-PPR analyses
source("ko-ppr-functions.ssc", synch = T)

## reset various options for ko-ppr analyses
## that are specified (and can be changed) in
## ko.ppr.control
ko.ppr.reset()
```

6.2 Read Swedish fertility data

The first step is to create a **KO data object** that contains all fertility data required for the calculation of tempo-adjusted parity progression measures in a list element `data`. In the Swedish case, this KO data object will be called `se.data`. When used as an argument in functions, we will refer to the KO data object by its generic name `all.data`.

The following lines read the Swedish data from the ASCII files (see above) into the KO data object using a function `ko.add.data`. The KO data object needs to be have the exact format (especially name of the list-elements) assigned by the function `ko.add.data` because the subsequent calculations will extract information from that list using the names and/or structure defined below. The syntax of this function and default value of its arguments are given by

```
ko.add.data(years, age, max.order = 4, period.age.data = !period.cohort.data, pe-
  riод.cohort.data = !period.age.data, last.rates.not.oerates = T, mat = NULL,
  nfx, par.dist = NULL)
```

and the specific implementation to read the data for Sweden is as follows:

```
se.data <-
  ko.add.data(years = 1970:1999,
             age = 15:44,
             max.order = 4,
             period.cohort.data = T,
             last.rates.not.oerates = T,
             mat = list(
               order.1 = ko.matread("se-mat-1.txt",
                                   data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
                                   set.na.to.zero=T),
               order.2 = ko.matread("se-mat-2.txt",
                                   data.years=1961:1999,keep.years=1970:1999,set.rows=1:2,
```

```

        set.na.to.zero=T),
order.3 = ko.matread("se-mat-3.txt",
    data.years=1961:1999,keep.years=1970:1999,set.rows=1:5,
    set.na.to.zero=T)
),
nfx = list(
    order.1 = ko.matread("se-nfx-1.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
        set.na.to.zero=T),
    order.2 = ko.matread("se-nfx-2.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=1:2,
        set.na.to.zero=T),
    order.3 = ko.matread("se-nfx-3.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=1:5,
        set.na.to.zero=T),
    order.4 = ko.matread("se-nfx-4plus.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=1:8,
        set.na.to.zero=T)),
par.dist = list(
    parity.0 = ko.matread("se-pardist-0.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
        set.na.to.zero=T),
    parity.1 = ko.matread("se-pardist-1.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
        set.na.to.zero=T),
    parity.2 = ko.matread("se-pardist-2.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
        set.na.to.zero=T),
    parity.3 = ko.matread("se-pardist-3plus.txt",
        data.years=1961:1999,keep.years=1970:1999,set.rows=NULL,
        set.na.to.zero=T)
)
)
)

```

The first arguments of the function `ko.add.data` provide a description of the data used in the analyses, as for instance the years (`years = 1970:1999`) and age range (`age = 15:44`), the format of the data (either `period.cohort.data = T` or `period.age.data = T`), and an indicator of whether the last category pertain to an open-ended category with births of order 4 and higher (as for Sweden, where we set `last.rates.not.oerates = T`).

The next arguments of the function `ko.add.data` provide the childbearing intensities (`mat`), age-specific fertility rates (`nfx`) and the parity distribution (`par.dist`) as lists, where the list elements are the respective parity specific information in the form of matrices. In order to read the data files (see above) and provide them in these list elements, we utilize an auxiliary function `ko.matread` with the arguments (and default values) given by


```

nfx = list(
  order.1 = ko.matread("se-nfx-1.txt",
    set.rows=NULL,
    set.na.to.zero=T),
  order.2 = ko.matread("se-nfx-2.txt",
    set.rows=1:2,
    set.na.to.zero=T),
  order.3 = ko.matread("se-nfx-3.txt",
    set.rows=1:5,
    set.na.to.zero=T),
  order.4 = ko.matread("se-nfx-4plus.txt",
    set.rows=1:8,
    set.na.to.zero=T)),
par.dist = list(
  parity.0 = ko.matread("se-pardist-0.txt",
    set.rows=NULL,
    set.na.to.zero=T),
  parity.1 = ko.matread("se-pardist-1.txt",
    set.rows=NULL,
    set.na.to.zero=T),
  parity.2 = ko.matread("se-pardist-2.txt",
    set.rows=NULL,
    set.na.to.zero=T),
  parity.3 = ko.matread("se-pardist-3plus.txt",
    set.rows=NULL,
    set.na.to.zero=T)
)
)

```

6.3 Calculate BF-adjusted total fertility rate

The next function `ko.ppr.tfr.adjustment`, which takes the syntax

