

# Formulating, Estimating, and Solving Dynamic Equilibrium Models: an Introduction

Jesús Fernández-Villaverde  
University of Pennsylvania

## Models

- Tradition in macroeconomics of using models:
  1. Policy Analysis.
  2. Forecasting.
  3. Counterfactuals.
- This course reviews the recent advances in macroeconometric model-building.

## Why Models?

- Organize our thinking.
- Reproducibility.
- Accumulation of capital within organizations.
- Ex-ante versus ex-post analysis.
- Counterfactuals.
- New circumstances.

## A Brief History of Macroeconometric Models: A Promising Beginning

- Old idea of economists: Quesnay, Walras.
- Pioneers: Tinbergen (1939, Noble Prize 1969), Haavelmo (1943, Noble Prize 1989).
- Klein-Goldberger model (1955).
- Big models of the 1960s: Brookings, MIT-FRB-Penn, Wharton.

## A Brief History of Macroeconometric Models: Disappointment

- 1970s and 1980s were the decades of disappointment
- Problems:
  1. Fit (Nelson, 1972).
  2. Nonstationarity (Granger, 1981).
  3. Economic Foundations (Lucas, 1972).

## A Brief History of Macroeconometric Models: Renewed Hopes

- 1980s: Real Business Cycle theory: advances and difficulties.
- Late 1990s, early 2000s: revival.
- Why?
  1. Advances in economic theory: second generation equilibrium models.
  2. Advances in econometrics: simulation methods.

## Why Dynamic Equilibrium Models?

- Explicit microeconomic foundation.
- Robust to Lucas critique.
- Welfare analysis.
- Design of optimal policy.

## A Brief History of Macroeconometric Models: References

- R.G. Bodking, L.R. Klein, and K. Marwah: *A History of Macroeconomic Model-Building*, Edward Elgar, 1991.
- T.J. Sargent: *Expectations and the Nonneutrality of Lucas*, Journal of Monetary Economics, 1996.



## Why Monetary Models?

- Is price stability a good idea? If so, at what price?
- How do we implement monetary policy?
- Does monetary policy have a role in stock market booms/busts?
- What do we do in the wake of a monetary crisis?
- Why did inflation take off during the 1970s?
- Was the Great Depression caused by bad monetary policy?

## Is Money Important?

- At a basic level yes.
- “Lenin is said to have declared that the best way to destroy the Capitalist System was to debauch its currency... Lenin was certainly right”.

J.M. Keynes (1919), *The Economic Consequences of the Peace*.

- Experiences of hyperinflation.

## Money and the Business Cycle

- At a more moderate level, the question is surprisingly difficult to answer.
- Two “observations”:
  1. Relation between Output and Money growth.
  2. Phillips Curve.
- Where do these observations come?

## “Conventional Wisdom”

- Old tradition in economics: Hume, *Of Money*, 1752.
- Friedman and Schwartz, *A Monetary History of the US*, 1963.
- Volcker’s recessions.
- Evidence from Structural Vector Autoregressions.

## Structural Vector Autoregression

- SVARs have become a common tool among economists.
- Introduced by Sims (1980)
- SVARs make explicit identifying assumptions to isolate estimates of policy and/or private agents behavior and its effects on the economy, while keeping the model free of the many additional restrictive assumptions needed to give every parameter a behavioral interpretation.

## Uses of SVARs

1. The effects of money on output (Sims and Zha, 2005).
2. The relative importance of supply and demand shocks on output (Blanchard and Quah, 1989),
3. The effects of fiscal policy (Rotemberg and Woodford, 1992).
4. The relation between technology shocks and hours (Galí, 1999).

## Economic Theory and the SVAR Representation

- Dynamic economic models can be viewed as restrictions on stochastic processes.
- An economic theory is a mapping between a vector of  $k$  economic shocks  $w_t$  and a vector of  $n$  observables  $y_t$  of the form

$$y_t = D(w^t)$$

- Interpretation.

## Mapping

- The mapping  $D(\cdot)$  is the product of the equilibrium behavior of the agents in the model, implied by their optimal decision rules and the consistency conditions like resource constraints and market clearing.
- The construction of the mapping  $D(\cdot)$  is the sense in which economic theory tightly relates shocks and observables.
- The mapping  $D(\cdot)$  can be interpreted as the impulse response of the model to an economic shock.



## A Linear Mapping

- We restrict our attention to linear mappings of the form:

$$y_t = D(L) w_t$$

where  $L$  is the lag operator.

- $w_t \sim N(0, \Sigma)$ .
- More involved structures?

## Invertibility

- If  $k = n$ , i.e., we have as many economic shocks as observables, and  $|D(L)|$  has all its roots outside the unit circle, we can invert the mapping  $D(L)$  (Fernández-Villaverde, Rubio-Ramírez, and Sargent, 2005).
- Meaning of invertibility.
- Example of non-invertibility:

$$y_t = w_t + 2w_{t-1}$$

## SVAR Representation

- We obtain:

$$A(L)y_t = w_t$$

where  $A(L) = A_0 - \sum_{k=1}^{\infty} A_k L^k$  is a one-sided matrix lag polynomial that embodies all the (usually non-linear) cross-equation restrictions derived by the equilibrium solution of the model.

- In general,  $A(L)$  is of infinite order.
- This representation is known as the SVAR representation.

## Reduced Form Representation

- Consider now the case where a researcher does not have access to the SVAR representation. Instead, she has access to the VAR representation of  $y_t$ :

$$y_t = B_1 y_{t-1} + B_2 y_{t-2} + \dots + a_t,$$

where  $E y_{t-j} a_t = 0$  for all  $j$  and  $E a_t a_t' = \Omega$ .

- This representation is known as the Reduced form representation.
- Can the researcher recover the SVAR representation using the Reduced form representation?

## Identification by Theory I

- Fernández-Villaverde, Rubio-Ramírez, and Sargent (2005) show that, given a strictly invertible economic model, there is one and only one identification scheme to recover the SVAR from the reduced form.
- In addition,  $a_t = A_0^{-1}w_t$ .
- Hence, if we knew  $A_0^{-1}$ , we could recover the SVAR representation from the reduced form representation noticing that  $A_j = A_0B_j$  for all  $j$  and  $w_t = A_0a_t$ .

## Identification by Theory II

- Fernández-Villaverde, Rubio-Ramírez, and Sargent's identification requires the specification of an explicit dynamic economic model.
- Can we avoid this step? Unfortunately, the answer is in general no, because knowledge of the reduced form matrices  $B_i$ 's and  $\Omega$  does not imply, by itself, knowledge of the  $A_i$ 's and  $\Sigma$ .
- Why: normalization and identification.

## Normalization

- Reversing the signs of two rows or columns of the  $A_i$ 's does not matter for the  $B_i$ 's.
- Thus, without the correct normalization restrictions, statistical inference about the  $A_i$ 's is essentially meaningless.
- Waggoner and Zha (2003) provide a general normalization rule that maintains coherent economic interpretations.

## Identification I

- If we knew  $A_0$ , each equation  $B_i = A_0^{-1}A_i$  would determine  $A_i$  given some  $B_i$ .
- But in  $\Omega = A_0\Sigma A_0'$  we have  $n(3n + 1)/2$  unknowns (the  $n^2$  distinct elements of  $A_0$  and the  $n(n + 1)/2$  distinct elements of  $\Sigma$ ) for  $n^2$  knowns (the  $n(n + 1)/2$  distinct elements of  $\Omega$ ).
- Thus, we require  $n^2$  identification restrictions.



