Bargaining Shocks and Aggregate Fluctuations
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Abstract

We argue that social and political risk causes significant aggregate fluctuations by changing workers’ bargaining power. Using a Bayesian proxy-VAR estimated with U.S. data, we show how distribution shocks trigger output and unemployment movements. To quantify the aggregate importance of these distribution shocks, we extend an otherwise standard neoclassical growth economy. We model distribution shocks as exogenous changes in workers’ bargaining power in a labor market with search and matching. We calibrate our economy to the U.S. corporate non-financial business sector, and we back out the evolution of workers’ bargaining power. We show how the estimated shocks agree with the historical narrative evidence. We document that bargaining shocks account for 28% of aggregate fluctuations and have a welfare cost of 2.4% in consumption units.

Keywords: Distribution risk, bargaining shocks, aggregate fluctuations, partial filter, historical narrative.

1 Introduction

In this paper, we argue that the social and political distribution risk between labor and capital is a quantitatively relevant source of aggregate fluctuations. To do so, we begin by documenting political distribution risk in the data, as one specific form of distribution risk. We use three empirical exercises. First, using the volatility of capital shares, we document considerable changes in how income was divided between capital and labor in France, the U.K., and the U.S. over the last century and a half. Second, we analyze the consequences for distribution of the adoption of right-to-work legislation by several U.S. states. We estimate that the introduction of right-to-work legislation in a state is followed, on average, by increases in the state capital share of 1.5-1.6 percentage points (pp.) relative to the U.S. five years after adoption. Third, we focus on the U.S. by estimating a Bayesian proxy-vector autoregression (VAR). We proxy the redistributive shock by legislated changes in the federal and state-level minimum wages. In different specifications, we document the significant effects of these distribution shocks on output, factor shares, and labor markets.

This evidence motivates us to augment a standard stochastic neoclassical growth model with labor search and matching à la Shimer (2010) with shocks to the bargaining power of workers. These shocks are a simple way to capture a central mechanism through which distribution risk operates. Formally, bargaining shocks can be interpreted as arising from social or political influences on the protocol of an underlying dynamic bargaining game (Binmore et al., 1986) through mechanisms such as collective bargaining rules, minimum wage regulations, etc. In our model, this risk is separate from changes due to endogenous movements in the bargaining position of workers and firms, since those movements are reflected in the outside values of the agents.

We identify our model by exploiting the differences in the responses of output and wages to shocks to productivity and the bargaining power of workers. After both a positive shock to productivity and a shock that lowers the bargaining power of workers, output grows. However, wages rise if the former shock occurs, but fall if the latter shock hits the economy. Hence, looking at the comovements of output and wages disentangles one shock from another.

With this identification, we calibrate our model to the U.S. non-financial corporate business sector and the labor market. As a baseline, we look at long-lived bargaining shocks with a half-life of 34 quarters. Thus, our approach deals not only with standard business cycles, but also with medium-term aggregate fluctuations. The half-life of 34 quarters is based on our reading of the changes in the social and political climate regarding the bargaining power of workers in the postwar U.S. and matches the average duration of control of the different branches of the federal government by each party after World War II.

We use U.S. data to back out the bargaining shocks implied by our quantitative model. We do so by applying the partial information filter recently introduced by Drautzburg et al. (2021). The central idea of the partial filter is to take one of the key optimality conditions of the model (in our case, the wage-setting equation), substitute the different conditional expectations of a product of variables by their conditional covariance plus the product of conditional expectations of single variables, and approximate those conditional covariances and expectations of single variables from
a Bayesian VAR (BVAR). Crucially, the backed-out shocks agree with our narrative evidence for the U.S. since WWII, with peaks at moments of significant labor union victories (e.g., the 1970 GM strike) and troughs at moments of weakness of unions (e.g., the early 2000s).

As a robustness check, we explore the behavior of the model with even more persistent bargaining shocks (half-life of 80 quarters) and with less persistent ones (half-life of 14 quarters, a standard business cycle persistence). The qualitative dynamic properties of the model are not significantly affected by this persistence. However, we find that the higher the persistence of bargaining power shocks, the more significant their impact on real fluctuations and the smaller their impact on the income distribution.

We solve our model using a third-order perturbation since we document how the non-linear features of the solution can be significant. These non-linearities also mean that we must calibrate the model to match the moments of its ergodic distribution and not its steady-state properties. To assess the properties of our economy, we compare it with a version of the model without bargaining shocks, with a benchmark real business cycle (RBC) model with productivity shocks, and with an RBC model augmented with factor share shocks in the production function (as in Danthine et al., 2006, Ríos-Rull and Santeaulàlia-Llopis, 2010, and Lansing, 2015).

Other significant findings of the paper are as follows. First, our model replicates the near acyclicality of wages highlighted by Lucas (1977) as an obstacle for equilibrium business cycle models that want to rely on movements in real wages as a source of fluctuations. In our economy, output can increase either because productivity grows, which raises wages, or because bargaining power shifts toward capital, which lowers wages. This finding allows us to discriminate between models. An RBC model with factor share shocks yields wages that are too pro-cyclical: a shock toward labor makes it more productive and, thus, raises wages.

Second, our model accounts reasonably well for the pro-cyclicality of the capital share and the net capital share (i.e., after depreciation). This stands in stark contrast to the version of the model without bargaining shocks and the RBC models (with and without factor share shocks). This result is robust to reasonable values of the elasticity of substitution between capital and labor in the production function.

Third, the bargaining shocks account for around 28% of the volatility of output, nearly all of the model-generated volatility of the gross capital share, and around 45% of the net capital share. When the model is calibrated to match observations from the U.S. labor market, the surplus of the labor relation is small. Minor variations in how this surplus is allocated induce substantial changes in the number of recruiters firms employ to hire new workers. This leads to lower output and employment but also higher wages. We illustrate this point as follows. The U.S. has benefited from a more stable capital share than other industrialized countries. For example, the overall volatility of the capital share is about 40% lower than in the U.K. If increased distribution risk would cause the capital share to become 40% more volatile, our model predicts that output and consumption volatility would be 15% higher. The welfare cost of bargaining shocks is a sizable 2.4% of consumption, much larger than in Lucas (1991).
Finally, we look at the dynamic effects of a bargaining power shock, and, in an Appendix, we perform an extensive battery of robustness exercises. We document, for example, how our main results are not affected when we partly endogenize the bargaining power process by having bargaining power or unemployment benefits depend directly on the business cycle. Nonetheless, the endogeneity of the bargaining shock (and how it reacts to issues such as technological change, globalization, inequality, and others) is a topic that deserves much further exploration than we can cover in this paper.

Our paper builds on a large literature. The recent evolution of the capital share has commanded much attention (e.g., among many others, Autor et al., 2017; Barkai, 2017; Berger et al., 2019; De Loecker and Eeckhout, 2017; Karabarbounis and Neiman, 2014; Koh et al., 2015). Some of the proposed explanations highlight technological change, the fall in the relative price of capital, increases in firm concentration, globalization, or the role of intellectual property products. For our investigation, we can remain agnostic about these mechanisms. Our point is not that all fluctuations in the capital share have a social or political origin: we only claim that part of them do. Also, we focus on fluctuations around a trend rather than on the trend (although we perform some high-persistence exercises). One should expect that the effects of technological change, increased market power, or structural transformation on factor shares would manifest themselves more clearly in the trend than in middle- and high-frequency movements.

Previous work has also focused on how changes in bargaining power affect factor shares. Examples include Blanchard (1997), Caballero and Hammour (1998), and Blanchard and Giavazzi (2003). The papers cited above are concerned with the trend decline in the labor share in Europe, whereas we focus on aggregate fluctuations. Gertler et al. (2008) and Liu et al. (2013) also allow, in passing, for time-varying bargaining power, but they do not study its implications in full. Foroni et al. (2017) have presented VAR evidence of the importance of bargaining supply shocks in employment fluctuations. Ríos-Rull and Santaüllània-Llopis (2010) and Danthine et al. (2006) interpret distribution shocks as technological shocks to the production function. As we will see later, our model outperforms an RBC model with factor share shocks in terms of matching important aggregate fluctuation statistics.

Our bargaining shocks resemble wage markup shocks such as those in Galí et al. (2012). There are some important differences, though. First, our model endogenously generates the equivalent to state-dependent markups over disutility from labor because the surplus of the match is time-varying. Second, we document the importance of higher-order effects related to labor market tightness. These effects are absent in models with markup shocks. Third, we link our shocks to a precise historical narrative. For instance, our partial information filter points out that, from 1950 to the late 1970s, the evolution of bargaining power mostly followed the fate of unions (except for the Kennedy-Johnson wage-posting guidelines). After 1980, changes such as Reagan’s regulation, Clinton’s welfare reform, immigration, and labor market policies such as minimum wages and unemployment extension played a bigger role.

We also link with papers dealing with wage bargaining and aggregate fluctuations. This literature
is too large to do justice to it in a few lines, but we can highlight the textbook treatment in Shimer (2010) and the references there. Interestingly, Shimer (2005) pointed out the potential of bargaining power shocks for resolving the unemployment volatility puzzle, but emphasized the need for clear identification. Our paper focuses much attention on identification and the link between historical evidence and bargaining power shocks.

Finally, our work is closely related to Danthine and Donalson (2002), who study the link between financial and labor markets. Their focus is to understand how time-varying labor shares affect the equity premium. In particular, they show that exogenous and stochastic variation in labor shares faced by workers and firms is essential to account for asset prices. The high welfare cost of the business cycle we find is the dual of the high equity premium documented by these authors. Relative to Danthine and Donalson (2002), first, we provide empirical evidence of the impact of changing bargaining power on the macroeconomy; second, we characterize the bargaining protocol between workers and firms; and third, we analyze the economy’s dynamics, including unemployment following a shock to bargaining power. Also, we show the welfare implications of this bargaining process.

The rest of the paper is organized as follows. Section 2 reviews the historical evidence on the variation in factor shares and political interventions in the U.S. Section 3 presents and computes our model. We take the model to the data in Section 4. Section 5 explains our partial information filter. Section 6 presents the quantitative results and Section 7 reports the dynamic effects of bargaining power shocks. Section 8 concludes. An extensive online appendix discusses further details.

2 Factor shares: Measurement and evidence

Factor shares change over time. Panel (a) of Figure 1 shows the evolution of the net corporate capital share for France, the U.K., and the U.S. since the mid 19th century. Panel (b) plots the same data, except in terms of the gross corporate capital share. The data reveal three facts. First, when we concentrate on the economy’s corporate sector –the object of interest in our model– the capital share does not display a trend, except for an increase at the end of the sample. Second, there has been, nevertheless, significant movement in the capital share over time, including larger changes before WWII. Third, the U.S. has exhibited the least volatile capital share among the three countries: 30% less volatile than in France and 40% less than in the U.K.

The top panel of Table 1 lists the raw and HP-filtered volatility of the gross capital shares for our three countries. The bottom panel shows that income shares are much more volatile in the U.K. than in the U.S. even after controlling for industry composition (and, hence, reducing the effect of structural transformations and technological change). For France, the same is true only in the raw data, but not after detrending. Lastly, while the standard deviation of the capital share of income fell in the U.S. in the post-WWII period by around 43%, it decreased around 65% in the U.K. and 73% in France.

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1See Appendix A for details. Notice that our model later in the paper can handle long-lived increases in capital shares (in one calibration, with half-lives up to 80 quarters).
The data on the U.K. include Ireland prior to its independence. Source: Piketty and Zucman (2014).

Figure 1: Net and gross corporate capital shares in the long run: The U.K., France, and the U.S.

Table 1: Changes in the gross labor share volatility across time and countries

<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td>USA</td>
<td>3.28</td>
<td>1.86</td>
<td>1.42</td>
<td>0.81</td>
<td>0.98</td>
</tr>
<tr>
<td>France</td>
<td>7.14</td>
<td>2.50</td>
<td>4.63</td>
<td>0.75</td>
<td>1.98</td>
</tr>
<tr>
<td>UK</td>
<td>10.16</td>
<td>2.72</td>
<td>7.44</td>
<td>1.14</td>
<td>1.30</td>
</tr>
<tr>
<td>Diff.: France – USA</td>
<td>3.85</td>
<td>0.64</td>
<td>0.94</td>
<td>-0.07</td>
<td></td>
</tr>
<tr>
<td>Diff.: UK – USA</td>
<td>6.88</td>
<td>0.86</td>
<td>0.65</td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
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(b) Within-industry volatility of the gross labor share

<table>
<thead>
<tr>
<th>Country</th>
<th>Raw Mean</th>
<th>Raw SE</th>
<th>HP-filtered Mean</th>
<th>HP-filtered SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>4.54</td>
<td>0.52</td>
<td>1.60</td>
<td>0.23</td>
</tr>
<tr>
<td>Difference: France – USA</td>
<td>2.78</td>
<td>1.00</td>
<td>-0.06</td>
<td>0.34</td>
</tr>
<tr>
<td>Difference: UK – USA</td>
<td>3.77</td>
<td>0.99</td>
<td>0.95</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The data for panel (a) come from Piketty and Zucman (2014) and for panel (b) from the EU KLEMS database and exclude agriculture, mining, and finance. Standard errors in panel (b) clustered by industry and country. HP-filtered with $\lambda = 6.25$.

Next, we argue that part of the variations in factor shares is accounted for by political redistribution by exploiting changes in the right-to-work legislation and minimum wages in the U.S.

2.1 Factor shares and right-to-work legislation in the U.S.

While the U.S. has avoided radical political events that one could use to identify exogenous drivers of labor regulation, there is direct evidence of the effect of policy changes on the labor share at the U.S. state level. Right-to-work legislation was aimed at limiting the bargaining power of unions by
allowing employees to opt out of union membership. Did it succeed?

We use data on states that were late to adopt right-to-work legislation to analyze its effects on the capital share. While most right-to-work states enacted the underlying laws in the first decade after 1945, seven states (Idaho, Louisiana, Oklahoma, Wyoming, Indiana, Michigan, and Wisconsin) adopted this legislation between 1963 and 2015, the period for which we have data on private-industry labor shares. Since Wisconsin adopted right-to-work only in March 2015, our most recent useful observations are for Indiana and Michigan, which passed right-to-work laws in 2012.

The data from Idaho, Louisiana, Oklahoma, and Wyoming indicate increases in the capital share after right-to-work legislation was passed, but with different dynamics. However, three to five years after the adoption of right-to-work legislation, the capital share in all four states has risen relative to that in the U.S. (see Figure 2; capital shares are computed as one minus the share of employees’ compensation in state GDP net of taxes and subsidies). For Indiana and Michigan, their capital share increased initially and then remained flat in absolute terms, while rising relative to that in the U.S. as a whole, but with only three observations, we must be cautious. Also, we cannot generally control for pre-trends in all states because of data limitations.

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Figure 2: Change in state private-industry capital shares after right-to-work adoption

To gauge the statistical significance of the previous numbers, we also use industry-level variation from these states (Table 2). In most specifications, the increase in the capital share following right-to-work legislation is positive, with two-sided \( p \)-values below 0.1 after five years. We can reject the null of decreases in the capital share after right-to-work legislation in most cases. Note that the permanent effect of the right-to-work legislation is ambiguous – potentially because eventually labor-intensive industries disproportionately flock to right-to-work states.

We also look at the effects of right-to-work legislation on state GDP. While the evidence on differences in private-sector real GDP growth is less pronounced, GDP growth weakly increases three to five years after a state adopts right-to-work legislation (see Figure C.6 and Table C.1 in

\[ \text{The point estimates are smaller when we also control for year fixed effects, but highly significant at the four- and five-year horizons. However, the standard errors may be unreliable in these cases and, therefore, we decided not to include the results in the table.} \]
Table 2: State-industry panel regression: Right-to-work laws and gross capital share

<table>
<thead>
<tr>
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<th>Controlling for state FE, and industry FE</th>
<th>Controlling for state FE, quadratic trend, and industry FE</th>
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<tbody>
<tr>
<td></td>
<td>Level</td>
<td>1y change</td>
</tr>
<tr>
<td>Right to Work</td>
<td>0.03</td>
<td>1.00</td>
</tr>
<tr>
<td>Change in RtW</td>
<td>(0.98)</td>
<td>(0.38)</td>
</tr>
</tbody>
</table>


This finding is consistent with Holmes (1998) and Alder et al. (2014). These papers analyze the effects of right-to-work legislation on the location of manufacturing and find positive effects on manufacturing activity. To provide more detail on the economic dynamics following a distribution shock, we now turn to a VAR estimated for the U.S. economy.

2.2 Factor shares and minimum wages in the U.S.

Besides changing the bargaining power of workers, the government can directly affect how the surplus is split between firms and workers by mandating a minimum wage. As Flinn (2006, p. 1014) points out: “Increases in the minimum wage can be viewed as a way to increase their [i.e., workers’] “effective” bargaining power.” Even though less than 10% of hours worked in the U.S. are paid at or below minimum wage (Autor et al., 2016), increases in statutory minimum wages often spill over to higher wage groups due to indexing or tournament wage structures. For the U.S., Lee (1999) estimates that the spillovers are big enough to account for more than 100% of the change in the ratio of the 50th to the 10th wage percentile after a decline in the real minimum wage. More recently, Autor et al. (2016) find spillovers in the order of 30-40%. Similar findings are reported by Harris and Kearney (2014), who argue that the “ripple effects” of an increase in the minimum wage will raise the wages of nearly 30% of the workforce.

Motivated by these results, we analyze how changes in the statutory federal minimum wage impact the capital income share. Figure 3 shows how the corporate non-financial capital share changed relative to the real change in the statutory minimum wage.\(^3\) Once we exclude the outlier in 1950, we find a robust negative relationship between changes in the statutory minimum wage and

\(^3\) We focus on the original federal minimum wage that covered only employees engaged in interstate commerce. From 1961 onward, other federal minimum wage rates were introduced. These were lower until 1978, when a single minimum wage replaced them. See [https://www.dol.gov/whd/minwage/chart.pdf](https://www.dol.gov/whd/minwage/chart.pdf). To convert the statutory change to 2000 USD, we divide the change by the level of the PCE deflator relative to the year 2000.
the capital share that explains about 18% of the variation in the observed capital share (reported p-values are based on White robust standard errors).\footnote{On January 25, 1950, the minimum wage increased from $0.40/hour to $0.75. However, given the limited coverage of minimum wage at the time (it covered only workers employed by firms engaged in interstate commerce) and the prevailing wages in the industry for low-skill workers (well above $0.40/hour), the real effects of this sharp increase in the minimum wage were likely limited. Furthermore, using a robust regression that drops any observation with Cook’s distance greater than 1 (a standard choice in the literature), only the 1950 observation is eliminated.}

To move from these simple scatter plots to a formal estimation, we use a Bayesian proxy-VAR (see Arias et al., 2018). Our proxy for the distribution shock is the legislated change in the federal minimum wage, converted to constant year 2000 USD - similar to the instrumental variables strategy in Autor et al. (2016). Under the assumption that the federal minimum wage legislation is uncorrelated with other shocks, this procedure identifies the response of the economy.

The assumption that statutory minimum wage changes are uncorrelated with other shocks allows us to identify a distribution shock without any additional restrictions, such as zero restrictions on impulse response functions (IRFs) common in the VAR literature. Technically, our proxy-VAR is a Bayesian version of the original frequentist approach in Stock and Watson (2012) and Mertens and Ravn (2013). We use the Bayesian implementation in Drautzburg (2016) with a flat prior over reduced-form parameters. Under the assumption that the proxy is a valid instrument, the proxy-VAR identifies the IRFs from the covariance of forecast errors and the proxy. The algorithm also allows us to purge the proxy from other predictors, which we exploit in robustness checks.

We estimate a parsimonious VAR that captures the labor market, the goods market, and the capital share. The VAR has four variables: (1) the real minimum wage, (2) the unemployment rate, (3) the net capital share in the non-financial corporate business sector, and (4) gross value added (GVA) in the overall non-farm business sector. Our data are quarterly and we include four lags. We begin our estimation in 1951, excluding the 1950 outlier and allowing us to keep the sample constant when we later extend the VAR. Here, we focus on specifications without any trend. Figure D.7 in
the Appendix shows that our results change little with a deterministic quadratic trend.

Figure 4: Responses to a 10% real minimum wage shock in VAR.

Figure 4(a) shows the IRFs to a 10% minimum wage increase in the full sample, a typical rise in the postwar period. Both for the full sample (top row) and the post-Volcker era (middle row), statutory minimum wage increases cause the real minimum wage to increase persistently – a redistribution effect. The capital share falls significantly on impact in the full sample by between 0.1 and 0.7 pp. with 68% posterior probability. Unemployment increases with a delay and peaks 0.3 to 0.9 pp. higher after about one year. GVA falls up to 2% one year after the shock. In the post-Volcker era, the effects are similar for the unemployment rate, with a more pronounced effect on the capital share that is delayed by one quarter. Except for the insignificant GVA response, the results are stronger for the post-Volcker sample. The lower share of hours worked at or below the federal minimum wage rate – a peak of 9% in 1980–81 and a low of 3% in 2002–06 (Autor et al., 2016, Table 1B) – may explain the delayed effect on the capital share as indirect equilibrium effects become more important than direct effects. In the post-Volcker sample, there is also some predictability in our shock proxy, i.e., the statutory minimum wage changes. The small-scale VAR does not capture this predictability. Controlling for the level and the square of the capital share, the minimum wage, and the unemployment rate (Figure 4(c)), the IRFs show a shorter, more plausible shape (see the next paragraph for the motivation for these controls).

One concern about our identification procedure could be that Congress increases minimum wages when the labor share is too low or the economy is performing well. However, if the information
set in the VAR is rich enough to capture the state of the economy well, our results cannot be explained by current economic conditions. Our estimation is based on the covariance between the statutory minimum wage changes and the VAR forecast errors. Because the forecast errors are orthogonal to the state of the economy as captured by the VAR, our results would not change if we further controlled for any linear combination of the lagged VAR variables in the proxy variable itself. Only variables outside our VAR or non-linear combinations of VAR variables could change our results. To check this, we consider additional non-linear controls to purge our proxy variable and we estimate, in Appendix E, a 10-variable VAR. We also estimate versions of the VAR that incorporate information on changes in state-level minimum wages and with different trends/subsamples. Our results are basically unchanged.

Similar results hold in state-level panel data. Specifically, in Appendix F, we regress outcomes on changes in the applicable nominal minimum wage, converted to real 2010 dollars. We include state fixed effects and consider variants with time fixed effects. Conditioning on state-years with changes in the minimum wage, we find that the capital share declines by a total of 0.8 pp. to 0.9 pp. in the year of and the year after a one-dollar minimum wage increase, before reverting to its mean. The timing of the estimates depends on the fixed effects, but is significant in both cases. Economic activity, measured as GDP growth or changes in the unemployment rate, tends to fall as the minimum wage rises. The estimates for real activity are, however, imprecise when we include year fixed effects and condition on minimum wage changes.

Previewing the results from our structural model in the next sections, we find results consistent with the empirical evidence. Backing out the path of bargaining shocks in the U.S. since the 1950s, we see that they are broadly consistent with our historical narrative and the time VAR. For example, our estimated model-driven bargaining power indicator increases following the defeat of proposed right-to-work laws in several states. Moreover, we uncover a positive correlation between the estimated bargaining process and both the real federal minimum wage and unemployment, which supports our VAR findings.

3 Model

We postulate a business cycle model with labor search and matching à la Andolfatto (1996), Merz (1995), and Shimer (2010) to think about how politics influences labor shares. We will have a representative household, a final good producer, and a government. Markets are complete. To make the model closer to the data in terms of income shares and their evolution, we add taxes on labor income and net corporate profits, adjustment costs in capital, and variable capacity utilization. We incorporate exogenous unemployment benefits and a government-mandated minimum wage to disentangle changes in those from the bargaining power shocks that we investigate.

Also, and following Shimer (2010), we will assume a labor matching technology that depends on the number of recruiting workers employed by the firm. This formulation has three advantages with respect to a standard model of vacancy posting. First, the number of recruiting workers will never go to zero: because of the concavity of the recruiting technology, firms always have a positive fraction
of recruiters. In comparison, models with vacancy posting can hit the “zero vacancy posting” condition, something that we never observe in the data. Second, our formulation is often easier to handle analytically. Lastly, by avoiding the occasionally binding inequality constraint on posting vacancies, our model can be computed efficiently and accurately using a third-order perturbation.\footnote{Petrosky-Nadeau and Zhang (2017) show how, in a partial equilibrium search-and-matching model with vacancy posting, we need a projection method to capture non-linearities adequately. Our recruiting technology avoids the occasionally binding constraint.}

We postulate social and political shocks to factor income shares as innovations to the bargaining power of workers. This single shock is a parsimonious way to capture various factors. For example, Binmore et al. (1986) show that variations in bargaining power can arise from changes in the bargaining procedure. These changes correspond in the data with innovations in labor law, judicial decisions, and, more generally, the social climate regarding collective bargaining. Also, shocks to the minimum wage and unemployment benefits are, to first order, equivalent to bargaining power shocks (Foroni et al., 2017). As we will see, the steady-state capital income is mostly pinned down by deep parameters other than bargaining power. In contrast, shocks to bargaining power will have significant transient effects. In the next subsections, we present the key model ingredients, while we relegate to Appendix G the full description of the model.

3.1 Households

There is a representative household formed by a continuum of individuals of measure 1. Individuals can be either employed or unemployed, but they are otherwise identical in terms of preferences. The household perfectly insures its members against idiosyncratic risk by equating marginal utilities. Under perfect insurance, the household problem can be recast in terms of the lifetime utility function:

\[
U_0 = E_0 \sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma} \left(1 + (\sigma - 1) \gamma n_{t-1}\right)^{\sigma - 1},
\]

and budget constraint:

\[
a_0 = E_0 \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} m_t \right) \left( c_t - (1 - \tau_n)(w_{h,t} n_{t-1} + \omega_t(1 - n_{t-1})) - T_t \right).
\]

In the utility function, $\beta$ is the discount factor, $c_t$ is the average consumption, and $n_{t-1} \in [0, 1]$ is the fraction of household members who are employed at time $t$. This fraction was determined in the previous period (and hence the subindex $t - 1$). The variable $w_{\zeta,t} = (1 - \zeta_0)w_{h,t} + \zeta_0 w_{l,t}$ is the average wage rate. A fraction $1 - \zeta_0$ of the employed receive the bargained wage $w_{h,t}$, while the remainder receives the minimum wage $w_{l,t}$. Unemployed workers receive unemployment benefits $\omega_t$. The parameter $\sigma$ determines the relative risk aversion and $\gamma$ the disutility of working.

In the budget constraint, $a_0$ is the household’s net worth at time 0 and $m_t$ is the stochastic discount factor, with $m_0 = 1$. Net expenditures are given by consumption less after-tax labor income and $T_t$, the lump-sum transfers from the government and the net profits from firms (capital
is owned directly by the firm; under complete markets, this is equivalent to capital being owned by
the household, but more convenient algebraically). The wage $w_{\zeta,t}$ is taxed at a rate $\tau_n$.

When making its decisions, the household considers that workers lose their jobs at rate $x$ and
find new jobs at rate $f(\theta_t)$, where $\theta_t$ is the recruiter-unemployment ratio that the representative
household takes as given. Thus, the fraction of household members employed next period will be:

$$n_t = (1 - x)n_{t-1} + f(\theta_t)(1 - n_{t-1}).$$  \hspace{2cm} (3.3)

The job finding rate, $f(\theta_t)$, is given by $f(\theta_t) = \xi \theta_t^{\eta}$, with matching efficiency $\xi$ and elasticity $\eta$.

### 3.2 Firms

A representative firm allocates the matched workers $n_{t-1}$ between recruiting (a fraction $\nu_t$) and
producing the final good (the remaining fraction $1 - \nu_t$). A fraction $1 - \zeta_0$ of workers have one
efficiency unit of labor, while $\zeta_0$ workers have $\zeta_1 < 1$ efficiency units. This lower efficiency, which
makes the government-mandated minimum wage binding, is revealed after recruiting and it is iid
over time, so the firm does not have any incentive to fire low-efficiency workers, since they have the
same expected future efficiency as current high-efficiency workers. The average number of efficiency
units is $1 - \bar{\zeta} = 1 - \zeta_0(1 - \zeta_1)$. The efficiency units of labor are combined with the capital owned
by the firm, $k_{t-1}$, to produce a final good with the technology:

$$y_t = \left( \alpha \left( \frac{1}{k_t} \right)^{1 - \frac{1}{\varepsilon}} + (1 - \alpha) \frac{1}{2} \left( \frac{i_t}{k_{t-1}} - \tilde{\delta} \right)^2 \right)^{\frac{\varepsilon}{\varepsilon - 1}},$$  \hspace{2cm} (3.4)

where $\varepsilon$ is the elasticity of substitution between capital and labor. For $\varepsilon \to 1$, we obtain a Cobb-
Douglas production function with capital share $\alpha$. A labor-augmenting trend productivity growth
is given by $g_z$ and a productivity shock by $z_t$. Finally, $u_t$ is capital capacity utilization.

