1 Reading

Read the proof of the Cantor's Theorem and start trying to understand it.

2 Response

Answers to these should be written down and turned in in class.

1. Go through the steps we talked about for analyzing a proof. In particular:
    - What is an example for which the statement is easy? An example for which it's tricky?
    - Can you work through the proof for a concrete set A and a concrete example of a hypothetical function $f : A \to \mathcal{P}(A)$?
    - Which steps could use more details to be clear enough?
    - What techniques could you use in a proof of your own?

Theorem 2.1 (Cantor's Theorem)**.** *For all sets A, $|A| < |\mathcal{P}(A)|$.*

Proof. Towards a contradiction, suppose $|A| = |\mathcal{P}(A)|$, so there is a bijection $f : A \to \mathcal{P}(A)$. Let $S \subseteq A$ be the set defined by $S = \{x \in A \mid x \notin f(x)\}$. Since $S \in \mathcal{P}(A)$ and f is surjective, there must be some $a \in A$ such that $f(a) = S$.

If $a \in S$ then, by the definition of S, $a \notin f(a)$, so $a \notin S$. This is a contradiction, so $a \notin S$. But then, by the definition of S, $a \in f(a)$, so $a \in S$, which is also a contradiction. So whether $a \in S$ or $a \notin S$, we get a contradiction.

These are both contradictions, so f does not exist. □