MIDTERM 2

Math 3A
11/16/2009

Name: _____

Signature: _____

Section: _____

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Whenever you invoke a theorem to justify a result, make sure to clearly identify all premises of the theorem, show that they are true, and specify which theorem you are using.

- Circle or otherwise indicate your final answers.

- Good luck!

1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (*20 points*) Find the specified derivatives of the following fuctions:

(a) $\frac{d}{dx} e^{\sin\sqrt{\tan x}}$

(b) $\frac{d}{dt} \frac{(t^2+t+7)e^t \sin t \cos t}{\sqrt{t+3}}$

(c) $\frac{d}{dx} f^{-1}(x)$ where $f(t) = \frac{\sin t}{1+t^2}$ (remember $f^{-1}(x)$ is the inverse of $f(t)$, so $f^{-1}(f(t)) = t$ and $f(f^{-1}(x)) = x$)

(d) $\frac{d^2}{dr^2}\left[r^7 + 7r^6 + 5r\right]$

(e) $\frac{d^{100}}{dx^{100}} xe^x$

2. (*20 points*) A spotlight is 30 feet from a wall. A 6 foot tall person is between the spotlight and the wall, walking towards the wall at 10 feet per minute. The person casts a shadow on the wall, caused by the spotlight. (Note that this question has a total of 5 parts.)

$Wall$ ——————— $Person$ ——————— θ
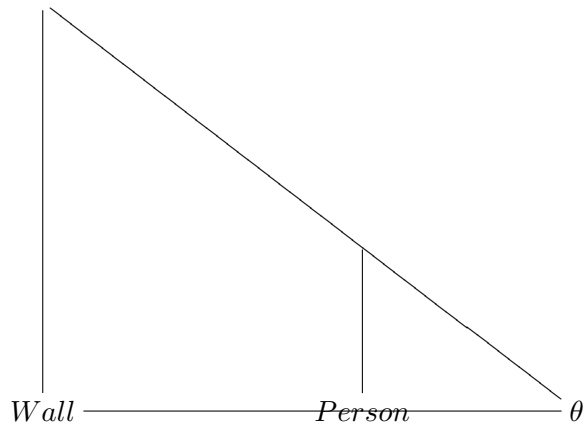
(**a**) Write an equation giving the relationship between the angle θ and the distance between the person and the spotlight.

(**b**) Write an equation giving the relationship between the angle θ and the height of the person's shadow.

(**c**) When the person's shadow is 18 feet tall, how far is the person from the spotlight?

(**d**)   Write an equation relating the rate of change of θ to the distance between the person and the spotlight and the rate the person is moving towards the wall.

(**e**)   How quickly is the angle θ changing when the person's shadow is 18 feet tall, and in which direction? (For full credit, do NOT leave unevaluated trig expressions in the answer.)

3. (*20 points*) Let c be a constant. Approximate the value of $e^{0.1 \cdot c}$ using a linearization.

4. (*20 points*)   (**a**)   Consider the function $f(x) = x^{2/3}$, so $f(-1) = f(1) = 1$, but there is no point where $f'(x) = 0$. Why doesn't this contradict the Mean Value Theorem?

(**b**)   Sketch an example of a function which is defined on the interval $[0, 1]$ and has both a local minimum and a local maximum in this interval, but has neither a global minimum nor a global maximum on this interval. (You don't need to be able to write down a formula for the function—you can make up any function as long as you can represent it in a sketch.)

5. (*20 points*) This problem concerns the function

$$f(x) = \frac{x+1}{x^2 - 2x + 1}$$

The following information may be useful:

- $f'(x) = -\frac{x+3}{(x-1)^3}$

- $f''(x) = \frac{2x+10}{(x-1)^4}$

    (**a**)   Identify the points where f is 0 or undefined, and the intervals where f is positive or negative.

    (**b**)   Identify the critical points of f, and the intervals where f is increasing or decreasing.

    (**c**)   Identify the inflection points of f, and the intervals where f is concave up or concave down.

(d)   Determine $\lim_{x \to 1^-} f(x)$

(e)   Determine $\lim_{x \to 1^+} f(x)$

(f)   Determine $\lim_{x \to \infty} f(x)$

(g)   Determine $\lim_{x \to -\infty} f(x)$

(h)   Describe all asymptotes (horizontal, vertical, or oblique) of f

(**i**)   Sketch a graph of f. Be sure to indicate all zeros, critical points, inflection points, and asymptotes of f.