For an investment $i_t$, capital $k_t$ evolves as:

$$k_t = (1 - \delta(u_t))k_{t-1} + \left( 1 - \frac{1}{2} \kappa \left( \frac{i_t}{k_{t-1}} - \tilde{\delta} \right)^2 \right) i_t,$$  \hspace{2cm} (3.5)

where $\tilde{\delta} \equiv g_z^{1/\alpha} - (1 - \delta(\bar{u}))$ depends on $g_z$ – the growth rate of $z_t$ – and average utilization $\bar{u}$. The utilization cost is:

$$\delta(u) = \delta_0 + \delta_1(u - 1) + \frac{1}{2} \delta_2(u - 1)^2.$$  \hspace{2cm} (3.6)

The firm’s value is determined by the discounted flow of post-tax revenue less investment and
labor payments:

$$J_0 = E_0 \left[ \sum_{t=0}^{\infty} \left( \prod_{s=1}^{t} m_t \right) \left( (1 - \tau_k)(y_t - w_{\zeta,t}n_t) - i_t + \tau_k \delta(\bar{u})\bar{q}k_{t-1} \right) \right],$$
where \(\tau_k\) is the tax on corporate profits net of a depreciation allowance and \(\bar{q}\) is the average Tobin’s \(q\). Production and capital follow from (3.4) and (3.5) and employment at the firm level satisfies:

\[
n_t = (\nu_t(1 - \bar{\zeta})\mu(\theta_t) + 1 - x)n_{t-1},
\]

where \(\mu(\theta_t) = f(\theta_t)/\theta_t\) is the hiring probability per efficiency unit of recruiter.

### 3.3 Wage determination

Households and firms determine the wage for high types under generalized Nash bargaining. Workers have bargaining power \(\phi_t\). Exogenous shifts in \(\phi_t\) capture social and political shocks (e.g., an administration that appoints more union-friendly board members to the NLRB or a major decision regarding labor relations by the Supreme Court of the United States), as well as other shocks (for example, a structural change in the economy such as a move from manufacturing factories to harder-to-organize services).\(^6\)

The equilibrium wage, thus, solves:

\[
wh_t = \arg \max \tilde{V}_{h,n,t}(\tilde{w}_t)\phi_t\tilde{J}_{h,n,t}(\tilde{w}_t)^{1-\phi_t},
\]

where \(\tilde{V}_{h,n,t}\) and \(\tilde{J}_{h,n,t}\) are, respectively, the marginal values of employment of a high-productivity worker for the worker and the firm given an arbitrary wage \(\tilde{w}_{h,t}\) for the current period and \(w_{h,t}\) thereafter. In equilibrium, \(\tilde{w}_{h,t} = w_{h,t}\). We derive \(\tilde{V}_{h,n,t}\) and \(\tilde{J}_{h,n,t}\) in the Appendix from the recursive formulation of the household and firm problems.

In comparison, the minimum wage \(w_{\ell,t}\) is fixed by government policy and grows at the same rate as labor productivity \(z_t\) (unemployment benefits also grow at this rate).\(^7\)

### 3.4 Exogenous processes

In our economy, two variables evolve exogenously: labor productivity \(z_t\) and bargaining power \(\phi_t\). Labor productivity follows \(\ln z_t = \rho z \ln z_{t-1} + \omega z\epsilon_{z,t}\), where \(|\rho_z| < 1\) and \(\epsilon_{z,t}\) is a normalized Gaussian shock. In the Appendix, we allow for the case \(\rho_z = 1\).

The transformation \(\ln \frac{\phi_t}{1-\phi_t}\) maps the level of bargaining power from \([0, 1]\) to \((-\infty, \infty)\). Then, bargaining power follows:

\[
\ln \frac{\phi_t}{1-\phi_t} = (1 - \rho_\phi) \ln \frac{\phi_t}{1-\phi_t} + \rho_\phi \ln \frac{\phi_{t-1}}{1-\phi_{t-1}} + \omega_\phi \epsilon_{\phi,t},
\]

\(^{6}\)Binmore et al. (1986) show that the static bargaining problem is the limit point of a sequential strategic bargaining model where \(\phi_t\) reflects asymmetries in the bargaining procedure or beliefs about the likelihood of a breakdown of negotiations. Their model provides a micro-foundation for how policies that change the bargaining procedure induce changes in \(\phi_t\) if the parties to the bargain are impatient.

\(^{7}\)In Appendix G, we include shocks to the minimum wage and unemployment insurance. Below, we will present an exercise where unemployment benefits and the share of hours worked under the minimum wage vary with the cycle. We could augment the model with endogenous separations and firing costs, such as those existing in Europe. Since we will calibrate the model to the U.S. economy, where these costs are small, we ignore them.
where $\bar{\phi}$ is the average value of the process. Again, $|\rho_\phi| < 1$ and $\epsilon_{\phi,t}$ is a normalized Gaussian shock. We hold the matching efficiency $\xi$ constant to isolate the effects of innovations to bargaining power.

### 3.5 Market clearing and equilibrium

Aggregate employment follows the law of motion for the representative household (3.3), where the recruiter-unemployment ratio is:

$$\theta_t = \frac{n_{t-1}}{1 - n_{t-1}} \nu_t.$$  

(3.8)

The production of the final good equals aggregate consumption and investment:

$$y_t = c_t + i_t.$$  

(3.9)

Finally, aggregate capital has to satisfy the capital law of motion for the representative firm (3.5). The equilibrium stochastic discount factor is:

$$m_{t+1} = \beta_t \frac{c_{t+1}^\sigma (1 + (\sigma - 1) \gamma n_t)^\sigma}{c_t^\sigma (1 + (\sigma - 1) \gamma n_{t-1})^\sigma}.$$  

(3.10)

In the Appendix, we derive a more general discount factor that allows for external habit formation.

In equilibrium, households choose consumption and employment optimally, taking the process for the wage rate, the real interest rate, and labor market tightness as given. Similarly, firms choose investment, utilization, capital, production, and recruiting optimally, taking the process for the wage rate, the stochastic discount factor, and labor market tightness as given. The goods market also clears.

In the spirit of Hosios (1990), Appendix G.5 shows that allocations along the balanced growth path (BGP) of the economy are constrained efficient if $\phi = 1 - \eta$, $\tau_n = \tau_k$, and $\zeta_0 = 0$. We will document that our departure from the Hosios condition is inconsequential for our results.

### 3.6 Mapping theory into data

The measures of the gross and net capital share in our economy are:

$$c_{st} = 1 - \frac{n_{t-1} \omega_{\zeta t}}{y_t},$$  

(3.11a)

$$nc_{st} = 1 - \frac{n_{t-1} \omega_{\zeta t}}{y^r_t} - \frac{\delta k_{t-1}}{y_t}.$$  

(3.11b)

In the model, depreciation changes with the endogenous utilization decisions. In comparison, the depreciation rate in NIPA varies only because the capital stock changes its composition. Since we have a single good in our economy, we measure the NIPA equivalent of the net capital share using
the steady-state depreciation rate. This computes the net capital share under the assumption of a constant service life of an asset.8

Finally, measured total factor productivity (TFP; hereafter, we will omit “measured,” but TFP should always be understood as such) is equal to

$$TFP = GDP_t - cs_t k_{t-1} - (1 - cs_t)n_{t-1}.$$ (3.12)

3.7 Solution

First, we HP-detrend all variables as needed. To compute business cycle statistics, we calculate quarterly averages and add the trend to all trending variables before HP filtering. In the model, labor productivity, capital, consumption, investment, the marginal value of employment, the marginal product of labor, and wages grow at the common rate $g_z$, while all other variables are stationary.

We use a third-order perturbation to solve for the equilibrium of the detrended economy. Thus, we can analyze some non-linear dynamics of interest while guaranteeing high accuracy. For example, the mean Euler equation errors are below 0.25% of consumption and the 99th percentile of Euler errors is below 2% (see Appendix G.11). To ensure stability, we apply the pruning method developed by Andreasen et al. (2018).

In the neighborhood of the calibrated, detrended steady state (see Section 4), the capital share is almost invariant to bargaining power. In the long run, the capital share is overwhelmingly determined by technology and preferences. In our baseline calibration, virtually all of the gross capital share is compensation for depreciation, impatience, and growth. Instead, employment moves to equate the marginal product of labor to the varying wage rate (adjusted for recruiting costs). For the same reason, the rental rate of capital and the marginal product of capital are also nearly invariant to bargaining power for a wide range of calibrations.9

4 Identification and calibration

As is customary in the labor-search literature, we calibrate the model to a monthly frequency. When mapping it to U.S. data, we first aggregate the model-generated data to a quarterly frequency. As described in Andreasen et al. (2018), we will match moments in the data to the corresponding moments of the model’s ergodic distribution, and not to their steady-state values. We do so because, under a non-linear solution, the latter may not summarize well the properties of ergodic distributions.

8Per capita consumption is the sum of real consumer non-durables and consumer services. Per capita investment is real gross domestic private investment plus real durable consumption. Because only nominal or quantity indices are available for private consumption expenditures, we compute real consumption expenditures in 2009 dollars as the product of the per capita quantity index times their average 2009 nominal expenditures. Per capita GDP is real per capita investment plus consumption. See Appendices A and G.8 for details.

9When we recalibrate the disutility of working to keep employment constant, the capital shares are independent of $\bar{\phi}$. The matching efficiency $\xi$ is calibrated for any given average employment level $\bar{n}$ to ensure that the BGP recruiter-employment ratio $\bar{\theta}$ is constant across parameterizations. Then, the optimality for recruiting and the production function imply that GDP, wages, the number of recruiters, and capital along the BGP are constant as well.
4.1 Technology and preferences

We select a Cobb-Douglas technology, with $\varepsilon = 1$, as in Cooley and Prescott (1995). Alternatively, we will consider values of $\varepsilon = 1 \pm 0.25$, reflecting the recent work of Karabarbounis and Neiman (2014) and Oberfield and Raval (2014). The former paper estimates an elasticity of substitution of 1.25 using long-run differences across countries. The latter estimates a macro-elasticity of substitution for the manufacturing sector of 0.7 based on a weighted average of micro-elasticities of substitution and demand. Also, after Cooley and Prescott (1995), we select $g_z$ to be 1.6% per year.\(^{10}\)

We choose the discount factor $\beta$, the capital share in production $\alpha$, and the depreciation rate $\delta$ to match three properties of the non-financial corporate business sector: (1) the gross capital share of 31.2%, (2) the ratio of depreciation to gross value added of 12.7%, and (3) the (annualized) ratio of capital to gross value added of 2.3. We calibrate the average corporate tax rate to 30%, the average in the data. Given our choice of $g_z$, the implied annual depreciation rate is 5.5% and the annualized discount rate is 0.976. In the Cobb-Douglas case, the capital share $\alpha$ is just 0.312. We normalize the average efficiency of investment $\bar{\chi}$ and the average detrended $\tilde{z}$ to 1.

We calibrate the labor market following Shimer (2010). The exogenous separation rate $x$ is 3.3% per month, the average unemployment rate is 5%, and the matching efficiency $\xi$ is such that one recruiter hires, on average, 25 employees per quarter. We set the matching elasticity $\eta$ and the average bargaining power $\bar{\phi}$ to 0.5. As in Prescott (2004), a labor income tax rate of 40% combines the consumption taxes and the actual labor tax rate. Choosing a conventional value of $\sigma = 2$ implies that the employed consume 30% more than the unemployed.

We calibrate the minimum wage to be one-third of the bargained wage rate along the BGP. In the U.S., the average ratio of the federally mandated minimum wage to the average hourly earnings of production workers and non-supervisory employees was 39.3% from 1964Q1 to 2018Q2, and 34.8% if we look at 2006Q1 to 2018Q2. Relative to all private-sector employees, for whom we have data since 2006Q1, the average ratio is 29.1%. Adjusting the post-1964 average for the composition difference leaves us with 33.8%. Thus, we set $\zeta_1 = 0.25$ and make it binding for 5% of workers following Autor et al. (2016, Table 1B). We also match the average replacement rate for the unemployed of 40%.

4.2 Stochastic processes

The stochastic processes for $z_t$ and $\phi_t$ are indexed by their persistences, $\rho_i$, and standard deviations, $\omega_i$, for $i \in \{z, \phi\}$. Following Cooley and Prescott (1995), we set $\rho_z = 0.95^{1/3}$. This leaves us three parameters to determine: $\rho_\phi$, $\omega_z$, and $\omega_\phi$. Given that we do not have direct observations of the bargaining shocks, we will proceed in two steps. First, we will describe the source of variation in the data that will give us identification of $\omega_z$ and $\omega_\phi$ given $\rho_\phi$. Second, we will explore a wide range of values of $\rho_\phi$ to capture different views on the persistence of bargaining shocks.

Figure 5 illustrates the first step by showing the response of the fraction of recruiters, unemploy-\(^{10}\)While the calibration of the model is not fully recursive, in practice, it is nearly so (i.e., we can calibrate one block of parameters, move to calibrate another one, and so on, and only have to do minor readjustments at the end). Thus, to simplify the exposition, we discuss each block separately.
Figure 5: Steady-state response to shocks.

In the top panels, we see that, after innovations to the shocks that deliver either higher bargaining power for workers, $\phi_t$, or lower productivity, $z_t$, the optimal recruiting decision for the firm that links the unemployment rate, $1 - n_t$, and the fraction of recruiters, $\nu_t$, moves to the right. In both cases, the firm finds it less profitable to recruit new workers: either the firm appropriates a smaller share of the match surplus or workers are less productive. Thus, $1 - n_t$ increases (and output falls) and $\nu_t$ falls after both innovations. The bottom panels of Figure 5 document that, in comparison, wages increase after an increase in bargaining power, but fall after a decrease in productivity. Higher bargaining power for workers lowers output and the match surplus, but because their share in the surplus rises enough, workers still take home a higher wage. In summary: higher worker bargaining power and lower productivity lower output, but have an opposite impact on wages.

Thus, we can exploit observations on wages, output, and TFP to pin down the size of bargaining power shocks and productivity shocks. More concretely, given $\rho$, we pick $\omega_z$ and $\omega_\phi$ to match the observed correlation between wages and GDP, and the volatility of TFP. Given $\omega_z$, a higher $\omega_\phi$ lowers the correlation of wages and GDP. This trade-off is illustrated by the left panel in Figure

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11To plot the four panels, we assume permanent shocks, keep consumption constant after each shock, and use our baseline calibration in Table 3. With the full transient dynamics, the panels would be harder to read. However, the intuition of the identification result remains unchanged.

12At a quarterly frequency, Cooley and Prescott (1995) find an autocorrelation of 0.95 and a standard deviation of 0.73% for TFP. Averaging monthly observations within quarters, their values correspond to a monthly autocorrelation of $0.95^{1/3}$ and a standard deviation of TFP productivity of 0.73%. We find a value of 0.79%.
The green horizontal line gives us the observed correlation between wages and GDP and the decreasing curves give us the model-implied correlation for different values of $\omega_\phi$ (with $\kappa$ that matches the volatility of investment at the preferred calibration).

6. We plot three lines, each corresponding to a different value of $\omega_z$ (and, as before, conditional on $\rho_\phi$). We can improve the sharpness of the identification if we simultaneously calibrate the standard deviation of productivity shocks, $\omega_z$, to match the volatility of TFP (middle panel of Figure 6), and the investment adjustment cost, $\kappa/\delta_0^2$, to match the relative volatility of investment (right panel). The horizontal lines in the figure indicate HP-filtered data moments (we also set the elasticity of utilization with respect to the marginal product of capital to $1/2$, i.e., $\delta_2 = 2\delta_1$). Intuitively, the larger the standard deviation of productivity, the larger the standard deviation of the bargaining shocks that matches the cyclicality of wages. Since productivity shocks are the primary driver of TFP, we can easily identify these parameters together. Last, the relative volatility of investment to GDP pins down the investment adjustment cost. Changing this parameter has next to no impact on the cyclicality of wages, but a large effect on investment. We match the relative volatility of investment, because recruitment is a quasi-investment activity that we discipline indirectly by getting capital investment right. Figure G.14 plots the additional bivariate relation among these three parameters and documents why we can ignore, for calibration purposes, the omitted plots.

Thus, it only remains to pick the persistence of the bargaining shock, $\rho_\phi$. Since we are agnostic about how persistent this shock is in the data, we select three cases. As a baseline medium-term case, we set $\rho_\phi = 0.981^{1/3}$. This value implies a half-life of the shock of 34 quarters. From 1948 to 2016, the average duration of a party’s control of the presidency was 30.2 quarters, of the Senate 38.9 quarters, and of the House of Representatives 24.7 quarters. Our numbers are slightly biased downward because of left- and right-censoring: in 1948, the Democrats were already in control of the presidency and the current spell of Republican control of the Senate lasted until 2020. Given

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13Note that 34 quarters is above the conventional 32-quarters cut-off of business cycle frequencies. Thus, we are dealing with a shock that generates medium-term business cycles. To allow a comparison with the literature, most of our quantitative results will focus on business cycle fluctuations. We will make some references, however, to the longer-lived effects of the shock.

14Coding the control of the Supreme Court is harder, as justices drift across time (John Paul Stevens started in 1975 as a Republican and ended in 2010 as a solid liberal) and across cases (think of Anthony Kennedy’s pivots).
that the changes in party control that resulted in small changes in policy (Truman followed by Eisenhower, the first Bush followed by Clinton) have been roughly the same as the changes that resulted in substantial policy shifts (Carter by Reagan, the second Bush by Obama), 34 quarters is a reasonable duration for the half-life of the middle-run political cycle in the U.S. after WWII. Given this $\rho_\phi = 0.98^{1/3}$, the monthly standard deviation of the bargaining power shock of 6.1 pp. (or 24.5% for the logit transform) and $\kappa = 0.095/\delta_0^2$. Besides, $\rho_\phi = 0.98^{1/3}$ replicates the features of the U.S. labor income share’s medium-term dynamics documented by Growiec et al. (2018). Table 3 summarizes this baseline calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion $\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Discount factor $\beta$</td>
<td>$0.976^{1/12}$</td>
</tr>
<tr>
<td>Disutility of working $\gamma$</td>
<td>such that $\bar{n} := 0.95$</td>
</tr>
<tr>
<td>Capital share $\alpha$</td>
<td>0.312</td>
</tr>
<tr>
<td>Elasticity of substitution $\varepsilon$</td>
<td>1</td>
</tr>
<tr>
<td>Depreciation $\delta_0$</td>
<td>0.055/12</td>
</tr>
<tr>
<td>Avg. efficiency of investment $\bar{x}$</td>
<td>1</td>
</tr>
<tr>
<td>Avg. detrended $\bar{z}$</td>
<td>1</td>
</tr>
<tr>
<td>Trend productivity growth $g_z$</td>
<td>1.016$^{1/12}$</td>
</tr>
<tr>
<td>Investment adjustment cost $\kappa$</td>
<td>$0.101 \times (\delta_0)^{-2}$</td>
</tr>
<tr>
<td>Capacity utilization cost $\delta_1$</td>
<td>such that $\bar{u} = 1$</td>
</tr>
<tr>
<td>Capacity utilization cost $\delta_2$</td>
<td>$2\delta_1$ (BGP ela. w.r.t. $\frac{mpk}{w}$ of $\frac{1}{2}$)</td>
</tr>
<tr>
<td>Bargaining power $\phi$</td>
<td>0.5</td>
</tr>
<tr>
<td>Matching elasticity $\eta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Matching efficiency $\xi$</td>
<td>2.3</td>
</tr>
<tr>
<td>Separation rate $x$</td>
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</tr>
<tr>
<td>Steady-state replacement rate $\bar{\omega}_{\bar{x}}$</td>
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</tr>
<tr>
<td>Probability of low productivity $\zeta_0$</td>
<td>0.05</td>
</tr>
<tr>
<td>Efficiency units of low-productivity workers $\zeta_1$</td>
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</tr>
<tr>
<td>Low-productivity wage $\bar{w}_{\bar{x}}$</td>
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</tr>
<tr>
<td>Labor income tax rate $\tau_n$</td>
<td>0.4</td>
</tr>
<tr>
<td>Corporate tax rate $\tau_k$</td>
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</tr>
<tr>
<td>Productivity shock persistence $\rho_z$</td>
<td>$0.95^{1/3}$</td>
</tr>
<tr>
<td>Bargaining shock persistence $\rho_\phi$</td>
<td>$0.98^{1/3}$</td>
</tr>
<tr>
<td>Bargaining power s.d. $\omega_\phi$</td>
<td>24.5%</td>
</tr>
<tr>
<td>Labor productivity s.d. $\omega_z$</td>
<td>0.79%</td>
</tr>
<tr>
<td>Implied gross capital share $c_s$</td>
<td>31.2%</td>
</tr>
<tr>
<td>Implied net capital share $\bar{n}c_s$</td>
<td>18.5%</td>
</tr>
</tbody>
</table>

Table 3: Parameter values for the baseline persistence of bargaining shock.

Second, as a high-persistence scenario, we choose $\rho_\phi = 0.9914^{1/3}$. This value gives a half-life of the bargaining shock of 80 quarters, which corresponds to long-run movements in the political climate related to party realignments, demographic changes, etc. We recalibrate all other parameters of the model accordingly.
Third, as a low-persistence scenario, we set $\rho = 0.95^{1/3}$, which yields a half-life of around 14 quarters. This low-persistence case matches the same persistence for the bargaining shock as for the productivity shock, and thus it embodies a degree of parsimony and “standard” business cycle properties. For this persistence, we recalculate all other parameter values.

4.3 An RBC model calibration

For comparison purposes in the results section, we formulate an RBC model à la Hansen (1985)-Rogerson (1988) where we eliminate the search and matching frictions. We calibrate the model using the same targets as for our baseline economy to pin down the $\omega_z$ and $\kappa$ (given the known properties of this model, we need to give up on matching the cyclical of wages). In that way, we can benchmark our model against the behavior of a well-understood environment. For better comparison, labor supply is determined one period in advance, but wages are set in spot markets. The counterpart model is described in detail in Appendix G.14 and its quantitative properties are reported in the rows labeled “RBC model” in Table 5. In Subsection 6.1, we will calibrate a version of this RBC model where we add exogenous shocks to the factor shares in the production function (as in Danthine et al., 2006, Ríos-Rull and Santaclaria-Llopis, 2010, and Lansing, 2015).

5 Quantitative results I: Historical bargaining power

This section argues that the data broadly confirm our empirical approach to the bargaining power process’s calibration. To do so, we use the partial information filter recently proposed by Drautzburg et al. (2021) to back out the bargaining power process. The partial filtering strategy exploits the structure of the key equations of our economy to deliver a simple statistical model that recovers bargaining power. Indeed, Drautzburg et al. (2021) call the filter partial because the researcher does not have to filter all the states: the partial information filter uses only some of the equilibrium conditions of the model. The main idea of this methodology is to move from unobserved expectations in the equilibrium conditions to conditional first and second moments that can be described using (potentially time-varying) VARs.

The partial information filter is an attractive alternative to a full-information particle filter—such as in Fernández-Villaverde et al. (2016)—when the latter is hard to implement. First, our model is a two-shock economy without the bells and whistles present in New Keynesian models designed to account for many observables. Second, our pruned non-linear solution features a large number of state variables.

With the partial information filter, we document that the bargaining power process’s statistical properties closely match our model. Also, we show that the implied bargaining power process covaries meaningfully with historical U.S. events. If anything, the model-implied bargaining power covaries too strongly at higher frequencies, a fact we use to inform one of our model extensions.
5.1 The partial information filter

Bargaining power enters into our model only through the wage-setting equation (G.32) (see Appendix G for the derivation of this equation). Hence, it is natural to use this equation to recover the historical bargaining power implied by the theory and data. The challenge is, however, that this equation features firms’ expectations. Specifically, we need to model the expectation of the discounted future value of employment to firms times their relative bargaining power.

We use an auxiliary statistical model and firms’ optimality conditions to model the expectations. Specifically, Appendix G.12 shows that we can re-write the wage-setting equation as:

\[
 w_{\zeta,t} = \zeta_0 w_{\ell,t} + \frac{e^{\phi_t}}{1 + e^{\phi_t}} m_{pl,t} \frac{1 - \zeta_0}{1 - \bar{\zeta}} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + \frac{1 - \zeta_0}{1 + e^{\phi_t}} \frac{1 - \zeta_0}{1 - \gamma_n(1 + (\sigma - 1)\gamma n_{t-1})} \gamma^\sigma \\
- (1 - x - f_t(\theta_t)) \frac{1}{1 + e^{\phi_t}} \left( T1(X_t, \hat{\phi}_t, \hat{\Sigma}, \hat{A}) + \frac{\zeta}{1 - \bar{\zeta}} T2(X_t, \hat{\phi}_t, \hat{\Sigma}, \hat{A}) - \zeta_0 T3(X_t, \hat{\phi}_t, \hat{\Sigma}, \hat{A}) \right),
\]

(5.1)

where \( m_{pl,t} \) is the marginal product of labor, \( \kappa_\phi \) is a constant, and the \( T \) functions depend on the state of the world (other than bargaining power) \( X_t \), bargaining power, and parameters of the statistical process: the covariance matrix \( \Sigma \) and the dynamic coefficients \( A \). When \( \zeta_0 = 0 \), so that \( \bar{\zeta} = 0 \), only \( T1 \) remains, which captures the expectation of the discounted future match surplus to the firm and its covariance with bargaining power.

To take equation (5.1) to the data, we exploit that, in our baseline calibration, \( m_{pl,t} \) is proportional to the average product \( \frac{y_t}{n_t} \). We use real GVA in the non-farm business sector to compute \( \frac{y_t}{n_t} \). Employment \( n_t \) is one minus the unemployment rate.\(^{15}\) The wage rate \( w_t \) is the real hourly compensation in the same sector. Consumption \( c_t \) equals real per capita expenditures on non-durables and services. The stochastic discount factor \( m_t \) is a function of consumption and employment. Labor market tightness \( \theta_t \) is the ratio of vacancies to the unemployed. Then, we use a Bayesian VAR that includes the discounted match surplus and the implied bargaining power to estimate this covariance.

A Gibbs sampler solves the problem of needing the bargaining power process to estimate the covariances and those, in turn, to back out bargaining power. The sampler starts with an arbitrary guess of the covariances and allows us to address the estimation uncertainty. More concretely, for \( d = 1, \ldots, D \), we iterate on the following steps:

1. Given the previous draw of the bargaining power sequence:
   
   (a) Draw \( \rho_\phi^{(d)} \) and the VAR coefficients given \( \Sigma^{(d-1)} \) from a normal standard SUR posterior.

\(^{15}\) In the model \( y_t \) excludes recruiting activities, whereas GVA does not. This discrepancy is, however, minimal.
(b) Draw the (inverse of the) covariance matrix $\Sigma^{(d)}$ from a Wishart distribution given $\rho^{(d)}_\phi$ and the corresponding VAR coefficients.

2. Given observables, $\Sigma^{(d)}$, $\rho^{(d)}_\phi$ and VAR coefficients, solve (5.1) period by period for \( \left( \frac{\phi_t}{1 - \phi_t} \right)^{(d)} \).

For details, see Appendix G.12.\(^{16}\) There we describe how we scale the observed variables to match the steady state of our model. By construction, we match the positive steady-state surplus to households and firms in our model, but these match surpluses would turn negative in some periods. We guarantee that the discounted match surplus is positive by first adjusting $mpl_t$ so that it lies weakly above the real wage. We then lower the average disutility of working, such that the implied bargaining power averages 0.5 in our sample when the covariance terms are set to zero. Last, we adjust the frequency of the model to quarterly data. We use a flat prior in the estimation.

5.2 Results

The blue solid line charted against the left axis in Figure 7 shows the path of the bargaining power process generated by our partial filter. We focus, first, on the medium-term fluctuations. Our sample starts at the time of a large victory for labor, the so-called “Reuther’s Treaty of Detroit,” a 5-year contract between General Motors (GM) and the United Automobile Workers (UAW), named after the UAW’s president, Walter Reuther. As one astute contemporary economist observed: “The inclusion of the modified union shop in a five-year contract and the conciliatory approach of the corporation in bargaining have finally convinced the union leadership, it appears, that GM has accepted the UAW on a realistic and permanent basis” (Harbison, 1950, p. 404). The Treaty of Detroit opened two decades of gains for U.S. workers across many industries and sectors (Levy and Temin, 2007). Even Eisenhower appointed Martin P. Durkin, a former union leader, as his first Secretary of Labor, and his second Secretary of Labor, James P. Mitchell, has been inducted into the Labor Hall of Honor. Proposed right-to-work laws were defeated in California, Ohio, Colorado, Idaho, and Washington (Dubofsky and Dulles, 2017, loc. 7891). In line with these events, our bargaining power increases from 0.5 to 0.9 by the end of the 1950s.

However, the early 1960s saw the start of a relative decline in workers’ power. Several factors contributed to it. First, the American Federation of Labor (AFL) and the Congress of Industrial Organizations (CIO) merged in 1955. The merger led, after a few years, to a more moderate attitude by the unions formerly associated with the CIO and a smaller effort to increase unionization rates. Richard Lester, a prominent Princeton labor economist at the time, labeled the new AFL-CIO a “sleepy monopoly.” Also, the merger of the two organizations facilitated the purge of the remaining communist sympathizers at the CIO. Second, many firms, worried by increasing foreign competition and rising costs, adopted the so-called “hard-line” position of 1958, which, after several years of industrial conflict (as in the steel industry in 1959), led to some victories for management. Third,

\(^{16}\)We initialize the process at our calibrated parameters and discard draws from a burn-in period. A time-varying VAR would yield a time-varying sequence of covariances that would match our non-linear model more closely. We abstract from this non-linearity because the covariance terms are small and the posterior uncertainty about the in-sample bargaining power is negligible.
the Landrum-Griffin Act of 1959, a response to corruption and racketeering in the labor movement, imposed additional guarantees regarding the internal behavior of unions and tightened the rules regulating secondary boycotts, “hot cargo” agreements, and recognitional picketing. These measures curbed the tactics of some unions, which called for fewer strikes in the early 1960s than in the late 1950s. As Walter Reuther put it in 1960: “We are going backward.”

The decline in workers’ bargaining power continued until the late 1960s, when it was at 0.4. This trend was helped by the wage-price guideposts started by Walter Heller at the Council of Economic Advisers in 1962 and pushed by Johnson as an essential tool of the economic policy of his administration. Although often remembered because of the fight between Kennedy and the steel industry in 1962, the wage-price guideposts slowed down wage increases relative to productivity growth (see the narrative in Slesinger, 1967).

