Central Bank Digital Currency: When Price and Bank Stability Collide

Linda Schilling,\textsuperscript{1} Jesús Fernández-Villaverde,\textsuperscript{2} and Harald Uhlig\textsuperscript{3}

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\textsuperscript{1}École Polytechnique

\textsuperscript{2}University of Pennsylvania

\textsuperscript{3}University of Chicago
A CBDC: Central banks as retail banks

Barrdear and Kumhof (2016, p.7):
By CBDC, we refer to a central bank granting universal, electronic, 24x7, national-currency-denominated and interest-bearing access to its balance sheet.

Bordo and Levin (2017):
“an account-based CBDC could be implemented via accounts held directly at the central bank” [p. 7] or “CBDC could be provided to the public via specially designated accounts at supervised commercial banks, which would hold the corresponding amount of funds in segregated reserve accounts at the central bank” [p. 8].
All of this has happened before...

- Historically, many central banks allowed deposits by and extended loans to firms and private citizens.


- The Bank War between Andrew Jackson and Nicholas Biddle was linked directly to the operations of the Second Bank of the United States with firms and merchants.

- Sometimes, the central banks were dominant players in the commercial banking sector.

- In 1900, the Bank of Spain (*Banco de España*), with 58 branches across the nation, held 68% of total financial assets and 75% of all checking accounts in the Spanish financial sector.

- Sharp distinction between a central bank operating only with primary depository institutions and commercial banks dealing with the public at large is mainly a post-WWII development.
A checkbook from the Banco de España, 1943
...and will it all happen again?

- Arrival of digital money has reopened the debate about the role of central banks.

- In particular, through the possibility of central bank digital currency (CBDC).

- Changes in technology may justify changes in the architecture of a financial system.

- Already relevant for policy: 2018 Swiss sovereign-money initiative (Vollgeld).

- Notice, however, that there is nothing inherently “digital” about this: for instance, postal banks.
Our goal

- In a previous paper *(Central Banking for All?, Review of Economic Dynamics)*, we study a model of CBDC in a real environment.

- We showed a (fragile) equivalence result: central banks can deliver the same level of maturity transformation than private banks.

- In this paper, we study the macroeconomic implications when CBDC accounts are akin to nominal deposit accounts.

- We build a nominal banking model for a central bank and its CBDC.

- Central bank faces competing objectives:
  1. Financial intermediation (optimal risk-sharing).
  2. Maturity transformation (proneness to runs).
• Central bank can always deliver on its nominal obligations.

• But: central bank runs can happen. Run on the price level (no rationing).

• The CBDC trilemma:

  1. The central bank can implement the social optimum and deter runs by threatening with inflation. In equilibrium: no runs $\Rightarrow$ no inflation.

  2. If the central bank is committed to keeping prices stable $\Rightarrow$ no runs take place, but allocation is inefficient.

  3. Allocation can be efficient if central bank is only partially committed to price stability, but then runs can happen.
The CBDC trilemma

- Run-proof contract
- Price stability
- Optimal risk-sharing
• Academic:
  • Allen and Gale (1998): system-wide run may alter price level.
  • Skeie (2008): nominal banking, keeping real resources fixed.
  • Allen, Carletti, and Gale (2014): nominal banking with central bank possibly adjusting price level and nom int rates in response to aggregate liquidity shocks.
  • Keister and Sanches (2019): Should central banks issue CBDC?

• Policy:
  • Barrdear and Kumhof (2016): The macroeconomics of CBDCs.
  • Adrian and Mancini-Griffoli (2019): The rise of digital money.
The model
The real Diamond-Dybvig block

- Time \( t = 0, 1, 2 \).

- Continuum \([0, 1]\) of agents:
  1. In \( t = 0 \): symmetric, endowed with one unit of a real good.
  2. In \( t = 1 \): types reveal. "Impatient" with probability \( \lambda \), "patient" with probability \( 1 - \lambda \).
    - Impatient agents only enjoy consumption \( u(x_1) \) in \( t = 1 \).
    - Patient agents only enjoy consumption \( u(x_2) \) in \( t = 2 \).
    - \( u(\cdot) \) strictly increasing, concave, and relative risk-aversion greater than one, \(-x \cdot u''(x)/u'(x) > 1\).

- Technologies, available to all:
  1. Long-term: yielding 1 in \( t = 1 \) or \( R > 1 \) in \( t = 2 \) for each unit invested in \( t = 0 \).
  2. Short-term: yielding 1 in \( t = 2 \) for each unit invested in \( t = 1 \) (storage).
Efficient allocation

• Optimal solution:

\[
\max \lambda u(x_1) + (1 - \lambda) u(x_2)
\]

such that

\[
\lambda x_1 + (1 - \lambda) \frac{x_2}{R} = 1
\]

• Unique solution, where:

\[
u'(x_1^*) = Ru'(x_2^*)
\]

• In this solution: \(x_1^* > 1\) and \(x_2^* < R\).