```
ko.ppr.tfr.adjustment(all.data)
```

where *all.data* is the generic name for the KO data object generated in the previous section. The function `ko.ppr.tfr.adjustment` adds a list element `tfr.adjustment` to the KO data object that contains the BF-adjusted total fertility rate for Sweden (along with estimates of r). The only difference in this calculation to Bongaarts and Feeney's original implementation is that we use the IRW smoothing methods, defined in `ko-smoother` and described in Section 4 above, to estimate the annual changes in the order-specific mean ages at birth (recall that the mean ages in the BF adjustment are calculated from standard age-specific fertility rates, or incidence rates, instead of childbearing intensities).

In the Swedish example, the syntax to perform the BF-adjustment of the *TFR* is

```
## calculate adjusted TFR
```

```
se.data <- ko.add.tfr.adjustment(se.data)
```

6.4 Estimation of mean- and variance changes in KO

The function `ko.ppr.add.fitted` performs the estimation of the parity-specific mean and variance changes of the childbearing intensity schedules, denoted $\gamma_j(t)$ and $\delta_j(t)$, and the level-effect $q_j(t)$ for all time periods t . The syntax of this function is given by

```
ko.ppr.add.fitted(all.data, calc.for.years)
```

where *all.data* is the function argument that takes the KO data object defined in Section 6.2 (which is called *se.data* in our Swedish example). The optional additional argument, *calc.for.years* provide a possibility to restrict the calculations to a subset of years (the default is to perform the analyses for all years include in *all.data*).

The tempo-adjusted childbearing intensities are not directly calculated by the above routines, and can be extracted from the KO data object using the following function:

```
ko.ppr.predict.adjusted(all.data, b.order, years, years.i)
```

where *all.data* is the generic name for the KO data object, and *b.order* specifies the birth order for which the tempo-adjusted intensities are calculated. The optional additional arguments, *years* and *years.i*, allow to restrict this calculation to specific years giving either directly the year (*years*) or the sequence in the range of years (*years.i*).

In the Swedish case, the estimation of mean- and variance changes in the schedules of childbearing intensities is implemented as follows:

```
## application of estimation
se.data <- ko.ppr.add.fitted(se.data)

## calculate the tempo-adjusted childbearing intensities
## for first births
## (Note: this object is not required for subsequent analyses
## and is not used as input by other functions)
se.adjusted.intensities <-
  ko.ppr.predict.adjusted(se.data, b.order = 1)
```

6.5 Estimation of tempo-adjusted period parity measures

The first application of our measures is the *description* of period fertility patterns based on fertility indicators for a synthetic cohort synthetic cohort that experiences the period fertility pattern (level, tempo and potentially tempo-change) during its life-course. Most analyses using tempo-adjusted parity progression measures will generally be based on the following fertility indicators:

- *period life-time birth probability of one, two,... children*: this measure reflects the probability that a woman in the synthetic cohort gives birth to at least one, two,... children conditional on (a) the parity-specific level of fertility in the reference year after removing tempo distortions and (b) the assumed postponement pattern during the life-course of the synthetic cohort.
- *period parity progression ratios (period PPRs)*: these period PPRs reflect the probability of progressing from parity j to parity $j + 1$ conditional on the level of fertility observed in the reference year and the assumed postponement pattern during the life-course of the synthetic cohort.
- *index of completed fertility*: this index reflects the expected number of children for women in the synthetic cohort conditional on the level of fertility observed in the reference year and the assumed postponement pattern during the life-course of the synthetic cohort.
- *period fertility index*: this is a special case of the above index of completed fertility calculated under the assumption that there is no further change in the timing of fertility during the life-course of the synthetic cohort; the period fertility index therefore assumes that women in the synthetic cohort are subject to the tempo-adjusted period childbearing intensities (for a detailed discussion of the period fertility index and its usefulness as a period fertility measure, see also Kohler and Ortega 2002a; Ortega and Kohler 2002a,b).

These measures (and some related period fertility indicators) are calculated by the function `ko.ppr.comparison` that takes the following arguments (along with its default values)