## Identification II

- Since we can set the diagonal elements of  $A_0$  equal to 1 by scaling, we are left with the need of  $n(n - 1)$  additional identification restrictions.
- Alternatively, we could scale the shocks such that the diagonal of  $\Sigma$  is composed of ones and leave the diagonal of  $A_0$  unrestricted.
- These identification restrictions are dictated by the economic theory being studied.

## Identification III

- What about identification restrictions compatible with a large class of models?
- Robustness versus efficiency.
- Discussion of trade-off.

## Identification IV

- Most common identification restrictions:  $\Sigma$  is diagonal.
- Why?
- Since this assumption imposes  $n(n - 1) / 2$  restrictions, we still require  $n(n - 1) / 2$  additional restrictions.

## Short-Run Identification I

- Sims (1980) pioneered the first approach when he proposed to impose zeroes on  $A_0$ .
- Why? Natural timing in the effect of economic shocks.
- Example: we can use the intuition that monetary policy cannot respond contemporaneously to a shock in the price level because of informational delays to place a zero on  $A_0$ .
- Similarly, institutional constraints, like the timing of tax collections, can be exploited for identification (Blanchard and Perotti, 2002).

## Short-Run Identification II

- Sims (1980) ordered variables in such a way that  $A_0$  is lower triangular.
- Sims and Zha (2005) present a non-triangular, identification scheme on an eight-variable SVAR.
- We will come back to Sims and Zha (2005).

## Long-Run Identification I

- Blanchard and Quah (1989).
- Restriction imposed on  $A(1) = A_0 - \sum_{k=1}^{\infty} A_k$ .
- Since  $A^{-1}(1) = D(1)$ , long-run restrictions are restrictions on the long-run effects of economic shocks, usually on the first difference of an observable.

## Long-Run Identification II

- Blanchard and Quah (1989): demand shock has no effect long-run effect on unemployment or output while supply shock has no long-run on unemployment, but may have a long-run effect on output.
- Galí (1999): technology shocks has no effect long-run effect on hours and but it has on productivity.

## New Identification Schemes

- Uhlig (2005): sign restrictions. Identifies a contractionary monetary policy shock as one that does not lead to an increase in prices or in nonborrowed reserves and does not lead to a decrease in the federal funds rate.
- Faust, Swanson, and Wright (2004) identify the effects if a monetary shock in a SVAR using the prices of federal funds futures contracts.



## Estimable Equation

- Why is the previous discussion of the relation between the reduced and structural form of a VAR relevant?
- Because the reduced form can be easily estimated.
- An empirically implementable version of the reduced form representation truncates the number of lags at the  $p$ -th order:

$$y_t = B_1^* y_{t-1} + \dots + B_p^* y_{t-p} + a_t^*$$

where  $Ea_t^* a_t^{*'} = \Omega^*$ .

## Effects of Truncation

- Bad: Chari, Kehoe, McGrattan (2005).
- Not too bad: Christiano, Eichenbaum, and Vigfusson (2005).
- Problems with infinite dimensional estimation: Faust and Leeper, (1997).

## Estimation

- Classical: GMM, ML, OLS.
- Bayesian.
- Why Bayesian? Overparametrization and priors.
- We will talk about these issues later.

## Estimation

- Point estimates  $B_i^*$  for  $i = 1, \dots, p$  and  $\Omega^*$
- Then, we find estimates of  $A_i$  and  $\Sigma$  by solving  $B_i^* = A_i^{-1}A_i$  for  $i = 1, \dots, p$ , and  $\Omega^* = A_0\Sigma A_0'$ .
- Finally,  $w_t = A_0\mathbf{a}_t^*$ .

## An Example: Sims and Zha (2005)

- A simple, model-based SVAR.
- U.S. Quarterly Data 1964-1994.
- 8 variables.
- We will need at least  $(8 * 7) / 2 = 28$  identification restrictions.

## Variables

*R*: federal funds rate.

*M*: M2.

*P<sub>y</sub>*: GNP deflator.

*y*: real GNP.

*W*: average hourly earnings of non-agricultural workers.

*P<sub>im</sub>*: producers' price index.

*Tbk*: bankruptcy filings, personal and business.

*P<sub>cm</sub>*: producer' price index for intermediate goods.

## Identification Scheme on $A_0$

	$R$	$M$	$Py$	$y$	$W$	$Pim$	$Tbk$	$Pcm$
$R$	×	×	×	×	0	0	0	0
$M$	×	×	0	0	0	0	×	×
$Py$	0	0	×	×	×	0	×	×
$y$	0	0	0	×	0	0	×	×
$W$	0	0	0	×	×	0	×	×
$Pim$	0	0	×	×	×	×	×	×
$Tbk$	0	0	0	0	0	0	×	×
$Pcm$	×	×	×	×	×	×	×	×

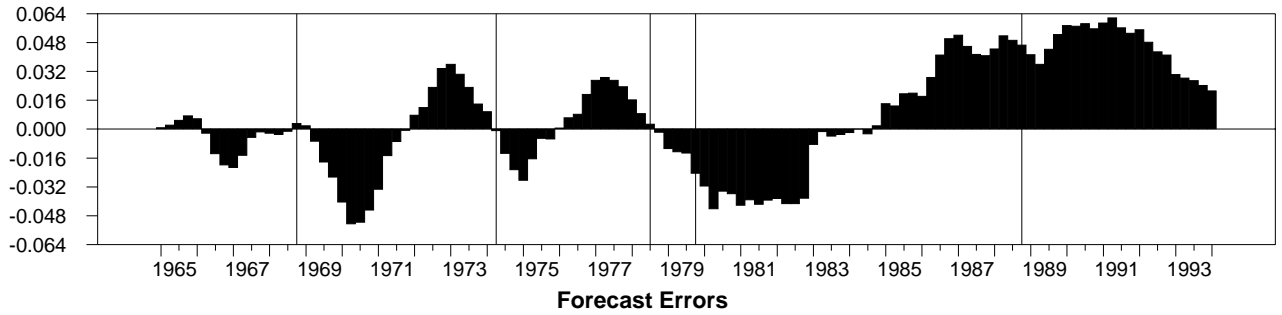
## Comments on Identification

- Sims and Zha impose exactly 28 identification restrictions.
- Motivated by an economic model.
- Intuition guides the choice of model.

