Practice Midterm

Math 340
9/26/2017

Name: ______________________

ID: ______________________

"I have adhered to the Penn Code of Academic Integrity in completing this exam."

Signature: ______________________

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.
- You may assume all graphs are simple (no multiple edges, no loops) and finite.
- You may use writing implements and a single 3"x5" notecard.
- You may use any result proved in class or in the textbook in your arguments.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	20	
2	10	
3	15	
4	10	
5	15	
6	10	
7	20	
Total	100	

1. (*20 points*)
You do not need to prove that the graphs have the specified properties; it suffices to draw the graphs.

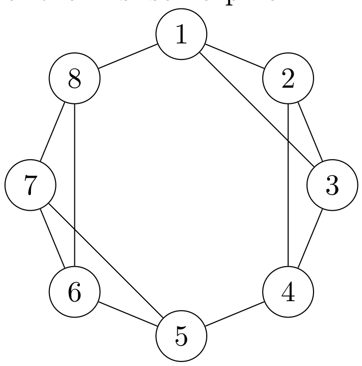
(**a**) Draw a non-planar graph with a Hamiltonian circuit.

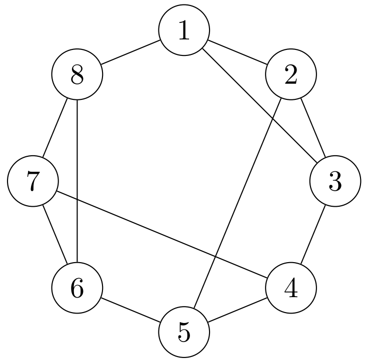
(**b**) Draw a planar graph with no Hamiltonian circuit.

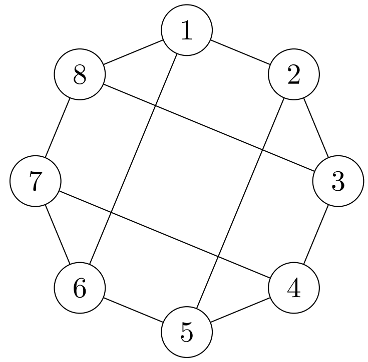
(**c**) Draw a bipartite graph with an Euler trail but no Euler cycle.

(**d**) Draw a bipartite, non-planar graph with a Hamiltonian circuit.

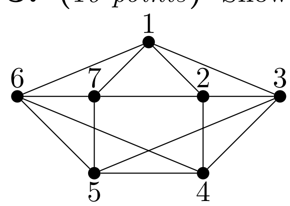
2. (*15 points*) All three of these graphs have 8 vertices, each of degree 3. Show that no pair of them is isomorphic.



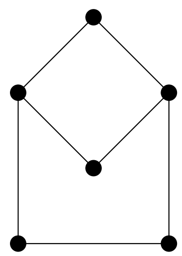




3. (*10 points*) Show that this graph is non-planar.



4. (*10 points*) Consider the following graph:



(**a**) Show that this graph can be colored using 3 colors.

(**b**) Show that this graph cannot be colored using 2 colors.

5. (*15 points*) $G = (V, E)$ is a graph with an Euler cycle. (In order for this to make sense, you may assume that $|V| \geq 3$.) Suppose I add some additional edges to obtain a new graph, $G' = (V, E \cup E')$. (So $E' \cap E = \emptyset$ and $|E'| > 0$; that is, I add at least one edge, and I only add new edges.) If G' also has an Euler cycle, prove that $|E'| \geq 3$. (That is, prove that if we add exactly 1 or exactly 2 edges, we can't get an Euler cycle. It matters that we're in a graph, not a multigraph—there are no loops or multiple edges.)

6. (*10 points*) Show that if n is even and $n \geq 4$ then there is a graph with n vertices such that every vertex has degree 3 and the graph has a Hamilton circuit.

7. (*20 points*) We want to prove that every finite connected graph has a connected *spanning* (containing every vertex) subgraph with no circuits. Let p_k be the statement "Every connected graph with k vertices has a spanning subgraph with no circuits."

(**a**) Prove p_1, that every connected graph with 1 vertex has a spanning subgraph with no circuits.

(**b**) Suppose that p_k is true. Prove p_{k+1}.

(**c**) Using the previous two parts, prove that *any* finite connected graph has a spanning subgraph with no circuits.