The late 1960s witnessed, in comparison, lower unemployment and an economy running at high utilization rates. It also saw the renewed strength of unions, often led by a younger, more militant generation that had experienced the civil rights struggles, the anti-war movement, and the radicalizing influence of the New Left.

One of these new union leaders, Leonard Woodcock (the successor of Walter Reuther as president of the UAW after Reuther’s untimely death), organized the nationwide UAW strike against GM in September 1970, which lasted for 67 days. This strike was one of the biggest victories of the post-WWII labor movement: UAW members received full cost-of-living wage adjustments and a “30-and-out” retirement plan (i.e., full pension after 30 years of work regardless of age). Our partial filter identifies this spike of labor’s power.

The early 1970s were an Indian summer for U.S. unions. The oil shocks and a changing political
climate eroded the bargaining power of workers by the late 1970s, perhaps best represented by the failure of the 1978 proposed reform of labor law, an endeavor in which the AFL-CIO had invested considerable political capital, and the peak in unionization coverage and membership percentages in 1979 (Hirsch and MacPherson, 2003).

This process sharply accelerated with Reagan’s arrival at the White House and his notorious firing of striking air-traffic controllers in 1981.Q3, the full repercussions of which were only felt in the second half of the 1980s. For the next two decades, our backed-out bargaining power series nearly uninterruptedly drops to well below 0.5. Reagan was followed by the first Bush and Clinton, a pro-business Democrat who worked with a Republican-controlled Congress to pass the Welfare Reform Act in 1996.Q3, lowering workers’ outside option at the time of bargaining. Simultaneously, the large immigration of the 1980s-2000s (spurred by the somewhat delayed effects of the 1965 Hart-Celler Act) created many service-sector workers with fewer links to the labor movement.

The late 1990s and early 2000s saw, according to our partial filter, a stop in the negative trend of workers' bargaining power. The 1997 Teamsters strike, the arrival of John Sweeney to the leadership of the AFL-CIO, and the creation of the Change to Win Federation in 2005 did not return organized labor to its former glory, but it did stabilize bargaining power. Bargaining power partially recovered in 2008.Q4, when the unemployment benefit extensions raised workers’ outside option (these benefits peaked in mid-2010), with the election of Obama –the most pro-union president in a generation–and later with the slowdown in immigration and early successes of the “fight for $15.”

While our stylized two-shocks economy has no hope of matching the richness of the data, our comparison of Figure 7 with the historical narrative above demonstrates a surprising level of agreement. In Appendix G.12, we compare our results with an alternative bargaining power index proposed by Levy and Temin (2007). The fact that both indexes display a correlation of 0.40 reinforces our positive assessment of our exercise.

Of course, there are aspects that we miss. For example, our filter shows increases in workers’ bargaining power during recessions (shaded lines in Figure 7). Without bargaining power increases, our model could not explain the relative stability of wages during recessions. While the model is consistent with the data, our baseline model may miss aspects such as unemployment benefits with endogenous duration that, in a richer model, would affect the outside option, but not necessarily bargaining power. Some measured increases in bargaining power may thus be partly systematic. We consider both possibilities in extensions below, without changing the main thrust of our results.17

Besides the previous narrative, the statistical properties of the bargaining power process are very close to the model; see Table 4. With a flat prior, the median posterior persistence is 0.9783 per quarter with a 90% credible set of (0.9541, 0.9949). The posterior for the conditional standard deviation is 0.2229 (0.2051, 0.2458). These values come close to our calibration of $\rho = 0.9800$ and $\omega = 0.2445$. Appendix G.12 shows that an alternative implementation of the filter that factors

17Specifically, we consider endogenous variation in the minimum wage and in the replacement rate in one extension, and a reduced-form policy rule for bargaining power, which matches the business cycle frequency correlation between bargaining power and unemployment rates coming from our filter. In the presence of additional heterogeneity with job-ladders or on-the-job learning, an interpretation of our results in Figure 7 is that the series represents the “aggregate” bargaining power that such models would imply.
the expectational terms differently produces a very similar result. In levels, the two series have a correlation of 0.98, and 0.88 in changes. Also, we find only small changes in our results when we use the wage of new hires or the employment-to-population ratio. The same appendix computes a counterfactual unemployment rate generated by our filtered bargaining shock and discusses the positive correlation of our bargaining power index with the federal minimum wage, in agreement with our VAR exercise in Section 2.2. For instance, the decline in the estimated bargaining power in the 1980s coincides with a sharp decline in the real federal minimum wage.

6 Quantitative results II: Business cycle statistics

We move now to assess the business cycle properties of the model. Table 5 compares U.S. business cycle statistics to those of our search and matching (S&M) model (with and without bargaining shocks) and its RBC counterpart. We focus on statistics that describe the volatility, cyclicality, and persistence of aggregate variables. All business cycle statistics are based on HP-filtered quarterly variables, averaged across three months in the model simulations. We take the log of level variables before filtering, but filter variables that are ratios as such. We construct GDP per capita as the sum of real per capita consumption and investment to match the data with our model.

In the data, the volatility of (log) GDP is nearly 2.0%. Investment is 3.28 times as volatile as GDP, whereas consumption is only 60% as volatile. With correlations of 0.91 and 0.84, both investment and consumption are highly pro-cyclical. Similarly, both the gross and the net capital share are moderately pro-cyclical, with a correlation of 0.57 and 0.36, respectively. All variables are very persistent, with quarterly autocorrelations of 0.67 to 0.90.

How does our model compare to the data? Recall that we calibrated the standard deviation of productivity and the capital adjustment cost to match the volatility of TFP (1.21%) and the relative volatility of investment (3.28) and consumption (0.58). Thus, it is not surprising that this calibration nearly replicates the level of GDP volatility (1.91). An important finding is that, if we eliminate the bargaining shock (but keep all the other parameters, including adjustment costs, at their baseline value), the volatility of output drops 29% to 1.35.18

Regarding capital shares, our baseline model can account for 31% of the volatility of the gross and 20% of the volatility of the net capital share in the data (leaving room for other factors, such as

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18 As a robustness check, we calibrated the model with productivity shocks and no bargaining shocks. Then, we measured the effects of adding bargaining shocks while keeping the other parameters constant. The results were nearly identical.
Table 5: Business cycle statistics: 1947Q1–2015Q2

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Y [%]</th>
<th>std(I) [%]</th>
<th>std(C) [%]</th>
<th>std(ncs) [%]</th>
<th>std(cs) [%]</th>
<th>std(w) [%]</th>
<th>std(u) [%]</th>
<th>std(TFP) [%]</th>
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<tbody>
<tr>
<td>U.S. data</td>
<td>1.99</td>
<td>3.28</td>
<td>0.58</td>
<td>1.07</td>
<td>0.86</td>
<td>0.95</td>
<td>0.83</td>
<td>1.21</td>
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<td>Models</td>
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<td></td>
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<td></td>
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<tr>
<td>S&amp;M model: baseline</td>
<td>1.91</td>
<td>3.28</td>
<td>0.62</td>
<td>0.33</td>
<td>0.17</td>
<td>1.45</td>
<td>1.89</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M model: no bargaining shock</td>
<td>1.35</td>
<td>3.33</td>
<td>0.57</td>
<td>0.18</td>
<td>0.01</td>
<td>1.13</td>
<td>0.24</td>
<td>1.20</td>
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<tr>
<td>RBC model: baseline</td>
<td>1.90</td>
<td>3.28</td>
<td>0.60</td>
<td>0.24</td>
<td>0.00</td>
<td>0.91</td>
<td>1.03</td>
<td>1.21</td>
</tr>
<tr>
<td>RBC model with factor share shock</td>
<td>1.76</td>
<td>3.28</td>
<td>0.60</td>
<td>0.31</td>
<td>0.11</td>
<td>0.88</td>
<td>0.85</td>
<td>1.21</td>
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<th>Y</th>
<th>I</th>
<th>C</th>
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<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
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<tbody>
<tr>
<td>U.S. data</td>
<td>1.00</td>
<td>0.91</td>
<td>0.84</td>
<td>0.57</td>
<td>0.36</td>
<td>0.19</td>
<td>-0.76</td>
<td>0.78</td>
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<tr>
<td>S&amp;M model: baseline</td>
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<td>0.96</td>
<td>0.97</td>
<td>0.80</td>
<td>0.13</td>
<td>0.19</td>
<td>-0.67</td>
<td>0.67</td>
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<td>S&amp;M model: no bargaining shock</td>
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<td>0.99</td>
<td>0.98</td>
<td>0.78</td>
<td>1.00</td>
<td>-0.95</td>
<td>1.00</td>
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<td>RBC model: baseline</td>
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<td>0.99</td>
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<td>NaN</td>
<td>0.97</td>
<td>-0.96</td>
<td>0.99</td>
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<tr>
<td>RBC model with factor share shock</td>
<td>1.00</td>
<td>0.98</td>
<td>0.99</td>
<td>0.94</td>
<td>0.57</td>
<td>0.97</td>
<td>-0.93</td>
<td>0.98</td>
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<th>Persistence</th>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>ncs</th>
<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
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<td>0.82</td>
<td>0.78</td>
<td>0.76</td>
<td>0.74</td>
<td>0.67</td>
<td>0.90</td>
<td>0.78</td>
</tr>
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<td>Models</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;M model: baseline</td>
<td>0.81</td>
<td>0.77</td>
<td>0.83</td>
<td>0.80</td>
<td>0.71</td>
<td>0.80</td>
<td>0.70</td>
<td>0.79</td>
</tr>
<tr>
<td>S&amp;M model: no bargaining shock</td>
<td>0.81</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.48</td>
<td>0.79</td>
<td>0.83</td>
<td>0.79</td>
</tr>
<tr>
<td>RBC model: baseline</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.82</td>
<td>NaN</td>
<td>0.77</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>RBC model with factor share shock</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.81</td>
<td>0.79</td>
<td>0.78</td>
<td>0.78</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

technological change or structural transformation, that might also drive these shares). The baseline model can also generate closer cyclicality and persistence of the capital shares than any of the other alternative models we consider. In comparison, the model without bargaining shocks fails to create any meaningful fluctuations in the gross capital share and explains only around 17% of the fluctuations in the net capital share. It also does considerably worse regarding the cyclicality of the capital shares, with correlations to output counterfactually close to 1. Without bargaining shocks, the only moves in the capital shares come from changes in capital and the outside value in the bargaining protocol, and both mechanisms are weak. That also means that the model with “no bargaining shock” does worse matching the autocorrelations of the net and gross capital shares.

By construction, our baseline model generates a nearly acyclical wage: the correlation between output and wages is 0.19 both in the model and in the data. The bargaining shocks compensate for the pro-cyclicality of wages in productivity-driven models (higher productivity increases the marginal productivity of labor and, with it, wages). A shock that lowers wages is also expansionary: it increases the returns to capital and, thus, investment in physical capital and recruiting and, with them, output in the following periods. This mechanism is sufficiently strong to wipe out the correlation between wages and output.
6.1 Comparison with an RBC model

Could a standard RBC model account for the same features of the data as our baseline model with bargaining shocks? The rows “RBC model” in Table 5 answer this question by reporting the quantitative properties of the RBC model à la Hansen-Rogerson introduced in Subsection 4.3 (see also Table G.12 for the corresponding numbers with non-unitary elasticities of substitution and Figures G.29 to G.31 for the corresponding generalized impulse-response functions, or GIRFs). The RBC model fails to account for the cyclicality of wages and unemployment and for the fluctuations in the net capital share induced by changes in depreciation (this RBC model does not generate any volatility in the gross capital share when \( \varepsilon = 1 \); we will relax this assumption below).

Early work by Danthine and Donalson (2002) demonstrated that distribution shocks in an RBC model deliver asset price dynamics consistent with the data.\(^{19}\) Hence, one could follow their approach (or that of Ríos-Rull and Santaeulàlia-Llopis, 2010, and Lansing, 2015) and introduce shocks to the factor shares in production within the RBC model. However, these shocks move real wages in the same direction as output. A negative shock to the factor share of capital lowers the productivity of capital and, via the marginal product of labor, the real wage. Thus, the real wage remains perfectly pro-cyclical even with the additional shock.

To illustrate this point, we calibrate a version of the RBC model with factor share shocks to generate the same volatility of TFP and relative volatility of investment as our baseline model (see rows “RBC model with factor share shock” in Table 5). Because both factor share shocks and productivity shocks generate pro-cyclical wages and highly volatile TFP, their volatility is not identified by our calibration strategy. Instead, we set the volatility of productivity to 0.05% and calibrate the volatility of the factor share to match the volatility of TFP. The correlation of the real wage is unchanged after such a shock relative to the standard RBC model. Figure G.32 shows that the real wage remains pro-cyclical in response to factor share shocks.

6.2 Possible endogeneity of bargaining power

One concern about our interpretation of bargaining shocks is that alternative labor market regulations could be the endogenous responses of the political process to changing unemployment rates. In Appendix G.16, we address this concern in two ways. First, we let bargaining power vary with the unemployment rate in a reduced form that matches the empirical correlation between the unemployment rate and bargaining power in the model and the partial filter. This version of the model explains a slightly larger share of the capital share volatility, but a somewhat smaller share of output volatility.

Second, we let the share of hours worked at the minimum wage and unemployment benefits vary with the unemployment rate, matching regression coefficients in the model with those in the data.

\(^{19}\)Danthine and Donalson (2002) model time-varying labor shares as a stochastic and exogenous wedge between the marginal utility of labor for workers and the marginal utility of labor for investors. Importantly, economic agents cannot react to the shifts in labor shares.
Here, we find that it diminishes the effect of bargaining shocks on the capital share, but increases the share of output volatility explained by bargaining power shocks.

While the topic of the endogeneity of bargaining power deserves further exploration, our reduced-form exercises suggest that bargaining power shocks are likely to remain significant even when accounting for the fact that the bargaining shocks react to issues such as inequality, technological change, demographic fluctuations, globalization, and demand effects.

### 6.3 Increased volatility

Within countries over time and across countries, we observe large differences in the volatility of the capital share. For example, the volatility of the HP-filtered gross capital share has fallen from 1.79% per year from 1929 to 1949 to 0.81% per year from 1950 to 2010 in the U.S., a drop to less than half. The U.K. and France have seen even larger reductions in volatility. At the same time, the U.K. has a much more volatile capital share than the U.S. post-1950; its HP-filtered capital share has been about 40% (0.31/0.81) higher than that in the U.S. (Table 1(a)). Controlling for industry composition, this difference amounts to 60% (Table 1(b)). What would the consequences be if the U.S. capital share became more volatile due to more social or political attempts to redistribute?

If increased distribution risk made bargaining power volatile enough to raise the capital share volatility by 40%, U.S. output (and consumption) would become 15% more volatile. Given that households prefer smooth consumption, the increased volatility reduces welfare. More volatile bargaining power shocks also lower welfare by reducing the ergodic mean of consumption.

Table 6 reports the sizable welfare effects of distribution risk, which are much larger than conventional measures of the welfare cost of the business cycle (Lucas, 1991). Welfare, expressed in consumption units, drops by 0.7% when we increase the volatility of distribution risk to make the capital share 40% more volatile. To compute consumption equivalents, we hold employment fixed at its ergodic mean. Doubling the volatility of the capital share through increased political risk, thus undoing the decline we saw in the U.S. post-1950, leads to a welfare loss of 1.5% of consumption. Eliminating all distribution risk would increase the welfare of the representative household by 2.4% of consumption.

Table 6: Welfare effects of increased or reduced distribution risk

<table>
<thead>
<tr>
<th>Specification</th>
<th>std(Y) [%]</th>
<th>Std(C)/Std(Y)</th>
<th>Std(cs) [%]</th>
<th>Std(n) [%]</th>
<th>Welfare: Δ baseline Consumption units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.91</td>
<td>0.62</td>
<td>0.17</td>
<td>1.89</td>
<td>0</td>
</tr>
<tr>
<td>40% higher capital share volatility</td>
<td>2.20</td>
<td>0.62</td>
<td>0.24</td>
<td>2.39</td>
<td>-0.7%</td>
</tr>
<tr>
<td>100% higher capital share volatility</td>
<td>2.62</td>
<td>0.62</td>
<td>0.34</td>
<td>3.02</td>
<td>-1.5%</td>
</tr>
<tr>
<td>No distribution risk</td>
<td>1.35</td>
<td>0.57</td>
<td>0.01</td>
<td>0.24</td>
<td>+2.4%</td>
</tr>
</tbody>
</table>

Appendix G.15 reports a set of robustness checks, in terms of the cyclical properties of the model and welfare, when we modify how far away the calibration of the model is from satisfying...
the Hosios rule. In particular, the main factor behind the welfare losses is the persistent bargaining power shock and not the steady-state deviations of the model with respect to the Hosios rule.

7 Quantitative results III: The dynamic effects of a bargaining power shock

In the previous section, we reported the unconditional properties of the model. In this section, we document the conditional responses to a bargaining shock. Figure 8 shows the response to a bargaining shock that strengthens workers’ bargaining power. In particular, we plot GIRFs, based on a third-order perturbation following the procedure outlined in Andreasen et al. (2018), to a two standard deviations shock.

![Graphs showing the response to a bargaining shock](image)

Figure 8: GIRFs to a two standard deviations shock raising workers' bargaining power (in months).
First, after the shock, both the gross and net capital shares fall. Second, output drops persistently: a lower capital share leads to less investment, either in capital or in recruiting, and a lower utilization rate. As recruiting efforts are scaled back, final goods production drops by less than output, but future employment also falls. Third, the value of the firm rises initially, if slightly, as firms shift workers from recruiting to production to raise the output of final goods to take advantage of the existing stocks of workers and capital (this movement also explains the initial increase in the marginal productivity of capital and of Tobin’s \( q \)). However, as firms reduce their recruiting efforts, employment falls and, with it, the marginal product of labor. This, together with the lower return on capital due to the bargaining shock, leads to lower investment and a declining capital stock. Both a lower capital and a lower share of the surplus contribute to a fall in firm value. Fourth, wages rise more than the marginal product of labor, again reflecting the change in bargaining power. Finally, market tightness decreases. Furthermore, the increase in unemployment and wages following the increase in workers’ bargaining power is in line with the VAR evidence in Figure 4.

![Graphs showing the response of economic variables to a bargaining power shock.](image)

**Note:** We compute the conditional IRFs by initializing the economy at the states associated with observing a capital share in the top, bottom, or middle 10% of the ergodic distribution.

**Figure 9:** State dependence in IRFs to a two standard deviations bargaining power shock with high vs. low initial capital share (in months).

Interestingly, the response of the economy displays a pronounced state dependence. In particular, the capital share at the moment of the shock matters. Figure 9 shows that a given shock to bargaining power has smaller effects on redistribution and causes larger drops in GDP and firm values when starting from a situation that already features a low capital share of income. When the capital share is small, further reductions in its bargaining power have a higher marginal cost to the

---

\(20\)In the short run, we have some non-monotonicities: the bargaining shock distributes what used to be profits to workers, lowering profits and keeping output initially fixed. The capital share drops. Next output drops, undoing much of the initial increase in the ratio. Then, marginal labor productivity and wages rise further, leading again to a lower capital share.
firm, but firms do not have space to redistribute much additional income to workers. In contrast, with a high capital share, the initial drop in the capital share is twice that of GDP and, after five years, the response of GDP is only twice that of the capital share. Similarly, drops in employment are more muted when the capital share is already high. In short, the non-linearities in our model imply that the price and quantity effects of bargaining power shocks depend on how polarized the income distribution is.

Our model generates large movements in output relative to those in bargaining power because the match surplus is small. If the model featured efficient bargaining in the presence of product market rents as in Blanchard and Giavazzi (2003), capital shares might become more volatile relative to output.

8 Conclusion

Capital shares of income can be volatile. For the three countries for which we have long historical time series—France, the U.K., and the U.S.—we observe substantial declines in the volatility of the capital share after 1950. This volatility also differs across countries. We argue that social and political factors can be important drivers of fluctuations in the factor income distribution. In particular, for the U.S., we find that capital shares rose after the introduction of right-to-work legislation.

We proceed by building a model where workers bargain with firms over the match surplus in the labor market, and their bargaining power is subject to shocks—which we interpret as social and political shocks. Filtering bargaining power from the data confirms our calibration and highlights the connection of bargaining shocks to social and political events. Also, our model matches the standard U.S. business cycle moments and the cyclicality of the capital share. Bargaining shocks are powerful in our model. Even though they explain only between 12% and 26% of the volatility of the gross capital share in the U.S. when calibrated to our baseline data, they can account for 28% to 46% of the volatility of output, depending on the elasticity of substitution between capital and labor. We use our model as a laboratory to ask what would happen if the U.S. capital share became more or less volatile due to increased political risk. Finally, we document that the dynamic effects of distribution shocks in our model are strong and non-linear.

The empirical and theoretical results suggest, therefore, that bargaining shocks might be a significant source of aggregate fluctuations.
References


Online appendix

This appendix includes details about the data, the empirical exercises, the model, and the quantitative results.

A Capital share data

A.1 U.S. data

While the definition of the capital share of income is conceptually straightforward, its measurement is challenging. For instance, we need to allocate ambiguous sources of income such as copyright royalties, deferred compensation, or proprietors’ income between labor and capital. Also, we must decide how to impute indirect taxes. Finally, to go from the gross to the net share, we need to pick depreciation rates.

We now overview different measurements of the capital income share in the U.S. economy. These alternative calculations agree among themselves regarding the behavior of capital income share over middle and business cycle frequencies (see Figure A.2). Thus, for our purposes, picking one measure or another in the U.S. case is inconsequential (Muck et al., 2015, make a similar point). On the other hand, across countries, our empirical statements depend on available data.

We construct the net capital share in the corporate business sector from BEA Table 1.14, “Gross Value Added of Domestic Corporate Business in Current Dollars and Gross Value Added of Nonfinancial Domestic Corporate Business in Current and Chained Dollars,” and focus on the data on non-financial corporate businesses. We compute the net capital share as compensation of employees (mnemonic A460RC1) relative to the sum of compensation and the net operating surplus (mnemonic W326RC1). Figure A.1 plots the resulting series.

![Figure A.1: Net capital share levels: Quarterly U.S. data.](image)

We also consider some alternative measures of the U.S. capital share for comparison:

1. We compute the capital share as the reciprocal of wages over net value added (mnemonic A457RC1), effectively treating taxes as coming out of the capital share only.

2. BLS data on the (reciprocal of the) labor share in the overall business sector (mnemonic PRS84006173), the non-farm business sector (mnemonic PRS85006173), and in the corporate
non-financial sector (mnemonic PRS88003173). The BLS defines the labor share as the ratio of current labor compensation paid to current dollar output, imputing a cost for labor services by proprietors. See p. 7 of http://www.bls.gov/lpc/lpcmethods.pdf for the definition and http://www.bls.gov/data/#productivity for the data.

3. Data on the capital share as the reciprocal of the U.S. labor share in the Penn World Table (Feenstra et al., 2013).

The different measures are reported in the two panels of Figure A.2. Figure A.3 compares the different measures of the labor share that are available in levels. The left panel shows the annual time series, and the right panel plots the shorter quarterly series. In both the annual and the quarterly data, there is no clear evidence of a trend in the labor share over the full sample period. However, most measures of the labor share are close to their minimum at the end of the sample period. In the quarterly data, adjusting for the share of taxes in corporate net value added only results in a roughly parallel shift of the labor share, whereas taking out net government production in the annual series changes the trend behavior. The different labor shares average between about 65% and 80%.\(^{21}\)

Extending the comparison to include the BLS data comes at the cost of losing the level information. Figure A.4 shows that the raw data, indexed to 100 in 2009, correlates positively at higher frequencies, but may exhibit different time trends. Figure A.2, therefore, uses HP-filtered data on the log-labor share. Eyeballing both the annual and the quarterly filtered time series suggests a high agreement. Correlation tables (not shown here) confirm this impression: Raw time series sometimes exhibit low correlations, but filtered correlations are above 0.6 for annual data and above 0.7 for quarterly data except for correlations between manufacturing sectors and broader measures.

A.2 International and U.S. state-level data

- Long-run capital share data: We downloaded the data in Piketty and Zucman (2014) from http://gabriel-zucman.eu/capitalisback/ and use the net capital share (“net profit share”)

\(^{21}\)Giandrea and Sprague (2017) show that 2 pp. of the recent 7 pp. decline in the BLS measure of the labor share is due to the self-employed, for whom the BLS imputes capital income. We use only the corporate non-financial labor share to sidestep this issue. In the Piketty and Zucman data in the main text, we find an increase of 7 pp. from 2001 to 2010 in the net capital share and of 6 pp. in the gross capital share. In our calculations, we find an increase of 10 pp. in the net corporate labor share over this period (see Figure A.1).
from the data sheets on “profits & wages in the corporate sector.”

• U.S. state capital share and GDP data: We use the Bureau of Economic Analysis Regional Accounts section “Annual Gross Domestic Product By State” from http://www.bea.gov/regional/ to obtain data on “compensation of employees,” “taxes on production and imports less subsidies” and “GDP in current dollars” to compute the gross capital share as one minus the compensation of employees over GDP minus taxes net of subsidies. All data are confined to (total) “private industry.” Since the five-year periods in the states we are studying do not include 1997, when the BEA switched from SIC to NAICS, we pool the changes in GDP growth and the capital share based on either underlying classification.

B Controlling for industry composition

To control for industry composition in the effect of capital income share movements in France, the U.K., and the U.S. as described in Section 2 of the main text, we use EU KLEMS data: http://www.euklems.net/. We compute the gross labor share as labor compensation relative to
We drop the following industries from our calculations, as the division between labor and capital income is less straightforward than in other industries:

- Agriculture (code: “AtB”).
- Mining (code: “C”).
- Government (code: “L”).
- Financial intermediation (code: “J”).

We keep the most disaggregated industries available, leaving a total of 27 industries with data available for the three countries.

![Figure B.5: Within-industry volatility of the gross labor share.](image)

C Additional evidence regarding right-to-work legislation

We repeat the same exercise as in Subsection 2.1, but now we look at real GDP growth instead of labor shares. Real GDP growth is computed using the change in state total private-sector GDP deflated by the national GDP deflator. Since the data start only in 1963, the year Wyoming adopted the new legislation, GDP growth in Wyoming is normalized to zero for the first year after adoption. Before 1997, we use private SIC industries. From 1997, we use private NAICS industries.

Figure C.6 reports the evolution of real state private industry GDP growth after the adoption of right-to-work legislation (in absolute levels and relative to the U.S.). Table C.1 presents a panel regression analysis of the data. Standard errors are clustered by state and industry, and two-sided $p$-values are in parentheses.
Figure C.6: Change in real state private industry GDP growth after right-to-work adoption.

Table C.1: State-industry panel regression: Right-to-work laws and real GDP growth

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<th>3y change</th>
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</table>
D Additional VAR results

Figure D.7 plots the IRFs from the small VAR with a quadratic trend.

Figure D.7: Responses to a 10% real minimum wage shock in small VAR, quadratic trend.
E Large VAR

To check the robustness of the VAR exercise in the main text, we now present results using a larger VAR. In addition to the labor market and the non-corporate business sector, this VAR captures asset prices, consumption, and investment. As a result, we arrive at the following ten-variable VAR: (1) the (log) of the federal minimum wage relative to the PCE deflator, (2) the net capital share in the corporate non-financial sector, (3) the average of the total returns of consumer and manufacturing firms, (4) the unemployment rate, (5) non-farm labor productivity in the business sector, (6) labor market tightness, (7) capacity utilization, (8) real private investment, (9) real private consumption, and (10) the average corporate tax rate. Instead of using the cumulative total return in Greenwald et al. (2014), we use the (unweighted) average of the cumulative total return in the consumer and manufacturing sectors based on the five-sector Fama-French industry classification because we expect the minimum wage to be more important in these sectors.\textsuperscript{22} Again, we use four lags in the estimation.

Minimum wage shocks are also clearly redistributive in this large VAR. Figure E.8 shows the IRFs to a typical minimum wage shock of 10%. Such a shock causes the capital share to drop by 0.25 to 0.5 pp. for two to five quarters with 68% posterior credibility. The labor market worsens, with unemployment rising by 0.5 to 1.5 pp. about a year after the initial shock. Labor productivity increases slightly with a delay, consistent with a selection effect. We also find that the stock market valuation drops significantly. Investment drops 5% at the peak and with it capacity utilization. Finally, there is a delayed decline in the average corporate tax rate. This decline may reflect the progressivity of the corporate tax code as corporate profits fall.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Figure E.8: Responses to a 10\% real minimum wage shock in extended VAR: 1951–2014.}
\end{figure}

Many states set minimum wages above the federal level, particularly in the second half of our full sample. Hence, we incorporate state minimum wage changes in our analysis. More concretely, prior to estimation, we aggregate minimum wages across states by weighting them with the relative populations of each state. This weighting is imperfect given that the unemployment rate in our VAR is labor force weighted and stock returns are weighted by market capitalization.\textsuperscript{23}

\textsuperscript{22}We use these sectors because our empirical model does not speak much to the other three sectors. We focus on the non-financial corporate business sectors and thus drop the “other” sector that includes financial firms. See for the source data: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Our results change little when we include only one of the sectors at a time.

\textsuperscript{23}We use the data from Autor et al. (2016). Their coverage of Washington, D.C., has a gap, so we drop it. For the other states, we compute the change in the maximum of the state and the federal minimum wage, quarter by quarter.
Combining state and federal minimum wage strengthens the redistributive effects we estimate; see Figure E.9. After a minimum wage shock, there is a drop in the capital share that lasts for three to four years and peaks -1 to -1.5 pp. after six quarters. With a delay, unemployment rises significantly after five quarters, while stock values, consumption, and utilization fall. The differences in the size and shape of the IRFs of this exercise are not due to the different sample period compared to our large VAR baseline.

