HW 4 Solutions

7.1 37. Let $u = ax^2+bx+c$, $\frac{du}{dx} = 2ax+b$, $du = 2ax+b\,dx$

$$\int \frac{2ax+b\,dx}{ax^2+bx+c} = \int \frac{1}{u}\,du = \ln|u| + k = \ln|ax^2+bx+c| + k$$

(k is a constant — c is already taken)

38. Let $u = ax+b$ $\frac{du}{dx} = a$, $dx = \frac{du}{a}$

$$\int \frac{1}{ax+b}\,dx = \int \frac{1}{ua}\,du = \frac{1}{a}\int \frac{1}{u}\,du = \frac{1}{a}\ln|u| + c = \frac{1}{a}\ln|ax+b| + c$$

41. $g'(x) = \frac{dg(x)}{dx}$, $dx = \frac{1}{g'(x)}\,d(g(x))$

$$\int g'(x)e^{-g(x)}\,dx = \int g'(x)e^{-g(x)}\frac{1}{g'(x)}\,d(g(x)) = \int e^{-g(x)}\,d(g(x)) = -e^{-g(x)} + c$$

42. $g'(x) = \frac{dg(x)}{dx}$, $dx = \frac{1}{g'(x)}\,d(g(x))$

$$\int \frac{g'(x)}{[g(x)]^2+1}\,dx = \int \frac{1}{[g(x)]^2+1}\,d(g(x)) = \tan^{-1}(g(x)) + c$$

56. Let $u = x^2+1$ $\frac{du}{dx} = 2x$ $dx = \frac{du}{2x}$

$$\int \frac{x}{(x^2+1)\ln(x^2+1)}\,dx = \frac{1}{2}\int \frac{1}{u\ln(u)}\,du$$ Let $w = \ln(u)$ $\frac{dw}{du} = \frac{1}{u}$, $du = u\,dw$

$$= \frac{1}{2}\int \frac{1}{w}\,dw = \ln|w| + c = \ln|\ln(u)| + c = \ln|\ln(x^2+1)| + c$$

7.2 7. Let $u = x$ $\quad v = e^x$

$du = dx$ $\quad dv = e^x dx$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + c$$

8. Let $u = 3x$ $\quad v = -2e^{-x/2}$
$du = 3\,dx$ $\quad dv = e^{-x/2}\,dx$

$$\int 3xe^{-x/2}\,dx = -6xe^{-x/2} + \int 6e^{-x/2}\,dx = -6xe^{-x/2} - 12e^{-x/2} + C$$

25. Let $u = \sin x$ $\quad v = e^x$
$du = \cos x\,dx$ $\quad dv = e^x\,dx$

$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$ $\quad$ Let $\tilde{u} = \cos x$ $\quad \tilde{v} = e^x$
$d\tilde{u} = -\sin x$ $\quad d\tilde{v} = e^x\,dx$

$\int e^x \cos x\,dx = e^x \cos x + \int e^x \sin x\,dx$. Combining these equations,

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$, so

$$\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int_0^{\pi/3} e^x \sin x\,dx = \frac{e^{\pi/3}(\frac{\sqrt{3}}{2}) - e^{\pi/3}(\frac{1}{2})}{2} - \frac{e^0(0) - e^0(1)}{2} = \frac{e^{\pi/3}(\sqrt{3}-1)+2}{4}$$

26. As above, $\int e^x \cos x\,dx = e^x \cos x + \int e^x \sin x\,dx$, and $\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$

So $\int e^x \cos x\,dx = e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$, $\int e^x \cos x\,dx = \frac{e^x \cos x + e^x \sin x}{2} + C$

$$\int_0^{\pi/6} e^x \cos x\,dx = \frac{e^{\pi/6}(\frac{\sqrt{3}}{2}) + e^{\pi/6}(\frac{1}{2})}{2} - \frac{e^0(1) + e^0(0)}{2} = \frac{e^{\pi/6}(\sqrt{3}+1) - 2}{4}$$

31. Let $u = \cos x$ $\quad v = \sin x$
$du = -\sin x\,dx$ $\quad dv = \cos x\,dx$

$$\int [\cos(x)]^2\,dx = \cos x \sin x + \int \sin^2 x\,dx = \cos x \sin x + \int 1 - \cos^2 x\,dx$$

$$\int \cos^2 x\,dx + \int \cos^2 x - 1\,dx = \cos x \sin x, \quad \int \cos^2 x\,dx = \frac{1}{2}\left[\cos x \sin x - \int dx\right]$$

$$= \frac{1}{2}\cos x \sin x - \frac{1}{2}x + C$$

32. Let $u = \sin x$ $\quad v = -\cos x$

$du = \cos x\,dx$ $\quad dv = \sin x\,dx$

$\int \sin^2 x\,dx = -\sin x\cos x + \int \cos^2 x\,dx = -\sin x \cos x + \int 1 - \sin^2 dx$