```
ko.ppr.comparison(all.data, calc.for.years = NULL, xx.values = seq(15,25,by = 10))
```

where *all.data* is the KO data object, *calc.for.years* is an optional additional argument to restrict the calculations to a subset of years (the default is to perform the analyses for all years include in *all.data*, and *xx.values* allows to specify the age of the cohort in the reference year for which the parity progression measures are calculated (the default is to perform the calculation for cohorts age 15 and 25 years old in the reference year, where the first cohort is of particular relevance since it is at the beginning of its childbearing years).

In the Swedish case, the function is implemented as follows:

```
## calculate period parity progression measures
se.ppr.comparison <- ko.ppr.comparison(se.data)
```

Although it is not necessary for the implementation of the estimation, it is useful for understanding the calculations that we briefly describe the key functions that are called by the function `ko.ppr.comparison` during the calculation of the tempo-adjusted period parity progression measures.

`ko.ppr.p(xx, yy, q.T = 1, mab.T, sdab.T, gamma = 0, delta = 0, period.T = 0, phi.points, postponement.continues = T, calc.from.observed = F)`: this function calculates the probability that a women the synthetic cohort, who is age xx in the reference year and at parity j at some age yy , experiences a birth of order $j + 1$. The additional arguments specify the mean age and variance of the adjusted intensity schedule in the reference year ($mab.T$ and $sdab.T$), $gamma$ and $delta$ specify the mean- and variance changes estimated in the reference year.

The arguments `phi.points` provide a discrete-time approximation of the tempo-adjusted childbearing intensities at parity j . In particular, we calculate the adjusted childbearing intensities at “mid-age” (e.g., at ages 15.5, 16.5, 17.5, ..., 44.5 in the Swedish case), using the estimated mean and variance change in the reference year (this calculation of the adjusted childbearing intensities is performed via the function `ko.ppr.predict.adjusted` described above). For ages between these “mid-ages”, we use a linear interpolation.

The remaining arguments specify whether the calculations are based on the postponement stops scenario (`postponement.continues = F`) or the postponement continues scenario (`postponement.continues = T`). If the postponement continues scenario is chosen, the cohort experiences the annual mean and variance changes given by the arguments $gamma$ and $delta$ during its life-course. Moreover, the option `calc.from.observed = T` allows to specify that the calculations are based on the observed instead of the tempo-adjusted childbearing intensities (this option can only be combined with the postponement stops scenario).

Based on the above information, the function `ko.ppr.p` performs the integral

$$\begin{aligned} & 1 - \exp \left[- \int_y^\omega m_j^s(a, T + a - x) da \right] \\ = & 1 - \exp \left[- \int_{y - R_j^s(y, T + (y - x))}^\omega m'_j(a, T) da \right], \end{aligned} \quad (2)$$

where $m_j^s(a, T + a - x)$ are the childbearing intensities by women in the cohort under consideration, and $R_j^s(y, T + (y - x))$ is the tempo change that is accumulated between the reference year T and the period $T + (y - x)$ when the women is at age y and exposed to a order $j + 1$ birth.

`ko.ppr.p.from.fitted(xx, yy, ii = 0, period.T = 0, all.data, postponement.continues = T, calc.from.observed = F, add.postponement.years = 0, overwrite.last.rates = NULL)`: helper function that calls `ko.ppr.p` for a given birth order $ii + 1$, using the information stored in the KO data object `all.data`.

`ko.ppr.DF(xx, ii, period.T, all.data, postponement.continues = T, calc.from.observed = F, add.postponement.years = 0)`: iteratively calculates the parity distribution, birth probability and related information for a synthetic cohort that is age xx and parity ii in the reference year, using the information stored in the KO data object `all.data`.

There are also some additional helper functions that iterate through all calendar years and parities for the calculations; the primary calculations for the parity progression measures, however, are performed by the functions listed above.

6.6 Completion of cohort fertility