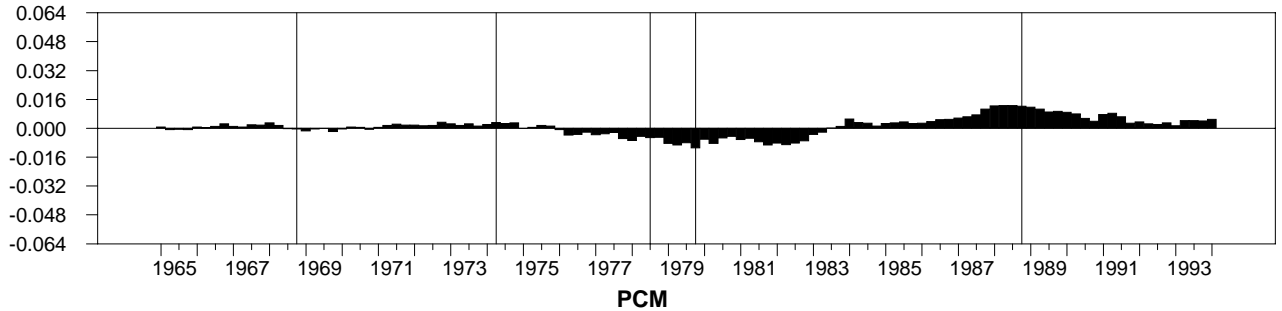




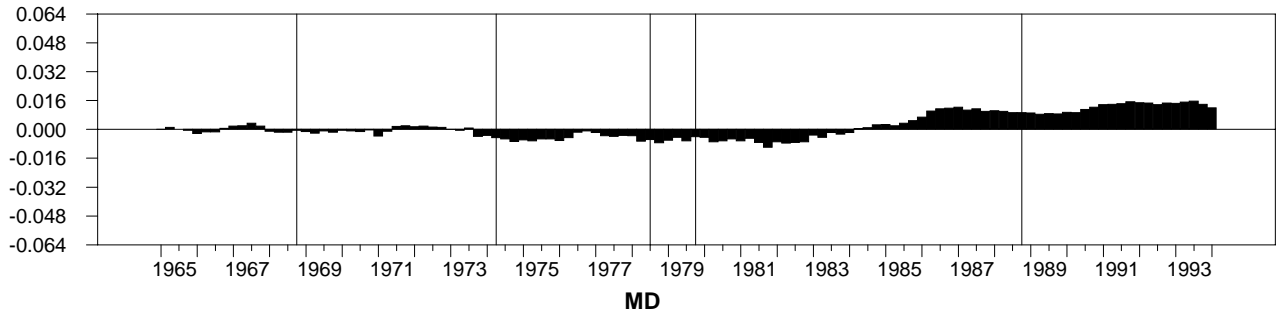
# M Decomposition, M2 Model



**Forecast Errors**

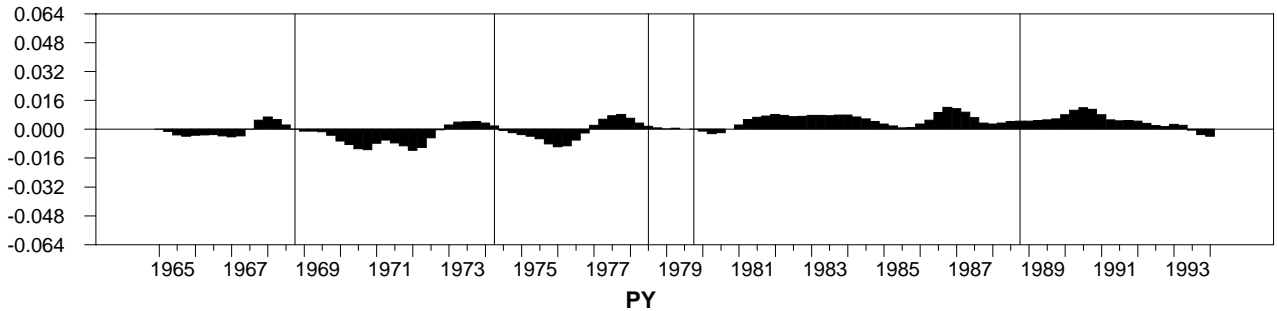
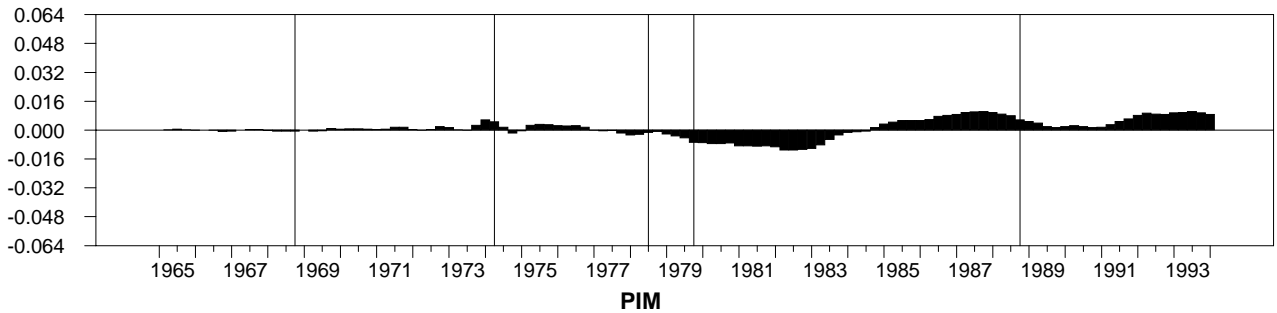
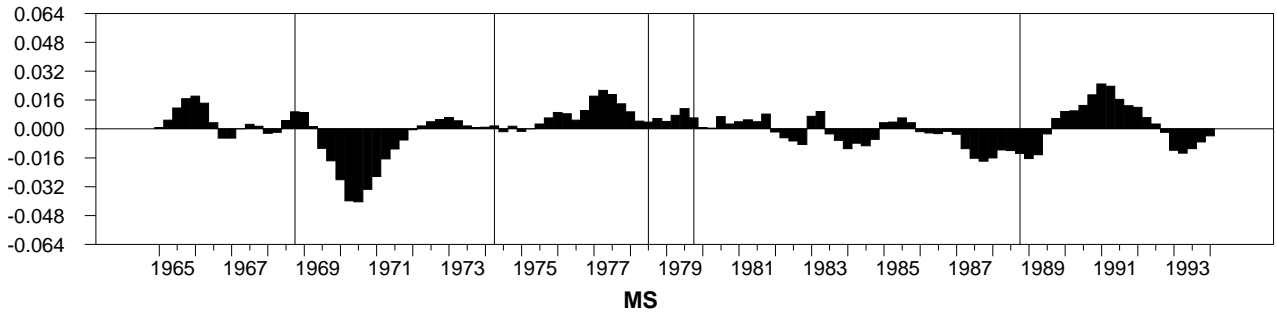