$\int 2\sin^2 x - 1\,dx = -\sin x\cos x, \int \sin^2 x\,dx = \frac{1}{2}\left[-\sin x\cos x + \int dx\right]$

$\int \sin^2 x\,dx = -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + c$

33. Let $u = \arcsin x$ $\quad v = x$

$du = \frac{dx}{\sqrt{1-x^2}}$ $\quad dv = dx$

$\int (\arcsin x)(1)\,dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

Let $w = 1 - x^2$ $\quad \frac{dw}{dx} = -2x$ $\quad dx = \frac{1}{-2x}\,dw$

$\int \arcsin x\,dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\left(\frac{1}{-2x}\right)dw = x\arcsin x + \frac{1}{2}\int w^{1/2}\,dw$

$= x\arcsin x + \frac{1}{2}\left(\frac{2}{3}\right)w^{3/2} + c = x\arcsin x + \frac{1}{3}\left(1-x^2\right)^{3/2}$

34. Let $u = \arccos x$ $\quad v = x$

$du = -\frac{dx}{\sqrt{1-x^2}}$ $\quad dv = dx$

$\int (\arccos x)(1)\,dx = x\arccos x + \int \frac{x}{\sqrt{1-x^2}}\,dx$ Pick w as above

$\int \arccos x\,dx = x\arccos x - \frac{1}{2}\int w^{1/2}\,dw = x\arccos x - \frac{1}{3}(1-x^2)^{3/2}$

36. (a) Let $u = x^n$ $\quad v = e^x$

$du = nx^{n-1}\,dx \quad dv = e^x\,dx$

$$\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$$

(b) $\int x^3 e^x\,dx = x^3 e^x - 3\int x^2 e^x\,dx = x^3 e^x - 3x^2 e^x + 6\int x e^x\,dx$

$= x^3 e^x - 3x^2 e^x + 6xe^x - 6\int e^x\,dx$

$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$

37. (a) Let $u = x^n$ $\quad v = \frac{1}{a} e^{ax}$

$du = nx^{n-1} \quad dv = e^{ax}\,dx$

$$\int x^n e^{ax}\,dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a}\int x^{n-1} e^{ax}\,dx$$

(b) $\int x^2 e^{-3x}\,dx = -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\int x\, e^{-3x}\,dx$

$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx\right]$

$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}\,dx$

$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$

46. Let $u = \sqrt{x+1}$ $\quad \frac{du}{dx} = \frac{1}{2}\frac{1}{\sqrt{x+1}}$ $\quad dx = 2\sqrt{x+1}\,du = 2u\,du$

$\int_1^2 e^{\sqrt{x+1}}\,dx = 2\int_{x=1}^{x=2} u e^u\,du$ $\quad$ Let $\tilde{u} = u$ $\quad v = e^u$

$d\tilde{u} = du \quad dv = e^u\,du$

$= 2ue^u\Big]_{x=1}^{x=2} - 2\int_{x=1}^{x=2} e^u\,du = 2ue^u - 2e^u\Big]_{x=1}^{x=2}$

When $x=1$, $u=\sqrt{2}$. When $x=2$, $u=\sqrt{3}$

$$\int_1^2 e^{\sqrt{x+1}}\,dx = 2\sqrt{3}e^{\sqrt{3}} - 2e^{\sqrt{3}} - 2\sqrt{2}e^{\sqrt{2}} + 2e^{\sqrt{2}}$$

41. Let $u = \sqrt{x}$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\quad dx = 2\sqrt{x}\,du = 2u\,du$

$$\int_1^4 \ln(\sqrt{x}+1)\,dx = \int_{x=1}^{x=4} \ln(u+1)\,2u\,du$$

Let $\tilde{u} = \ln(u+1)$ $\quad v = u^2$

$d\tilde{u} = \frac{1}{u+1}\,du$ $\quad dv = 2u\,du$

$$\int_{x=1}^{x=4} \ln(u+1)\,2u\,du = u^2\ln(u+1)\Big]_{x=1}^{x=4} - \int_{x=1}^{x=4} \frac{u^2}{u+1}\,du$$

$$= u^2\ln(u+1)\Big]_{x=1}^{x=4} - \int_{x=1}^{x=4} \frac{u^2-1}{u+1}\,du - \int_{x=1}^{x=4} \frac{1}{u+1}\,du$$

$$= u^2\ln(u+1)\Big]_{x=1}^{x=4} - \int_{x=1}^{x=4} u-1\,du - \int_{x=1}^{x=4} \frac{1}{u+1}\,du$$

$$= u^2\ln(u+1) - u^2 + u - \ln(u+1)\Big]_{x=1}^{x=4}$$ when $x=1, u=1$ when $x=4, u=2$

$$= 4\ln(3) - 4 + 2 - \ln(3) - \ln(2) - 1 + 1 - \ln(2)$$
$$= 3\ln(3) - 2\ln(2) - 2$$
$$= \ln(108) - 2$$