• Intuition.
Nominal banking via CBDC

**Definition**

A **central bank policy** is a triplet \((M, y(\cdot), i(\cdot))\), where \(y : [0, 1] \rightarrow [0, 1]\) is the central bank’s liquidation policy for every observed fraction \(n\) of early withdrawing agents and \(i : [0, 1] \rightarrow [-1, \infty)\) is the nominal interest rate policy.

**A CBDC contract \((M, i(\cdot))\)**

- \(t = 0\): Agent gets a CBDC account, promising \(M\) units of “cash” (i.e., CBDC balance) in \(t = 1\) or \(M(1 + i(n))\) in \(t = 2\) for each unit of good delivered now.

- \(t = 1\): Learns type. Withdraws \(M\) or not.

- \(t = 2\): If “not withdrawal” in \(t = 1\): Withdraws \(M(1 + i(n))\).
Implementation of the central bank policy

Given policy \((M, y(\cdot), i(\cdot))\), the central bank:

- \(t = 0\): Invests all received real goods in long-term technology.

- \(t = 1\):
  - Observes aggregate spending \(n\) by agents in \(t = 1\), who wish to withdraw \(M\).
  - Liquidates fraction \(y = y(n) \in [0, 1]\) of long-term investment.
  - Sells these goods to spending agents at market clearing price \(P_1\).

- \(t = 2\):
  - Return \(R(1 - y)\) on long-term investment.
  - Sells these goods to spending agents at market-clearing price \(P_2\).
Market clearing

• In $t = 1$ and $t = 2$ (i.e., a simple version of Fisher’s Equation):

$$nM = P_1y(n)$$

$$(1 - n)(1 + i(n))M = P_2R(1 - y(n))$$

• Thus, we get the price levels:

$$P_1(n) = \frac{n}{y(n)}M$$

$$P_2(n) = \frac{(1 - n)(1 + i(n))}{R(1 - y(n))}M$$

• For given spending behavior $n$, the central bank’s liquidation policy $y(n)$ and nominal interest rate $i(n)$ pin down $(P_1, P_2)$. 
Implied real contract

- For an agent, spending in $t = 1$ delivers:
  \[ x_1 = \frac{M}{P_1} \]
  Impatient agent consumes the good right away, while patient agent stores until $t = 2$.

- For an agent, spending in $t = 2$:
  \[ x_2 = \frac{(1 + i(n))M}{P_2} \]

- The fraction of early withdrawals $n$, the liquidation policy $y(n)$ fully determine the goods allocation:
  \[ x_1(n) = \frac{y(n)}{n} \]
  \[ x_2(n) = \frac{1 - y(n)}{1 - n} R \]
The document contains a diagram illustrating the flow of real CBDC balances and real investment over time. The text and annotations on the diagram are as follows:

- **Real supply (individual)**
- **Real supply (aggregate)**
- **Real investment (aggregate)**
- **Nominal CBDC balances**
- **Real CBDC value (individual)**

The diagram shows the following steps:

1. **Real CBDC value (individual)**: **M/P1** → **M** (real storage) → **M(1+i)** → **M(1+i)/P2**
2. **Nominal CBDC balances**
   - **M** (deposit in CB)
   - spend → **M(1+i)**
   - not spend → **M(1+i)**
3. **Real investment (aggregate)**
   - **1** → **(1-y)R/(1-n)**
4. **Real supply (aggregate)**
   - **y ∈ (0,1)**
   - measure 'n' agents spend CBDC early
5. **Real supply (individual)**
   - **(1-y)R**

The diagram also includes the notation for time:

- **Time t**
  - 0 → 1 → 2
Equilibrium
A commitment equilibrium is an initial money supply $M$, a liquidation policy $y : [0, 1] \rightarrow [0, 1]$, a nominal interest rate policy $i : [0, 1] \rightarrow [-1, \infty)$, aggregate spending behavior $n \in [0, 1]$, and price levels $(P_1, P_2)$ such that:

1. The individual consumer's spending decisions are optimal, given aggregate spending $n$, the central bank's policy $(M, y(\cdot), i(\cdot))$, and the price level sequence $(P_1, P_2)$.
2. Given the aggregate spending realization $n$, the central bank liquidates $y(n)$ and sets the nominal interest rate $i(n)$.
3. Given the realization $(n, y(n), i(n))$ and $M$, the price levels $(P_1, P_2)$ clear the goods market in each period.