We also report several robustness exercises. First, in Figure E.10, we plot the IRFs from the large VAR in the post-1974 sample. Second, in Figure E.11, we plot the IRFs from the same VAR, but now with a quadratic trend and using shocks to the real effective state-level minimum wage. Third, in Figure E.12, we plot the IRFs of the same VAR with a quadratic trend for the full 1951-2014 sample.

Figure E.9: Responses to a 10% real effective state-level minimum wage shock in extended VAR: 1974–2014.

Figure E.10: Responses to a 10% real minimum wage shock in extended VAR: 1974–2014.

We deflate this nominal increase and average it across states using annual population weights.
Figure E.11: Responses to a 10% real effective state-level minimum wage shock in extended VAR: 1974–2014, quadratic trend.

Figure E.12: Responses to a 10% real minimum wage shock in extended VAR: 1951–2014, quadratic trend.
State minimum wage changes

Here we examine the relationship between changes in the maximum of statutory state minimum wage and federal minimum wages, deflated to constant 2010 real dollars, and three outcomes: (1) changes in the gross capital share, (2) changes in the unemployment rate, and (3) real GDP growth per capita. The state-level data are the same as in Section 2.1, except that we obtain the unemployment rate from the BLS via Federal Reserve Economic Data (FRED). We regress the outcome variable on the current or lagged changes in the applicable nominal minimum wage, converted to 2010 dollars. In all specifications, we include state year fixed effects and cluster the standard errors by state. We also report variants that also include year fixed effects. For each specification, we report results for the full sample, and the sample of state-years with actual changes in the minimum wage.

Our results are the strongest for the capital share. Figure F.13 documents a significant negative relationship between changes in state minimum wages and the gross capital share within states in the specification that considers only state-years with changes in the minimum wage and includes year fixed effects. Table F.2 includes the detailed regression result that corresponds to the figure. Panel (a) reports regressions using the full sample, where columns (a1) through (a4) include state fixed effects, while columns (a5) through (a8) also include year fixed effects. Here, the point estimates point to a decline of the capital share in the year of the minimum wage increase and the year after, with a reversal after two years. The declines are, however, not significant with the year fixed effects. When we condition on changes in the minimum wage only and use only state fixed effects, we find no significant change in the capital share on impact (column (b1)), but a decline of 0.8 pp. with a one-year delay (column (b2)), and a partial reversal two years after (column (b3)). The results are similar when we estimate the impact and lagged effects simultaneously (column (b4)). With year fixed effects, we find an impact decline in the capital share of 0.42 (column (b5)) and a further decline of 0.46 in the year after (column (b6)), resulting in a cumulative decline of around 0.9 pp. for a one-dollar increase, similar to the estimate without year fixed effects. Two years after, this effect is partially reversed (column (b7)). Jointly estimating the effects yields similar signs, but smaller magnitudes and no statistical significance (column (b8)). Using all state-years for the estimation yields results similar to those with only state fixed effects, but insignificant results once we also include year fixed effects.

Tables F.3 and F.4 show the analogous results for economic activity, measured as changes in the unemployment rate or the real per capita GDP growth rate. For all state-years and with only state fixed effects, we find significant increases in the unemployment rate and decreases in real GDP growth on impact and two years after, as columns (a1) through (a4) show. A one-dollar increase in the minimum wage is associated with an increase in the unemployment rate of 0.7 pp. and a decrease in the GDP growth rate of 2.2 pp. in the year of the increase. These results weaken, however, once we introduce year fixed effects, in which case only the effect after two years remains significant for both the unemployment increase and the reduction in GDP growth (columns (a7) and (a8)). Conditioning on years with changes, our results for GDP growth are very similar: We find a significant drop on impact and after two years (columns (b1), (b3) and (b4)), but with year fixed effects the results become largely insignificant, except in column (b8), which also points to a decrease in GDP growth with a two-year lag. For the unemployment rate, the results are more subtle when we consider only years with changes in the minimum wage. With state fixed effects only, we estimate a 0.3 pp. drop in the unemployment rate on impact (column (b1)), followed by increases of 0.4 pp. and 0.8 pp. (columns (b2) and (b3)). Estimating the current and lagged effects jointly points to statistically and economically significant drops on impact and with lags of one and two years (column (b4)). With year fixed effects, however, the results are largely insignificant.

The ticker symbols are AKURN, ALURN, ..., downloadable from http://research.stlouisfed.org/fred2/.
## Figure F.13: Change in state statutory minimum wage and change in private-sector capital share: Contemporaneous and lagged relationship.

### Table F.2: State panel regression: Statutory state minimum wage changes and gross capital share

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<th>(a2)</th>
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<th>(a5)</th>
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<td>(-0.572***)</td>
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Standard errors are clustered by state. t-statistics are in parentheses. * p < 0.1 ** p < 0.05 *** p < 0.01
Table F.3: State panel regression: Statutory state minimum wage changes and unemployment

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</tr>
<tr>
<td>L2. $\Delta$ statutory min. wage (real USD)</td>
<td>0.873***</td>
<td>0.418***</td>
</tr>
<tr>
<td></td>
<td>(10.80)</td>
<td>(4.31)</td>
</tr>
</tbody>
</table>

R-squared | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| R-sq. within | 0.04 | 0.12 | 0.06 | 0.14 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Observations | 1770 | 1773 | 1776 | 1672 | 1770 | 1773 | 1776 | 1672 | 1770 | 1773 | 1776 | 1672 | 1770 | 1773 | 1776 | 1672 |
| States | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 |
| State FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year FE | No | No | No | No | No | No | No | No | No | No | No | No | No | No | No | No |
| Sample | All | All | All | All | All | All | All | All | All | All | All | All | All | All | All | All |

R-squared | 0.04 | 0.05 | 0.04 | 0.12 | 0.73 | 0.74 | 0.75 | 0.71 | 0.00 | 0.01 | 0.03 | 0.11 | 0.00 | 0.00 | 0.00 | 0.01 |
| R-sq. within | 0.00 | 0.01 | 0.03 | 0.11 | 0.73 | 0.74 | 0.75 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Observations | 734 | 741 | 790 | 1096 | 730 | 737 | 787 | 1095 | 734 | 741 | 790 | 1096 | 734 | 741 | 790 | 1096 |
| States | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 |
| State FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year FE | No | No | No | No | No | No | No | No | No | No | No | No | No | No | No | No | No |
| Sample | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes | Changes |

Standard errors are clustered by state. $t$-statistics are in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$
Table F.4: State panel regression: Statutory state minimum wage changes and real per capita GDP growth

(a) All state-years

<table>
<thead>
<tr>
<th>Growth (%)</th>
<th>(a1)</th>
<th>(a2)</th>
<th>(a3)</th>
<th>(a4)</th>
<th>(a5)</th>
<th>(a6)</th>
<th>(a7)</th>
<th>(a8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ statutory min. wage (real USD)</td>
<td>-2.163***</td>
<td>-2.132***</td>
<td>0.089</td>
<td>-0.061</td>
<td>(-6.61)</td>
<td>(-6.45)</td>
<td>(0.20)</td>
<td>(-0.13)</td>
</tr>
<tr>
<td>L.Δ statutory min. wage (real USD)</td>
<td>-1.835***</td>
<td>-0.875***</td>
<td>-0.246</td>
<td>-0.133</td>
<td>(-7.38)</td>
<td>(-3.25)</td>
<td>(-0.81)</td>
<td>(-0.51)</td>
</tr>
<tr>
<td>L2.Δ statutory min. wage (real USD)</td>
<td>-1.405***</td>
<td>-1.452***</td>
<td>-0.533*</td>
<td>-0.544*</td>
<td>(-5.41)</td>
<td>(-5.47)</td>
<td>(-1.86)</td>
<td>(-1.87)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
<td>0.17</td>
<td>0.38</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>R-sq, within</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Observations</td>
<td>1220</td>
<td>1223</td>
<td>1226</td>
<td>1122</td>
<td>1220</td>
<td>1223</td>
<td>1226</td>
<td>1122</td>
</tr>
<tr>
<td>States</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>State FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
</tr>
</tbody>
</table>

(b) State-years with minimum wage changes only

<table>
<thead>
<tr>
<th>Growth (%)</th>
<th>(b1)</th>
<th>(b2)</th>
<th>(b3)</th>
<th>(b4)</th>
<th>(b5)</th>
<th>(b6)</th>
<th>(b7)</th>
<th>(b8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ statutory min. wage (real USD)</td>
<td>-1.958***</td>
<td>-2.760***</td>
<td>0.147</td>
<td>-0.317</td>
<td>(-3.81)</td>
<td>(-7.20)</td>
<td>(0.29)</td>
<td>(-0.65)</td>
</tr>
<tr>
<td>L.Δ statutory min. wage (real USD)</td>
<td>-0.690</td>
<td>-1.026***</td>
<td>-0.122</td>
<td>-0.271</td>
<td>(-1.49)</td>
<td>(-3.48)</td>
<td>(-0.28)</td>
<td>(-0.90)</td>
</tr>
<tr>
<td>L2.Δ statutory min. wage (real USD)</td>
<td>-2.331***</td>
<td>-2.093***</td>
<td>-0.648</td>
<td>-0.838***</td>
<td>(-4.55)</td>
<td>(-7.46)</td>
<td>(-1.40)</td>
<td>(-2.70)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.18</td>
<td>0.13</td>
<td>0.15</td>
<td>0.23</td>
<td>0.53</td>
<td>0.54</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>R-sq, within</td>
<td>0.04</td>
<td>0.00</td>
<td>0.05</td>
<td>0.13</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Observations</td>
<td>520</td>
<td>482</td>
<td>488</td>
<td>733</td>
<td>518</td>
<td>480</td>
<td>487</td>
<td>733</td>
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<td>States</td>
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<td>51</td>
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<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>State FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
</tr>
</tbody>
</table>

Standard errors are clustered by state. t-statistics are in parentheses. * p < 0.1 ** p < 0.05 *** p < 0.01
G Model appendix

Our business cycle model with search frictions in the labor market is in the spirit of those in Andolfatto (1996) and Merz (1995) and builds on the formulation of Shimer (2010, ch. 3). Relative to the notation in Shimer (2010), we change the timing convention so that capital $k_t$ and employment $n_t$ are time $t$ measurable, but not time $t-1$ measurable.

G.1 The household

There is a representative household that perfectly ensures its members against idiosyncratic risk. The following utility function represents its preferences:

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_{e,t} - h c_{e,t-1}^{a})^{1-\sigma}(1 + (\sigma - 1)\gamma)^\sigma - 1}{1 - \sigma} n_{t-1} + \frac{(c_{u,t} - h c_{u,t-1}^{a})^{1-\sigma} - 1}{1 - \sigma} (1 - n_{t-1}) \right),
$$

(G.1)

where $c_{e,t}$ and $c_{u,t}$ are the consumption of the employed and unemployed household members, respectively, and $n_{t-1}$ denotes the fraction of employed households. The parameter $h \in [0,1)$ controls the strength of the external habit.

After matching with firms, employed household members draw an iid type. With probability $\zeta_0,t$, they have only $\zeta_1 \in (0,1)$ efficiency units of labor – otherwise they have one efficiency unit. For clarity, we denote variables referring to the high types with subscripts $h$ and, for low types, by subscripts $\ell$. We allow these employed members to receive a wage $w_{\ell,t}$, different from $w_{h,t}$ that productive workers receive. Minimum wages may imply that $w_{\ell,t} > \zeta_1 w_{h,t}$ in equilibrium. We denote variables with subscript $\zeta$ as weighted averages. For example, $w_{\zeta,t} \equiv (1 - \zeta_0,t) w_{h,t} + \zeta_0,t w_{\ell,t}$. Because we end up assuming that all employed household members with the low type receive the minimum wage $w_{\ell}$, it may also have the interpretation of other welfare payments. For example, if $\zeta_1 = 0$, then $w_{\ell}$ might better be thought of as mandatory sick pay. The household perfectly insures the employed against variations in their type.

The household faces a lifetime budget constraint given the stochastic discount factor $m_t$:

$$
a_{-1} = E_0 \sum_{t=0}^{\infty} \prod_{s=0}^{t} m_t \left( c_{e,t} n_{t-1} + c_{u,t} (1 - n_{t-1}) - (1 - \tau_n) (1 - (1 - \zeta_0,t) w_{h,t} + \zeta_0,t w_{\ell,t}) n_{t-1} - (1 - \tau_n) \omega_t (1 - n_{t-1}) - T_t \right),
$$

where the present discounted value of consumption equals the beginning of the period financial wealth $a_{-1}$ plus net labor income $(1 - \tau_n) w_{h,t}$ for the fraction $1 - \zeta_0,t$ of workers who are productive and $(1 - \tau_n) w_{\ell,t}$ for the workers who are unproductive. We also define $w_{\zeta,t} = (1 - \zeta_0,t) w_{h,t} + \zeta_0,t w_{\ell,t}$. The household receives unemployment benefits $\omega_t$ and lump-sum transfers $T_t$.

Finally, when making its decisions, the household considers that workers lose their jobs at rate $x$ and find new jobs at rate $f(\theta_t) = \xi \theta_t^\eta$, where $\theta_t$ is the recruiter-unemployment ratio that the household takes as given. Thus, the fraction of household members employed next period will be:

$$
n_t = (1 - x) n_{t-1} + f(\theta_t) (1 - n_{t-1}).
$$

(G.2)
G.1.1 Aggregation

Under perfect insurance within the household, a necessary condition for the household’s optimality is that consumption of the employed and unemployed satisfies:

\[
\beta^t(c_{e,t} - h c_{e,t-1}^a)^{-\sigma} (1 + (\sigma - 1)\gamma)^\sigma = \beta^t(c_{u,t} - h c_{u,t-1}^a)^{-\sigma} = \lambda m_t,
\]

where \(\lambda\) is the Lagrangian multiplier associated with the budget constraint. If \(h = 0\) or given the initial condition that \(c_{e,t-1}^a = c_{u,t-1}^a(1 + (\sigma - 1)\gamma)\), it follows that:

\[
c_{e,t} = c_{u,t}(1 + (\sigma - 1)\gamma)
\]

and

\[
c_{t} \equiv c_{e,t} n_{t-1} + c_{u,t}(1 - n_{t-1})
\]

\[
c_{u,t} = \frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}}
\]

\[
c_{e,t} = \frac{c_t(1 + (\sigma - 1)\gamma)}{1 + (\sigma - 1)\gamma n_{t-1}}.
\]

Hence, the utility function can be simplified as:

\[
U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( c_t - h c_{e,t-1}^a \right)^{1-\sigma} \left( 1 + (\sigma - 1)\gamma n_{t-1} \right)^\sigma - 1
\]

and the budget constraint becomes:

\[
a_{-1} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} m_t \right) \left( c_t - (1 - \tau_n)w_{t+1} - (1 - \tau_t)\omega_t(1 - n_{t-1}) - T_t \right).
\]

With \(h > 0\), the household partially internalizes that increasing employment changes the size of habit one period ahead. Setting \(h = 0\) recovers equation (3.1) in the main text.

G.1.2 Equilibrium conditions

We start the analysis of the labor market by writing the household problem using a recursive formulation:

\[
V(a_{-1}, n_{-1}; S) = \max_{a(S'), c, n} \frac{(c - \hat{h}(n_{-1}) c_{-1}^a)^{1-\sigma} (1 + (\sigma - 1)\gamma n_{-1})^\sigma - 1}{1 - \sigma} + \beta \mathbb{E}[V(a(S'), n; S') | S]
\]

subject to:

\[
n = (1 - x)n_{-1} + f(\theta)(1 - n_{-1})
\]

\[
c = a_{-1} + (1 - \tau_n)w_{t+1} - (1 - \tau_t)\omega_t(1 - n_{t-1}) + T_t - \mathbb{E}[m(S') a(S') | S]
\]

and where:

\[
\hat{h}(n_{-1}) = \frac{h}{1 + (\sigma - 1)\gamma n_{-1}^a_2}.
\]
Complete markets ensure that the household can pick next period’s assets as a function of the future state $S'$.

The equilibrium conditions for an interior equilibrium are:

$$
\lambda = (c - \hat{h}(n_{-1})e_{-1})^{-\sigma}(1 + (\sigma - 1)\gamma n_{-1})^\sigma
$$  

$$
\lambda m(S') = \beta V_u(a(S'), n; S') = \beta(c(S') - \hat{h}(n)e^\sigma)(1 + (\sigma - 1)\gamma n)^\sigma. 
$$  

Thus, the stochastic discount factor of the economy is:

$$
m(S') = \beta \frac{(c(S') - \hat{h}(n)e^\sigma)(1 + (\sigma - 1)\gamma n)^\sigma}{(c - \hat{h}(n_{-1})e_{-1})^{-\sigma}(1 + (\sigma - 1)\gamma n_{-1})^\sigma}. 
$$  

In equilibrium, $c^\sigma = c$. In what follows, we use $m_t$ as shorthand for $m(S_t)$ with $m_0 = 1$. 

The marginal value of employment (after the type $i$ is realized) is given by:

$$
V_{i,n}(a_{-1}, n_{-1}; S) = \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma}(1 - \tau_n)(w_i - \omega)
- \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{1-\sigma} \gamma \left(\sigma + (\sigma - 1)\frac{\hat{h}(n_{-1})e_{-1}}{c - \hat{h}(n_{-1})e_{-1}}\right)
+ \beta(1 - x - f(\theta))\mathbb{E}\left[(1 - \zeta_0)V_{h,n}(a(S'), n; S') + \zeta_0'\nu_{\ell,n}(a(S'), n; S')|S]\right] 
$$  

and

$$
V_{\ell,n}(a_{-1}, n_{-1}; S) = V_{h,n}(a_{-1}, n_{-1}; S) + \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma}(1 - \tau_n)(w_\ell - w_h). 
$$  

Because all terms are independent of $i$ except for the wage rate, the ex ante marginal value or the average marginal value is simply:

$$
V_{\zeta,n}(a_{-1}, n_{-1}; S) = \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma}(1 - \tau_n)(w_\zeta - \omega)
- \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{1-\sigma} \gamma \left(\sigma + (\sigma - 1)\frac{\hat{h}(n_{-1})e_{-1}}{c - \hat{h}(n_{-1})e_{-1}}\right)
+ \beta(1 - x - f(\theta))\mathbb{E}\left[V_{\zeta,n}(a(S'), n; S')|S]\right] . 
$$  

For intuition, note that:

$$
V_{\zeta,n}(a_{-1}, n_{-1}; S) = V_{h,n}(a_{-1}, n_{-1}; S) + \left(\frac{c - \hat{h}(n_{-1})e_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma}(1 - \tau_n)\zeta_0\nu_\ell(w_\ell - w_h) . 
$$  

The average marginal value of having an extra person employed is that of having the high type employed plus the (negative) expected value of the wage differential to the low type.

For future reference, it is useful to have a dynamic expression that uses (G.14) to write the $V_{h,h}$
The constant elasticity of substitution between effective capital and labor in production is given by

\[
V_{h,n}(a_{-1}, n_{-1}; S) = \left( \frac{c - \hat{h}(n_{-1})e_{a-1}}{1 + (\sigma - 1)\gamma n_{-1}} \right)^{-\sigma} (1 - \tau_n)(w_h - \omega) 
- \left( \frac{c - \hat{h}(n_{-1})e_{a-1}}{1 + (\sigma - 1)\gamma n_{-1}} \right)^{1-\sigma} \gamma \left( \sigma + (\sigma - 1) \frac{\hat{h}(n_{-1})e_{a-1}}{c - \hat{h}(n_{-1})e_{a-1}} \right) 
+ \beta(1 - x - f(\theta))E \left[ V_{h,n}(a, n; S') + \left( \frac{c' - \hat{h}(n)e_{a}}{1 + (\sigma - 1)\gamma n} \right)^{-\sigma} (1 - \tau_n)\zeta_0'(w'_\ell - w'_h)|S \right].
\] (G.15)

A useful equilibrium object is the value of having a worker employed at an arbitrary wage \( \tilde{w} \) this period and at the equilibrium wage:

\[
\tilde{V}_{i,n}(a, n_{-1}; S) = \left( \frac{c - \hat{h}(n_{-1})e_{a-1}}{1 + (\sigma - 1)\gamma n_{-1}} \right)^{-\sigma} (1 - \tau_n)(\tilde{w}_i - w_i) + V_{i,n}(a_{-1}, n_{-1}; S).
\] (G.16)

\( \tilde{V}_{i,n} \) differs from the marginal value of an extra worker employed at the equilibrium wage both this period and thereafter, i.e., \( V_{i,n} \), by the marginal utility of income times the difference in the net wage income.

In what follows, we write \( \mathbb{E}_t[\cdot] \) for the conditional expectation \( \mathbb{E}_t[\cdot|S_t] \) and similarly index the value function instead of explicitly carrying the state vector and its other arguments.

G.2 The firm

There is a representative firm with \( n_{-1} \) workers and capital \( k_{-1} \). It assigns a fraction \( \nu_i, i = h, \ell \) of its \( (1 - \zeta_0)n_{-1} \) type \( h \) and \( \zeta_0n_{-1} \) workers to recruiting and the remainder to production. Because the marginal product of type \( \ell \) is \( \zeta_1 \) both in production and in hiring, in equilibrium \( v_h = \nu_h \) is optimal. Thus, we can drop the type \( i \) subscript and just write \( \nu \), so that \( n_{-1}(1 - \nu) \) workers are producing. The firm produces the final good with the technology:

\[
y_t = \left( \alpha^{1/\varepsilon}(u_tk_{t-1})^{1-1/\varepsilon} + (1 - \alpha)^{1/\varepsilon}(\epsilon^{\eta,\ell}(1 - \tilde{\zeta}_t)z_t\nu_{t-1}(1 - \nu_t))^{1-1/\varepsilon} \right)^{-\frac{1}{\varepsilon - 1}} 
\equiv \omega(u_tk_{t-1}, z_t(1 - \tilde{\zeta}_t)\nu_{t-1}(1 - \nu_t))
\] (G.17)

where \( 1 - \tilde{\zeta}_t \equiv 1 - \zeta_0, + \zeta_0, \zeta_1 = 1 - \zeta_0, (1 - \zeta_1) \) is the average number of available efficiency units. The constant elasticity of substitution between effective capital and labor in production is given by \( \varepsilon \), labor-augmenting growth trend \( g_z \), and the productivity process \( z_t \) that follows, for the moment, the AR(1) specified in the main text.

The law of motion for capital is:

\[
k_t = (1 - \delta(u_t))k_{t-1} + \chi i_t \left( 1 - \frac{1}{2} \kappa \left( \frac{i_t}{k_{t-1}} - \tilde{\delta} \right)^2 \right), \] (G.18)

where \( \tilde{\delta} \equiv g_z - (1 - \delta(u)) \), \( \chi \) is the marginal efficiency of investment, and

\[
\delta(u) = \delta_0 + \delta_1(u - 1) + \frac{1}{2} \delta_2(u - 1)^2. \] (G.19)
The firm’s value is given by:

\[ J(n_{-1}, k_{-1}) = \mathbb{E} \sum_{t=0}^{\infty} \left( \prod_{s=1}^{t} m_t \right) ((1 - \tau_k)(y_t - w_t n_t) + \tau_k \delta(uq_{k-1} - i_t)), \]

where production and capital follow from equations (G.17) and (G.18) and employment growth satisfies:

\[ n_t = (\nu_t \mu(\theta_t)(1 - \bar{\zeta}_t) + 1 - x)n_{t-1}, \]

where \( \mu(\theta_t) \equiv f(\theta_t)/\theta_t \) is the hiring probability per efficiency units of recruiters. Given a LLN, this value function holds both before and after learning the current types of individual workers.

The firm’s value can be expressed recursively as:

\[ J(n_{-1}, k_{-1}) = \max_{\nu, \alpha, k, I} \left( (1 - \tau_k) \left( \omega(u_{k-1}, z(1 - \bar{\zeta}_t)n_{-1}(1 - \nu), \Psi) - n_{-1}(w_{\zeta}) \right) + \tau_k \delta k_{t-1} - I \right. \\
+ q \left( -k + (1 - \delta(u))k_{t-1} + \chi I \left( 1 - \frac{1}{2} k \left( \frac{I}{k_{t-1} - \delta} \right) ^2 \right) \right) \\
+ \mathbb{E} \left[ mJ(n_{-1}(\nu \mu(\theta)(1 - \bar{\zeta}_{t+1}) + 1 - x), k) \right]. \] (G.20)

### G.2.1 Firm Optimality

At an interior solution for the share of recruiters, the following optimality condition holds:

\[ (1 - \tau_k) \left( 1 - \bar{\zeta}_t \right) \left( 1 - \alpha \right) \frac{Y_t}{z_t(1 - \bar{\zeta}_{t+1})n_{t-1}(1 - \nu_t)} \left( \frac{1}{2} \right) = \mu(\theta_t)(1 - \bar{\zeta})\mathbb{E}[m_{t+1}J_{\zeta,n}(n_t, k_t)]. \] (G.21)

There is no analogous FOC with \( \mathbb{E}[J_{h,n}] \) or \( \mathbb{E}[J_{x,n}] \) on the RHS because one period ahead, the two types are identical.

Thus, the marginal value of overall employment is given by:

\[ J_{\zeta,n}(n_{t-1}, k_{t-1}) = (1 - \tau_k) \left( mpl_t \times (1 - \nu_t) - w_{\zeta,t} \right) + (\nu_t \mu(\theta_t)(1 - \bar{\zeta}_t) + 1 - x) \mathbb{E}[m_{t+1}J_{\zeta,n}(n_t, k_t)] \]

\[ = (1 - \tau_k) \left( mpl_t \left( 1 + \frac{1 - x}{\mu(\theta_t)(1 - \bar{\zeta}_{t+1})} \right) - w_{\zeta,t} \right), \] (G.22)

using equation (G.21) to substitute for \( \mathbb{E}[m_{t+1}J_{\zeta,n}(n_t, k_t)] \). The constant taxes \( \tau_k \) do not distort the recruiting decision because they affect costs and benefits proportionally.

Define as \( n_{t,-} \equiv \zeta_{0,0}n_{-} \) and \( n_{h,-} \equiv (1 - \zeta_{0,0})n_{-} \) and \( n_{\zeta,-} = \zeta_1 n_{t,-} + n_{h,-} = (1 - \bar{\zeta})n_{-} \). \( y_t \) in (G.17) depends only on \( n_{\zeta,-} \) and then:

\[ mpl \equiv \frac{\partial y}{\partial n_{-}} = \frac{\partial y}{\partial n_{\zeta,-}} \frac{\partial n_{\zeta,-}}{\partial n_{-}} = \frac{\partial y}{\partial n_{\zeta,-}}(1 - \bar{\zeta}) \]

\[ mpl_t \equiv \frac{\partial y}{\partial n_{t,-}} = \frac{\partial y}{\partial n_{\zeta,-}} \frac{\partial n_{\zeta,-}}{\partial n_{t,-}} = \frac{\partial y}{\partial n_{\zeta,-}} \zeta_1 = \frac{mpl}{1 - \bar{\zeta}} \]

\[ mpl_h \equiv \frac{\partial y}{\partial n_{h,-}} = \frac{\partial y}{\partial n_{\zeta,-}} \frac{\partial n_{\zeta,-}}{\partial n_{h,-}} = \frac{mpl}{1 - \bar{\zeta}}. \]
Using the type-specific marginal products, analogous equations hold for the marginal value of individual types. This reflects the linearity of the production and recruiting technologies. More concretely,

\[
J_{\ell,n}(n_{t-1},k_{t-1}) = (1 - \tau_k) \left( mpl_t \times (1 - \nu_t) \frac{\zeta_1}{1 - \zeta_t} - w_{\ell,t} \right) + (\nu_t \mu(\theta_t) \zeta_1 + 1 - x) \mathbb{E} [m_{t+1} J_{\cdot,n}(n_t, k_t)]
\]

\[
= (1 - \tau_k) \left( mpl_t \times (1 - \nu_t) \frac{\zeta_1}{1 - \zeta_t} - w_{\ell,t} \right) + \frac{\zeta_1}{1 - \zeta_t} \left( \nu_t + \frac{1 - x}{\mu(\theta_t) \zeta_1} \right) \mu(\theta_t)(1 - \zeta) \mathbb{E} [m_{t+1} J_{\cdot,n}(n_t, k_t)]
\]

\[
= (1 - \tau_k) \left( mpl_t \frac{\zeta_1}{1 - \zeta_t} \left( 1 + \frac{1 - x}{\mu(\theta_t) \zeta_1} \right) - w_{\ell,t} \right),
\]

and replacing \( \zeta_i \) with 1 and \( \ell \) with \( h \):

\[
J_{h,n}(n_{t-1},k_{t-1}) = (1 - \tau_k) \left( mpl_t \frac{1}{1 - \zeta_t} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) - w_{h,t} \right),
\]

As in the household case, \( J_{\zeta,n} = (1 - \zeta_{0,t}) J_{h,n} + \zeta_{0,t} J_{\ell,n} \). We can also write

\[
J_n(n_{t-1},k_{t-1}) = J_{h,n}(n_{t-1},k_{t-1}) - (1 - \tau_k) mpl_t \frac{\zeta_t}{1 - \zeta_t} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) \mathbb{E} [w_{h,t} - w_{\ell,t}].
\]

\[
\iff J_{h,n}(n_{t-1},k_{t-1}) = J_n(n_{t-1},k_{t-1}) + (1 - \tau_k) mpl_t \frac{\zeta_t}{1 - \zeta_t} - (1 - \tau_k) \zeta_0 (w_{h,t} - w_{\ell,t}).
\]

The difference between the values of having an average instead of a high type is, all else equal, negative because of the lower MPL of the average type, but lower if the average type receives a lower wage rate than the high type.