59. Let $u = x^2+3$ $\quad \frac{du}{dx} = 2x$ $\quad dx = \frac{1}{2x}\,du$

$$\int \frac{x}{x^2+3}\,dx = \int \frac{x}{x^2+3}\,\frac{1}{2x}\,du = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(u) + C$$
$$= \frac{1}{2}\ln(x^2+3) + C$$

60. $\int \frac{x+2}{x^2+2}\,dx = \int \frac{x}{x^2+2} + \frac{2}{x^2+2}\,dx$ let $u = x^2+2$ $\quad \frac{du}{dx} = 2x$ $\quad dx = \frac{1}{2x}\,du$

$w = \frac{1}{\sqrt{2}}x$ $\quad \frac{dw}{dx} = \frac{1}{\sqrt{2}}$ $\quad dx = \sqrt{2}\,dw$

$$\int \frac{x+2}{x^2+2}\,dx = \int \frac{x}{x^2+2}\,\frac{1}{2x}\,du + \int \frac{2}{x^2+2}\sqrt{2}\,dw$$

$$= \frac{1}{2}\int \frac{1}{u}\,du + \sqrt{2}\int \frac{2}{2w^2+2}\,dw = \frac{1}{2}\int \frac{1}{u}\,du + \sqrt{2}\int \frac{1}{w^2+1}\,dw$$

$$= \frac{1}{2}\ln(u) + \sqrt{2}\arctan(w) + C = \frac{1}{2}\ln(x^2+2) + \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + C$$

62. Let $t = -x^2/2$ $\quad \frac{dt}{dx} = -x \quad dx = \frac{-1}{x}\,dt$

$\int x^3 e^{-x^2/2}\,dx = \int x^3 e^t \left(-\frac{1}{x}\right) dt = -\int x^2 e^t\,dt = 2\int t e^t\,dt$

Let $u = t$ $\quad v = e^t \quad 2\int te^t\,dt = 2te^t - 2\int e^t\,dt$

$du = dt \quad dv = e^t\,dt$

$\int x^3 e^{-x^2/2} = 2te^t - 2e^t + c = -x^2 e^{-\frac{x^2}{2}} - 2e^{-\frac{x^2}{2}} + c$

63. Let $u = (x-2)^{1/4}$, $x = u^4 + 2$, $\frac{du}{dx} = \frac{1}{4}(x-2)^{-3/4} = \frac{1}{4}u^{-3}$, $dx = 4u^3\,du$

$\int x(x-2)^{1/4}\,dx = \int (u^4+2)\,u\,4u^3\,du = \int 4u^8 + 8u^4\,du$

$= \frac{4}{9}u^9 + \frac{8}{5}u^5 + c = \frac{4}{9}(x-2)^{9/4} + \frac{8}{5}(x-2)^{5/4} + c$

7.3 1.

$$
\begin{array}{r|l}
 & 2x + 1 \quad R -1 \\
x+2 & 2x^2 + 5x + 1 \\
 & -(2x^2 + 4x) \\
 & \quad x + 1 \\
 & \quad -(x+2) \\
 & \quad -1
\end{array}
$$

$\frac{2x^2+5x+1}{x+2} = 2x + 1 - \frac{1}{x+2}$

2.

$$
\begin{array}{r|l}
 & x - 3 \quad R\ 4 \\
x-1 & x^2 - 4x - 1 \\
 & -(x^2 - x) \\
 & \quad -3x - 1 \\
 & \quad -(-3x+3) \\
 & \quad 4
\end{array}
$$

$\frac{x^2-4x-1}{x-1} = x - 3 + \frac{4}{x-1}$

7. $\dfrac{4x^2-14x-6}{x(x-3)(x+1)} = \dfrac{A}{x} + \dfrac{B}{x-3} + \dfrac{C}{x+1}$

$4x^2-14x-6 = A(x-3)(x+1) + Bx(x+1) + Cx(x-3)$

Setting $x=0$, $-3A=-6$, so $A=2$

Setting $x=3$, $36-42-6=-12=12B$, so $B=-1$

Setting $x=-1$ $4+14-6=12=C(-1)(-4)$ so $C=3$

$$\frac{4x^2-14x-6}{x(x-3)(x+1)} = \frac{2}{x} + \frac{-1}{x-3} + \frac{3}{x+1}$$

8. $\dfrac{16x-6}{(2x-5)(3x+1)} = \dfrac{A}{2x-5} + \dfrac{B}{3x+1}$

$A(3x+1)+B(2x-5)=16x-6$

$3Ax+2Bx=16x$, $3A+2B=16$

$A-5B=-6$, $A=5B-6$

$3(5B-6)+2B=16$

$17B=34$, $B=2$

$A=5(2)-6=4$

$$\frac{16x-6}{(2x-5)(3x+1)} = \frac{4}{2x-5} + \frac{2}{3x+1}$$