

The Numbers Game

The school of Pythagoras; a cult of secrecy & the music of numbers



Mr Ruche was convinced that there was more to Grosrouvre's letter than met the eye. He knew that if only he could read between the lines, he could decode it. Everything seemed to revolve around Pythagoras. Why had Grosrouvre mentioned him, and what was he trying to tell Mr Ruche?

Mr Ruche realized he would have to make a study of the life and work of Pythagoras and the Greek thinkers who were part of his school. What was the *akousmata* he mentioned in the letter, and why were they all sworn to secrecy? Why was the discovery of irrational numbers so 'incredible' and how could it be important enough to cause the death of Hippasus of Metapontum? Did it have anything to do with Pythagoras' famous theorem? Mr Ruche had flirted with some of these ideas as a young man, but had only a vague recollection of them now. Grosrouvre had been right when he said that Mr Ruche had never really been interested in Pythagoras and his school. He had found them too mystical, too religious for his taste.

Mr Ruche went into the Rainforest Library and wheeled himself along the shelves to the section on Greek maths. He took the long pincers he used for reaching books and brought down *The Life of Pythagoras* by Iamblicus, written in the second century AD. He wheeled himself over to the desk he had installed in a corner of the studio – a beautiful old leather-topped escritoire with carved legs and balled feet.

He read *The Life of Pythagoras* at one sitting. It was as gripping as a novel. The cover was worn; it must have been consulted often by Grosrouvre and its previous owners. Some of the pages were

dog-eared, and these he read with particular attention. He took out his notebook and his glass fountain pen and wrote:

Pythagoras invented the word 'philosophy'.

As in the case of Thales, read Mr Ruche, no written work of Pythagoras survives. Even the dates of his birth and death are unknown. What is known, however, is that he was born on the island of Samos in the Aegean sea in the 6th century BC, and died in the town of Crotona in southern Italy. He was only eighteen when he participated in the Olympic Games, where he won every single boxing tournament. After that, he decided to travel. He went first to nearby Ionia, where he spent some years with Thales and his pupil Anaximander. Then he travelled to Syria, where he stayed with the Phoenician sages. From there he went to Mount Carmel in what we now call the Lebanon, and then to Egypt, where he stayed for twenty years. In the temples on the banks of the Nile he learned the wisdom of the Egyptian high priests.

When the Persians invaded Egypt, he was taken prisoner and brought to Babylon. He spent twelve years there in the capital of Mesopotamia, where he learned much from the scribes and from the wise men. He returned to Samos forty years after he had left, older and wiser.

At that time, Samos was ruled by the tyrant Polycrates and, unable to bear his tyranny, Pythagoras left once again. This time he travelled west, towards the coast of Greece. From there he went to Sybaris in southern Italy – famous throughout the ancient world as a city of pleasure – but it was nearby in the town of Crotona, that Pythagoras made his home and founded his 'school', which lasted for one hundred and fifty years. In its time, it nurtured 218 Pythagoreans, including Hippocrates of Chios, Theodorus of Cyrene, Philolaus, Archytas of Tarentum and, of course, Hippasus.

Mr Ruche put aside the biography and opened the other books dealing with Pythagoras' mathematical work and that of his disciples. Hippasus was among the first of them; he was the leader of the *akousmata* – the 'acousmaticians' were candidates to be initiated into the school, while Pythagoras was leader of the 'mathematicians' – those who had been initiated.

Hippasus is associated with the discovery of the third type of

mean. Means are numbers which indicate the average of (originally) two numbers. To the arithmetic mean and the geometric mean was added a third, called the harmonic mean. It would become the basis of the study of harmonics in music. Pythagoras was right: numbers truly were everywhere.

