FINAL EXAM

Math 3A
12/10/2009

Name: _____

Signature: _____

Section: _____

Read all of the following information before starting the exam:

- NO CALCULATORS!
- Check your exam to make sure all pages are present.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Whenever you invoke a theorem to justify a result, make sure to clearly identify all premises of the theorem, show that they are true, and specify which theorem you are using.
- Circle or otherwise indicate your final answers.
- Good luck!

1	20	
2	20	
3	20	
4	30	
5	30	
6	25	
7	25	
8	30	
Total	200	

1. (*20 points*) Find the following derivatives:

(**a**) $\frac{d}{dx} x(\ln x)$

(**b**) $\frac{d}{dx} \ln(\tan(e^{\sin^{-1} x}))$

(**c**) $\frac{d}{dx} \frac{(x^2+x)e^x \sin x}{(x^4+1)\cos x}$

(**d**) $\frac{d^{1000}}{dx^{1000}} (\sin x + \cos x)$

2. (*20 points*) Indicate the type of indeterminate form of the following limits, if any, and find the value of the limit.

(**a**) $\lim_{x\to\infty} \frac{x \ln x}{e^x}$

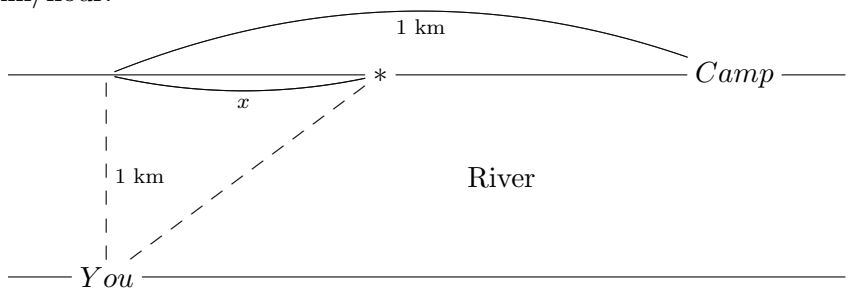
(**b**) $\lim_{x\to\infty} x^{\frac{1}{x^2}}$

(**c**) $\lim_{x\to 2^+} \frac{x-3}{x-2}$

3. (*20 points*) (**a**) Find the general antiderivative of $x^3 + \sec^2(2x)$.

(**b**) Find the particular antiderivative of $f(x) = \frac{1}{x^3}$ such that $F(1) = 1$.

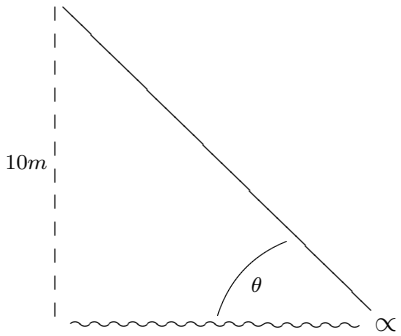
4. (*30 points*) You are on one side of a river 1 km wide. You wish to reach your camp, which is on the other side of the river and 1 km away. You can swim at 3 km/hour and walk at 5 km/hour.



(**a**) Suppose you swim to a point $*$ on the far side of the river (a horizontal distance of x km), and then walk the rest of the way ($1 - x$ km) to the camp. What is the total time it takes you to reach the camp, as a function of x? (Remember that the time taken is given by $\frac{\text{distance}}{\text{rate}}$.)

(**b**) What are the candidates (both critical points and endpoints) for the value of x which minimizes the time taken?

5. **(*30 points*)** A fish (on the surface of the water) is being reeled in from a point $10m$ above the surface of the water. The fishing line being shortened at a rate of $2m/s$.



(**a**) Give an equation relating the angle θ between the fishing line and the water with the length of the fishing line.

(**b**) When the fishing line is $25m$ long, what is $\cos\theta$?

(**c**) How quickly is θ changing when the fishing line is $25m$ long?

6. (*25 points*) Let $f(x) = x^6 + 4x^4 + x^2 - 7$.

(**a**) Show that $f(x)$ has at least two distinct real roots.

(**b**) Suppose $f(x)$ had three distinct roots, $a < b < d$; conclude that $f'(x)$ must have two distinct real roots.

(**c**) Show that f' cannot have two distinct real roots.

7. (*25 points*) A cylinder with height exactly 2 m has volume $V(r) = 2\pi r^2$ where r is the radius of the base.

(**a**) Give an approximation for the relative error in the volume $V(r)$ in terms of the relative error in r.

(**b**) If the relative error in r is 0.02, what is the relative error in $V(r)$?

8. (*30 points*) This problem concerns the function

$$f(x) = \frac{x^2 + 4x - 5}{x - 2}.$$

(Note that the rest of the exam—parts (a) through (i) over three pages—are all referring to this function.) The following information may be useful:

- $f'(x) = \frac{x^2-4x-3}{(x-2)^2}$
- $f''(x) = \frac{14}{(x-2)^3}$
- $f(x) = x + 6 + \frac{7}{x-2}$

(**a**) Identify the points where f is 0 or undefined, and the intervals where f is positive or negative. (You may indicate intervals in any intelligible format you like.)

(**b**) Identify the critical points of f, and the intervals where f is increasing or decreasing.

(**c**) Identify the inflection points of f, and the intervals where f is concave up or concave down.

(**d**) Determine

$$\lim_{x \to 2^-} f(x)$$

(**e**) Determine

$$\lim_{x \to 2^+} f(x)$$

(**f**) Determine

$$\lim_{x \to \infty} f(x) \text{ and } \lim_{x \to \infty} [f(x) - (x + 6)]$$

(**g**) Determine

$$\lim_{x \to -\infty} f(x) \text{ and } \lim_{x \to -\infty} [f(x) - (x + 6)]$$

(**h**) Describe all asymptotes (horizontal, vertical, or oblique) of f.

(**i**) Sketch a graph of f. Be sure to indicate all zeros, critical points, inflection points, and asymptotes of f.