**Remark:** The price levels flexibly adjusts to $(n, M, y, i)$. Nominal rigidities? How do we clear markets?
A run on the central bank occurs if $n > \lambda$ (i.e., some or all patient agents withdraw; $n < \lambda$ cannot occur).

Recall:

$$x_1(n) = \frac{y(n)}{n}$$
$$x_2(n) = \frac{1 - y(n)}{1 - n} R$$

Given the central bank policy $(M, y(\cdot), i(\cdot))$,

1. $n = \lambda$ is an equilibrium only if $x_1(\lambda) \leq x_2(\lambda)$.

2. A central bank run with $n = 1$ is an equilibrium if and only if $x_1(1) \geq x_2(1)$.

Why are central bank runs inefficient?
Implementing the social optimum

The central bank policy \((M, y(\cdot), i(\cdot))\) implements the social optimum \((x_1^*, x_2^*)\) in equilibrium if:

i) for any \(n = \lambda\): it sets \(y(\lambda) = y^* = \lambda x_1^* > \lambda\).

ii) for all \(n > \lambda\): it sets a liquidation policy that implies \(x_1(n) < x_2(n)\).

Definition

We call a liquidation policy \(y(\cdot)\) “run-deterring” if it satisfies

\[
y^d(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in (\lambda, 1]
\]

This liquidation policy implies that “roll over” is ex-post optimal, i.e., \(x_1(n) < x_2(n)\), even though patient agents are withdrawing \(n \in (\lambda, 1]\).

Remark: In equilibrium, agents anticipate the central bank’s policy. Thus, a run does not occur, and the threat \(y^d(n)\) is never executed.
Run-deterring liquidation policy
**The trilemma**

### Definition

Every policy choice \((M, y(\cdot), i(\cdot)), n \in [0, 1]\) with \(y(\lambda) = y^*\) and

\[
y^d(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in (\lambda, 1],
\]

(1)

deters central bank runs and implements the social optimum in dominant strategies.

Such a deterring policy choice requires the interim price level \(P_1(n)\) to exceed the withdrawal-dependent bound:

\[
P_1(n) > \frac{1 + n(R - 1)}{R} M, \quad \text{for all } n \in (\lambda, 1].
\]

(2)

**Remark:** This looks like a version of “suspension of convertibility”, but not quite. The bank does not stop customers from spending their nominal CBDC accounts. Rather, the supply of goods traded against these accounts is restricted.
So far:

- In our model, the central bank can always deliver on its nominal obligations.
- To deter runs, the central bank threatens with a high price level ("inflation") for $t = 1$, that would make running ex-post suboptimal.

Price target:

- Central banks usually wish to keep prices stable (for reasons outside this model)!
  ⇒ Time-inconsistency.
- What, if the central bank is constrained by price stability objectives?
- We add an **exogenous price stability objective**.
Price stability
Central bank constraint: full price stability

Definition

i) A central bank policy is \( P_1 \)-stable at level \( \bar{P} \), if it achieves \( P_1(n) \equiv \bar{P} \) for the price level target \( \bar{P} \), at all spending fractions \( n \in [\lambda, 1] \).

ii) A central bank policy is price-stable at level \( \bar{P} \), if it achieves \( P_1(n) = P_2(n) \equiv \bar{P} \) for the price level target \( \bar{P} \), for all spending fractions \( n \in [\lambda, 1] \).

- Recall that, by market clearing:

\[
P_1(n) = \frac{n}{y(n)} M
\]

\[
P_2(n) = \frac{(1 - n)(1 + i(n))}{R(1 - y(n))} M
\]

- Then price stability at level \( \bar{P} \) requires a central bank policy that satisfies:

\[
\bar{P} = \frac{n}{y(n)} M = \frac{(1 - n)(1 + i(n))}{R(1 - y(n))} M
\]
Price stability

Definition

Given a price goal $\bar{P}$, a **price-stable equilibrium** is a central bank policy $(M, y(\cdot), i(\cdot))$, aggregate spending $n \in [0, 1]$, and price levels $(P_1, P_2)$ such that:

1. The individual consumer’s spending decisions are optimal, given aggregate spending $n$, the central bank’s policy $(M, y(\cdot), i(\cdot))$, the price goal $\bar{P}$, and the price level sequence $(P_1, P_2)$.
2. Given the aggregate spending realization $n$, money supply $M$, and the price goal $\bar{P}$, the central bank liquidates assets $y$ such that $P_1 \equiv \bar{P}$.
3. Given the aggregate spending realization $n$, money supply $M$, the price goal $\bar{P}$, and asset liquidation $y$, the central bank sets the nominal interest rate $i$ such that $P_2 \equiv \bar{P}$.
Characterizing price-stable central bank policies