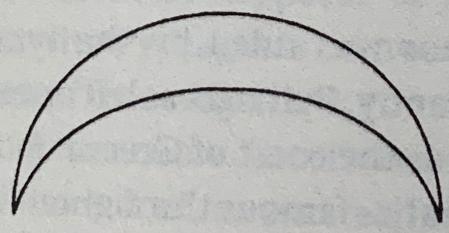
The arithmetic mean is usually just called the mean. It is half the sum of two numbers: 'The difference between the first number and the second is exactly equal to the difference between the second and the third.' Mr Ruche wrote down the formula:

$$a - b = b - c$$

b is the arithmetic mean of a and c :

$$b = \frac{1}{2}(a + c)$$

From Hippasus, Mr Ruche moved on to another of Pythagoras' pupils, Hippocrates. Hippocrates studied crescents, known in maths as *lunes*. It was Hippocrates who first established the quadrature of lunes - quadrature, or 'squaring', involved calculating a square which would be equal to the surface area of the crescent. Hippocrates' lune was the first quadratic figure of a curved object. Mr Ruche began a new page and drew the following:



He made a note in the margin:

The three great problems of Greek mathematics: squaring the circle (quadrature), duplication of the cube and the trisection of an angle.

As a young man, Hippocrates had squandered all his money. As an old man, he was thrown out of the Pythagorean school for 'demonstrating geometry for money'. It was precisely what Grosrouvre had refused to do - accept money for his mathematical proofs. If he had, he would still be alive today. Grosrouvre wasn't prepared to give his work away, as Hippasus had, or sell it, as Hippocrates had.

Mr Ruche continued town of Crotona in the named Cylon who lived Pythagorean school, man: he was not interested.

Mr Ruche stopped remembered Grosrouvre from these men if they He went back to to take his revenge regularly to discuss porters set fire to died in the blaze.'

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Mr Ruche continued reading: 'The school was founded in the town of Crotona in the tip of southern Italy. A rich, powerful man named Cylon who lived in the town wanted to be admitted to the Pythagorean school, but was rejected. Cylon was an autocratic man: he was not in the habit of being refused something he wanted.'

Mr Ruche stopped. The last sentence seemed familiar. Then, he remembered Grosrouvre's letter: '*It would be hard to keep something from these men if they have decided they want it.*'

He went back to his book. 'Furious at his refusal, Cylon decided to take his revenge. The members of the Pythagorean school met regularly to discuss city politics. One night, Cylon and his supporters set fire to the meeting house. All but one of those inside died in the blaze.'

Mr Ruche shuddered. This was no coincidence. Grosrouvre's 'business partners', like Cylon, had burned what they could not possess. Mr Ruche was angry now. Perhaps Perrette was right, and the fire in Manaus had been set deliberately. If so, he had to find the 'Cylon' who had murdered Grosrouvre. To do that he had to concentrate on the maths Grosrouvre mentioned in his letters - he was convinced all of the answers were in there.

He picked up the biography again. 'One man survived the fire in Crotona: Philolaus. An astronomer and cosmographer, Philolaus had proposed an astounding theory about the world two thousand years before Copernicus and Galileo. Not only did the earth turn, according to Philolaus, but it was not the centre of the universe. Philolaus further suggested that there was a fire at the centre of the universe around which the earth, the planets and even the sun turned.' Mr Ruche wondered whether the theory came before or after the fire from which he had miraculously escaped.

He was astonished by the next sentence: 'Across a small strait from Crotona in southern Italy was the town of Tarentum, where Archytas invented the number 1.' Mr Ruche paused for a moment. Surely the number 1 had always existed? According to this book, it seemed that Greek thinkers believed numbers began with 2. To them, there was 'one' and 'more than one'. They believed that 'one' was not a quantity but a statement of existence, whereas numbers were about multiplicity: 'one is that which is'. Mr Ruche

was excited – this was pure philosophy. In taking the singularity of 'one' and its epistemology, Archytas made a number of it.

Mr Ruche continued to take copious notes. Archytas was not only the 'father of one' but the 'first engineer'. Marshalling his knowledge of mathematics and geometry, he developed a theory of mechanics and is reputed to have made a mechanical bird – a wooden dove which could fly, powered by a small engine.

There was something else: Archytas was the first 'graffiti artist' in history. He wrote graffiti because he could not bring himself to swear. When he felt he absolutely had to, he simply wrote the offending word on a nearby wall. He reminded Mr Ruche of Max, who never swore. It was as if words were too important for him to waste them.