**PCM**

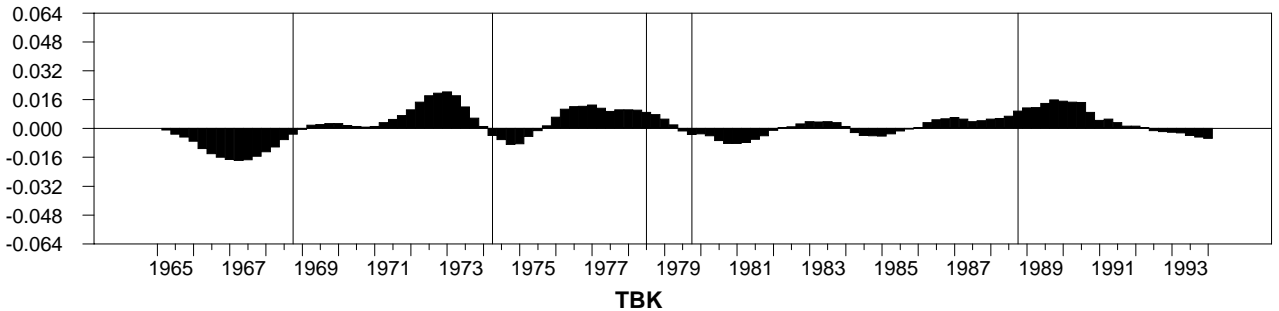
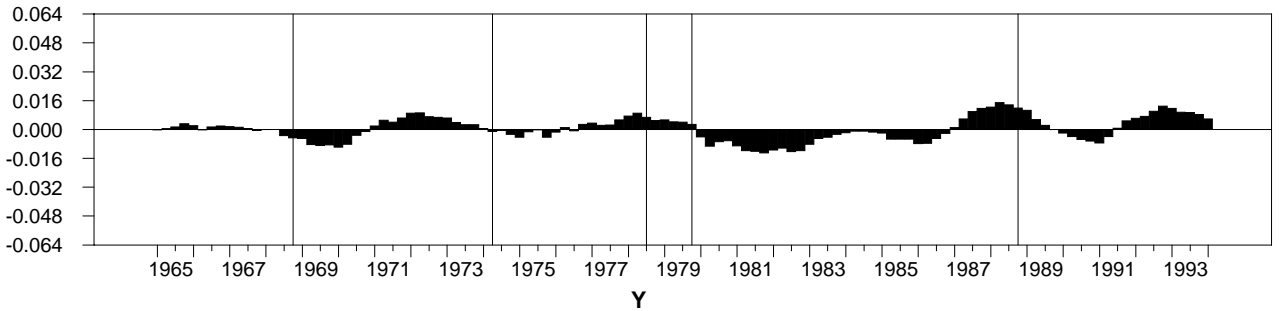
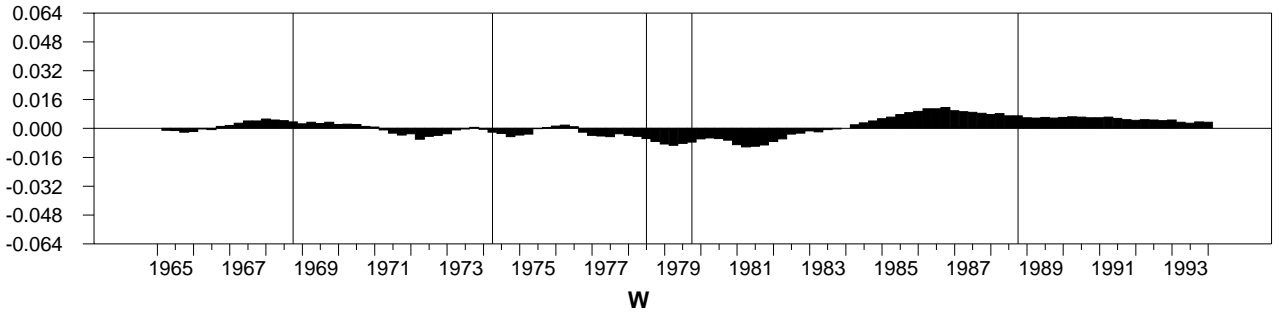


**MD**

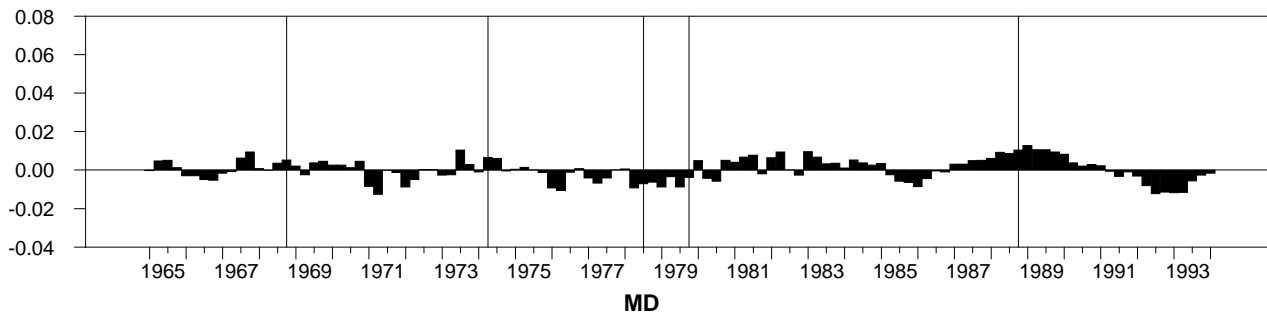
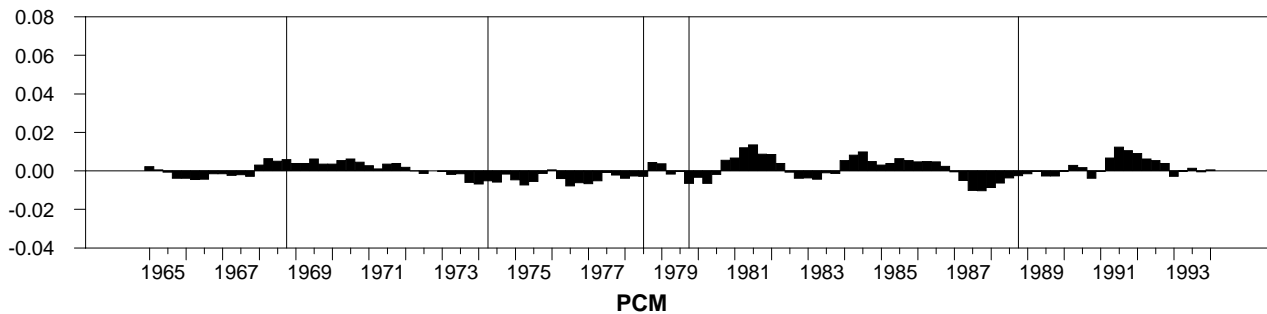
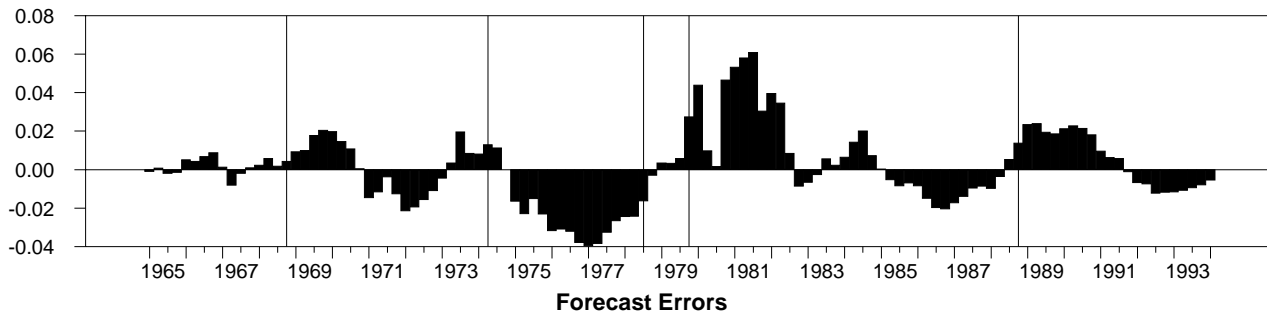
# M Decomposition, M2 Model