**Proposition**

A central bank policy is:

i) $P_1$-stable at level $\bar{P}$, if and only if its liquidation policy satisfies, for all $n \in [0, 1]$:

\[
y(n) = \frac{M}{\bar{P}} n
\]

\[
x_1(n) \equiv \bar{x}_1 = \frac{M}{\bar{P}} \leq 1 < x_1^*
\]

ii) A central bank policy is price-stable at level $\bar{P}$, if and only if its liquidation policy satisfies equation above and its interest policy satisfies:

\[
i(n) = \frac{\bar{P}}{M} - n R - 1 \quad \text{and} \quad \bar{P} \geq M
\]
Full price stability: liquidation policy

\[ y(n) \text{ for full price stability} \]
Full price stability: liquidation policy vs. run-deterring bound

Full price stability

\[ y(n) \]

\[ \text{eff.all.} \]
$P_1$-stable central bank policies are inefficient

**Corollary: The trilemma redux**

If the central bank commits to a $P_1$-stable policy, then:

i) The socially optimal allocation is not implemented.

ii) There is a unique equilibrium where only impatient agents spend, $n^* = \lambda$, i.e., there are no central bank run equilibria.

iii) If the central bank commits to a price-stable central bank policy, then the nominal interest rate is non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$. Furthermore, the interest rate $i(n)$ is increasing in $n$. 
A weaker target: partial price stability
Partial price stability

Definition

1. A central bank policy is partially $P_1$-stable at level $\bar{P}$, if either it achieves $P_1(n) = \bar{P}$ for some price level target $\bar{P}$, or the central bank fully liquidates real investment $y(n) = 1$.

2. A central bank policy is partially price-stable at level $\bar{P}$, if either it achieves $P_1(n) = P_2(n) = \bar{P}$ for some price level target $\bar{P}$, or the central bank fully liquidates real investment $y(n) = 1$.

• Resembles some central bank policies in the past.
Proposition

Suppose that $M > \bar{P} \geq \lambda M$. A central bank policy is partially $P_1$-stable at level $\bar{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \min \left\{ \frac{M}{\bar{P}} n, 1 \right\}$$
Full vs. partial price stability

- Full price stability
  - Liquidation policy

- Partial price stability
  - $y(n)$ for full price stability
  - $y(n)$ for partial price stability

Graph:
- $y(n)$ vs. $n$
  - $y(n)$ for partial price stability
  - $y(n)$ for full price stability
  - $\lambda x_1^*$
  - $\lambda x_1$
  - $\lambda$
  - $n_c$
  - $1$

Mathematical expressions:
- $y(n)$ for full price stability
- $y(n)$ for partial price stability
- $\lambda x_1$
- $\lambda x_1^*$
- $\lambda$
Comparing liquidation policies

eff.all.
Suppose that $\bar{P} \in [\lambda M, M]$ and consider a partially $P_1$-stable central bank policy at level $\bar{P}$. Define the critical aggregate spending level:

$$n_c \equiv \frac{\bar{P}}{M}$$

For all $n \leq n_c$, the price level is stable at $P_1(n) = \bar{P}$.

For all $n > n_c$ (full liquidation):

1. Price level **not stable**: $P_1(n)$ proportionally increasing with $n$: $P_1(n) = Mn$.
2. Real goods per agent: $x_1(n) = 1/n$, $x_2 = 0 \Rightarrow$ runs occur in equilibrium + negative real interest rate.
Corollary

Suppose that central bank policy is partially price-stable at $\bar{P} \in [\lambda M, M]$:

1. Then, runs on the central bank can occur (multiple equilibria) $n^* \in \{\lambda, 1\}$.

2. Given no run: then the social optimum and the price goal are attained.

3. Given a run: the social optimum and the price goal are not attained.
We have several extensions:

1. Voluntary participation in CBDC.
2. Competition by private banks.
3. Token-based CBDC.
4. Synthetic CBDC and retail banking.
5. Cash.
Conclusions

- We have shown the CBDC trilemma:

  1. Implementation of the social optimum $x_1^* > 1$ requires the threat of inflation to deter runs (price stability lost).

  2. Full price stability requires giving up the social optimum, $x_1 \leq 1$. But runs do not occur.

  3. Under partial price stability, runs can occur (multiple equilibria). But absent a run, the social optimum can be implemented.

- There is a deeper point at play. Efficiency is probably not achieved in most environments when price stability is a requirement for the central bank.

- In some sense, we already knew this (e.g., Friedman’s rule), but the presence of a CBDC makes the point more salient.