For Mr Ruche, however, Archytas' crowning glory was that he had saved Plato from Denys, the tyrannical ruler of Syracuse who wanted the philosopher assassinated. Archytas dispatched a battalion of soldiers to Syracuse ordering Denys to release Plato immediately. Fearful of a war, Denys agreed and Plato was released.

Mr Ruche read back over his notes, then taking his pen wrote:

The world of mathematics was greatly expanded by the Pythagoreans. They brought to it disciplines such as mechanics. Their often mystical view of numbers did not stop them from establishing arithmetic as a science. The first proofs in the history of maths were developed by them. They proved that the square root of 2 was an irrational number. It had been irrational numbers that had proved Hippasus' downfall. In geometry, they demonstrated that the sum of the angles in any triangle is 180° .

Mr Ruche was satisfied, he had more than enough for his next session on Pythagoras & Co. He put away his notebook and wheeled himself towards the door of the Rainforest Library.

Two days later, after much secretive preparatory work by Max and Mr Ruche, Jonathan and Lea came into the screening room – the empty studio beside the library – for their presentation on the school of Pythagoras. The room was dark; they could just make out some chairs and Albert, sitting in the far corner.

After a long silence, a light glowed faintly through the curtain

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which divided the studio. There was a tinkling, like musical chimes. On the other side of the curtain Max had placed four identical vases on a small table. The first was empty, the second half-filled with water, the third one-quarter filled, the fourth filled to one-third. Max held two small hammers. He tapped gently on the empty vase, then on the vase which was half-full. Two distinct notes rang out. Then he tapped the two together, making a more harmonious sound than either alone.

'An octave!' squawked Sidney.

There was a moment's silence, before Max tapped the empty vase and the one that was one-third full. They rang out.

'A fifth!' said Sidney.

Another silence, then Max tapped the empty vase again, this time with the one that was one-quarter full.

'A fourth!' said Sidney.

Max could barely make out the sounds himself, but he was determined to carry out the experiment. Jonathan and Lea listened without really understanding what was going on. Albert listened too, but didn't try to understand. Mr Ruche wondered whether a taut string would have produced more distinctive notes. He worried that he had chosen the more spectacular experiment, rather than the most useful, but it was too late now.

'Pythagoras saw numbers ever...' The parrot's voice trailed off, and Jon-and-Lea heard a fluttering of wings before he cleared his throat and began again:

'Pythagoras saw numbers everywhere. Everything that is can be enumerated. He first found numbers in music.' The bird's voice trailed off again and Mr Ruche took over:

'Using a simple device, Pythagoras had made an extraordinary discovery: that the difference in musical notes is a difference between two numbers. The octave you heard is produced by the two vases, one empty, one half-filled; it is defined as a ratio of one-half; a musical fifth is two-thirds and a musical fourth is three-quarters. Can you think of a series of numbers simpler than that?'

Mr Ruche went on, 'So the relationship between numbers could be used to describe musical harmonies. In fact, harmony is simply a series of sounds based on numerical ratios. The notes are numbers, and the music is mathematics.'

From behind the curtain, a woman's voice began to sing Bach's *Ich habe Genug* a cappella. As the soprano's voice swooped lower and lower, Mr Ruche continued. 'Pythagoreans believed that the whole universe was a series of harmonies, that the heavens themselves were regulated by a musical scale. They called it the Music of the Spheres. Pythagoras invented a word for it: the *cosmos*. Order and beauty: to him the history of the universe was the struggle between *cosmos* and *chaos*.'

Mr Ruche looked down at the text he had prepared. 'Those three notes – mathematical harmonies – sounded out for Pythagoras the first mathematical law of nature. After that, he began to find numbers everywhere.'

'Pythagoras and his school were determined to discover the mathematical laws inherent in the way nature regulated itself. To do this, they had to study numbers themselves. This was the beginning of *arithmetic* – the science of numbers, which they distinguished from *logistic* – pure calculation. In separating these disciplines, they elevated arithmetic above mere counting.'