The second application of our period parity progression measures is the completion of cohort fertility. This completion is possible because the measures calculated in the previous section provide conditional projections for the period- T cohorts at all ages and parities. This allows us to take the parity distribution *observed* in the reference year T as the starting point for the calculation. We then use period parity progression measures to fill in the ‘missing’ part of a cohort’s childbearing pattern conditional on future trends in the tempo and parity-specific level of fertility. In the cohort completions that we present in KO, the projection of future fertility is based on the parity-specific level of fertility $q_j(T)$ in the reference year and the parity-specific postponement pattern given by γ_j^s and δ_j^s for all parities $j = 0, \dots$. The completed fertility of birth cohorts then follows via a weighted average of the conditional total fertility indexes across all parities in the reference year T , where the weights are equal to the observed parity distribution of the cohort in the reference year T .

This calculation is conditional on a specific postponement scenario. Particularly interesting and appealing for cohort completion, and therefore implemented via the below functions, are the postponement stops ($\gamma^s = 0, \delta^s = 0$) and the postponement continues ($\gamma^s = \gamma^T, \delta^s = \delta^T$) scenarios.

While the completed cohort fertility is a centrally important indicator of cohort fertility, the analysis with our period parity progression measures is not restricted to this measure. Other cohort fertility measures, such as the final parity distribution or the mean age at birth (overall or at some specific birth order), can also be obtained by taking corresponding weighted averages. For instance, the cohort mean age at birth of order j can be calculated via a weighted average between the mean age of women at order- j births prior to the year T and the order-specific mean age at order- j births that are projected to occur in the future.

The completion of cohort fertility is implemented via two functions that perform (a) the calculation of the cohort parity distribution at the end of the reference year and (b) the projection of cohort fertility via a weighted average of the conditional total fertility indexes across all parities in the reference year T , where the weights are equal to the observed parity distribution of the cohort in the reference year T . The respective functions are as follows:

`ko.calc.cohort.summary`(*all.data*, *first.cohort* = *NULL*, *use.initial.par.dist*=*F*):
 this function calculates the completed fertility and final childlessness (for cohorts that have completed childbearing) and the cumulated fertility of cohorts still in childbearing years. For these cohorts, the function also calculates the order-specific mean age at birth for births that have so far occurred to the cohort, and it calculates the parity distribution on Dec 31 of each year. The function `ko.calc.cohort.summary` takes the KO data element as first argu-

ment, and the optional additional arguments specify the first cohort for which the calculations are performed *first.cohort* and whether the parity distribution supplied in the KO data object should be used to “fill in” parts of a cohorts fertility history that are not available in the *nfx*-matrices supplied in the KO data matrix.

```
ko.ppr.complete.fert.3.scenarios(all.data, period.T, age.mid = NULL, par.dist
= NULL, cum.nfx = NULL, cum.mab = NULL): calculates the cohort projection under the postponement stops and postponement continues scenario as well as based on the observed childbearing intensities. The arguments are the KO data object, and several optional arguments that allow to specify mid-age age.mid, the parity distribution par.dist, the cumulated fertility cum.nfx and the mean age at birth (prior to reference year) cum.mab.
```

The information for these optional arguments in `ko.ppr.complete.fert.3.scenarios` is calculated from the KO data object via the above function `ko.calc.cohort.summary`.

In the Swedish case, the commands to for the completion of cohort fertility are as follows:

```
## cohort summary for nfx cohort data
se.cohort.summary <-
  ko.calc.cohort.summary(se.long.data,
                        first.cohort = 1930,
                        use.initial.par.dist = T)

