FINAL EXAM

Math 340
12/12/2012

Name: ______________________

ID: ______________________

"I have adhered to the Penn Code of Academic Integrity in completing this exam."

Signature: ______________________

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.
- You do not need to simplify answers. You may include factorials, $P(n, k)$, $\binom{n}{k}$, etc. in your answers. Do not use $\sum$ in your answers.
- You may use writing implements and a single 3"x5" notecard.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	17	
2	20	
3	10	
4	12	
5	15	
6	12	
7	14	
Total	100	

1. (*17 points*) A class has 24 students. On the exam, 9 of them use black pens, 6 use blue pens, 6 use green pens, and 3 use red pens.

(**a**) The students all sit in a long row in the front of the classroom. How many possible orders are there for the students to sit in?

(**b**) When the students finish, each one leaves the pen at their desk. Assuming that the pens are identical except for color, how many possible orders are there for the pens?

Continuing the problem from the previous page, involving 9 black pens, 6 blue pens, 6 green pens, and 3 red pens.

(**c**) The professor takes 5 of the pens home. How many possible combinations of colors are there for the pens the professor takes? (Here we only distinguish the quantity of each color.)

(**d**) How many ways are there for the professor to talk 5 of the pens home with at most two of each color. (Again, we only distinguish the quantity of each color.)

2. (*20 points*) (**a**) Find the general solution to the recurrence relation

$$a_{n+2} = -6a_{n+1} - 9a_n.$$

(**b**) Find the particular solution to the recurrence relation

$$a_{n+2} = -6a_{n+1} - 9a_n$$

such that $a_0 = 1$ and $a_1 = 6$.

(c) Find *a* particular solution to the recurrence relation

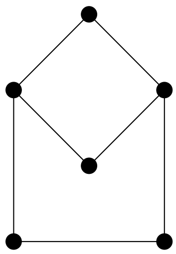
$$a_{n+2} = -6a_{n+1} - 9a_n + 1.$$

(d) Find *a* particular solution to the recurrence relation

$$a_{n+2} = -6a_{n+1} - 9a_n + (-3)^n.$$

3. (*10 points*) Show that if n is even and $n \geq 4$ then there is a graph with n vertices such that every vertex has degree 3 and the graph has a Hamilton cycle.

4. (*12 points*) Consider the following graph:



(**a**) Show that this graph can be colored using 3 colors.

(**b**) Show that this graph cannot be colored using 2 colors.

5. (*15 points*) Prove (using induction on the number of vertices) that if every vertex in G has degree at most k then it is possible to color G using $k+1$ colors.

6. (*12 points*) Consider the field $\mathbb{Z}_3[x]/(x^3+2x^2+x+2)$. Find the following sum and product. In all cases, give the (unique) representative of the equivalence class which is a polynomial of degree less than 3 with all coefficients integers 0, 1, or 2.

(**a**) $[x^2+x]+[2x+2]$

(**b**) $[x^2+x]\cdot[2x+2]$

(**c**) Find the multiplicative inverse of $[x^2+x]$.

7. (*14 points*) A professor gives an exam with three problems, and needs to decide how many points each problem is worth. The first problem needs to be worth an odd number of points, the second problem needs to be worth a number of points which is a multiple of 5, and the third problem needs to be worth between 2 and 6 points. The three problems should add to $n \geq 8$ points. (Of course, all problems are worth a positive number of points!)

(**a**) Give a generating function for the numbers of ways to assign points to the three problems; your answer may include $\sum$.

(**b**) Give the same generating function without using $\sum$.