Now, define the marginal profit of employing a worker at an arbitrary (off-equilibrium) wage \( \tilde{w} \) and at the equilibrium wage from then on, given employment and capital at the firm:

\[
\tilde{J}_i,n(n,k) = (1 - \tau_k) (w_{i,t} - \tilde{w}) + J_i,n(n,k).
\]

The optimality condition for the utilization rate is:

\[
\delta'(u_t) q_t k_{t-1} = (\delta_1 + \delta_2(u_t - 1)) q_t k_{t-1} = (1 - \tau_k) \left( \frac{\alpha - y_t}{u_t k_{t-1}} \right)^{1/\varepsilon} k_{t-1} \equiv (1 - \tau_k) \frac{mpl_t}{u_t},
\]

and for investment:

\[
1 = q_t \chi \left( 1 - \frac{1}{2} \kappa \left( \frac{\tilde{\delta}}{k_{t-1}} \right)^2 \right) - \kappa \left( \frac{\tilde{\delta}}{k_{t-1}} \right)^{\frac{1}{\kappa}}.
\]

The optimality condition for capital \( k' \) is given by:

\[
q_t = \mathbb{E} [m_{t+1} J_k(n_t, k_t)]
\]

\[
q_t = \mathbb{E} \left[ m_{t+1} \left( mpl_{t+1} (1 - \tau_k) + \tau_k \tilde{\delta} + (1 - \delta(u_{t+1})) + \chi \kappa \left( \frac{i_{t+1}}{k_t} \right)^2 \left( \frac{i_{t+1}}{k_t} - \tilde{\delta} \right) q_{t+1} \right) \right],
\]
where the marginal product of physical capital is:

\[ m p k_{t+1} \equiv u_{t+1} \left( \alpha \frac{Y_{t+1}}{u_{t+1} k_t} \right)^{\frac{1}{\gamma}} \].

(G.30)

**G.3 Wage determination**

Under Nash bargaining, the equilibrium wage for type \( h \) solves, for a generic time-varying \( \phi_t \):

\[ w_{h,t} = \arg \max_w \tilde{V}_{h,n,t}(\tilde{w})^\phi_t \tilde{J}_{h,n,t}(\tilde{w})^{1-\phi_t}. \]

The solution of this bargaining problem requires that, after plugging in from equations (G.26) and (G.31), the following condition holds:

\[
(1 - \phi_t)(1 - \tau_k) \frac{c_t - \hat{h}_{t-1} c_{t-1}^h}{1 + (\sigma - 1) \gamma n_{t-1}} = \phi_t (1 - \tau_n) \frac{J_{h,n}(n_{t-1}, k_{t-1})}{J_{h,n,t}}.
\]

We use this expression to simplify equation (G.15), after multiplying (G.15) through by \((1 - \tau_k)\) and dividing by the current marginal utility of consumption. We multiply and divide within the expectation operator:

\[
(1 - \phi_t)(1 - \tau_k) V_{h,n,t} \left( \frac{c_t - \hat{h}_{t-1} c_{t-1}^h}{1 + (\sigma - 1) \gamma n_{t-1}} \right)^{\sigma} = (1 - \phi_t)(1 - \tau_k)(w_{h,t} - \omega_t) - (1 - \phi_t)(1 - \tau_k) \left( \frac{c_t - \hat{h}_{t-1} c_{t-1}^h}{1 + (\sigma - 1) \gamma n_{t-1}} \right)^{\gamma} \left( \sigma + (\sigma - 1) \frac{\hat{h}_{t-1} c_{t-1}^h}{c_t - \hat{h}_{t-1} c_{t-1}^h} \right) + (1 - x - f_t(\theta_t)) \mathbb{E}_t \left[ \beta_t \left( \frac{c_{t+1} - \hat{h}_{t+1} c_{t+1}^h}{c_{t+1} - \hat{h}_{t+1} c_{t+1}} \right)^{\sigma} \left( 1 - \phi_t + 1 \right) \right] \times (1 - \phi_{t+1}) \left( \frac{c_{t+1} - \hat{h}_{t+1} c_{t+1}^h}{1 + (\sigma - 1) \gamma n_t} \right)^{\sigma} (1 - \tau_n) V_{h,n,t+1} + (1 - \tau_n)(1 - \tau_k) \xi_{0,t+1}(w_{t,t+1} - w_{h,t+1})].
\]

Next, we substitute from equation (G.31), taking care to keep track of the future bargaining power terms:

\[
\phi_t (1 - \tau_n) J_{h,n,t} = (1 - \phi_t)(1 - \tau_k)(1 - \tau_n)(w_{h,t} - \omega_t) - (1 - \phi_t)(1 - \tau_k) \left( \frac{c_t - \hat{h}_{t-1} c_{t-1}^h}{1 + (\sigma - 1) \gamma n_{t-1}} \right)^{\gamma} \left( \sigma + (\sigma - 1) \frac{\hat{h}_{t-1} c_{t-1}^h}{c_t - \hat{h}_{t-1} c_{t-1}^h} \right) + (1 - x - f_t(\theta_t)) \mathbb{E}_t \left[ \frac{1 - \phi_t}{1 - \phi_{t+1}} m_{t+1}(1 - \tau_n) (\phi_{t+1} J_{h,n,t+1} + (1 - \tau_k)(1 - \phi_{t+1}) \xi_{0,t+1}(w_{t,t+1} - w_{h,t+1})) \right].
\]

Then, we substitute from equation (G.24) for current \( J_{h,n} \) on the LHS and for future \( J_{h,n} \) from (G.25). Also, divide by \((1 - \tau_k)\):

\[
\phi_t (1 - \tau_n) \left( m p_{t+1} \frac{1}{1 - \xi_t} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) - w_{h,t} \right) = (1 - \phi_t)(1 - \tau_n)(w_{h,t} - \omega_t) - (1 - \phi_t) \left( \frac{c_t - \hat{h}_{t-1} c_{t-1}^h}{1 + (\sigma - 1) \gamma n_{t-1}} \right)^{\gamma} \left( \sigma + (\sigma - 1) \frac{\hat{h}_{t-1} c_{t-1}^h}{c_t - \hat{h}_{t-1} c_{t-1}^h} \right).
\]
\[ + (1 - x - f_t(\theta_t))E_t \left[ \frac{1 - \phi_t}{1 - \phi_{t+1}} m_{t+1}(1 - r_n) \left( \phi_{t+1} \frac{J_{\xi,n,t+1}}{1 - r_k} + \phi_{t+1} \frac{\bar{\zeta}_{t+1}}{1 - \zeta} m_{pl,t+1} + \zeta_0 w_{t+1} - w_{h, t+1} \right) \right] \]

(G.32)

If \( \phi_t \) were constant, we could substitute out for future \( J_{\xi,n} \) conveniently from the recruiting optimality condition (G.21).

**G.4 Market clearing**

Market clearing involves, first, the resource constraint of the economy:

\[ y_t \equiv \left( \alpha^{1/\varepsilon} (u_{t} k_{t-1})^{1-1/\varepsilon} + (1 - \alpha)^{1/\varepsilon} (z_t(1 - \tilde{\zeta}_t)n_{t-1}(1 - \nu_t))^{1-1/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon - 1}} = c_t + i_t. \]  

(G.33)

Second, the law of motion of capital:

\[ k_t = (1 - \delta(u_t))k_{t-1} + \chi i_t \left( 1 - \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \tilde{\delta} \right)^2 \right). \]  

(G.34)

Third, the law of motion for employment:

\[ n_t = (1 - x)n_{t-1} + f_t(\theta_t)(1 - n_{t-1}). \]  

(G.35)

Finally, the recruiter-unemployment ratio (analogous to market tightness) is:

\[ \theta_t = \frac{\nu n_{t-1}(1 - \tilde{\zeta}_t)}{1 - n_{t-1}}. \]  

(G.36)

**G.5 Efficiency**

Following Hosios (1990), we assess the allocative efficiency of the decentralized equilibrium. We consider a social planner’s problem that is subject to the same set of distortionary taxes as the equilibrium allocation but recognizes the externalities embodied in the matching function. Because the external habit would introduce an additional externality, we set habit \( h = 0 \) in this section to derive a cleaner result. We also eliminate type heterogeneity by setting \( \zeta_0 = 0 \). This implies that \( w_\zeta = w_h \) and \( J_{\xi,n} = J_n \). For simplicity, we use this simpler notation in this section.

The planner solves:

\[ W(n_{-1}, k_{-1}; S) = \max_{x, i, k, n, \nu, u} \frac{e^{1-\sigma} (1 + (\sigma - 1)\gamma n_{-1})^{\sigma - 1}}{1 - \sigma} + \beta \mathbb{E}[W(n, k; S')|S] \]  

(G.37)

subject to:

\[ c + i = (1 - r_n)wn_{-1} + (1 - r_n)\omega(1 - n_{-1}) + (1 - r_k)(y - n_{-1}w) + r_k \tilde{\delta} k_{-1} + T \]  

(G.38a)

\[ k = (1 - \delta(u))k_{-1} + \chi i_t \left( 1 - \frac{\kappa}{2} \left( \frac{i_t}{k_{-1}} - \tilde{\delta} \right)^2 \right) \]  

(G.38b)

\[ n = (1 - x)n_{-1} + \xi(\nu n_{-1})^{\eta}(1 - n_{-1}). \]  

(G.38c)

Let \( \lambda_b \) be the multiplier on the budget constraint (G.38a), \( \lambda_k \) the multiplier on the law of motion for capital (G.38b), and \( \lambda_n \) the multiplier on the law of motion for employment \( n \).
The optimality conditions for \( c, u, \nu, i, n, \) and \( k \) are, respectively:

\[
\lambda_b = \left( \frac{c}{1 + (\sigma - 1)\gamma_{n-1}} \right)^{-\sigma} \tag{G.39a}
\]

\[
\lambda_k k_{-1} \delta'(u) = \lambda_b \frac{m pk}{u} k_{-1} (1 - \tau_k) \tag{G.39b}
\]

\[
\lambda_n \eta \xi \left( \frac{\nu n_{-1}}{1 - n_{-1}} \right)^{\eta - 1} = \lambda_b (1 - \tau_k) m p l \times n_{-1} \tag{G.39c}
\]

\[
\lambda_b = \lambda_k \chi \left( 1 - \frac{\kappa}{2} \left( \frac{i}{k_{-1} - \delta} \right)^2 - \kappa \frac{i}{k_{-1} - \delta} \left( \frac{i}{k_{-1} - \delta} \right) \right) \tag{G.39d}
\]

\[
\lambda_n = \beta E \left[ W_n(S') | S \right] \tag{G.39e}
\]

\[
\lambda_k = \beta E \left[ W_k(S') | S \right]. \tag{G.39f}
\]

We also have two envelope conditions with respect to \( n_{-1} \) and \( k_{-1} \):

\[
W_n = \lambda_n \left( 1 - x + \eta \nu \xi \left( \frac{\nu n_{-1}}{1 - n_{-1}} \right)^{\eta - 1} - (1 - \eta) \left( \frac{\nu n_{-1}}{1 - n_{-1}} \right)^{\eta} \right) \equiv \mu(\theta) \tag{G.40a}
\]

\[
\lambda_b (1 - \tau_n)(w - \omega) - \lambda_b (1 - \tau_k) w + \lambda_b (1 - \tau_k) m p l (1 - \nu) - \lambda_b \frac{\gamma \sigma c}{1 + (\sigma - 1)\gamma_{n-1}} \tag{G.40b}
\]

We now guess and verify that, when we appropriately choose a constant bargaining power \( \phi \), the allocation of the planner’s problem and the decentralized equilibrium coincide. Define:

\[
q \equiv \frac{\lambda_k}{\lambda_b} \tag{G.41a}
\]

\[
m \equiv \beta \frac{\lambda_k}{\lambda_b} \tag{G.41b}
\]

\[
J_n \equiv \frac{\eta W_n}{\lambda_b} \tag{G.41c}
\]

\[
\phi \equiv 1 - \eta. \tag{G.41d}
\]

Guessing that allocations are the same, we verify that we also obtain the private-sector optimality conditions for utilization, recruiting, investment, and capital. From equation (G.39) and the equilibrium for capital (G.40b) along with the optimality condition for employment (G.39e):

\[
q \delta'(u) = \frac{mpk}{u} (1 - \tau_k) \tag{G.39b'}
\]

\[
(1 - \tau_k) m p l = \mathbb{E} \left[ m' \frac{W_n'}{\lambda_b'} \bigg| S \right] \eta \mu(\theta) = \mathbb{E}[m' J_n' | S] \mu(\theta) \tag{G.39c'}
\]
\[ q = \chi^{-1} \left( 1 - \frac{\kappa}{2} \left( \frac{i}{k_{l-1}} - \hat{\delta} \right)^2 - \kappa \frac{i}{k_{l-1}} \left( \frac{i}{k_{l-1}} - \hat{\delta} \right) \right)^{-1} \]  
\[ q = \mathbb{E} \left[ m' \left( q'(1 - \delta(u)) + q' \left( \frac{i}{k_{l-1}} \right)^2 \kappa \chi \left( \frac{i}{k_{l-1}} - \hat{\delta} \right) + \tau_k \delta + mpk' \right) | S \right]. \]

(G.39d')

(G.39f')

Therefore, we checked that the guess satisfies all the optimality conditions and the equilibrium condition for capital. We now check the remaining condition, the equilibrium condition for employment, using equation (G.39c'):

\[ \eta^{-1} J_n = \left( \left( 1 + \frac{1 - x}{\mu(\theta)} \right) mpn - w \right) (1 - \tau_k) + (1 - x - f(\theta))\mathbb{E}[m' J'_n | S] \frac{1 - \eta}{\eta} \frac{\gamma \sigma c}{1 + (\sigma - 1) \gamma n_{t-1}}. \]

(G.40a')

Plug in from equation (G.22) for \( \left( 1 + \frac{1 - x}{\mu(\theta)} \right) mpn - w \):

\[ \frac{1 - \eta}{\eta} J_n = (1 - \tau_n) (w - \omega) + \frac{\gamma \sigma c}{1 + (\sigma - 1) \gamma n_{t-1}} + (1 - x - f(\theta))\mathbb{E}[m' J'_n | S] \frac{1 - \eta}{\eta}. \]

(G.40a'')

Compare this to equation (G.32) with constant \( \phi \) and dividing that equation through by \( 1 - \phi \) and substituting from equation (G.22):

\[ \frac{\phi}{1 - \phi} J_n \frac{1 - \tau_n}{1 - \tau_k} = (1 - \tau_n) (w - \omega) - \left( \frac{c}{1 + (\sigma - 1) \gamma n_{t-1}} \right) \gamma \sigma + (1 - x - f(\theta))\mathbb{E} \left[ m' J'_n | S \right] \frac{\phi}{1 - \phi} \frac{1 - \tau_n}{1 - \tau_k}. \]

(G.32')

Comparing this equation to equation (G.40a'') shows that the two equations are equal with \( \phi = 1 - \eta \) if and only if \( \tau_n = \tau_k \).

G.6 Detrended economy

In this subsection, we augment the model by allowing for a stochastic trend in \( z_t \) by allowing for the growth rate \( g_z \) to be stochastic:

\[ \ln g_{z,t} = \ln(g_z) + \epsilon_{p,t}, \]

(G.43)

where \( \epsilon_{p,t} \) is the permanent shock to productivity. Thus, we replace \( g_z^t \) by \( \prod_{s=1}^t g_{z,s} \) in the production function. When \( \epsilon_{p,t} = 0 \) for all \( t \), we recover the deterministic growth process from the main text.

Capital, consumption, investment, the marginal value of employment, and wages grow with \( z_t \), while all other variables are stationary. We denote detrended variables by \( \sim \). To simplify notation, define the (detrended) marginal products of capital and labor as:

\[ \widetilde{mpk}_t \equiv u_t \left( \alpha \frac{\bar{y}_t g_{z,t}}{u_t k_{t-1}} \right)^{1/2} = mpk_t \]

(G.44)
\[ \overline{mpl}_t \equiv \frac{z_t(1 - \zeta_t)}{(1 - \theta_t)\zeta_t(1 - \zeta_t)n_{t-1}(1 - \nu_t)} \left( \frac{\tilde{y}_t}{\zeta_t(1 - \zeta_t)n_{t-1}(1 - \nu_t)} \right)^{\frac{1}{\tau}}. \] (G.45)

We substitute out for the number of recruiters by using the definition for market tightness:
\[ n_{t-1} - \nu_{t-1}n_{t-1} = n_{t-1} - \frac{\theta_{t-1}}{1 - \zeta_t}(1 - n_{t-1}). \] (G.46)

Similarly, for the capital law of motion:
\[ \tilde{k}_t = (1 - \delta(u_t))g_{z,t}^{-1}\tilde{k}_{t-1} + \chi\tilde{r}_t \left( 1 - \frac{\kappa}{2} \left( \frac{\tilde{r}_t}{\tilde{g}_{z,t} - \delta} \right)^2 \right), \] (G.47)
the resource constraint
\[ \left( \alpha^{1/\varepsilon}(u_t\tilde{k}_{t-1}g_{z,t}^{-1})^{1/\varepsilon} + (1 - \alpha)^{1/\varepsilon}(\tilde{z}_t(1 - \tilde{\zeta})n_{t-1}(1 - \nu_t))^{1-1/\varepsilon} \right)^{\frac{1}{1-\eta}} = \tilde{v}_t + \phi_t, \] (G.48)
and the firm value with equilibrium choices for investment, capital, utilization, and recruiting:
\[ \tilde{J}_t = \left( (1 - \tau_k)(\tilde{y}_t - n_{t-1}\tilde{w}_{\zeta,t}) - \tilde{v}_t + \delta\tilde{k}_{t-1}/g_{z,t} \right) 
+ q_t \left( -\tilde{k}_t + (1 - \delta(u))g_{z,t}^{-1}\tilde{k}_{t-1} + \chi\tilde{r}_t \left( 1 - \frac{1}{2} \kappa \left( \frac{\tilde{r}_t}{\tilde{g}_{z,t} - \delta} \right)^2 \right) \right) + \mathbb{E}_t \left[ m_{t+1}g_{z,t+1}\tilde{J}_{t+1} \right]. \]

Since the constraint on capital accumulation binds, firm value is simply the present discounted value of the cash flow:
\[ \tilde{J}_t = \left( (1 - \tau_k)(\tilde{y}_t - n_{t-1}\tilde{w}_{\zeta,t}) \right) - \tilde{v}_t + \tau_k\delta\tilde{k}_{t-1}/g_{z,t} + \mathbb{E}_t \left[ m_{t+1}g_{z,t+1}\tilde{J}_{t+1} \right]. \] (G.49)

We also have the marginal value of employment
\[ \tilde{J}_{\zeta,n,t} = (1 - \tau_k) \left( \overline{mpl}_t \left( 1 + \frac{1 - x}{\mu(\theta)(1 - \zeta)} \right) - \tilde{w}_{\zeta,t} \right), \] (G.50)
and the recruiting optimality condition:
\[ (1 - \tau_k)\overline{mpl}_t = \mu(\theta)(1 - \zeta)\mathbb{E}_t \left[ m_{t+1}g_{z,t+1}\tilde{J}_{\zeta,n,t+1} \right]. \] (G.51)

Here, we use that \( \mu(\theta) = \xi\theta^{n-1} \). It is useful to note that \( f(\theta)(1 - n_{-1}) = \xi(1 - n_{-1})^{1-\eta}((1 - \zeta)n_{-1}1 - n_{-1}1) = (1 - \zeta)n_{-1}1 - n_{-1}\mu(\theta) \). This implies that the equilibrium laws of motion perceived by the household and the firm are, actually, identical.

Wage setting implies:
\[ \phi_t(1 - \tau_n) \left( \overline{mpl}_t \frac{1}{1 - \zeta} \left( 1 + \frac{1 - x}{\mu(\theta)} \right) - \tilde{w}_{h,t} \right) = (1 - \phi_t)(1 - \tau_n)(\tilde{w}_{h,t} - \tilde{w}_t) - (1 - \phi_t) \left( \frac{c_t - \tilde{h}_{t-1}c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma \left( \sigma + (\sigma - 1)\frac{\tilde{h}_{t-1}c^a_{t-1}}{\tilde{c}_t - \tilde{h}_{t-1}c^a_{t-1}} \right). \]
\begin{align*}
+ (1 - x - f_t(\theta_t)) & E_t \left[ \frac{1 - \phi_t}{1 - \phi_{t+1}} m_{t+1}g_{t+1}(1 - \tau_n) \left( \phi_{t+1} \frac{\tilde{J}_{\zeta,n,t+1}}{1 - \tau_k} + \phi_{t+1} \frac{\tilde{\zeta}}{1 - \zeta} mp_{t+1} + \zeta_0(\tilde{w}_{t,t+1} - \tilde{w}_{h,t+1}) \right) \right], \\
& \text{(G.52)}
\end{align*}

where \( \tilde{h}_{t-1} = \hat{h}_{t-1}/g_{z,t} \) incorporates trend growth. Specifically, in equilibrium with \( n^a_{t-2} = n_{t-2} \):
\begin{equation}
\tilde{h}_t = h_{g_{z,t}} \frac{1}{1 + (\sigma - 1)\gamma n_{t-1}}. \tag{G.53}
\end{equation}

In our solution, we perturb \( \ln(1 - \tilde{h}_{t-1}) \) and use the identity that \( \tilde{h}_t = 1 - \exp(\ln(1 - \tilde{h}_{t-1})) \).

Other equilibrium conditions are optimal utilization:
\begin{equation}
(\delta_1 + \delta_2(u_t - 1)) q_t = (1 - \tau_k) \frac{mpk_t}{u_t}, \tag{G.54}
\end{equation}

optimal capital:
\begin{equation}
q_t = E_t \left[ m_{t+1} \left( (1 - \tau_k)mpk_{t+1} + \frac{\tilde{k}_{t-1}}{g_{z,t}} \left( (1 - \delta(u_{t+1})) + \kappa \left( \frac{\tilde{t}_{t+1}}{k_t} g_{z,t+1} \right)^2 \left( \frac{\tilde{t}_{t+1}}{k_t} g_{z,t+1} - \tilde{\delta} \right) q_{t+1} \right) \right) \right], \tag{G.55}
\end{equation}

optimal investment:
\begin{equation}
q_t = \left( \left( 1 - \frac{1}{2} \kappa \left( \frac{\tilde{t}_t}{k_{t-1}} g_{z,t} - \tilde{\delta} \right)^2 \right) - \kappa \left( \frac{\tilde{t}_t}{k_{t-1}} g_{z,t} - \tilde{\delta} \right) \left( \frac{\tilde{t}_t}{k_{t-1}} g_{z,t} \right) \right)^{-1}, \tag{G.56}
\end{equation}

and the stochastic discount factor:
\begin{equation}
m_{t+1} = \beta g_{z,t+1} \left( \frac{c_{t+1} - \tilde{h}_{t-1}c_{t-1}^{\sigma}}{c_{t-1} - \tilde{h}_t c_t^{\sigma}} 1 + (\sigma - 1)\gamma n_t \right)^\sigma. \tag{G.57}
\end{equation}

Equations (G.47) to (G.57) determine:

1. Detrended capital \( \tilde{k}_t \) from equation (G.47).
2. Detrended consumption \( \tilde{c}_t \) from the resource constraint (G.48).
3. Detrended firm value \( \tilde{J} \) from the Bellman equation (G.49).
4. Detrended marginal value of employment \( \tilde{J}_{\zeta,n} \) from the envelope condition (G.50).
5. Recruiting intensity \( \nu_t \) from equation (G.51).
6. Detrended wages \( \tilde{w}_t \) from the Nash bargaining equation (G.52).
7. The utilization rate \( u_t \) from the utilization equation (G.54).
8. The shadow price of capital \( q_t \) from the capital equation (G.55).
9. Detrended investment \( \tilde{i}_t \) from the investment equation (G.56).
10. The stochastic discount factor \( m_{t+1} \) from equation (G.57).
In addition, the following variables and equations matter:

11. Employment $n_t$ is determined from equation (G.35).

12. Market tightness $\theta_t$ (or the number of recruiters) from equation (G.46).

And, for completeness, we add a few definitions:

13. The (detrended) marginal product of capital $\tilde{mp}k_t$ from equation (G.44).

14. The (detrended) marginal product of labor $\tilde{mpl}_t$ from equation (G.45).

15. Final goods production $\tilde{y}_t$

\[
\tilde{y}_t \equiv \left( \alpha^{1/\varepsilon}(u_t\tilde{k}_{t-1}g_{z,t}^{-1})^{1/\varepsilon} + (1 - \alpha)^{1/\varepsilon}(\tilde{z}_t(1 - \tilde{z})n_{t-1}(1 - \nu_t))^{1-1/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon - 1}}. \tag{G.58}
\]

16. The gross capital share $cs_t$ from equation (G.59):

\[
 cs_t \equiv 1 - \frac{n_{t-1}w_{\zeta,t}}{\tilde{y}_t}. \tag{G.59}
\]

17. The net capital share $ncs_t$ from equation (G.60):

\[
 ncs_t \equiv 1 - \frac{n_{t-1}w_{\zeta,t}}{\tilde{y}_t} - \delta_t\tilde{k}_{t-1}g_{z,t}. \tag{G.60}
\]

18. To compute welfare in the presence of trend growth, we first shift the location of the value function $V_t$ in (G.5) to $\hat{V}_t \equiv (1 - \beta)V_t + \frac{1}{1-\sigma}$ for $\sigma \neq 1$:

\[
\hat{V}_t = (1 - \beta)V_t + \frac{1}{1-\sigma} = (1 - \beta)\left( c_t - \hat{h}(n_{t-1})c_{t-1}^\sigma(1 + (\sigma - 1)\gamma n_{t-1})^\sigma \right) + \beta E_t \left( (1 - \beta)V_{t+1} + \frac{1}{1-\sigma} \right) \\
= (1 - \beta)\left( c_t - \hat{h}(n_{t-1})c_{t-1}^\sigma(1 + (\sigma - 1)\gamma n_{t-1})^\sigma \right) + \beta E_t [(1 - \beta)V_{t+1}]. \tag{G.61}
\]

Let $\tilde{V}_t = \prod_{s=0}^t g_s^{1-\sigma} \tilde{V}_t$, so that $\tilde{V}_t$ is the detrended version of the welfare measure:

\[
\tilde{V}_t = (1 - \beta)\left( c_t - \tilde{h}(n_{t-1})c_{t-1}^\sigma(1 + (\sigma - 1)\gamma n_{t-1})^\sigma \right) + \beta g^{1-\sigma} E_t [\tilde{V}_{t+1}].
\]

When $\sigma > 1$, we find it useful to actually compute

\[
\hat{V}_t = \tilde{V}_t \frac{1 - \beta g^{1-\sigma}}{1 - \beta} - \frac{1}{1 - \sigma} = (1 - \beta g^{1-\sigma})\left( c_t - \tilde{h}(n_{t-1})c_{t-1}^\sigma(1 + (\sigma - 1)\gamma n_{t-1})^\sigma - 1 \right) + \beta g^{1-\sigma} E_t \left( \tilde{V}_{t+1} \frac{1 - \beta g^{1-\sigma}}{1 - \beta} - \frac{1}{1 - \sigma} \right) \\
= (1 - \beta g^{1-\sigma})\left( c_t - \tilde{h}(n_{t-1})c_{t-1}^\sigma(1 + (\sigma - 1)\gamma n_{t-1})^\sigma - 1 \right) + \beta g^{1-\sigma} E_t [\tilde{V}_{t+1}]. \tag{G.62}
\]
In this version of the model, there are the following exogenous processes:

\[ \log \phi_t = (1 - \rho_\phi) \log(\bar{\phi}) + \rho_\phi \log \phi_{t-1} + \epsilon_{\phi,t}. \] (G.63)

20. Stationary labor productivity
\[ \log z_t = (1 - \rho_z) \log(\bar{z}) + \rho_z \log z_{t-1} + \epsilon_{z,t}. \] (G.64)

21. Permanent labor productivity
\[ \log(g_{z,t}) = \log(g_z) + \epsilon_{p,t}. \] (G.65)

22. Minimum wage
\[ \log(\tilde{w}_{\ell,t}) = (1 - \rho_{w,\ell}) \log(\bar{w}_{\ell}) + \rho_{w,\ell} \log(\tilde{w}_{\ell,t-1}) + \epsilon_{w,\ell,t}. \] (G.66)

23. Unemployment benefits
\[ \log(\tilde{\omega}_{\ell,t}) = (1 - \rho_{\omega}) \log(\bar{\omega}) + \rho_{\omega} \log(\tilde{\omega}_{t-1}) + \epsilon_{\omega,\ell,t}. \] (G.67)

G.7 Balanced growth path and data matching
Along the BGP of the economy, the discount factor becomes:
\[ \bar{m} = \beta g_z^{-\sigma}. \] (G.68)
and the number of recruiters is given from (G.69):
\[ (1 - \zeta) \nu \bar{n} = \bar{\theta}(1 - \bar{n}) \iff \bar{n} - \nu \bar{n} = \bar{n} - \frac{\bar{\theta}}{1 - \zeta} (1 - \bar{n}). \] (G.69)

In an initial calibration, we can normalize capacity utilization to be 1 along the BGP to get:
\[ \bar{u} = 1 \] (G.70)
\[ \delta_1 = (1 - \tau_k) \frac{mpk}{\bar{u}}. \] (G.71)
If \( \delta_1 \) is given, rather than calibrated, utilization solves:
\[ (\delta_1 + \delta_2(\bar{u})) \bar{q} = (1 - \tau_k) \frac{mpk}{\bar{u}}. \] (G.72)
Clearly, if \( \bar{u} = \bar{u} = 1 \), equation (G.71) holds.