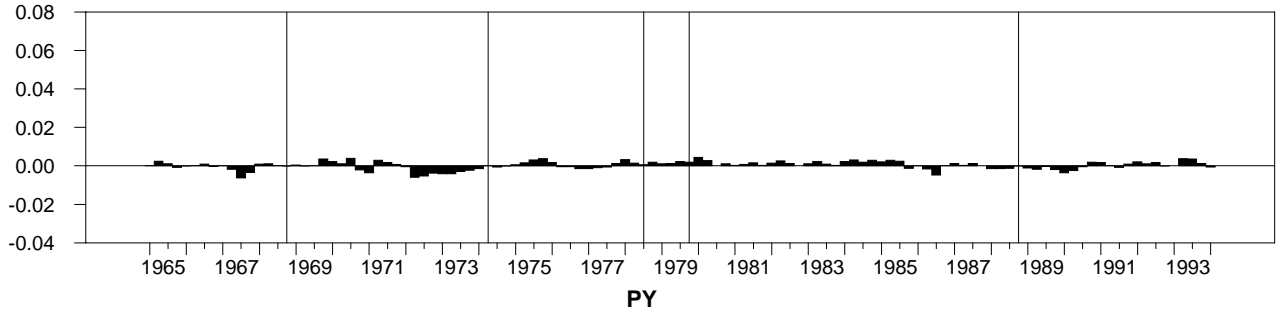
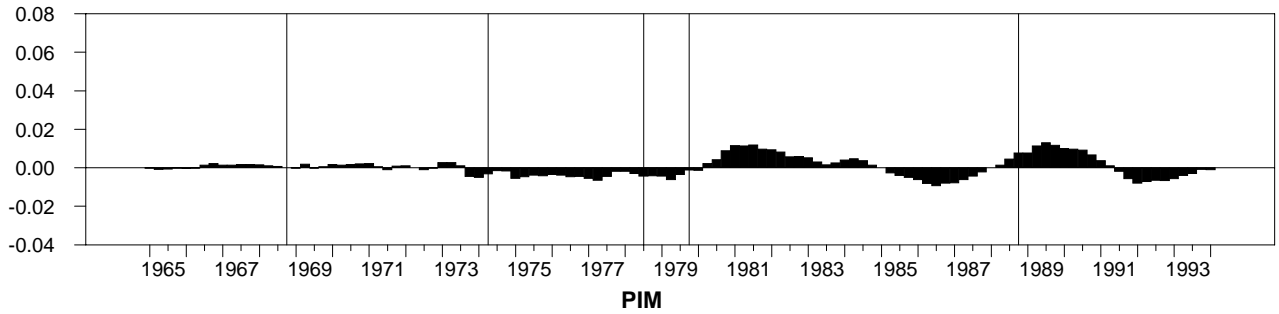
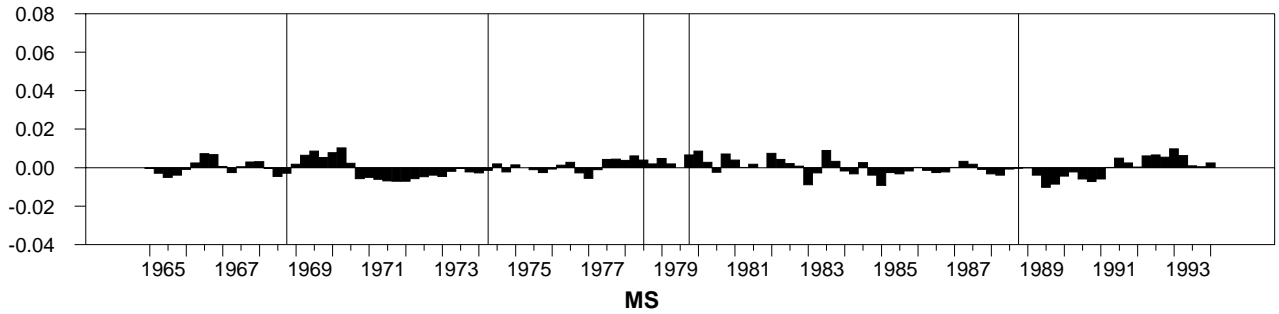
# M Decomposition, M2 Model



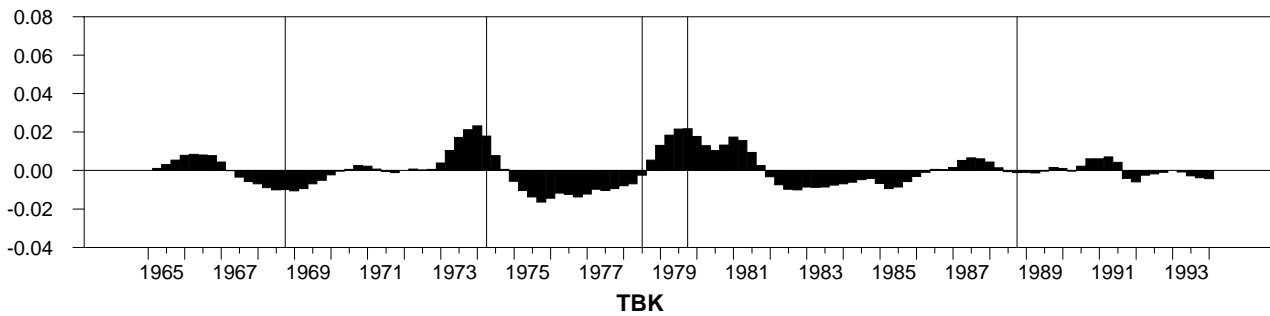
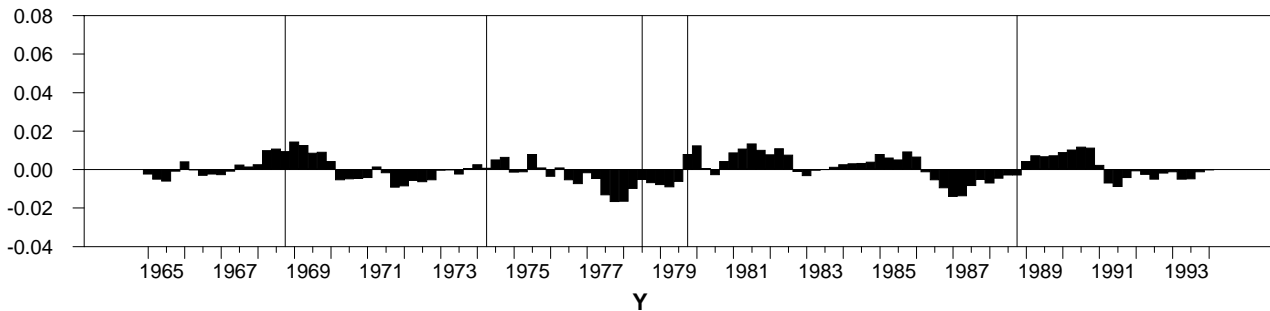
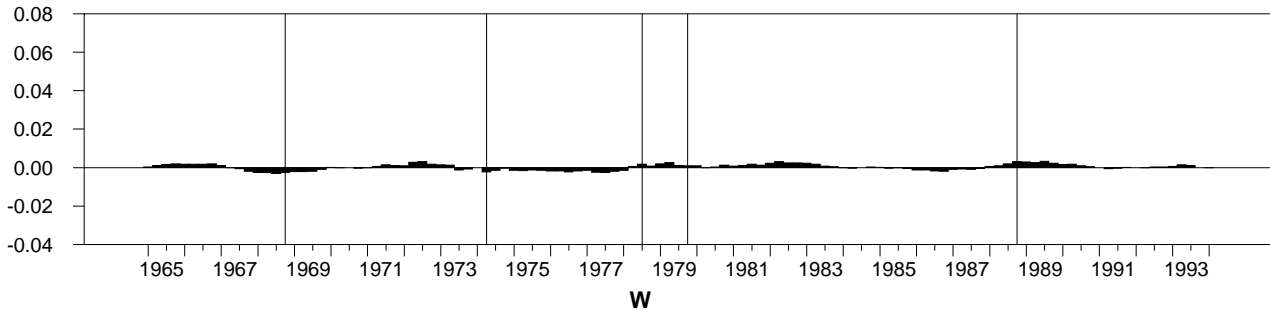
# R Decomposition, M2 Model