Mr Ruche switched on the tape recorder and a voice boomed out through the loudspeaker: 'Attention! Attention! Listeners are asked to move to the other side of the curtain. The other side of the curtain.' Jon-and-Lea stood up (noting that they were listeners rather than viewers) lifted the curtain and went through. Here, three spotlights glowed in the darkness. One illuminated Max, who was sitting at a low table on which a number of objects – including the four musical vases – were arranged. The second spotlight was on Sidney, on his perch in front of a partition. The third and most powerful spotlight shone on Mr Ruche, who was on a small podium, surrounded by records and tapes and a hi-fi system with two powerful speakers. Mr Ruche picked up a sheet of paper and began to read:

'Pythagoras began by creating a catalogue of numbers, beginning with 1 – which seems so natural to us now that we assume it has always existed. He divided whole numbers into odd and even numbers: those which are divisible by 2 and those which aren't. He then went on to establish the rules of calculation...'

Sidney chipped in, 'Even plus even equals even; odd plus odd equals even; odd plus even equals odd.'

Mr Ruche continued, '...and the rules of multiplication.'
The parrot chipped in again, 'even times even equals even, odd times odd equals odd; odd times even equals even.'

The door opened on the other side of the curtain and a fresh breeze blew into the studio. Perrette slipped quietly into the room just as Jon-and-Lea applauded. She spotted Albert and sat down near him.

The loudspeaker broke the silence again: 'Attention! Attention! This is a revelation, this is a revel...'

Mr Ruche cut the sound and picked up the story. 'I have an important announcement: Pythagoras' theorem is not, I repeat not, by Pythagoras.' There was a scattering of applause at this revelation. Lea didn't quite know why she was so impressed; Jonathan was indifferent.

'Credit where credit is due', Mr Ruche continued. 'Before Pythagoras was born, the Egyptians and the Babylonians had discovered a link between triple numbers. That was the link made famous by Pythagoras. A collector named Plimpton acquired a Babylonian tablet on which a scribe had engraved a dozen triplets, indicating definitively that the sum of the squares of the first two was equal to the square of the third.'

Mr Ruche signalled to Sidney. The parrot arched himself on his perch. Max stood up.

'Three blocks of wood', Sidney announced. Max picked up three pieces of wood from the table. Sidney went on, 'The first is 3 units long, the second 4 and the third 5.' With his hands, Max indicated three units, then four, then five.

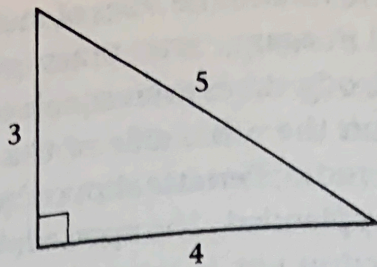
'Looks like they've been rehearsing!' moaned Lea.

'Yeah', said Jonathan. 'Max looks like a flight attendant.'

It was certainly true that Max had a fixed smile, and his mechanical hand gestures did look a little like a steward showing passengers how to put on their life jackets.

The parrot continued, '3 squared is 9; 4 squared is 16, added together they make 25, which is 5 squared. A triangle with sides of these lengths is a right-angled triangle.'

As Sidney spoke, Max traced the equation $3^2 + 4^2 = 5^2$, then he placed the three pieces of wood together to form a right-angled triangle:



'So, what does the theorem tell us?' asked Mr Ruche. 'It tells us that there is a relationship between the lengths of the sides of a right-angled triangle which can be expressed as $a^2 + b^2 = c^2$.' Mr Ruche poured himself a glass of water and took a sip.

On the other side of the curtain, Perrette stretched out her legs and kicked off her shoes. She was tired from her long day in the bookshop. She could hear everything, but couldn't see what was going on. More importantly, she thought, she couldn't see what all this had to do with Grosrouvre's letter and the questions it posed. Jonathan, meanwhile, interrupted Mr Ruche: 'I'm not trying to defend Pythagoras, but...' (in fact, that was precisely what he wanted to do: for some reason, perhaps his long hair and his Grecian nose, he identified with this Greek traveller) '...but you said yourself that there's a difference between giving examples of something and proving it. The Egyptians and the Babylonians gave examples, but were they able to prove the theorem?'

'Apparently not', admitted Mr Ruche.

'In that case it is Pythagoras' theorem. Credit where credit's due!' said Jonathan triumphantly.

Lea asked Mr Ruche, 'So what was the curtain for?'