The BGP optimality condition for capital can be written as:
\[ 1 = \bar{m} \left( (1 - \tau_k) \frac{mpk}{\bar{u}} + (1 - (1 - \tau_k) \delta(\bar{u})) \bar{q} + \kappa \left( \frac{i}{k/g_z} \right)^2 \left( \frac{i}{k/g_z} - \delta \right) \bar{q} \right) \]
\[ \iff \frac{\bar{q} / \bar{m} - (1 - (1 - \tau_k) \delta(\bar{u})) - \kappa \left( \frac{i}{k/g_z} \right)^2 \left( \frac{i}{k} - \delta \right)}{1 - \tau_k} = \frac{mpk}{\bar{u}}. \] (G.73)
If \( \bar{u} = 1 \) holds, the marginal product of capital does not depend on adjustment costs in the steady state.

Investment along the BGP is given by:

\[
\frac{\bar{i}}{g_z} = \frac{(g_z - (1 - \delta(\bar{u})))}{1 - \frac{1}{2} \kappa \left( \frac{\bar{i}}{k/g_z} - \tilde{\delta} \right)^2 g_z}, \quad (G.74)
\]

where \( \tilde{\delta} \equiv 1 - \frac{1}{1 - \bar{g}_z} \).

The steady-state price of capital is:

\[
\bar{q} = \frac{1}{1 - \frac{1}{2} \left( \frac{\bar{i}}{k/g_z} - \tilde{\delta} \right)^2 - \left( \frac{\bar{i}}{k/g_z} - \tilde{\delta} \right)^2} \quad (G.75)
\]

If we cannot calibrate the adjustment costs in investment and utilization, then \( \bar{i}/k/g_z, \bar{u}, \bar{q}, \) and \( \bar{mpk} \) are jointly determined by equations (G.72), (G.73), (G.78), and (G.75). If \( \bar{q} = \bar{u} = 1 \), then \( \bar{i}/k/g_z \) and \( \bar{mpk} \) are available in closed form.

Using the recruiting optimality condition (G.51), the wage equation (G.52) becomes:

\[
\bar{\phi}(1 - \tau_n) \left( \frac{mpl}{1 - \zeta} \left( 1 + \frac{1 - x}{\mu(\bar{t})} \right) - \bar{w}_{h,t} \right)
\]

\[= (1 - \bar{\phi})(1 - \tau_n)(\bar{w}_h - \bar{\omega}) - (1 - \bar{\phi}) \left( \frac{\bar{c} - \bar{h}\bar{c}^a}{1 + (\sigma - 1)\bar{\gamma}\bar{n}} \right) \gamma \left( \sigma + (\sigma - 1) \frac{\bar{h}\bar{c}^a}{\bar{c} - \bar{h}\bar{c}^a_{t-1}} \right)
\]

\[+ (1 - x - \bar{f}(\bar{\theta}))(1 - \tau_n) \left( \bar{\phi} \frac{mpl}{1 - \zeta} + \bar{m}\bar{g}\bar{\phi} \frac{\bar{\zeta}}{1 - \zeta} mpl + \bar{m}\bar{g}\bar{\zeta}_0 (\bar{w}_t - \bar{w}_h) \right).
\]

Using \( \bar{c}_a = \bar{c} \), we have that \( (\bar{c} - \bar{h}\bar{c}^a) \left( \sigma + (\sigma - 1) \frac{\bar{h}\bar{c}^a}{\bar{c} - \bar{h}\bar{c}^a_{t-1}} \right) = \bar{c}(\sigma - \bar{h}) \). With this result, canceling \( 1 - \tau_n \) and using that \( f(\theta_t) \equiv \theta_t \mu(\theta_t) = \bar{\xi}\theta^n \) and that \( 1 = 1 - x - \bar{f} + \frac{\bar{f}}{\bar{n}} \) yields:

\[
\frac{\bar{\phi} mpl}{1 - \zeta} (1 + \bar{\theta} - \bar{\zeta}_0 (1 - x - f) \bar{m}\bar{g})
\]

\[= \bar{w}_h (1 - (1 - x - f) \bar{m}\bar{g}\bar{z}_0) + (1 - x - f) \bar{m}\bar{g}\bar{\zeta}_0 \bar{w}_t - (1 - \bar{\phi}) \bar{\omega} - \frac{1 - \bar{\phi}}{1 - \tau_n} \left( \frac{\bar{c}(\sigma - \bar{h})}{1 + (\sigma - 1)\bar{\gamma}\bar{n}} \right) \gamma. \quad (G.76)
\]

We solve this equation for \( \gamma \).

The marginal product of labor along the BGP is:

\[
\bar{mpl} = (1 - \bar{\zeta}) \left( (1 - \alpha) \frac{\bar{y}}{n - \bar{\theta} / (1 - \zeta)} (1 - \bar{n}) \right)^{1/\varepsilon}.
\]

Note that we can rewrite the definition of \( \bar{mpk} \) as:

\[
\frac{k}{g_z} = \bar{n}(1 - \bar{\nu}) \left( \left( \frac{\alpha}{1 - \alpha} \right)^{1/\varepsilon} \left( \frac{\bar{mpk}/\bar{u}}{\alpha} \right)^{z-1} - 1 \right)^{-\varepsilon / z+1} \bar{n}(1 - \bar{\nu}) \alpha \left( \frac{\bar{mpk}/\bar{u}}{\alpha} \right)^{z-1} = \frac{1}{1 - \alpha}.
\]

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This expression is useful to express output in terms of $\overline{mpk}$ and employment. Recall the expression for detrended production net of recruiting services:

$$\bar{y} = \left( \alpha^{1/\varepsilon} (u_t \bar{k}_{t-1} g_z^{-1})^{1/\varepsilon} + (1 - \alpha)^{1/\varepsilon} (\bar{\zeta}(1 - \bar{z}) n_{t-1} (1 - \nu_t))^{1-1/\varepsilon} \right)^{\varepsilon^{-1}}$$

$$\bar{y} = \left( \frac{mpk}{\alpha} \bar{k} \bar{u}^{1-\varepsilon} = \bar{n}(1 - \bar{\nu})(1 - \bar{\zeta}) \left( \frac{mpk / \bar{u}}{\alpha} \right)^\varepsilon \left( \frac{\alpha}{1 - \alpha} \right)^{1/\varepsilon} \left( \frac{mpk^{\varepsilon - 1}}{\alpha} - 1 \right) \right)^{-\varepsilon^{-1}}$$

$$\varepsilon \rightarrow 1 \bar{n}(1 - \bar{\nu}) (mpk / \bar{u})^{-\frac{\alpha}{1 - \alpha}} \alpha.$$

The law of motion for capital gives us:

$$\bar{c} = 1 - \left( 1 - \frac{1 - \delta_0}{g_z} \right) \frac{\bar{k}}{yyg_z} = 1 - \left( 1 - \frac{1 - \delta_0}{g_z} \right) \frac{\bar{k} g_z}{yyg_z mpk} \bar{u}^{\varepsilon - 1}.$$  

The law of motion for employment implies:

$$\bar{n} = \frac{f(\bar{\theta})}{x + f(\bar{\theta})}. \quad (G.79)$$

If we combine equation (G.21) with (G.22):

$$\bar{w}_\zeta = \overline{mpl} \left( 1 - \frac{1 - (1 - x) \bar{m} g_z}{\bar{m} g_z \mu(\bar{\theta})(1 - \bar{\zeta})} \right). \quad (G.80)$$

We use this equation to set $w_\zeta$.

For a given calibration target (e.g., $w_\ell = \frac{1}{3} w_\zeta$), we have:

$$\bar{w}_h = \frac{\bar{w}_\zeta - \bar{w}_\ell}{1 - \zeta_0}. \quad (G.81)$$

Per definition:

$$\mu(\bar{\theta}) = \frac{f(\bar{\theta})}{\bar{\theta}} = \xi \bar{\theta}^{\eta - 1}.$$

In general, we have the following unknowns and equations:

1. Employment $\bar{n}$ from the law of motion (G.79).
2. Capital $\bar{k}$ from the first-order condition (G.73).
3. Investment from the capital law of motion (G.78).
4. Capacity utilization, which follows from equation (G.70) when $\delta_1$ is calibrated or, more generally, from (G.72).
5. The derivative of capacity utilization along the BGP $\delta_1$ from equation (G.71).
6. The price of capital, which follows from equation (G.75).
7. Consumption $\bar{c}$ from the resource constraint (G.78).
8. Number of recruiters $\bar{n} \bar{\nu}$ from the definition of market tightness (G.69).
9. The stochastic discount factor $\bar{m}$ from no arbitrage (G.68).
10. Production $\bar{y}$ per definition (G.77).

and with these variables we can find market tightness $\bar{\theta}$ and the wages. In our calibration, we set the production function parameters as follows:

- Capital share: $\alpha = (\text{NIPA capital share})^{\varepsilon} \left( \frac{\bar{y}}{k} \right)^{1-\varepsilon}$.
- Average depreciation rate: $\delta_0 = \frac{\text{NIPA depreciation}}{\bar{y}} \times \frac{\bar{y}}{k}$.
- Rate of time preference: $\bar{\beta} = \bar{g}^{\sigma} \left( 1 - \delta_0 (1 - \tau_k) + (1 - \tau_k) \left( \alpha \frac{k}{\bar{y}x} \right)^{1/\varepsilon} \right)^{-1}$.

We can also fix $\bar{n}$ and choose $\gamma$:
1. Preference for leisure $\gamma$ given $n$ from wage setting.
2. Tightness $\bar{\theta}$ from the law of motion (G.79)

$$\bar{\theta} = \left( \frac{\bar{n}x}{\xi \times (1 - \bar{n})} \right)^{1/\eta}.$$ (G.79')

3. Capital-to-production ratio $\frac{\bar{k}}{\bar{y}}$ from the first-order condition (G.73).
4. Investment-to-production ratio from the law of motion of capital (G.78).
5. Capacity utilization, which follows from equation (G.70) when $\delta_1$ is calibrated or, more generally, from (G.72).
6. The derivative of capacity utilization along the BGP $\delta_1$ from equation (G.71).
7. The price of capital, which follows from equation (G.75).
8. Consumption-to-production ratio $\frac{\bar{c}}{\bar{y}}$ from the resource constraint (G.78).
9. Number of recruiters $\bar{n} \bar{\nu}$ from the definition of market tightness (G.69).
10. The stochastic discount factor $\bar{m}$ from equation (G.68).
11. Production $\bar{y}$ per definition (G.77).

The additional variables and the exogenous processes follow directly from the detrended economy.

**G.8 U.S. business cycle data**

To map observations into variables in the model we proceed as follows. First, we compute consumption as the sum of real services and non-durable consumption, divided by the civilian non-institutionalized population above 16. Specifically:

$$C_t = \frac{\text{DSERRA3Q086SBEA}_{t, 2009} \times \text{PCESVC96 in 2009} + \text{DGOERA3Q086SBEA}_{t, 2009} \times \text{PCNDGC96 in 2009}}{\text{CN16OV}_{t}}.$$. 

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We multiply the base year (2009 average) value of the real consumption expenditure by the corresponding quantity index to obtain dollar amounts for longer horizons, i.e., before 1999.

We compute investment as the sum of consumer durables and gross private domestic investment, divided by the civilian non-institutionalized population above 16. Specifically:

\[ I_t = \frac{\text{GDPIC96}_t + \frac{\text{DDURRA3Q09SBEA}}{\text{DDURRA3Q09SBEA in 2009}} \times \text{PCDGCC96 in 2009}}{\text{CN16OV}_t}. \]

Real GDP per capita is defined as the sum of real per capita investment and consumption:

\[ Y_t = C_t + I_t. \]

**G.9 Introducing product market power**

An interesting extension of the model is to introduce market power for firms. To do so, we need to differentiate among firms. There is a representative final goods producing firm that produces aggregate output \( \bar{y}_t \) as a CES aggregate of intermediate goods \( y_t(i) \) with elasticity \( \iota > 1 \):

\[
\bar{y}_t = \left( \int_0^1 y_t(i)^{1-1/\iota} di \right)^{\iota-1}. \tag{G.82}
\]

Let \( p_t(i) \) denote the price of each individual variety and \( \bar{p}_t \) the optimal aggregate price index. Standard cost minimization for the representative final goods firm then implies that demand for variety \( i \) is given by:

\[
y_t(i) = \bar{y}_t \left( \frac{p_t(i)}{\bar{p}_t} \right)^{-\iota}. \tag{G.83}
\]

Each variety is produced according to the following production function:

\[
y_t(i) = \left( \alpha^{1/\iota} (u_t(i) k_{t-1}(i))^{1-1/\iota} + (1 - \alpha)^{1/\iota} (z_t n_{t-1}(i)(1 - \nu_t(i)))^{1-1/\iota} \right)^{-1/\iota} \Phi_t
\equiv \psi(u_t(i) k_{t-1}(i), z_t n_{t-1}(i)(1 - \nu_t(i)); \Phi_t), \tag{G.84}
\]

where \( \Phi_t \geq 0 \) is the fixed cost of operating. Along the BGP, it grows at the rate of labor productivity.

The intermediate goods producing firm takes its demand schedule (G.83) into account and has revenues of \( p_t(i) \left( \frac{p_t(i)}{\bar{p}_t} \right)^{-\iota} \bar{y}_t \). Equivalently, revenue as a function of quantities becomes:

\[
\bar{p}_t y_t(i)^{1-1/\iota} \bar{y}_t^{-1/\iota}.
\]

In a symmetric equilibrium, each firm sets the same price so that \( \bar{y}_t = y_t(i) \) and \( \bar{p}_t = p_t(i) \) for all \( i \).

We choose the final good as the numeraire in the period.

With market power, as firms consider employing an extra worker or unit of capital, they take into account that the marginal revenue product is smaller than the marginal product. Importantly, the functional form for the match surplus \( J_m(n, k) \) is unchanged but, as (G.22') shows, the marginal value of employment that enters into it reflects the lower marginal revenue product.
To see this, note that now the following optimality condition holds for recruiting:

\[
(1 - \tau_k) (1 - 1/\iota) z_t \left( \frac{Y_t}{z_t(1 - \zeta)n_{t-1}(1 - \nu_t)} \right)^{1/\varepsilon} = \mu(\theta_t) E[m_{t+1} J(n_t, k_t)]. \tag{G.21'}
\]

Thus, the marginal value of employment is given by:

\[
J_{\zeta,n}(n_{t-1}, k_{t-1}) = (1 - \tau_k) (mrpl_t \times (1 - \nu_t) - w_t) + (\nu_t(1 - \tilde{\zeta}_t) \mu(\theta_t) + 1 - x) E[m_{t+1} J_{\zeta,n}(n_t, k_t)]
\]

\[
= (1 - \tau_k) \left( mrpl_t \left( 1 + \frac{1 - x}{\mu(\theta_t)(1 - \zeta)} \right) - w_t \right), \tag{G.22'}
\]

using equation (G.21') to substitute for \( E[m_{t+1} J_{\zeta,n}(n_t, k_t)] \).

The optimality condition for the utilization rate becomes:

\[
\delta'(u_t) q_t k_{t-1} = (\delta_1 + \delta_2(u_t - 1)) q_t k_{t-1} = (1 - \tau_k)(1 - 1/\iota) \left( \frac{\eta_t}{u_t k_{t-1}} \right)^{1/\varepsilon} \equiv (1 - \tau_k) \frac{mrp k_{t-1}}{u_t}. \tag{G.27'}
\]

The optimality condition for capital \( k' \) becomes:

\[
q_t = E \left[ m_{t+1} \left( mrpk_{t+1}(1 - \tau_k) + \tau_k \tilde{\delta} + \left( 1 - \delta(u_{t+1}) \right) + \chi \kappa \left( \frac{i_{t+1}}{k_t} \right)^2 \left( \frac{i_{t+1}}{k_t} - \tilde{\delta} \right) q_{t+1} \right) \right]. \tag{G.29'}
\]

The marginal revenue product of physical capital is:

\[
mrpk_{t+1} \equiv u_{t+1}(1 - 1/\iota) \left( \frac{\eta_{t+1}}{u_{t+1} k_t} \right)^{1/\varepsilon}. \tag{G.30'}
\]

Market power also has an impact on the calibration. Monopolistic competition is an extra source of profits in the economy: In the detrended economy, the flow profit is \( \tilde{y}/\iota \) along the BGP. We consider two variants for calibrating the model with market power that keep the aggregate capital share in the economy unchanged:

1. No fixed cost, lower capital share in production. Here, we set the fixed cost of production \( \Phi_t \) to zero. Then, we calibrate \( \iota \) and adjust \( \alpha \) so that the gross capital share in the economy is unchanged. Specifically, we target a capital share in production of \( 1 - (1 - 0.31)(1 - 1/\iota)^{-1/\varepsilon} \).

2. Fixed cost, same capital share in production. Here, we set the detrended fixed cost of production equal to the share of profits from monopolistic competition: \( \tilde{\Phi}_t = \tilde{y}/\iota \).

G.10 Identification: Additional relationships

Recall that we use three moments to pin down three parameters: \( \omega_z, \omega_\phi \), and \( \kappa/\delta_0^2 \). In the main text, we show the three bivariate plots that show the important interaction terms among these three parameters. For completeness, we show here in Figure G.14 the additional bivariate plots. It is clear from this figure that the required standard deviations vary little with the adjustment cost and the adjustment cost depends little on \( \omega_\phi \).
Finally, the three panels in Figure G.15 complete this discussion by showing the explained standard deviation of GDP and the gross capital share as a function of the standard deviation of the bargaining and productivity shocks and the investment adjustment. Only the size of the bargaining shocks has noticeable effects on the volatility of the capital share.

\[ EE(s_t) = 1 - \frac{u_c^{-1} (E_t [\beta_t z_t^{-\sigma} u_c(c(s_{t+1}); n(s_{t+1})) R_t(s_{t+1}) ; n(s_t))]}{c(s_t)}. \]  \hspace{1cm} (G.85)\]

Here:

\[ R_{t+1}^c = (1 - \tau_k) \bar{m} p_{t+1} \mu(\theta_t) g_{z,t+1} \tilde{J}_{n,t+1} \]
\[ R_{t+1}^k = q_{t+1}^{-1} \left( \bar{m} p_{k+1} (1 - \tau_k) + \bar{\delta} \tau_k + (1 - \delta(u_{t+1})) + \chi_{t+1} \left( \frac{\tilde{I}_{t+1}}{k_t} g_{z,t+1} \right)^2 \left( \frac{\tilde{I}_{t+1}}{k_t} g_{z,t+1} - \tilde{\delta} \right) \right) q_{t+1} \]

Note: The vertical lines indicate HP-filtered data moments.

Figure G.15: Explained standard deviation as a function of calibrated parameters.

G.11 Euler equation errors

Our model has two Euler equations: (1) The recruiting optimality condition (G.51) and (2) the capital optimality condition (G.55). We transform the Euler equation error to consumption units. To do so, take an Euler equation with a generic return \( R_{t+1}^c \). Following Fernández-Villaverde and Rubio-Ramírez (2006), the Euler equation error in state \( s_t \) is:

\[ EE(s_t) = 1 - \frac{u_c^{-1} (E_t [\beta_t z_t^{-\sigma} u_c(c(s_{t+1}); n(s_{t+1})) R_t(s_{t+1}) ; n(s_t))]}{c(s_t)}. \]  \hspace{1cm} (G.85)\]

Here:

\[ R_{t+1}^c = (1 - \tau_k) \bar{m} p_{t+1} \mu(\theta_t) g_{z,t+1} \tilde{J}_{n,t+1} \]
\[ R_{t+1}^k = q_{t+1}^{-1} \left( \bar{m} p_{k+1} (1 - \tau_k) + \bar{\delta} \tau_k + (1 - \delta(u_{t+1})) + \chi_{t+1} \left( \frac{\tilde{I}_{t+1}}{k_t} g_{z,t+1} \right)^2 \left( \frac{\tilde{I}_{t+1}}{k_t} g_{z,t+1} - \tilde{\delta} \right) \right) q_{t+1} \]
\[ u_c^{-1}(\tilde{u}_c;n) = \tilde{u}_c^{-\frac{1}{\sigma}} \times (1 + (\sigma - 1)\gamma n). \]

The difficulty in our setup is that, because of the pruning, the state in terms of the endogenous observables is not uniquely defined: Any given level of capital can be reached by different combinations of the first-, second-, and third-order components of the solution. Thus, as in Andreasen et al. (2018), we resort to Monte Carlo integration (with a burn-in of 1,000 simulations). The pseudo-code below outlines the algorithm.

**Pseudo-code for Monte Carlo integration**

1. Simulate the model for 6,000 periods.
2. Discard the first 1,000 periods and save the remaining 5,000 draws for the state \( s_t \) as \( \{s_t^{(\ell)}\}_\ell \).
3. For \( \ell = 1, \ldots, 5,000 \):
   (a) Compute the vector of current policies and stack it with the state vector:
   \[ s_t^{(\ell)} \]
   (b) For \( m = 1, \ldots, 1,000 \):
       i. Draw \( \epsilon_{t+1}^{(m)} \sim \mathcal{N}(0, I) \).
       ii. Compute \( s_t^{(\ell,m)} = f(s_t^{(\ell)}, \epsilon_{t+1}^{(m)}) \),
   (c) Average over \( d \):
   \[ EE(s_t^{(\ell)}) = \left| 1 - u_c^{-1} \left( 1,000^{-1} \sum_{m=1}^{1,000} \left( \beta_t g z^{-\sigma} u_c \left( c_t^{(\ell,m)} ; n_t^{(\ell,m)} \right) R_t^{(\ell,m)} ; n(s_t^{(\ell)}) \right) \right) \right| \] .
4. Compute moments of \( EE(s_t) \).

We find that the implied Euler equation errors are reasonably small for both the capital and recruiting Euler equations. Table G.5(a) reports the mean of the Euler equation errors for both Euler equations along with their distribution. The average Euler equation error is below \( 10^{-2} \), implying that agents would pay less than 1% of their period consumption to avoid the approximation error. The 99th percentile of approximation is below 2%. This is only a bit worse than the RBC analogue of our search model, as panel (c) shows. Errors in the search and matching model without bargaining shocks in panel (b) are smaller than in the RBC model.

Figure G.16 shows the mean, minimum, and maximum Euler equation errors also as a function of the endogenous state of the economy, i.e., capital and employment. The dependence is weak, though, except for some extreme values of employment and, even in this case, they still average below 1% of consumption.

**G.12 Partial filter for bargaining power**

**G.12.1 Derivation**

We derive the partial filter from (G.32) using the definition that:
\[ w_{\zeta,t} \equiv (1 - \zeta_0) w_{h,t} + \zeta_0 w_{\ell,t} \quad \Leftrightarrow \quad w_{h,t} - w_{\ell,t} = \frac{w_{\zeta,t} - w_{\ell,t}}{1 - \zeta_0} \]
### Table G.5: Euler equation errors (expressed as log_{10}): Mean and distribution.

<table>
<thead>
<tr>
<th>Model</th>
<th>Euler Equation</th>
<th>Mean</th>
<th>Min</th>
<th>p1</th>
<th>p5</th>
<th>Median</th>
<th>p95</th>
<th>p99</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Baseline search &amp; matching model</td>
<td>Capital EE</td>
<td>-3.03</td>
<td>-7.87</td>
<td>-5.67</td>
<td>-4.99</td>
<td>-3.86</td>
<td>-2.42</td>
<td>-1.77</td>
<td>-1.26</td>
</tr>
<tr>
<td></td>
<td>Recruiting EE</td>
<td>-2.61</td>
<td>-7.43</td>
<td>-4.55</td>
<td>-3.81</td>
<td>-2.77</td>
<td>-2.20</td>
<td>-1.73</td>
<td>-1.16</td>
</tr>
<tr>
<td>(b) Search &amp; matching model without bargaining shocks</td>
<td>Capital EE</td>
<td>-4.34</td>
<td>-7.80</td>
<td>-6.15</td>
<td>-5.47</td>
<td>-4.41</td>
<td>-3.95</td>
<td>-3.83</td>
<td>-3.62</td>
</tr>
<tr>
<td></td>
<td>Recruiting EE</td>
<td>-3.54</td>
<td>-7.46</td>
<td>-5.42</td>
<td>-4.70</td>
<td>-3.64</td>
<td>-3.11</td>
<td>-2.95</td>
<td>-2.52</td>
</tr>
<tr>
<td>(c) Hansen-Rogerson RBC model</td>
<td>Capital EE</td>
<td>-3.96</td>
<td>-8.04</td>
<td>-5.99</td>
<td>-5.30</td>
<td>-4.27</td>
<td>-3.45</td>
<td>-2.92</td>
<td>-2.29</td>
</tr>
<tr>
<td></td>
<td>Labor supply EE</td>
<td>-3.25</td>
<td>-7.18</td>
<td>-5.53</td>
<td>-4.83</td>
<td>-3.75</td>
<td>-2.66</td>
<td>-2.11</td>
<td>-1.56</td>
</tr>
</tbody>
</table>

Figure G.16: Euler equation errors as a function of capital and employment: Mean, maximum, and minimum.
First, solve (G.32) for \( w_{h,t} \). Here, we also set the habit parameter to zero:

\[
\begin{align*}
    w_{h,t} &= \phi_t m_{l,t} \frac{1}{1-\zeta} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + (1 - \phi_t)\omega_t + \frac{1 - \phi_t}{1 - \tau_n} \left( \frac{c_t}{1 + (\sigma - 1)\gamma_{n-1}} \right) \gamma^\sigma \\
    & \quad - (1 - x - f_t(\theta_t)) E_t \left[ \frac{1 - \phi_t}{1 - \phi_{t+1}} m_{l,t+1} \left( \phi_{t+1} J_{\zeta,n,t+1} + \phi_{t+1} \frac{\tilde{\zeta}}{1 - \zeta} m_{l,t+1} + \zeta_0 (w_{\ell,t+1} - w_{h,t+1}) \right) \right].
\end{align*}
\]

Multiply by \( (1 - \zeta_0) \) and add \( \zeta_0 w_{\ell,t} \) and plug in for \( w_{h,t+1} - w_{\ell,t+1} \):

\[
\begin{align*}
    w_{\zeta,t} &= \zeta_0 w_{\ell,t} + (1 - \zeta_0) \phi_t m_{l,t} \frac{1}{1-\zeta} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + (1 - \zeta_0)(1 - \phi_t)\omega_t + (1 - \zeta_0) \frac{1 - \phi_t}{1 - \tau_n} \left( \frac{c_t}{1 + (\sigma - 1)\gamma_{n-1}} \right) \gamma^\sigma \\
    & \quad - (1 - x - f_t(\theta_t)) (1 - \phi_t) E_t \left[ (1 - \zeta_0) \frac{\phi_{t+1}}{1 - \phi_{t+1}} m_{l,t+1} \left( \phi_{t+1} J_{\zeta,n,t+1} + \phi_{t+1} \frac{\tilde{\zeta}}{1 - \zeta} m_{l,t+1} - \zeta_0 (w_{\zeta,t+1} - w_{\ell,t+1}) \right) \right] \\
    & \quad + \zeta_0 (1 - x - f_t(\theta_t)) (1 - \phi_t) E_t \left[ \frac{1}{1 - \phi_{t+1}} m_{l,t+1} (w_{\zeta,t+1} - w_{\ell,t+1}) \right].
\end{align*}
\]

Below, we model covariances and first moments using a VAR that includes \( \tilde{\phi}_t, \ln m_t, \ln w_{\zeta,t}, \ln w_{\ell,t}, \ln m_{l,t}, \) and \( \ln \left( 1 + \frac{1 - x}{\mu(\theta_{t+1})(1 - \zeta)} \right) \equiv \tilde{\theta}_{t+1} \).

Write the VAR as:

\[
\begin{align*}
    X_{t+1} &= \mu_X + AX_t + B\epsilon_{t+1} \\
    \epsilon_{t+1} &\sim N(0, I), \\
    \Sigma &\equiv BB'.
\end{align*}
\]

We use selection vectors (Kronecker deltas) \( e_m \) to select \( m_{t+1} = e_m X_{t+1} \) and analogously for other variables. We can, then, write:

\[
\begin{align*}
    E_t[m_{t+1}] &= e^{e_m(\mu_X + AX_t) + \frac{1}{2} e_m \Sigma e_m'} \\
    \text{Cov}_t[m_{t+1}, w_{\zeta,t+1}] &= E_t[e^{\ln m_{t+1}} \text{Cov}_t[\ln m_{t+1}, \ln w_{\zeta,t+1}] E_t[e^{\ln w_{\zeta,t+1}}] \\
    &= \exp(e_m(\mu_X + AX_t) + \frac{1}{2} e_m \Sigma e_m') e_m \Sigma e_{w,\zeta} \exp(e_{w,\zeta}(\mu_X + AX_t) + \frac{1}{2} e_{w,\zeta} \Sigma e_{w,\zeta}' ) \\
    &= \exp((e_m + e_{w,\zeta})(\mu_X + AX_t) + \frac{1}{2} (e_m \Sigma e_m' + e_{w,\zeta} \Sigma e_{w,\zeta}')) e_m \Sigma e_{w,\zeta}. \quad (G.86)
\end{align*}
\]

Below, we use analogues to both expressions repeatedly and exploit log-normality to rewrite the expectational terms.

Last, we use our distributional assumption on the bargaining power process. To make sure that the bargaining power process remains bounded, we model the logistic transform of the bargaining power in the paper. This proves convenient here too:

\[
\tilde{\phi}_t \equiv \ln \frac{\phi_t}{1 - \phi_t} = (1 - \rho_\phi) \ln \frac{\bar{\phi}}{1 - \phi} + \rho_\phi \ln \frac{\phi_{t-1}}{1 - \phi_{t-1}} + \omega_\phi \epsilon_{t,\phi}
\]
For future reference, note that \( 1 + e^{\tilde{\phi}_t} = \frac{1}{1 - \tilde{\phi}_t} \).