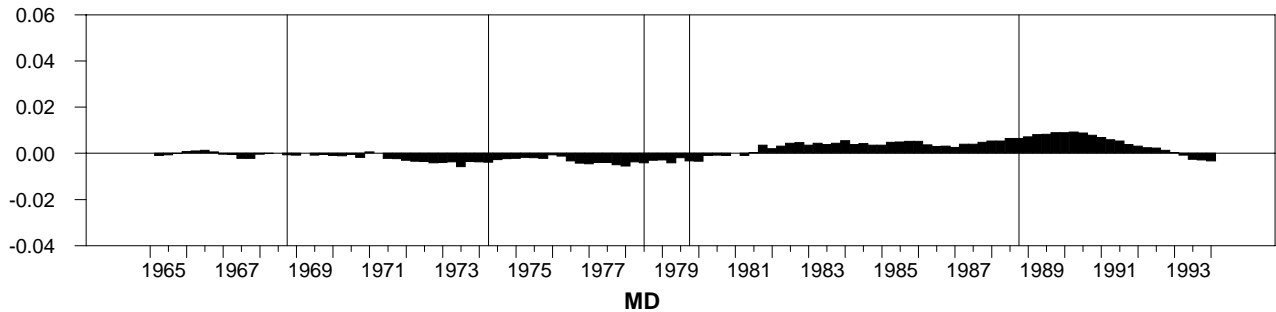
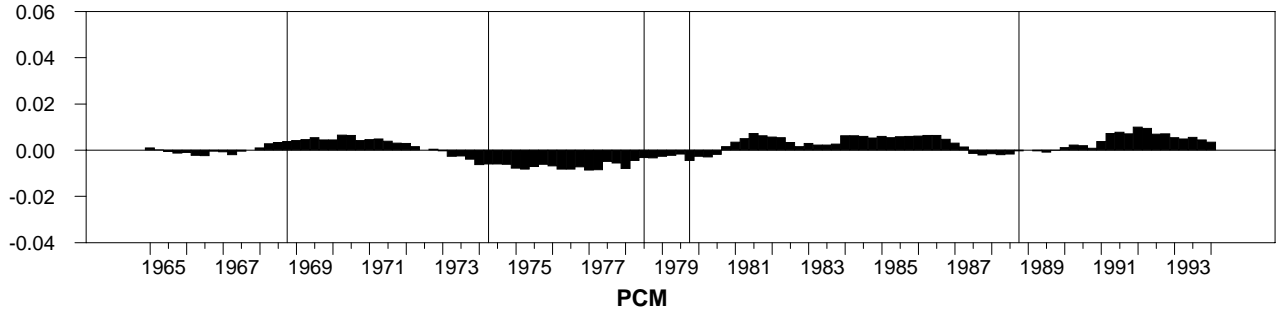
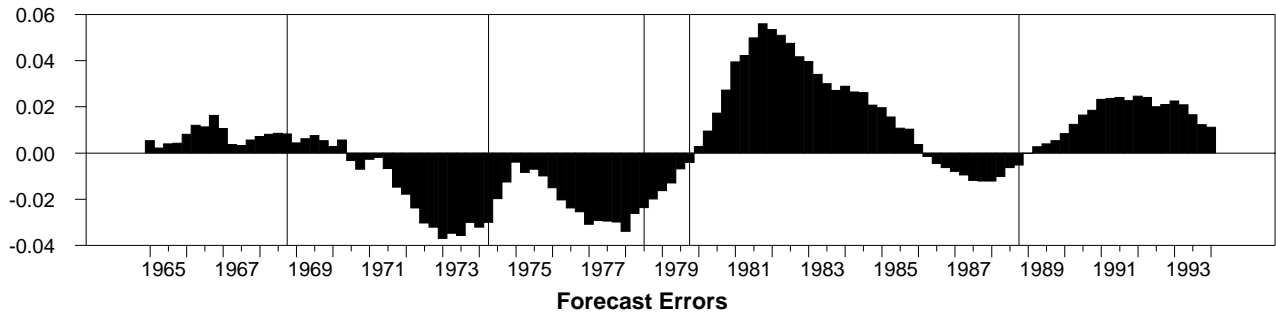
# R Decomposition, M2 Model



# R Decomposition, M2 Model

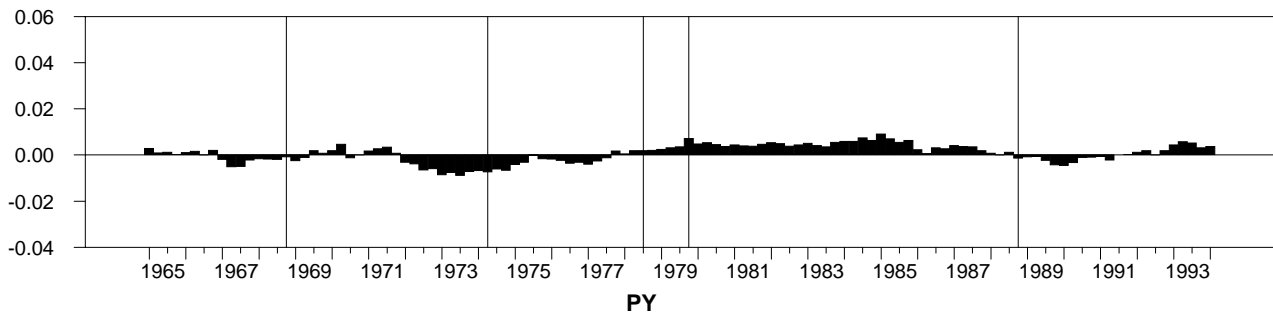
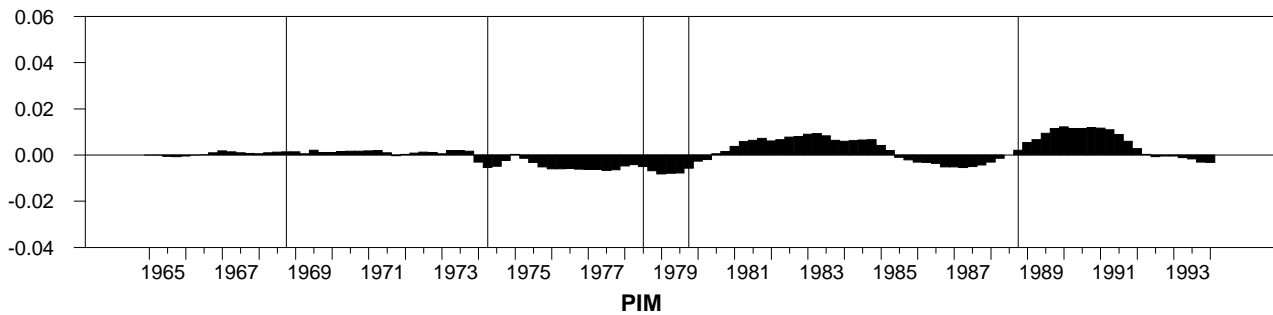
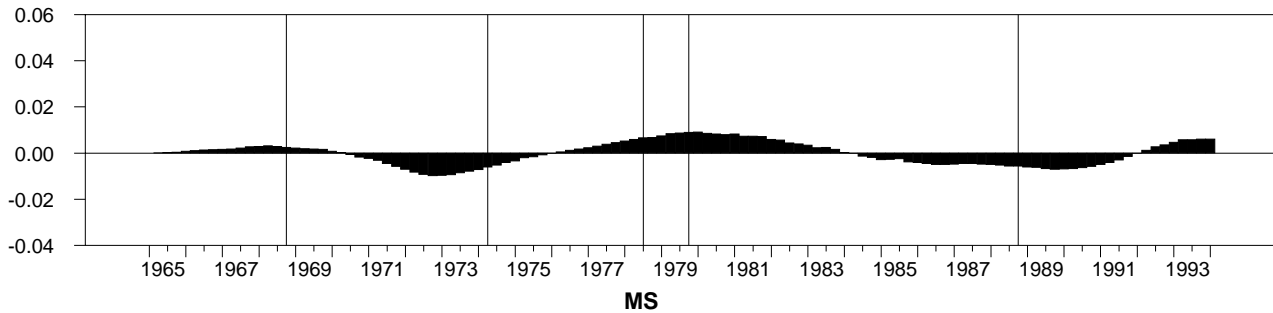