'I'm surprised you didn't ask before. I'm a bit worried that you're learning to be patient', Mr Ruche said sarcastically. 'The curtain was there so that you would know how it felt to be an apprentice applying to join Pythagoras' school. Pythagoras was rigorous in his selection. He began by seeing whether the disciple could "hold his tongue" - his words, not mine. Could he remain silent during lessons and, more importantly, keep secret what he learned? The first test depended not on what the disciple said, but on what he did not say. The schoolroom was divided into two by a curtain. Pythagoras would sit on one side and the candidates on the other.

They could listen, but couldn't hear.
Lea was outraged: 'Five years.'

'Aren't we entitled to learn anything if we were behind the curtain?'

Mr Ruche could tell what Max was fuming too. 'Sorry, Max, that's beyond it meant that one of the curtain were called Pythagoras were esoteric.'

'So when you brought it up, we were worth asking in unison.'

'Precisely', said Mr Ruche. 'Why?'

'Because you managed to hold the curtain both managed to hold it.'

'So it was a trick?' Lea asked.

'Not a trick, a test', Mr Ruche said.

'And if we hadn't known, we would have stayed silent.'

'You would have stayed silent. I decided that was fair.'

Hearing his name, Max stood up and walked around the room. He tried to open it, but only succeeded in falling on top of himself.

Perrette was sitting quietly on the other side of the curtain.

'How long have you been there?'

'Since the bit about the curtain.'

Albert shifted in his seat, laughing, but even so, he looked like an actor trying to hide his nerves.

They could listen, but could not see. This period lasted for five years.'

Lea was outraged: 'Five years! Don't look, just shut up and listen - it sounds like a cult.'

Max was fuming too. 'What about deaf people?' he thought. 'Aren't we entitled to learn? How could we be expected to find out anything if we were behind a curtain? I know I wouldn't stand for it.'

Mr Ruche could tell what was bothering him and said apologetically, 'Sorry, Max, that's the way it was.' Then he continued. 'The curtain played an important role in the Pythagorean school. To go beyond it meant that one had passed the tests. Those on the far side of the curtain were called *exoterics* and those who sat with Pythagoras were *esoterics*. They alone could hear and see him.'

'So when you brought us behind the curtain it was because you thought we were worthy of being esoterics, is that it?' Jon-and-Lea asked in unison.

'Precisely', said Mr Ruche.

'Why?'

'Because you managed to keep your mouths shut the whole time you were on the other side. I must admit I was surprised, but you both managed to hold your tongues.'

'So it was a trick?' Lea gave Jonathan a knowing look.

'Not a trick, a test', Mr Ruche corrected.

'And if we hadn't kept our mouths shut?'

'You would have stayed on the other side of the curtain. Max and I decided that was fair, and Sidney agreed.'

Hearing his name, the parrot took off from his perch and flew around the room. He flew straight into the curtain. Max tried to open it, but only succeeded in pulling the whole contraption down on top of himself. It was only then that they noticed Perrette sitting quietly on the other side. No one had heard her come in.

'How long have you been there, Mum?'

'Since the bit about Pythagoras' theorem', she smiled at him.

Albert shifted in his seat. He was sound asleep. They all burst out laughing, but even that didn't wake him. Mr Ruche carried on, like an actor trying to remain calm as the scenery falls down around him.

'The writings of Pythagoras' school were also secret and were deliberately written to have a double meaning: one which anyone could understand, but the other comprehensible only to the initiated. Pythagoreans often spoke of *sumbola* and *ainigmata*: symbols and enigmas.' As he said this, he thought of Grosrouvre's letter which, he was sure, was a real Pythagorean text, full of symbols and codes.

'Most of their knowledge was passed on by word of mouth. This further divided the school into *acousmaticians*, who were told the results of a proof, but not the proof itself, and *mathematicians*, who knew both the proofs and the results.' Mr Ruche wondered aloud who Grosrouvre's loyal friend was – the person to whom he confided his proofs after he had burned them. Like Pythagoras' disciples, his friend would have had to learn the proofs by heart, though he did not need to understand them – only to be able to repeat them. The friend did not have to be a mathematician, just an acousmatician, as Pythagoras called them.