Now, start with the last term in the wage setting equation:

\[
T3_t \equiv E_t \left[ \frac{1}{1 - \phi_{t+1}} m_{t+1} (w_{\zeta, t+1} - w_{t, t+1}) \right] \\
= E_t \left[ (1 + e^{\tilde{\phi}_t}) e^{\ln m_{t+1} + \ln w_{\zeta, t+1}} \right] - E_t \left[ (1 + e^{\tilde{\phi}_t}) e^{\ln m_{t+1} + \ln w_{t, t+1}} \right] \\
= e^{(e_m + e_{w, \zeta})(\mu_X + AX_t) + \frac{1}{2}(e_m + e_{w, \zeta})\Sigma(e_m + e_{w, \zeta})} + e^{(e_\omega + e_{w, \zeta})(\mu_X + AX_t) + \frac{1}{2}(e_\omega e_m + e_{w, \zeta})\Sigma(e_m + e_{w, \zeta})} - e^{(e_\omega + e_{w, \zeta})(\mu_X + AX_t) + \frac{1}{2}(e_\omega e_m + e_{w, \zeta})\Sigma(e_m + e_{w, \zeta})}.
\]

When \( w_{t, t} \) is constant, \( e_{w, t} \mu_X \neq 0 \), but \( e_{w, t} \Sigma = 0 \).

Similarly:

\[
T2_t \equiv E_t \left[ \frac{\phi_{t+1}}{1 - \phi_{t+1}} m_{t+1} m_{t+1} \right] \\
= E_t \left[ e^{\tilde{\phi}_{t+1} + \ln m_{t+1} + \ln m_{t+1}} \right] - E_t \left[ e^{\tilde{\phi}_{t+1} + \ln m_{t+1} + \ln m_{t+1}} \right] \\
= e^{(e_m + e_{m, t})(\mu_X + AX_t) + \frac{1}{2}(e_m + e_{m, t})\Sigma(e_m + e_{m, t})}.
\]

For the term involving the future marginal value of employment to firms, we can use economic theory to plug in for the future value of employment to the firm:

\[
T1_t \equiv E_t \left[ \frac{\phi_{t+1}}{1 - \phi_{t+1}} m_{t+1} \J_{\zeta, n, t+1} \right] \\
= E_t \left[ \frac{\phi_{t+1}}{1 - \phi_{t+1}} m_{t+1} \left( 1 + \frac{1 - x}{\mu(\theta_{t+1})(1 - \zeta)} - w_{\zeta, t+1} \right) \right] \\
= E_t \left[ e^{\tilde{\phi}_{t+1} + \ln m_{t+1} + \ln \J_{\zeta, n, t+1}} \right] - E_t \left[ e^{\tilde{\phi}_{t+1} + \ln m_{t+1} + \ln w_{\zeta, t+1}} \right] \\
= e^{(e_\omega + e_{m, t})(\mu_X + AX_t) + \frac{1}{2}(e_\omega + e_{m, t})\Sigma(e_\omega + e_{m, t})} + e^{(e_\omega + e_{w, \zeta})(\mu_X + AX_t) + \frac{1}{2}(e_\omega + e_{w, \zeta})\Sigma(e_\omega + e_{w, \zeta})} - e^{(e_\omega + e_{w, \zeta})(\mu_X + AX_t) + \frac{1}{2}(e_\omega + e_{w, \zeta})\Sigma(e_\omega + e_{w, \zeta})}.
\]

Plugging \( \phi_t \) for \( \tilde{\phi}_t \)-terms and for the three terms, we have:

\[
w_{\zeta, t} = \zeta_0 w_{t, t} + \frac{e^{\phi_t} m_{t, t}}{1 + e^{\phi_t}} m_{t, t} \left( 1 - \frac{\phi_0}{1 - \zeta} \right) + \frac{1 - \z_0}{1 + e^{\phi_t}} \omega_t + \frac{c_t}{1 + (\sigma - 1) \gamma n_{t-1}} \gamma \sigma \\
- (1 - x - f_t(\theta_t)) \frac{1}{1 + e^{\phi_t}} \left( 1 - \frac{\phi_0}{1 - \zeta} \right) m_{t, t} \left( \frac{J_{\zeta, n, t+1}}{1 - \zeta} + \frac{\zeta}{1 - \zeta} m_{t, t} \right) \\
- \z_0(1 - x - f_t(\theta_t)) \frac{1}{1 + e^{\phi_t}} \left( 1 - \frac{\phi_0}{1 - \zeta} \right) w_{t, t} \left( \frac{1}{1 - \phi_{t+1}} m_{t, t} - w_{t, t+1} \right) \\
= \zeta_0 w_{t, t} + \frac{e^{\phi_t} m_{t, t}}{1 + e^{\phi_t}} \left( 1 - \frac{\phi_0}{1 - \zeta} \right) \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + \frac{1 - \z_0}{1 + e^{\phi_t}} \omega_t + \frac{c_t}{1 + (\sigma - 1) \gamma n_{t-1}} \gamma \sigma.
\]
Given VAR estimates and noting that $X_t$ contains $\tilde{\phi}_t$, we can solve (G.87) for $\tilde{\phi}_t$ using data on:

- The marginal product of capital $mpl_t$,
- Labor market tightness $\theta_t$,
- The average (mean) wage rate $w_{\zeta,t}$
- The stochastic discount factor $m_t$,

taking the minimum wage rate $w_{\ell,t}$ and unemployment benefits $\omega_t$ as constant.

When $\zeta_0 = 0 \Rightarrow \bar{\zeta} = 0, T2_t = 0$ and $\omega_t = 0$, then equation (G.86) simplifies to:

$$w_{\zeta,t} = \frac{e^{\tilde{\phi}_t}}{1 + e^{\tilde{\phi}_t}}mpl_t \frac{1}{1 - \bar{\zeta}} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + \frac{1}{1 + e^{\tilde{\phi}_t}} \frac{1}{1 - \tau_n} \left( \frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma^\sigma$$

$$- (1 - x - f_t(\theta_t)) \frac{1}{1 + e^{\tilde{\phi}_t}} \left( e^{(e_{\tilde{\phi}} + e_n + m_{mpl} + e_0)(\mu_{X} + AX_t)} + \frac{1}{2}(e_{\tilde{\phi}} + e_n + m_{mpl} + e_0)^T \Sigma(e_{\tilde{\phi}} + e_n + m_{mpl} + e_0) \right).$$

We estimate the VAR in the demeaned variables and add the model-implied mean.

### G.12.2 Sampler

In our baseline model, we assume that the bargaining power is exogenous to the state of the economy and is driven by an AR(1) model. This implies an exclusion restriction on the estimated VAR. The exclusion restriction allows us to pull $e_{\tilde{\phi}}(\mu_{X} + AX_t)$ out and write it simply as $\kappa_{\tilde{\phi}} + \rho_{\tilde{\phi}} \tilde{\theta}_t$ in (G.87).

Under the joint normality of the forecast errors, together with a flat prior this gives rise to a standard SUR algorithm for inference.$^{25}$ To begin, stack the model as follows:

$$Y_{SUR} = X_{SUR} \beta_{SUR} + v_{SUR}, \quad v_{SUR} \sim \mathcal{N}(0, V \otimes I_T), \quad (G.88)$$

where $Y_{SUR} = \{vec(\tilde{\phi})', vec(Y)\}'$ and similarly for $v_{SUR}$. In addition, we have the definitions:

$$V = \begin{bmatrix} \Sigma_{\tilde{\phi},\tilde{\phi}} & \Sigma_{\tilde{\phi},Y'} \\ \Sigma_{Y,\tilde{\phi}} & \Sigma_{YY} \end{bmatrix} \quad \beta_{SUR} = \begin{bmatrix} \rho_{\tilde{\phi}} \\ A_{Y} \end{bmatrix},$$

$$X_{SUR} = \begin{bmatrix} \tilde{\phi}_{-1} \\ 0_{T \times ny} \end{bmatrix} \otimes Y_{X} \quad X_y = \begin{bmatrix} \tilde{\phi}_{-1} \\ Y_{-1} \end{bmatrix} [I_{T}]_{l=0}^{T-1} \begin{bmatrix} 1_T \end{bmatrix}. \quad (G.89)$$

With these definitions, we transform the model to yield standard normal residuals: $\tilde{v} = \tilde{Y} - \tilde{X} \beta \sim \mathcal{N}(0, I)$. The transformed model gives rise to standard conditional Normal-Wishart posterior distributions. To implement the transformation, define $U$ as the Cholesky decomposition of $\Sigma$ such that $U'U = \Sigma$.

$$\tilde{X} = ((U^{-1})' \otimes I_T) X_{SUR} \quad \tilde{Y} = ((U^{-1})' \otimes I_T) \begin{bmatrix} \tilde{\phi} \\ Y \end{bmatrix}$$

$$N_{XX}(\Sigma) = \tilde{X}' \tilde{X} \quad N_{XY}(V) = \tilde{X}' \tilde{Y}$$

$$S_T(\beta) = \frac{1}{\nu_0 + T} \begin{bmatrix} (\tilde{\phi} - \rho_{\tilde{\phi}} \tilde{\theta}_{-1})' \\ (Y - XB)' \end{bmatrix} \begin{bmatrix} (\tilde{\phi} - \rho_{\tilde{\phi}} \tilde{\theta}_{-1}) \\ (Y - XB) \end{bmatrix} + \frac{\nu_0}{\nu_0 + T} S_0.$$
Here, $S_0$ is a prior over $\Sigma$, i.e., $\Sigma^{-1} \sim \mathcal{W}(\nu_0 S_0^{-1}, \nu_0)$. Given a prior $\beta \sim \mathcal{N}(\tilde{\beta}_0, N_0)$, the conditional posterior distributions are given by:

$$
\begin{align*}
\beta | \Sigma, Y^T & \sim \mathcal{N}(\tilde{\beta}_T(V), (N_{XX}(\Sigma) + N_0)^{-1}), \\
\Sigma^{-1} | \beta, Y^T & \sim \mathcal{W}(S_T(\beta)^{-1}/(\nu_0 + T), \nu_0 + T),
\end{align*}
$$

where $\tilde{\beta}_T(V) = (N_{XX}(V) + N_0)^{-1}(N_{XY}(V) + N_0\tilde{\beta}_0)$.

The algorithm, then, is:

1. Initialize $\Sigma^{(0)} = S_T(\bar{\beta}_T)$ and $\{\tilde{\phi}_t^{(0)}\}_t$.

2. Repeat for $i = 1, \ldots, n_G$:

   (a) Draw $\beta^{(i)} | \Sigma^{(i-1)}, \{\tilde{\phi}^{(0)}_t\}_t$ from (G.90a).

   (b) Draw $\Sigma^{(i)} | \beta^{(i)}, \{\tilde{\phi}^{(0)}_t\}_t$ from (G.90b).

   (c) Draw $\{\tilde{\phi}^{(i)}_t\}_t | \beta^{(i)}, \Sigma^{(i)}$ from (G.87).

### G.12.3 Derivation with an alternative VAR

Running a VAR in a linear combination allows us to also use a model-implied FOC to factor the terms within the expectations. When $\zeta_0 = 0$, we would then only have to estimate the covariance term.

We begin our derivation as above (also with $h = 0$), but now use once that $E_t[XY] = \text{Cov}_t[X, Y] + E_t[X]E_t[Y]$ to rewrite the term involving the future value of employment.

$$
\begin{align*}
w_{\zeta,t} = \zeta_0 w_{\ell,t} + (1 - \zeta_0)\phi_t\text{mpl}_t \left[ \frac{1}{1 - \zeta} \left( 1 + \frac{1 - x}{\mu(\theta_t)} \right) + (1 - \zeta_0)(1 - \phi_t)\omega_t + (1 - \zeta_0)^2 \frac{1 - \phi_t}{1 - \tau_n} \left( \frac{c_t}{1 + (\sigma - 1)\gamma_{m_{t-1}}} \right) \gamma \sigma \\
- (1 - x - f_t(\theta_t))(1 - \phi_t)E_t \left[ (1 - \zeta_0) \frac{\phi_{t+1}}{1 - \phi_{t+1}} m_{t+1} \left( \frac{J_{\zeta, n, t+1}}{1 - \tau^k} + \frac{\tilde{\zeta}}{1 - \zeta} \text{mpl}_{t+1} \right) \right] \\
+ \zeta_0(1 - x - f_t(\theta_t))(1 - \phi_t)E_t \left[ \frac{1}{1 - \phi_{t+1}} m_{t+1}(w_{\zeta, t+1} - w_{\ell, t+1}) \right] \\
\end{align*}
$$

Below, we model covariances and first moments using a VAR that includes $\tilde{\phi}_t$, $\ln(m_t \times \text{mpl}_t)$, $\ln \left( m_t \times \left( (1 + \theta_t)\text{mpl}_t - w_{\zeta,t} \right) \right)$, and $\ln (m_t \times (w_{\zeta,t} - \tilde{w}_{\ell,t}))$.
Plugging in from (G.21) and the law of motion for $\tilde{\phi}_t$:

$$w_{\zeta,t} = \zeta_0 w_{\ell,t} + (1 - \zeta_0)\phi_t mpl_t \frac{1}{1 - \zeta} \left(1 + \frac{1 - x}{\mu(\theta_t)}\right) + (1 - \zeta_0)(1 - \phi_t)\omega_t + (1 - \zeta_0)\frac{1 - \phi_t}{1 - \tau_n} \left(\frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}}\right) \gamma \sigma$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t)(1 - \zeta_0) \left(Cov_t \left[e^{\tilde{\phi}_{t+1}} m_{t+1} J_{\xi,n,t+1} \frac{1}{1 - \tau_k} - \omega_t \right] + e^{r_s + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t) E_t \left[(1 - \zeta_0) e^{\tilde{\phi}_{t+1}} m_{t+1} \frac{1}{1 - \zeta} \right]$$

$$+ \zeta_0(1 - x - f_t(\theta_t))(1 - \phi_t) E_t \left[(1 + e^{\tilde{\phi}_{t+1}}) m_{t+1} (w_{\zeta,t+1} - w_{t,t+1})\right].$$

Now, plugging in from (G.22) and using Stein’s Lemma:

$$w_{\zeta,t} = \zeta_0 w_{\ell,t} + (1 - \zeta_0)\phi_t mpl_t \frac{1}{1 - \zeta} \left(1 + \frac{1 - x}{\mu(\theta_t)}\right) + (1 - \zeta_0)(1 - \phi_t)\omega_t + (1 - \zeta_0)\frac{1 - \phi_t}{1 - \tau_n} \left(\frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}}\right) \gamma \sigma$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t)(1 - \zeta_0) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t) E_t \left[(1 - \zeta_0) e^{\tilde{\phi}_{t+1}} m_{t+1} \frac{1}{1 - \zeta} \right]$$

$$+ \zeta_0(1 - x - f_t(\theta_t))(1 - \phi_t) E_t \left[(1 + e^{\tilde{\phi}_{t+1}}) m_{t+1} (w_{\zeta,t+1} - w_{t,t+1})\right].$$

Define:

$$X_{1,t} \equiv \tilde{\phi}_t$$

$$X_{2,t} \equiv \ln \left(m_{t+1} \left( mpl_t \left(1 + \frac{1 - x}{\mu(\theta_t)(1 - \zeta)}\right) - w_{\zeta,t}\right)\right)$$

$$X_{3,t} \equiv \ln \left(m_{t+1} mpl_t\right)$$

$$X_{4,t} \equiv \ln \left(m_{t+1} (w_{\zeta,t+1} - w_{t,t+1})\right).$$

This gives:

$$w_{\zeta,t} = \zeta_0 w_{\ell,t} + (1 - \zeta_0)\phi_t mpl_t \frac{1}{1 - \zeta} \left(1 + \frac{1 - x}{\mu(\theta_t)}\right) + (1 - \zeta_0)(1 - \phi_t)\omega_t + (1 - \zeta_0)\frac{1 - \phi_t}{1 - \tau_n} \left(\frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}}\right) \gamma \sigma$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t)(1 - \zeta_0) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t)(1 - \zeta_0) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t)(1 - \zeta_0) \frac{1}{1 - \zeta} \left(e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$+ \zeta_0(1 - x - f_t(\theta_t))(1 - \phi_t) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$+ \zeta_0(1 - x - f_t(\theta_t))(1 - \phi_t) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$= \zeta_0 w_{\ell,t} + (1 - \zeta_0) e^{\tilde{r}_t + \phi_t} \omega_t + \frac{1}{2} \omega_r^2 \frac{mpl_t}{\mu(\theta_t) 1 - \zeta}\right)$$

$$- (1 - x - f_t(\theta_t))(1 - \phi_t) T 1_{\ell}$$
Finally, we proceed as above, with (G.91) taking the role of (G.86).

G.12.4 Measurement

To implement our filter, we need data on: (1) the real wage, (2) the marginal product of labor, (3) labor market tightness, (4) the unemployment rate, and (5) consumption. The data sources are the same as for our main model, so we just discuss the mapping into model variables.

1. We use raw consumption, real wages, and real GVA.

2. We set the counterpart to unemployment benefits to 40% of the initial wage rate and let this grow at the average wage growth rate over the entire sample to account for balanced growth. Similarly, we set the initial minimum wage according to the model steady state and then let it grow at the average wage growth rate over the entire sample.

3. We re-scale the average real wage (an index) to the steady-state wage in the model.

4. We implement our model for the Cobb-Douglas case of the production function. Thus, the average product of labor is proportional to the marginal product of labor.\(^{26}\) We consider two different measures:
   - Real GVA in the business sector divided by (1-unemployment) and re-scaled.
   - Real GVA in the business sector population ratio instead of 1-unemployment.

We first re-scale the average marginal product of labor to the steady-state marginal product of labor in the model. Second, we shift the marginal product of labor up so that it lies weakly above the real wage.

5. We compute the monthly job finding rate implied by labor market tightness as \(f_m(\theta_t)\). We then adjust the job finding rate and the separation rate \(x\) for the quarterly data frequency in the following way: The quarterly separation rate is \(x_q = (1 + x_m/100)^3 - 1\). The quarterly job finding rate is \(f_q = f_m + (1 - f_m)f_m + (1 - f_m)^2f_m\). This neglects within-quarter separations.

6. Given the real-wage rate, the static component of the household surplus turns negative in the 1990s. We shift the average disutility of working up until the implied average bargaining power in the data equals 0.5 for a first burn-in period, as in the model.

7. When we use the employment-to-population ratio to compute labor productivity, we also use the employment-to-population ratio to compute the disutility from working. However, our model is calibrated to an average employment-to-participation ratio of 0.95. To avoid

\(^{26}\)Unfortunately, there is no easy way to differentiate in the data the marginal productivity of production workers (the object of interest in the model) from the marginal productivity of recruiters. Our prior is that the bias induced by considering the aggregate marginal productivity of both types of workers is negligible.
having the data counterpart to \( n_t \) in the model exceed unity, we re-scale the employment-to-participation ratio so that it averages 0.95 and has the same range (max – min) as the unemployment rate.

**Wages and MPL**

**Firm-side surplus: Adj. MPL - w**

**Quarterly job finding rate \( f(\theta) \)**

**Household surplus: w - disutility**

Figure G.17: Variables entering the filter

**G.12.5 New hire wages**

Since the wages of new hires are much more procyclical than the wages of continuing workers, Pissarides (2009) argues that wage rigidity à la Hall (2005) or Gertler and Trigari (2009) is unlikely to be a realistic way to make unemployment more volatile in search and matching models of the labor market. Given that all wages adjust every period in our model, wage stickiness is irrelevant in our setup. However, the cyclicity of real wages is a central calibration target and may affect our historical estimates.

As a robustness check, we consider a measure of the wage of new hires. Specifically, we use median hourly wages in the private sector from Haefke et al. (2013), extended by the Federal Reserve Bank of Philadelphia.\(^{27}\) This series begins in 1979.Q1. The left panel of Figure G.18 shows the change in the real wages from Haefke et al. (2013) and the updated series for when they overlap. The correlation is very high at 0.85. The right panel of Figure G.18 plots the nominal series. We splice the nominal series together by adjusting the updated series down in the last quarter for which we have data from Haefke et al. (2013). We deflate the series with the PCE deflator.

Once we HP-filter this series, we find that the wage of new hires is as acyclical with respect to GDP (constructed, on a model-consistent basis, as the sum of real consumption and investment),

\(^{27}\)We are grateful to Paul van Vliet for sharing his data.
similar to the composition-adjusted employment cost index, which we also consider; see Table G.6. Our finding is in line with Gertler et al. (2016). These authors report that wages of new hires from unemployment are no more procyclical than aggregate wages, based on SIPP data, once they control for match quality. Our finding differs somewhat from Haefke et al. (2013), who document that their measure of composition-adjusted real wage growth of new hires correlates much more strongly with labor productivity growth than the analogous hourly wage for all workers. While we replicate these authors’ results (which are subject to large sample uncertainty) with their sample period, we find that the cyclicality of new wages and continuing wages is close in the extended sample. Haefke et al. (2013) exclude the early Volcker years (up to the end of 1983) from their baseline analysis and their sample ends in 2006.Q1. In that sample— but with new productivity data— we find a point estimate ($t$-statistics) of 0.56 (1.08) for the new hire wage, relative to 0.40 (3.79) for the overall wage rate and 0.41 (3.49) for the BLS wage rate. Adding the three observations raises the coefficient on the BLS wage to 0.46 (3.84) and lowers it to 0.46 (0.86) for the new hire wage. Adding 1983 and earlier years yields similar results.

Given that our series for new wages has acyclical properties similar to those of the other measures of wages, it is not a surprise that our backed-out bargaining power is not materially affected. Figure G.19 compares the implied bargaining power. While the bargaining power implied by the new hire wage data is noisier (the new wage data, based on survey data, are themselves noisier), it closely mimics our baseline measure. Both backed-out bargaining power series have a high degree of comovement and identify similar peaks and troughs. The levels of the series are not comparable, because we calibrate the bargaining power to average 0.5 over the entire available sample, but not over the sub-sample shown for our baseline measure.

---

Note that, in Table G.6, we change the length of the sample of aggregate variables in each row to make it consistent with the corresponding wage sample. That is why the statistics of these aggregate variables slightly vary across rows.
Table G.6: Business cycle statistics: Different wage measures and samples

<table>
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<th></th>
<th>Volatility</th>
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<th>Persistence</th>
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<td></td>
<td>Y</td>
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<td>std(C)</td>
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<td>2.93</td>
<td>0.52</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline wage &amp; sample</td>
<td>1.00</td>
<td>0.91</td>
<td>0.84</td>
</tr>
<tr>
<td>ECI wage &amp; sample</td>
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<td>0.95</td>
<td>0.81</td>
</tr>
<tr>
<td>New hire wage &amp; sample</td>
<td>1.00</td>
<td>0.95</td>
<td>0.83</td>
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</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. 
Figure G.19: Bargaining power process implied by the baseline calibration: Baseline wage measure and new hire wage
G.12.6 Additional results

When we use an alternative measure of labor productivity or detrend non-stationary variables prior to filtering, we find only small changes in the implied moments: See Table G.7(a) to (c).

<table>
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<tr>
<th>Table G.7: Implied bargaining power process moments</th>
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<td>(a) Productivity based on complement of the unemployment rate, alternative VAR and filter</td>
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<td>In-sample autocorrelation</td>
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<tr>
<td>Posterior autocorrelation</td>
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<tr>
<td>In-sample AR(1) st.dev.</td>
</tr>
<tr>
<td>Posterior AR(1) st.dev.</td>
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<td>In-sample average bargaining power</td>
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</table>

<table>
<thead>
<tr>
<th>(b) Productivity based on employment-to-population ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
</tr>
<tr>
<td>In-sample autocorrelation</td>
</tr>
<tr>
<td>Posterior autocorrelation</td>
</tr>
<tr>
<td>In-sample AR(1) st.dev.</td>
</tr>
<tr>
<td>Posterior AR(1) st.dev.</td>
</tr>
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<td>In-sample average bargaining power</td>
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</table>

<table>
<thead>
<tr>
<th>(c) Productivity based on BLS labor productivity</th>
</tr>
</thead>
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<td>Median</td>
</tr>
<tr>
<td>In-sample autocorrelation</td>
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<td>Posterior autocorrelation</td>
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<tr>
<td>In-sample AR(1) st.dev.</td>
</tr>
<tr>
<td>Posterior AR(1) st.dev.</td>
</tr>
<tr>
<td>In-sample average bargaining power</td>
</tr>
</tbody>
</table>

G.12.7 A comparison with an alternative bargaining power index

Levy and Temin (2007) propose to measure bargaining power as the inverse of the real unit labor cost. They call this measure the “bargaining power index.” We compute their measure for our longer sample as the real hourly compensation divided by the real hourly output, both measured in the non-farm business sector.\textsuperscript{29} This measure exhibits a pronounced downward trend, perhaps due to changes in the underlying industry or occupation mix. To compare our measures, we remove a quadratic trend from (the log of) their measure and do the same for our measure. Figure G.20 shows the resulting time series. Despite the very different methodological approaches, the two series move together. The overall correlation between the detrended series is 0.40. They track each other particularly well from the beginning of our sample period to the mid-1970s and again during the 2000s.

Quarterly changes in our filtered bargaining power tend to move with the changes in (log of) the Levy and Temin (2007) bargaining index, as Figure G.21 shows. The overall correlation is 0.28, and both measures pick up on the increased bargaining power due to the extension of unemployment

\textsuperscript{29}Levy and Temin (2007) use the median, not the mean compensation. We prefer the mean because, due to demographic changes, who the median worker is has noticeably changed over the last few decades.
benefits in late 2008 and the reversal during several periods after the Great Recession. While both measures may contain measurement error, it is reassuring that the 10 lowest and highest changes in our bargaining power index also tend to be classified as such in the bargaining power index. The figure dates these data points, and their correlation is 0.38.

We have also argued in the main text that increases in the real minimum wage resemble increases in bargaining power, as pointed out by Flinn (2006). Figure G.23 plots the real minimum wage alongside our bargaining power – the real federal minimum wage (solid green line) and the real effective minimum wage, that is, the population-weighted average of the maximum of the state and federal minimum wage. Overall, the correlation between the real federal minimum wage and our bargaining power is a low 0.08. However, from 1974 on, when the federal minimum wage is unified and has the broadest coverage, the correlation is 0.53. For most of the period since 1974, we also have data on state minimum wages from Autor et al. (2016), and the correlation with the effective minimum wage is 0.41. Given that our filtered bargaining power was high before 1974 and the minimum wage was less broadly applicable, we view this evidence as consistent with the notion that our bargaining power index reflects redistributive measures such as the minimum wage when bargaining power is low.
We demean the log real minimum wage measures, divide them by 100, and re-center them at 0.5 to ease the comparison.

Figure G.23: Filtered bargaining power and real minimum wage measures
G.12.8 Counterfactual unemployment

Our final exercise in this subsection is to filter the historical bargaining power with our calibrated stochastic process. Specifically, we set $\epsilon_{\phi,1} = \omega^{-1}_\phi (\tilde{\phi}_1 - (1 - \rho_\phi) \log \frac{\tilde{\phi}}{1-\phi})$ and for subsequent observations $\epsilon_{\phi,t} = \omega^{-1}_\phi (\tilde{\phi}_t - \rho_\phi \tilde{\phi}_{t-1} - (1 - \rho_\phi) \log \frac{\tilde{\phi}}{1-\phi})$.

With this series, we can compute the implied unemployment rate coming out of our model. More concretely, for each realization of the bargaining power shock, we take one draw of the labor productivity shock from its distribution and compute the difference to a world where the bargaining power shocks are set to zero, but the same productivity shocks. Figure G.24 displays the result for the unemployment rate. We also plot the simulated level of bargaining power to confirm that our procedure is consistent.

The bargaining power shock explains 38.3% of the historical fluctuations in the unemployment rate. Its correlation with the historical unemployment rate is 0.25, and its relative standard deviation is 1.53. Multiplying these two numbers gives a share of the explained variance of 0.383. HP-filtered, the correlation is 0.86, and the relative standard deviation 1.29.\(^{30}\)

---

\(^{30}\)The correlation is $\frac{\text{Cov}[x,y]}{\text{Var}[x]^{1/2} \text{Var}[y]^{1/2}}$. Multiplying this correlation by the ratio of standard deviations gives the covariance relative to the variance. Because the variance is, up to first order, the sum of the variance due to bargaining shocks and the remainder due to all other shocks, the reported measure is the historical variance accounted for by the bargaining power shock.

Figure G.24: Historical counterfactuals implied by bargaining power process
Figure G.25: GIRFs to a negative two standard deviation shock to labor productivity with Cobb-Douglas technology.
Figure G.26: GIRFs to a two standard deviation shock to workers' bargaining power with Cobb-Douglas technology.
Figure G.27: GIRFs to a two standard deviation shock to workers’ bargaining power with CES $\varepsilon = 0.75$. 