##complete fertility in the years 1975,1999
se.complete.fert.1975 <-
  ko.ppr.complete.fert.3.scenarios(se.data,1975,
  par.dist =
    se.cohort.summary$par.dist[,se.cohort.summary$years==1975,],
  cum.nfx =
    se.cohort.summary$cum.nfx[,se.cohort.summary$years==1975,],
  cum.mab =
    se.cohort.summary$cum.mab[,se.cohort.summary$years==1975,]
  )
se.complete.fert.1999 <-
  ko.ppr.complete.fert.3.scenarios(se.data,1999,
  par.dist =
    se.cohort.summary$par.dist[,se.cohort.summary$years==1999,],
  cum.nfx =
    se.cohort.summary$cum.nfx[,se.cohort.summary$years==1999,],
  cum.mab =
    se.cohort.summary$cum.mab[,se.cohort.summary$years==1999,]
  )
```

7 Output: figures for KO analysis

The following commands draw a variety of figures that are reported in Kohler and Ortega (2002a) and Kohler and Ortega (2002b).

```
## make some assignments that allow a flexibly change of
## countries
ko.per.data <- se.data
ko.ppr.data <- se.ppr.comparison
ko.coh.data <- se.cohort.summary
ko.refyrs <- c(1975,1999)
ko.coh.complete <- list(ref.yr.1975 = se.complete.fert.1975,
                        ref.yr.1999 = se.complete.fert.1999)

## these lines define a set of options that specifies the
## position of legends, etc, for the graphs
ko.p.opt <-
  list(
    ## lifetime birth prob graphs
    ltb.key.xpos=c(1970,1970,1970),
    ltb.key.ypos=c(.74,.8,.38),
    ## parity progression rates
    ppr.key.xpos=c(1970,1970),
    ppr.key.ypos=c(.93,.5),
    ## adjusted tfr graph
    tfr.1.key.xpos = 1970,
    tfr.1.key.ypos = .96,
    tfr.1.ylim =c(.6,1.1),
    tfr.a.key.xpos = 1970,
    tfr.a.key.ypos = 2.3,
    tfr.a.ylim = c(1.4,2.4),
    tfr.comp.1.key.xpos = 1970,
    tfr.comp.1.key.ypos = .45,
    tfr.comp.a.key.xpos = 1977,
    tfr.comp.a.key.ypos = 1,
    ## projections
    proj.complete.age = 44,
    proj.cohort.lim =c(1930,1985),
    proj.ylim.1 = c(.08,.28),
    proj.ylim.2 = c(1.4,2.1),
    proj.ctext.ypos = list(c(.27,.25,1.48,1.43),
                          c(.27,.25,1.48,1.43)),
    ## mean variance changes
    mvc.mablim = list(c(25.5,31.5),c(25.5,30.5),c(25.5,31.5)),
    mvc.mab.key.xpos = 1970,
    mvc.sdlim = list(c(4.85,5.45),c(4.85,5.45),c(4.6,5.2)),
    mvc.gammalim = list(c(-.05,.25),c(-.05,.25),c(-.05,.25)),
```

```

    mvc.deltalim = list(c(-.005,.01),c(-.005,.01),c(-.005,.001))
  )

## this script file reproduces the figures in KO
source("ko-ppr-tokyo-figures.ssc")

```

8 Customization

The script file `ko-ppr-functions.ssc` defines a object `ko.ppr.control` that contains various defaults for parameters used in the calculations that can be changed by the user (although caution is warranted). The default setting is as follows:

```

ko.ppr.control <-
  list(
    ## smoothing for BF calculations
    bf.smooth.par.firstbirths = .1,
    bf.smooth.reduction.parity = 1,
    ## smoothing parameters for KO application
    ko.smooth.par.firstbirths = .1,
    ko.smooth.reduction.parity = 1,
    ko.smooth.reduction.variance = 1,

    ## criteria for iterative fitting
    iteratefit.mean.crit = .000001,
    iteratefit.max.crit = .00001,
    iteratefit.max.iter = 40,

    ## for PPR calculations
    max.age = 70,
    age.step = 1
  )

```

It is possible to re-set `ko.ppr.control` to its default values using

```
ko.ppr.reset()
```

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