5.4.4

Let l be the length of the rectangular area and let h be its height. Then the perimeter is $p = 2l + 3h$. The area is $A = lh = 384$, so $l = 384/h$. Therefore $p = 2 * 384/h + 3h$, so

$$\frac{dp}{dh} = -\frac{2 * 384}{h^2} + 3.$$

Setting $\frac{dp}{dh} = 0$, we have $3 = \frac{2*384}{h^2}$, so

$$h^2 = \frac{2 * 384}{3}.$$

Solving for h, we get $h = \pm 16$. Since h must be positive, the critical point is $h = 16$. Finally, checking the second derivative, we have

$$\frac{d^2p}{dh^2} = \frac{4 * 384}{h^3} > 0$$

so this graph is always concave up when $h > 0$, so $h = 16$ must be a minimum, and therefore the minimum perimeter is 96.

5.4.12

We wish to minimize $d = \sqrt{x^2 + y^2}$ given that $y = 1 + 2x$. It suffices to minimize $D = d^2 = x^2 + y^2$. Substituting for y, we have $D = x^2 + (1 + 2x)^2 = x^2 + 1 + 4x + 4x^2 = 5x^2 + 4x + 1$.

$$\frac{dD}{dx} = 10x + 4.$$

Setting this equal to 0, we have $10x + 4 = 0$, so $x = -2/5$. There are no endpoints, and $\frac{d^2D}{dx^2} = 10 > 0$, so the graph is always concave up, so $x = -2/5$ is a minimum.

5.4.22

We have $b = 20 - a$ and wish to maximize $m = ab = a(20 - a) = 20a - a^2$. $\frac{dm}{da} = 20 - 2a$, so setting $20 - 2a = 0$, $a = 10$, so $b = 10$ and $m = ab = 100$.

5.5.8

$\lim_{x\to 0} x \sin x = 0$, $\lim_{x\to 0} 1 - \cos x = 0$, so this is an indeterminate form of type 0/0, so it is equal by LH to

$$\lim_{x\to 0} \frac{x \cos x + \sin x}{\sin x}.$$

By LH again, this is equal to

$$\lim_{x\to 0} \frac{\cos x - x \sin x + \cos x}{\cos x} = 2.$$

5.5.16

Top and bottom are both going to 0, this has type 0/0, so by LH, this is equal to

$$\lim_{x\to 0} \frac{5^x \ln 5}{7^x \ln 7} = \frac{\ln 5}{\ln 7}.$$

5.5.26

This is equal to $\lim_{x\to\infty} \frac{x^2}{e^x}$, which by LH is equal to

$$\lim_{x\to\infty} \frac{2x}{e^x} = [\lim_{x\to\infty} \frac{2}{e^x} = 0.$$

5.5.28

By the same method as the previous part, after n derivatives applied to $\frac{x^n}{e^x}$, the top becomes $n \cdot (n-1) \cdots$ while the bottom remains e^x, so this is equal to 0.

5.5.56

The top goes to 0 while the bottom goes to 1, so the answer is 0.