PRACTICE MIDTERM

Math 570

Name: ______________________________

ID: ______________________________

"I have adhered to the Penn Code of Academic Integrity in completing this exam."

Signature: ______________________________

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.
- You may use any result proved in class or in the textbook in your arguments.
- Circle or otherwise indicate your final answers.
- Good luck!

1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (*20 points*) One of the following is valid, and the other is not. Give a deduction of the valid one and prove that there is no deduction of the other.

$$\forall x\phi \to \exists x\phi, \qquad \exists x\phi \to \forall x\phi.$$

2. (*20 points*) Consider a language with a unary function symbol **g** and a binary predicate symbol **P**. Give two examples of models which are not elementarily equivalent.

3. (*20 points*) Consider a language with a single constant symbol $\mathbf{e}$, a binary function symbol $\mathbf{f}$, and a unary predicate symbol $\mathbf{Q}$. Let $\mathfrak{A}$ be a model with:

- $|\mathfrak{A}| = \mathbb{Q}$,
- $\mathbf{e}^{\mathfrak{A}} = 0$,
- $\mathbf{f}^{\mathfrak{A}}(q, r) = q - r$,
- $\mathbf{Q}^{\mathfrak{A}} = \{1/z \mid z \in \mathbb{Z} \setminus 0\}$.

Let $s(v_i) = 1/i$.

(**a**) What is $\overline{s}(e)$?

(**b**) What is $\overline{s}(\mathbf{f}v_2v_3)$?

(**c**) Does $\models_{\mathfrak{A}} \mathbf{Q}fv_2v_3[s]$ hold?

(**d**) Does $\models_{\mathfrak{A}} \forall v_4(\mathbf{Q}v_4 \rightarrow \mathbf{Q}fev_4)[s]$ hold?

4. (*20 points*) You have a computer terminal into which you can type 0's and 1's. At any point (that is, after you have typed some finite sequence), it may reject the input with an error. (For instance, you might type 0, then 1, and then 0, at which point it gives an error, so it rejects the sequence 010.) Note that if a sequence is rejected, all longer sequences are also rejected (since if you tried to type them, you'd get stopped along the way).

Suppose that for every possible length, there is at least one input of that which is not rejected. (And therefore all initial segments must also not be rejected.) Prove that there is a sequence of infinite length which is not rejected. (That is, every finite initial segment of this infinite sequence is not rejected.)

5. (*20 points*) Consider a language with a single unary function symbol f, a single unary predicate symbol P, and $=$. Let Σ be the sentences:

- $\forall x f f x = x$,
- $\forall x P x \leftrightarrow \neg P f x$
- $\exists x_1 \exists x_2 x_1 \neq x_2$,
- $\exists x_1 \exists x_2 \exists x_3 x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3$,
- $\cdots$

(The last two sentences and $\cdots$ collectively say there are infinitely many elements.) Prove that $Cn\Sigma$ is complete.