There was silence for a moment. Then Lea smiled and said, "In search of a Rainforest Acousmatician." That sounds like a good title for our end-of-term paper.'

'What about us? Are we acousmaticians or mathematicians?' asked Jonathan.

'That depends on how well you understand the proofs. Only time will tell.'

'Every pupil of Pythagoras had to train his memory', Mr Ruche picked up where he had left off, 'Every morning, before getting up, he had to remember the precise events of the day before – everything he had seen or said or done and everyone he had met.'

'What happened to the ones who weren't accepted?' Lea asked.

'When a disciple first put himself forward to the school, he had to give all his worldly goods to the community.'

'Like I said, it sounds just like some sect', said Lea.

'With one condition,' Mr Ruche went on, 'that anyone who was not accepted be given twice the value of the goods he had brought.'

'You mean the stupid ones left with more money than they came with?' Jonathan asked incredulously. 'You don't find religious sects doing that nowadays. They're more likely to bleed you dry!'

'He was rewarded financially for what he was not able to gain in

knowledge', added Mr Ruche. 'But...' (he left a pause for emphasis) '...as soon as it was decided that he should leave, a grave was dug for him.'

'They didn't kill him, did they?' shouted Max.

'It was supposed to be symbolic, Max', said Lea. Perrette looked up, her eyes shining. 'The death was symbolic, but the grave was real enough. If someone had come along and seen it, they would have assumed that he was really dead. So it is possible to believe you have proof that someone is dead, even though he's still alive.'

'What is she going on about?' Lea wondered.

Max moved closer to his mother, and all the others listened carefully.

'You're talking about Grosrouvre, aren't you?' asked Mr Ruche. 'But they found a...' (he couldn't bring himself to say 'a corpse') '...they found his body. It's one thing to find a grave, but I think you're confusing a tomb with a body.'

'I'm not confusing anything', said Perrette.

'So what are you saying?' Mr Ruche sounded angry.

'What I'm saying is, how do we know that the body in the ruins of your friend's house was really his?'

This possibility hadn't occurred to them before now. Mr Ruche turned to her: 'Perrette, the commissioner told us in the letter that it was Grosrouvre's body.'

'I don't understand you, Mr Ruche. Do you want your friend to be alive or don't you?'

'What do you mean, "Do I want him to be alive?" What I want doesn't make any difference. It won't bring him back to life.'

'And if you don't have any proof that he is dead there's no reason to kill him off.' Perrette was angry now too.

'Just a minute...what do you mean, kill him? Are you saying I killed Grosrouvre?'

'Calm down a minute. I'm not saying that at all. I'm just saying that we don't have any proof that he's dead.'

'No proof?' Mr Ruche was furious now. 'What about the charred body they found in the ruins - isn't that proof enough?'

'No. All the body proves is that the person in the house when it burned down is dead. It doesn't tell us who that is or even whether

they died in the fire. I mean, did anyone identify the body? Was there an autopsy?

'May I remind you Perrette, that you're the one who suggested Grosrouvre was murdered.'

'I'm not trying to contradict what I said before. Let's just consider every possibility.'

'Is anyone hungry?' Lea interrupted.

'But if it isn't Grosrouvre, then who is it?' asked Mr Ruche.

'Let's just try to find out first whether it is Grosrouvre', said Perrette.

'Well, you lot may not be hungry, but I'm starving,' said Lea.

'OK, OK, let's leave it there', said Mr Ruche. 'We can talk again after we've eaten. We can do a...what is it you kids call it?'

'An all-nighter?'

'That's it. If we have to, we can do an all-nighter.'

At this, Albert woke up. His cap had slipped off, but his cigarette was still stuck to his lower lip.

'I think I might have dropped off', he said. 'I did an all-nighter last night out at the airport. It's good money, airport runs, but it takes it out of you.'

'What about Albert, Mr Ruche?' asked Lea. 'Is he an esoteric too? He didn't say anything and you said that was the rule.'

'Albert,' declared Mr Ruche, 'you have just been made an esoteric. You now belong to the ranks of the Pythagoreans.'

'I do not. I don't belong to nothing. I've always been a bit of a loner. You won't catch me joining associations or parties or unions. I wouldn't join a bowling team, me!'