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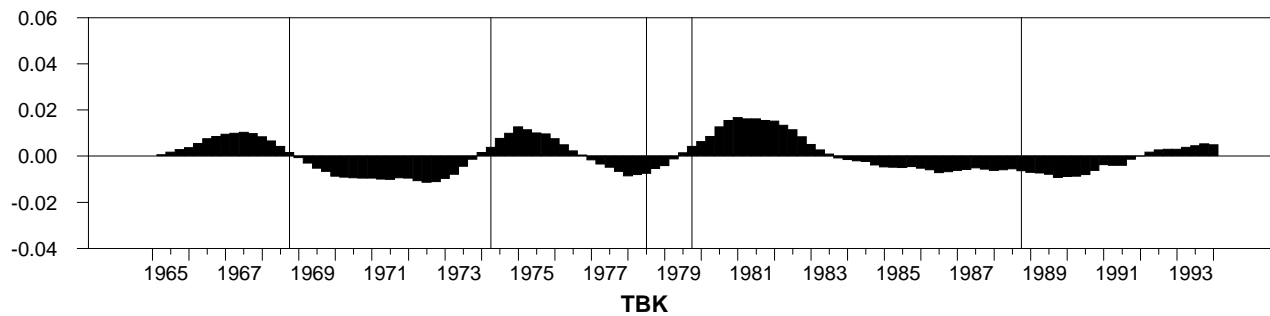
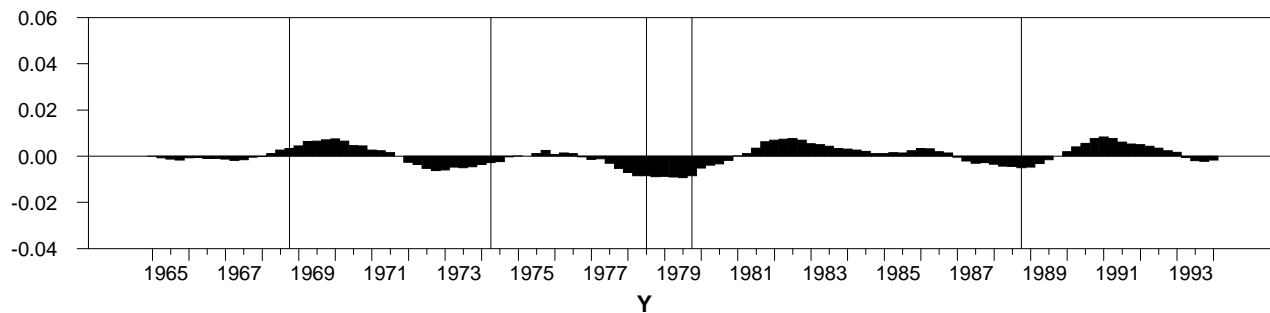
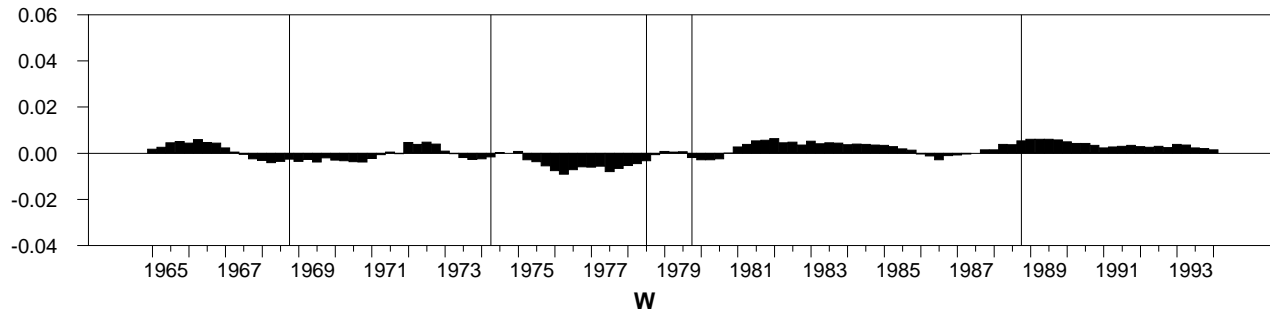




