MIDTERM

Math 340
9/26/2017

Name: ______________________________

ID: ______________________________

"I have adhered to the Penn Code of Academic Integrity in completing this exam."

Signature: ______________________________

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.
- You may assume all graphs are simple (no multiple edges, no loops) and finite.
- You may use writing implements and a single 3"x5" notecard.
- You may use any result proved in class or in the textbook in your arguments.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	20	
2	15	
3	15	
4	15	
5	15	
6	20	
Total	100	

1. (*20 points*)
You do not need to prove that the graphs have the specified properties; it suffices to draw the graphs.

(**a**) Draw a bipartite graph with eight vertices where each vertex has degree 2.

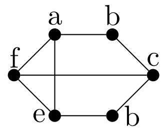
(**b**) Draw a connected planar graph where every vertex has degree 4.

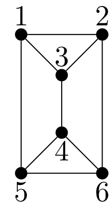
(**c**) Draw a graph which has an Euler cycle that is also a Hamilton circuit.

(**d**) Draw a planar graph with a Hamiltonian circuit, an Euler trail, but no Euler cycle.

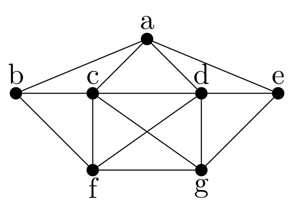
2. (*15 points*) For each of the following pairs of graphs, explain how you can be sure the pair is not isomorphic.

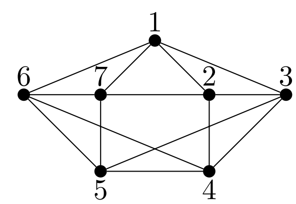
(**a**)



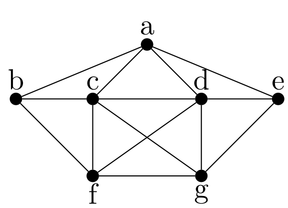


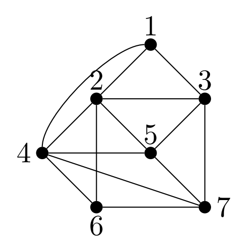
(**b**)





(**c**)





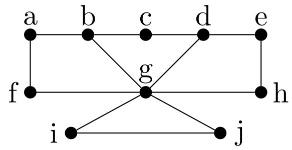
3. (*15 points*) $G = (V, E)$ is a connected graph in which all vertices have even degree. Show that if we remove one edge from G, the graph remains connected.

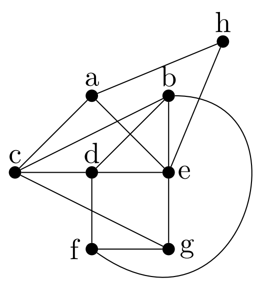
4. (*15 points*) Suppose $G = (V, E)$ is a connected graph with v vertices (that is, $|V| = v$). Let $L = \{w \in V \mid deg(w) \geq 11\}$: the set of vertices with degree at least 11.

(**a**) Suppose $|L| \geq v/2$. Show that $\sum_{w \in V} deg(w) \geq 6v$.

(**b**) Show that if G is planar, $|L| < v/2$.

5. (*15 points*) For each of the following, either write down a Hamilton circuit or show the graph does not have one.

(**a**) 



(**b**)

6. (*20 points*) We want to prove that if G is a finite graph with no cycles then G can be 2-colored. (The case where G is empty is trivial, so we'll only worry about graphs with at least one vertex.) Let p_k be the statement "Every graph with k vertices and no cycles can be 2-colored."

(**a**) Prove p_1.

(**b**) Suppose that p_k is true. Prove p_{k+1}. You may find the following fact useful: any finite graph with no cycles has a vertex of degree ≤ 1.

(**c**) Prove that *any* finite graph with no cycles can be 2-colored.