[Graphs showing various economic indicators such as output, consumption, investment, marginal product of capital, Tobin’s q, firm value, employment, market tightness, end of period capital, gross/net capital share, wages, MPL, and bargaining power.]
Figure G.28: GIRFs to a two standard deviation shock to workers’ bargaining power with CES $\varepsilon = 1.25$. 
G.14 GIRF comparison: Search and matching vs. RBC model

We benchmark our model against an RBC analogue to our economy. Since our baseline model features indivisible labor, its RBC analogue is closest to Hansen (1985) and Rogerson (1988). In keeping with our timing convention, however, labor is also hired and paid one period in advance. Also, employed and unemployed agents have the same consumption and hence the period utility function is simply:

\[ U_t = \frac{(c_t - h\tilde{c}_{t-1})^{1-\sigma} - 1}{1 - \sigma} - \gamma \tilde{n}_{t-1}. \]

Compared to the solution of the search model, this implies the following changes:

- The detrended habit function \( \tilde{h}(\cdot) \) in (G.53) is constant at \( \tilde{h} = hg - \frac{1}{1-\alpha} \).
- The law of motion for employment (G.6) drops out as well as the recruiting optimality condition (G.21) – the fraction of recruiters \( \nu_t \) and labor market tightness \( \theta_t \) are not defined.
- There are alternative ways of setting wages that allow us to retain the assumption that labor is set one period in advance. We pick a structure where labor supply is predetermined:
  - The equation (G.50) for the marginal value of employment \( J_n \) is replaced by
    \[ \tilde{mpl}_t - w_t = 0. \]
    In words, the wage rate equals the marginal product of labor state by state – keeping the labor share of income constant with a Cobb-Douglas production function.
  - Wage-setting is replaced by an indifference condition for the household:
    \[ E_t \left[ m_{t+1}g^{1-\alpha} \left( (1 - \tau_n)w_t - \sigma \gamma \frac{c_{t+1} - h\tilde{c}_t}{1 + (\sigma - 1)\gamma \tilde{n}_t} \right) \right] = 0. \]
    Households choose labor supply one period in advance so that, on expectation, they are indifferent between leisure and work.

We can then compare the responses to the common productivity shock, using the same deep parameters that we calibrated for our baseline model – except that we also recalculate \( \gamma \) to make sure the employment levels in both models are the same.

We show the comparison of GIRFs from this model and our baseline model in Figures G.29 (for unitary elasticity of substitution), G.30 (for \( \epsilon = 0.75 \)), and G.31 (for \( \epsilon = 1.25 \)). Finally, in Figure G.32, we show the comparison of GIRFs with the RBC model with factor share shocks.
Figure G.29: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with Cobb-Douglas production function.
Figure G.30: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with CES $\varepsilon = 0.75$. 
Figure G.31: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with CES $\varepsilon = 1.25$. 

Output $y, yr$, Investment $I$, Consumption $c$, Wages $w$ and $mpl$, End of period employment $n$, Gross / net capital share $cs, ncs$.
G.15 The Hosios rule and the welfare cost of political risk

The Hosios condition holds in our model without recipients of the minimum wage, i.e., when $\zeta_0 = 0$ and when $\tau_n = \tau_k$ and $\phi = 1 - \eta$. Below we compare our baseline calibration to one without minimum wage recipients but with unequal tax rates, and a calibration that also has equal tax rates so that the Hosios condition holds in the steady state.

We re-calibrate the model for each parameter combination. Table G.8 shows the implied business cycle statistics. The violation of the Hosios condition in the steady state is immaterial for our results. Without minimum wages, we find a smaller role for bargaining power shocks for the capital share than in our baseline model. Equalizing the tax rates has no perceivable effect on the role of bargaining power shocks. The welfare implications, however, are unchanged up to the first digit; see Table G.9.

Intuitively, the actual bargaining power in our model is highly persistent, so that deviations from the Hosios condition are long-lasting independent of the steady-state calibration.\(^{31}\) Thus, steady-state efficiency may have little effect on efficiency in the stochastic economy.

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\(^{31}\)We also re-calibrate the steady state of the model to attain the same calibration targets, including the employment level, independent of whether the Hosios condition holds.
Table G.8: Business cycle statistics: 1947Q1–2015Q2

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<th></th>
<th>Y</th>
<th>std(I)</th>
<th>std(C)</th>
<th>std(ncs)</th>
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<th>std(w)</th>
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<td>-0.95</td>
<td>1.00</td>
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<th>std(w)</th>
<th>std(u)</th>
<th>std(TFP)</th>
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<tr>
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<td>0.87</td>
<td>0.82</td>
<td>0.78</td>
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<td>0.67</td>
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<tr>
<td>Baseline</td>
<td>0.81</td>
<td>0.77</td>
<td>0.83</td>
<td>0.80</td>
<td>0.71</td>
<td>0.80</td>
<td>0.70</td>
<td>0.79</td>
</tr>
<tr>
<td>No bargaining shock</td>
<td>0.81</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.48</td>
<td>0.79</td>
<td>0.83</td>
<td>0.79</td>
</tr>
<tr>
<td>No minimum wage ζ₀ = 0</td>
<td>0.82</td>
<td>0.78</td>
<td>0.83</td>
<td>0.77</td>
<td>0.48</td>
<td>0.79</td>
<td>0.74</td>
<td>0.79</td>
</tr>
<tr>
<td>Same, no bargaining shock</td>
<td>0.80</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.45</td>
<td>0.79</td>
<td>0.83</td>
<td>0.79</td>
</tr>
<tr>
<td>No minimum wage ζ₀ = 0, τₙ = τₖ = 0.3</td>
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<td>0.78</td>
<td>0.51</td>
<td>0.79</td>
<td>0.74</td>
<td>0.79</td>
</tr>
<tr>
<td>Same, no bargaining shock</td>
<td>0.80</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.47</td>
<td>0.79</td>
<td>0.83</td>
<td>0.79</td>
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Note: Quarterly data, HP-filtered with smoothing parameter λ = 1,600. We average the monthly model-generated data first within quarters before HP-filtering.
Table G.9: Welfare effects of increased or reduced political distribution risk

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<tr>
<th>Specification</th>
<th>std(Y)</th>
<th>Std(C)</th>
<th>Std(cs)</th>
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<th>Welfare change</th>
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<tr>
<td>Baseline</td>
<td>1.91</td>
<td>0.62</td>
<td>0.17</td>
<td>1.89</td>
<td>0</td>
</tr>
<tr>
<td>No bargaining shock</td>
<td>1.35</td>
<td>0.57</td>
<td>0.01</td>
<td>0.24</td>
<td>+2.40%</td>
</tr>
<tr>
<td>No minimum wage recipients: Hosios fails</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No minimum wage $\zeta_0 = 0$</td>
<td>1.96</td>
<td>0.62</td>
<td>0.11</td>
<td>1.82</td>
<td>0</td>
</tr>
<tr>
<td>Same, no bargaining shock</td>
<td>1.37</td>
<td>0.57</td>
<td>0.01</td>
<td>0.23</td>
<td>+2.29%</td>
</tr>
<tr>
<td>No minimum wage recipients, equal taxes: Hosios condition holds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No minimum wage $\zeta_0 = 0, \tau_n = \tau_k = 0.3$</td>
<td>1.96</td>
<td>0.62</td>
<td>0.11</td>
<td>1.82</td>
<td></td>
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<tr>
<td>Same, no bargaining shock</td>
<td>1.37</td>
<td>0.56</td>
<td>0.01</td>
<td>0.23</td>
<td>+2.28%</td>
</tr>
</tbody>
</table>
G.16 Sensitivity analysis

Here we provide a sensitivity analysis of the quantitative properties of the model. In each version of the model, we recalibrate the model to hit our target moments. Figure G.33 provides an overview. It decomposes the standard deviation of GDP and the gross capital share into the contributions from productivity and bargaining power shocks. The contribution to GDP of bargaining power shocks varies from 5% to 73%, while the contribution to the gross capital share varies from 5% to 37%, compared with our baseline result of 28% and 19%, respectively. Most model variants roughly match the overall volatility of output but underestimate the volatility of the gross capital share. Below, we document these results in detail.

(a) Decomposing the model-implied GDP volatility

(b) Decomposing the model-implied gross capital share volatility

Figure G.33: Decomposing GDP and capital share volatility across model variations and calibrations.
G.16.1 Matching the correlation of bargaining power and unemployment

To introduce partly endogenous fluctuations in bargaining power, we let bargaining power depend directly on the unemployment rate. While reduced-form, this dependence captures unmodeled factors that vary with the cycle and affect wages, but not labor productivity. To discipline this channel, we calibrate the loading on the unemployment rate, $\omega$, so that regressions in the model match the regression coefficient of 0.08 of the HP-filtered bargaining power with the HP-filtered unemployment rate $(1 - n_{t-1})$ in the partial filter. Formally:

$$\ln \frac{\phi_t}{1 - \phi_t} = (1 - \rho_\phi) \left( \ln \frac{\bar{\phi}}{1 - \phi} - \omega_\phi (n_{t-1} - \bar{n}) \right) + \rho_\phi \ln \frac{\phi_{t-1}}{1 - \phi_{t-1}} + \omega_\phi \epsilon_{\phi,t},$$

The recalibrated model now matches 40% of the observed variation in the gross capital share, about double the value implied by the baseline calibration. Most of the variation in the observed bargaining power is due to the exogenous political distribution shocks. Intuitively, political distribution causes much larger unemployment fluctuations than TFP shocks, so that the main driver of the bargaining power process remains the bargaining shock, despite the systematic component. The systematic component does, however, diminish the effect of bargaining power shocks on GDP slightly, from 28% to 20%. See Figure G.10, rows labeled “with endogenous bargaining power”.

G.16.2 Endogenizing hours worked below the minimum wage and unemployment benefits

To further explore how bargaining shocks affect the labor market and aggregate fluctuations, we augment our model with additional institutional features of the U.S. labor market directly. We do so by endogenizing the share of hours worked below the minimum wage and the generosity of unemployment benefits.

Both of these variables are countercyclical. In the data, regressing the share of hours worked below the minimum wage on the unemployment rate yields a regression coefficient of 0.5. When the unemployment rate goes up 1 pp., the share of hours worked at or below the minimum wage increases by 0.5 pp. Moreover, the replacement rate increases. When the unemployment rate is 1 pp. higher, the replacement rate increases, in reduced-form regressions, by 0.1 pp. To capture these movements in our model, we let the fraction of workers employed at the minimum wage, $\zeta_0$, and the unemployment benefits, $\omega$, linearly depend on the unemployment rate. We calibrate the loading on the unemployment rate to match these coefficients in regressions run in the model, while still matching the countercyclicality of the wage rate and other targeted moments.

Endogenizing hours worked below the minimum wage and unemployment benefits raises the effects of bargaining power on output, but diminishes its effect on the capital share. The fraction of explained output volatility rises 30% to 40%, while the share of the capital share volatility drops from 20% of the observed variation to 7%. See Figure G.10, rows labeled “with varying $\zeta_0, \omega$”.
Table G.10: Business cycle statistics: 1947Q1–2015Q2

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Y [%]</th>
<th>std(I)</th>
<th>std(Y)</th>
<th>std(ncs)</th>
<th>std(cs)</th>
<th>std(w)</th>
<th>std(u)</th>
<th>std(TFP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>1.99</td>
<td>3.28</td>
<td>0.58</td>
<td>1.07</td>
<td>0.86</td>
<td>0.95</td>
<td>0.83</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Models

| S&M model: baseline | 1.91  | 3.28  | 0.62  | 0.33    | 0.17   | 1.45  | 1.89  | 1.21    |
| S&M model: no bargaining shock | 1.35  | 3.33  | 0.57  | 0.18    | 0.01   | 1.13  | 0.24  | 1.20    |
| S&M model w/ end. barg. power | 1.71  | 3.28  | 0.60  | 0.48    | 0.35   | 1.59  | 1.64  | 1.21    |
| S&M model w/ end. barg. power w/o shock | 1.32  | 3.10  | 0.61  | 0.20    | 0.03   | 1.14  | 0.18  | 1.21    |
| S&M model w/ varying \( \zeta_0, \omega \) | 2.06  | 3.28  | 0.69  | 0.29    | 0.06   | 1.06  | 1.67  | 1.21    |
| S&M model w/ varying \( \zeta_0, \omega \) w/o shock | 1.22  | 3.71  | 0.50  | 0.16    | 0.01   | 0.96  | 0.24  | 1.07    |

Cyclicality

<table>
<thead>
<tr>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>ncs</th>
<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>1.00</td>
<td>0.91</td>
<td>0.84</td>
<td>0.57</td>
<td>0.36</td>
<td>0.19</td>
<td>-0.76</td>
</tr>
</tbody>
</table>

Models

| S&M model: baseline | 1.00 | 0.96 | 0.97 | 0.80 | 0.13 | 0.19 | -0.67 | 0.67 |
| S&M model: no bargaining shock | 1.00 | 0.99 | 0.99 | 0.98 | 0.78 | 1.00 | -0.95 | 1.00 |
| S&M model w/ end. barg. power | 1.00 | 0.98 | 0.99 | 0.71 | 0.36 | 0.19 | -0.66 | 0.71 |
| S&M model w/ end. barg. power w/o shock | 1.00 | 0.99 | 0.99 | 0.97 | 0.85 | 1.00 | -0.89 | 1.00 |
| S&M model w/ varying \( \zeta_0, \omega \) | 1.00 | 0.90 | 0.93 | 0.94 | 0.19 | 0.19 | -0.69 | 0.90 |
| S&M model w/ varying \( \zeta_0, \omega \) w/o shock | 1.00 | 0.99 | 0.99 | 0.97 | 0.80 | 0.99 | -0.94 | 0.99 |

Persistence

<table>
<thead>
<tr>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>ncs</th>
<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.87</td>
<td>0.82</td>
<td>0.78</td>
<td>0.76</td>
<td>0.74</td>
<td>0.67</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Models

| S&M model: baseline | 0.81 | 0.77 | 0.83 | 0.80 | 0.71 | 0.80 | 0.70 | 0.79 |
| S&M model: no bargaining shock | 0.81 | 0.81 | 0.80 | 0.80 | 0.48 | 0.79 | 0.83 | 0.79 |
| S&M model w/ end. barg. power | 0.77 | 0.76 | 0.78 | 0.76 | 0.72 | 0.78 | 0.77 | 0.79 |
| S&M model w/ varying \( \zeta_0, \omega \) | 0.84 | 0.75 | 0.84 | 0.86 | 0.64 | 0.79 | 0.81 | 0.81 |
| S&M model w/ varying \( \zeta_0, \omega \) w/o shock | 0.81 | 0.81 | 0.81 | 0.81 | 0.49 | 0.79 | 0.83 | 0.80 |

Note: Quarterly data, HP-filtered with smoothing parameter \( \lambda = 1,600 \). We average the monthly model-generated data first within quarters before HP-filtering. “S&M model w/ end. barg. power” matches the regression coefficient of HP-filtered \( \phi \) on HP-filtered \( u \) implied by the partial filter (which uses the baseline calibration) by making the bargaining power depend on the unemployment rate. “S&M model w/ varying \( \zeta_0, \omega \)” makes the share of minimum wage workers \( \zeta_0 \) vary over time to replicate a regression coefficient of the share of hours worked below the minimum wage on the unemployment rate and another regression coefficient of the replacement rate on the unemployment rate.
G.16.3 The role of persistence

If distribution shocks are short-lived, agents face little incentive to adjust their investment decisions to the realizations of the shock. Thus, when we calibrate shocks to be less persistent, there is a larger price effect (wages and the capital share become more volatile), but a smaller quantity effect. Recall that we consider two alternative calibrations. In one, the distribution shock is a business cycle shock with a half-life of 3.5 years. In the other, it has a half-life of 20 years. The lower half-life is just below the posterior 5th percentile, while the higher half-life is below the 95th percentile implied by our filtering of U.S. data. When we re-calibrate the model, the differences with our baseline calibration are minor. With all persistences, bargaining shocks account for around 30% of the variation in GDP. The business cycle shock explains 26% of the gross capital share, compared with 19% in the baseline and 17% in the long-run calibration. See Table G.11 for details.

First, we consider different values of the persistence of the bargaining power shock in addition to the baseline value of $\rho_\phi = 0.98^{1/3}$. For the low persistence, we choose $\rho_\phi = 0.95^{1/3}$. For the high persistence, we choose $\rho_\phi = 0.9914^{1/3}$. For each value, we re-calibrate the model. In short, the output effects of bargaining power shocks are roughly invariant to the persistence. In contrast, with shorter-lived shocks, the bargaining power shock explains more variation in the capital share. This is unsurprising, given that, as argued in the main text, steady-state changes in bargaining power have virtually no effects on capital shares.
Table G.11: Business cycle statistics with different persistence for the bargaining power shock and re-calibrated persistence and investment adjustment cost: 1947Q1–2015Q2.

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<tr>
<th>Volatility</th>
<th>Y</th>
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<th>std(C)</th>
<th>std(ncs)</th>
<th>std(cs)</th>
<th>std(w)</th>
<th>std(u)</th>
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<td>[%]</td>
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<tr>
<td>U.S. data</td>
<td>1.99</td>
<td>3.28</td>
<td>0.58</td>
<td>1.07</td>
<td>0.86</td>
<td>0.95</td>
<td>0.83</td>
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<td></td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.95$</td>
<td>2.01</td>
<td>3.28</td>
<td>0.60</td>
<td>0.37</td>
<td>0.23</td>
<td>1.46</td>
<td>1.98</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.95$: no barg. shock</td>
<td>1.32</td>
<td>3.17</td>
<td>0.60</td>
<td>0.18</td>
<td>0.01</td>
<td>1.13</td>
<td>0.20</td>
<td>1.18</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$ (baseline)</td>
<td>1.91</td>
<td>3.28</td>
<td>0.62</td>
<td>0.33</td>
<td>0.17</td>
<td>1.45</td>
<td>1.89</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$: no barg. shock</td>
<td>1.35</td>
<td>3.33</td>
<td>0.57</td>
<td>0.18</td>
<td>0.01</td>
<td>1.13</td>
<td>0.24</td>
<td>1.20</td>
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<td>S&amp;M, $\rho_3^3 = 0.9914$</td>
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<td>0.16</td>
<td>1.44</td>
<td>1.88</td>
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<td>0.18</td>
<td>0.01</td>
<td>1.12</td>
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<th>cs</th>
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<td>0.84</td>
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<td>0.69</td>
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<td>0.98</td>
<td>0.76</td>
<td>1.00</td>
<td>-0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$ (baseline)</td>
<td>1.00</td>
<td>0.96</td>
<td>0.97</td>
<td>0.80</td>
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<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$: no barg. shock</td>
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<td>0.99</td>
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<td>0.98</td>
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<th>ncs</th>
<th>cs</th>
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<th>u</th>
<th>TFP</th>
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<tbody>
<tr>
<td></td>
<td>[]%</td>
<td>[pp.]</td>
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<tr>
<td>S&amp;M, $\rho_3^3 = 0.95$</td>
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<td>0.79</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.95$: no barg. shock</td>
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<td>0.80</td>
<td>0.80</td>
<td>0.79</td>
<td>0.45</td>
<td>0.79</td>
<td>0.81</td>
<td>0.79</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$ (baseline)</td>
<td>0.81</td>
<td>0.77</td>
<td>0.83</td>
<td>0.80</td>
<td>0.71</td>
<td>0.80</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.98$: no barg. shock</td>
<td>0.81</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.48</td>
<td>0.79</td>
<td>0.83</td>
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<tr>
<td>S&amp;M, $\rho_3^3 = 0.9914$</td>
<td>0.80</td>
<td>0.74</td>
<td>0.83</td>
<td>0.71</td>
<td>0.42</td>
<td>0.79</td>
<td>0.57</td>
<td>0.79</td>
</tr>
<tr>
<td>S&amp;M, $\rho_3^3 = 0.9914$: no barg. shock</td>
<td>0.81</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
<td>0.50</td>
<td>0.79</td>
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</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.
Figure G.34: GIRFs of output, capital share, real wage, and firm value with the model calibrated to different levels of persistence.
G.16.4 The role of the elasticity of substitution

We vary the elasticity of substitution between capital and labor to $\varepsilon = 1 \pm 0.25$. This range includes the estimates in both Oberfield and Raval (2014) and Karabarbounis and Neiman (2014) that we described in Section 4.

When we depart from $\varepsilon = 1$ (see Table G.12 for the complete results), the RBC model produces fluctuations in the capital share, but it cannot match the low but positive cyclicality of the gross capital share. With a low elasticity $\varepsilon = 0.75$, the RBC model generates a correlation of the gross capital share with output of 0.98 vs. 0.36 in the data. For a high elasticity $\varepsilon = 1.25$, the same correlation is −0.98. Similarly, the search and matching model with “no bargaining shocks” can produce sizable fluctuations in the gross capital share, but it fails to account for the cyclicality of capital shares, either getting the sign wrong ($\varepsilon = 1.25$) or considerably overstating it ($\varepsilon = 0.75$).

In comparison, our baseline model with $\varepsilon = 1.25$ matches well the cyclicality of the net capital share (0.65 vs. 0.57 in the data), but it misses the cyclicality of the gross share (−0.45 vs. 0.36). With $\varepsilon = 0.75$, the model can neither match the cyclicality of the gross capital share (0.73 vs. 0.36 in the data) nor the cyclicality of the net share (0.92 vs. 0.57 in the data). In both cases the bargaining shock accounts for between 30% and 42% of output fluctuations.

Table G.12 documents some properties of the model as we change the elasticity of substitution.

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<tr>
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<td>std(C)</td>
<td>std(ncs)</td>
<td>std(cs)</td>
<td>std(w)</td>
<td>std(u)</td>
<td>std(TFP)</td>
<td></td>
</tr>
<tr>
<td>U.S. data</td>
<td>1.99</td>
<td>3.28</td>
<td>0.58</td>
<td>1.07</td>
<td>0.86</td>
<td>0.95</td>
<td>0.83</td>
<td>1.21</td>
</tr>
<tr>
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<td>0.21</td>
<td>1.28</td>
<td>1.72</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M: no bargaining shock</td>
<td>1.29</td>
<td>3.34</td>
<td>0.56</td>
<td>0.21</td>
<td>0.11</td>
<td>1.00</td>
<td>0.17</td>
<td>1.18</td>
</tr>
<tr>
<td>RBC</td>
<td>1.66</td>
<td>3.28</td>
<td>0.59</td>
<td>0.25</td>
<td>0.13</td>
<td>0.84</td>
<td>0.65</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M: with bargaining shock</td>
<td>1.29</td>
<td>3.30</td>
<td>0.57</td>
<td>0.11</td>
<td>0.06</td>
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</tr>
<tr>
<td>RBC</td>
<td>1.66</td>
<td>3.28</td>
<td>0.59</td>
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<td>0.13</td>
<td>0.84</td>
<td>0.65</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Note: Quarterly data, HP-filtered with smoothing parameter \( \lambda = 1,600 \). We average the monthly model-generated data first within quarters before HP-filtering.
G.16.5 The role of market power

In our baseline model, the profits largely go to physical capital. Market power introduces another source of profits: markups. Therefore, we augment our model to encompass market power; see Appendix G.9 for details. If these markups represent pure profits due to inelastic demand in a model of monopolistic competition, our analysis is virtually unchanged. If these markups compensate for the fixed cost of operating, we find a more volatile capital share. Re-calibrating the extended model, we set the elasticity of substitution between varieties to 10, corresponding to an 11% markup. We find (see Table G.13) that with market power these contributions of bargaining shocks to the capital share variation are slightly lower than in our baseline, while the contributions to the output variation are higher. Given that markups are constant, the RBC model cannot generate fluctuations in the capital share, unless there are fixed costs. With fixed costs, the RBC model can only account for 15% of the variation in the capital share. Table G.13 summarizes our findings on the role of market power.

<table>
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<tr>
<th></th>
<th>Y [%]</th>
<th>std(I) [std(Y)]</th>
<th>std(C) [std(Y)]</th>
<th>std(ncs) [pp.]</th>
<th>std(cs) [pp.]</th>
<th>std(w) [%]</th>
<th>std(u) [%]</th>
<th>std(TFP) [%]</th>
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<tbody>
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<td>U.S. data</td>
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<tr>
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<td></td>
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<td></td>
</tr>
<tr>
<td>S&amp;M: with bargaining shock</td>
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<td>0.09</td>
<td>0.98</td>
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<tbody>
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<td></td>
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</tr>
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</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.
G.16.6  The role of exogenous shocks

Table G.14 reports business cycle statistics with endogenous policy changes.


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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
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<td>S&amp;M model: policy rule</td>
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<td>S&amp;M model: no bargaining shock</td>
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<td>0.99</td>
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<td>0.79</td>
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<td>-0.96</td>
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<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
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<tbody>
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<td>0.62</td>
<td>0.79</td>
<td>0.71</td>
<td>0.79</td>
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<td>0.79</td>
<td>0.83</td>
<td>0.79</td>
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</tbody>
</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.
G.16.7 Alternative calibrations

We report two alternative calibrations: matching the industry- and occupation-adjusted wage rate (Table G.15) and matching the unemployment rate volatility and business cycle statistics (Table G.16).

Specifically, we calibrate our baseline model to the relative cyclicality of real compensation per hour in the non-farm business sector for the post-war sample. Alternatively, we can calibrate the model to the industry- and occupation-adjusted wage rate from the employment cost index, deflated by the PCE deflator. To form the longest possible sample, we splice together the SIC-based measure for wages and salaries and the NAICS-based measure for total compensation. The resultant sample covers 1980 to 2014, and we recompute the target moments: First, wages are now moderately countercyclical, with a correlation of -0.25 instead of +0.19. (For the baseline measure, the cyclicality is -0.02 for the subsample.) Second, investment is smoother, with a relative volatility of 3.06 instead of 3.28. Third, the average tax rate is lower (20% rather than 30%). Fourth, the share of depreciation is slightly higher (13.6% rather than 12.7%). Last, the volatility of TFP is 20% lower over the subsample. Hence, we recalibrate the model using the same strategy as before. Intuitively, we find that bargaining shocks are more important because wages are now more countercyclical. Bargaining shocks account for 73% of the volatility of GDP and 36% of the volatility of the gross capital share. See Table G.15 for more information.

Interestingly, our baseline model yields excess volatility of unemployment. The HP-filtered average quarterly unemployment rate has a standard deviation of 0.83% in the data, but of 1.89% in our calibration. This excess volatility is in stark contrast to the basic search model with only productivity shocks, which is well-known for failing to replicate the volatility of unemployment (Shimer, 2005). Cutting the volatility of the bargaining power shock by about 40%, however, matches the volatility of the unemployment rate. Therefore, bargaining power shocks can be a simple and empirically relevant way to reconcile search and matching dynamic macro models with the data. But as Table G.16 shows, in this case, our model explains only 6% of the volatility of the gross capital share, almost entirely because of bargaining power shocks. Only 5% of output volatility is due to bargaining shocks, but more than 70% of employment fluctuations are due to bargaining power shocks. Moreover, this calibration implies wages that are too pro-cyclical.

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<td>0.95</td>
<td>3.06</td>
</tr>
<tr>
<td></td>
<td>RBC model: baseline</td>
<td>1.51</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Note: Quarterly data, HP-filtered with smoothing parameter λ = 1,600. We average the monthly model-generated data first within quarters before HP-filtering.
Table G.16: Matching the unemployment rate volatility and business cycle statistics: 1947Q1-2015Q2.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Y</th>
<th>std(I)</th>
<th>std(C)</th>
<th>std(ncs)</th>
<th>std(cs)</th>
<th>std(w)</th>
<th>std(u)</th>
<th>std(TFP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[%]</td>
<td>[pp.</td>
<td>[pp.</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
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<tr>
<td>U.S. data</td>
<td>1.99</td>
<td>3.28</td>
<td>0.58</td>
<td>1.07</td>
<td>0.86</td>
<td>0.95</td>
<td>0.83</td>
<td>1.21</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>S&amp;M model: matching std(u)</td>
<td>1.47</td>
<td>3.28</td>
<td>0.60</td>
<td>0.21</td>
<td>0.05</td>
<td>1.20</td>
<td>0.83</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;M model: no bargaining shock</td>
<td>1.37</td>
<td>3.32</td>
<td>0.58</td>
<td>0.19</td>
<td>0.01</td>
<td>1.15</td>
<td>0.24</td>
<td>1.21</td>
</tr>
<tr>
<td>RBC model: baseline</td>
<td>1.90</td>
<td>3.28</td>
<td>0.60</td>
<td>0.24</td>
<td>0.00</td>
<td>0.91</td>
<td>1.03</td>
<td>1.21</td>
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<table>
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<th>Cyclical</th>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>ncs</th>
<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>1.00</td>
<td>0.91</td>
<td>0.84</td>
<td>0.57</td>
<td>0.36</td>
<td>0.19</td>
<td>-0.76</td>
<td>0.78</td>
</tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;M model: matching std(u)</td>
<td>1.00</td>
<td>0.98</td>
<td>0.99</td>
<td>0.95</td>
<td>0.27</td>
<td>0.77</td>
<td>-0.58</td>
<td>0.91</td>
</tr>
<tr>
<td>S&amp;M model: no bargaining shock</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
<td>0.78</td>
<td>1.00</td>
<td>-0.95</td>
<td>1.00</td>
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<tr>
<td>RBC model: baseline</td>
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<td>0.99</td>
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<td>NaN</td>
<td>0.97</td>
<td>-0.96</td>
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<table>
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<th>Persistence</th>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>ncs</th>
<th>cs</th>
<th>w</th>
<th>u</th>
<th>TFP</th>
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</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.87</td>
<td>0.82</td>
<td>0.78</td>
<td>0.76</td>
<td>0.74</td>
<td>0.67</td>
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<td></td>
</tr>
<tr>
<td>S&amp;M model: matching std(u)</td>
<td>0.81</td>
<td>0.80</td>
<td>0.81</td>
<td>0.81</td>
<td>0.65</td>
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<td>0.79</td>
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<tr>
<td>S&amp;M model: no bargaining shock</td>
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<td>0.81</td>
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<td>0.80</td>
<td>0.48</td>
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<td>0.79</td>
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<td>0.80</td>
<td>0.82</td>
<td>NaN</td>
<td>0.77</td>
<td>0.79</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.
References