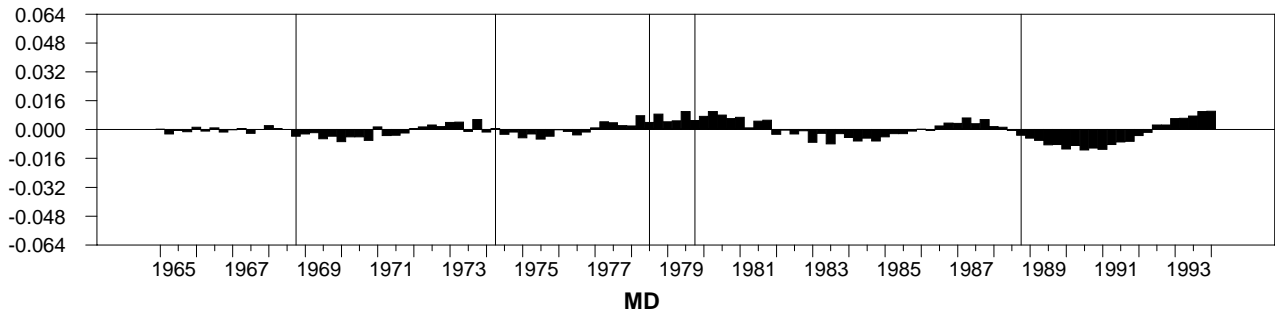
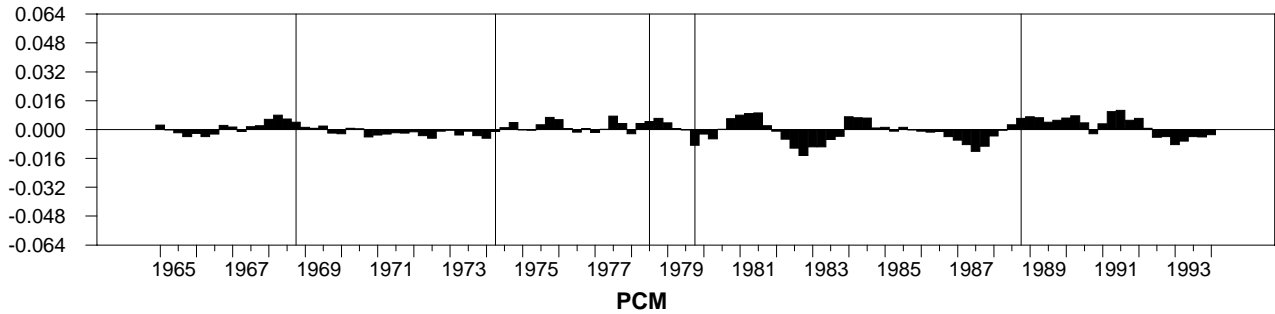
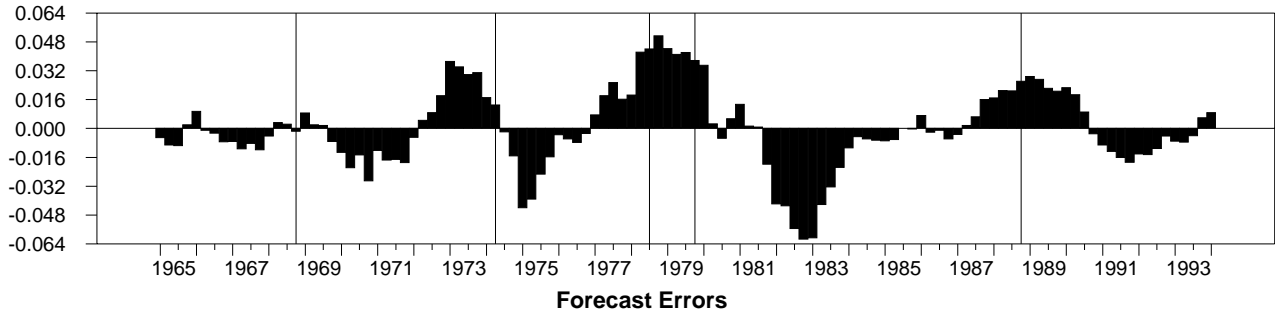
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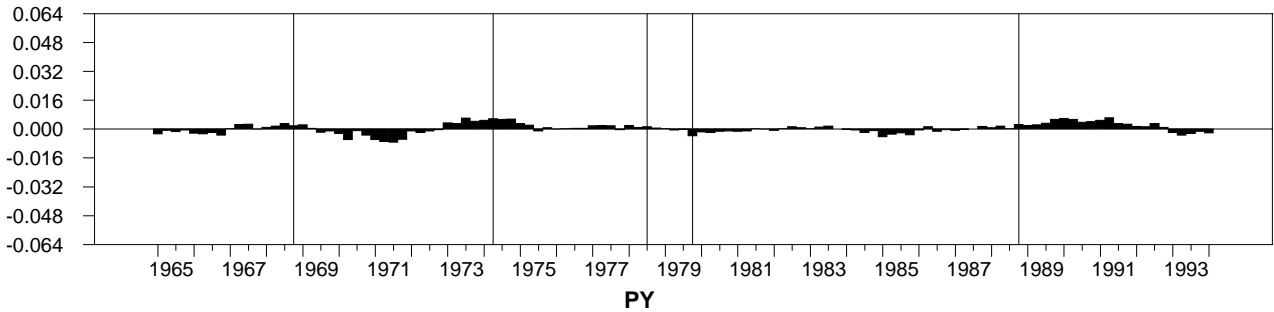
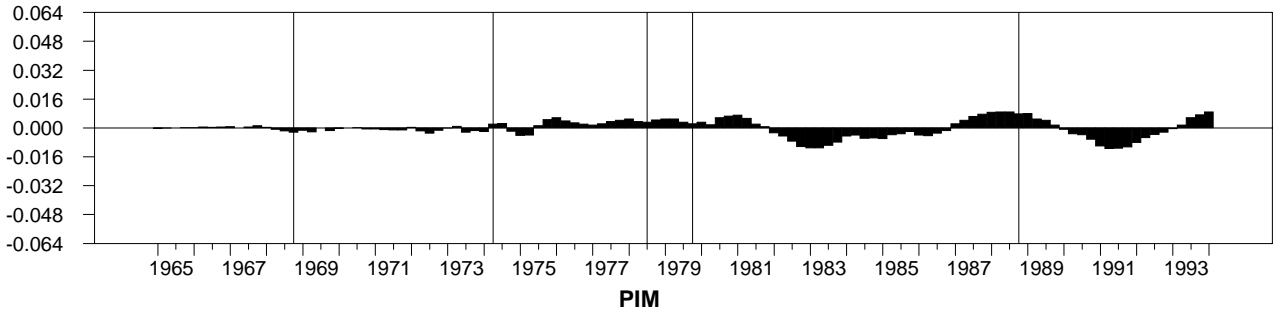
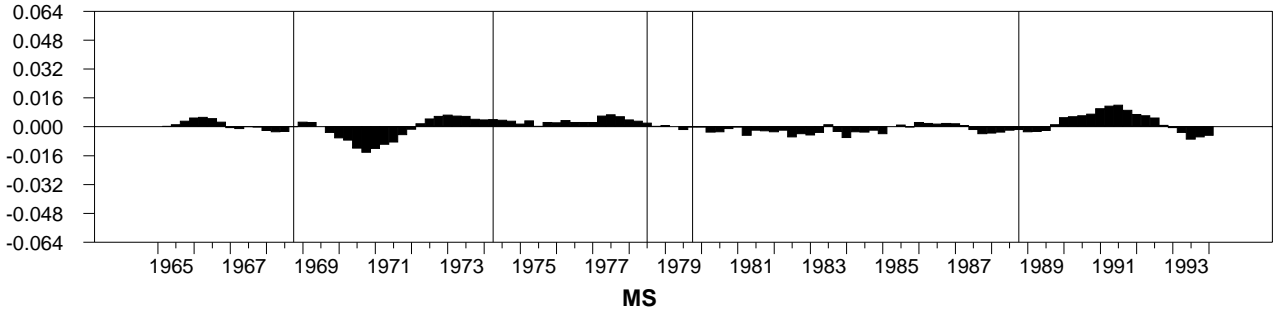
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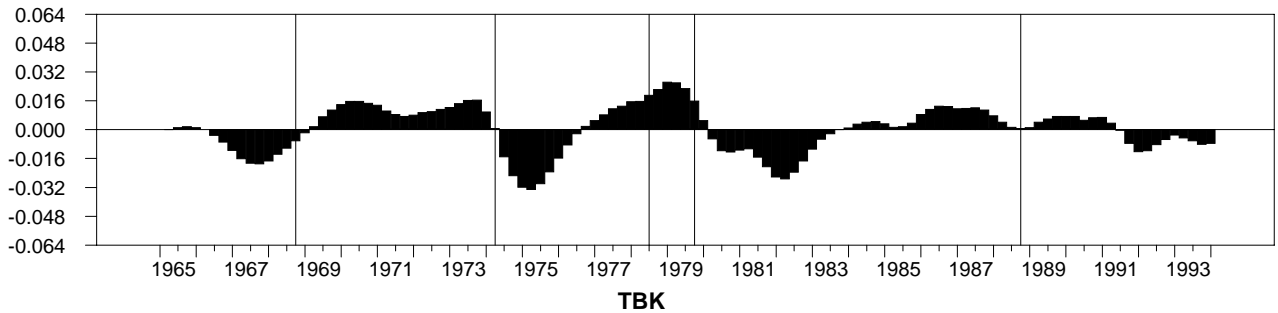
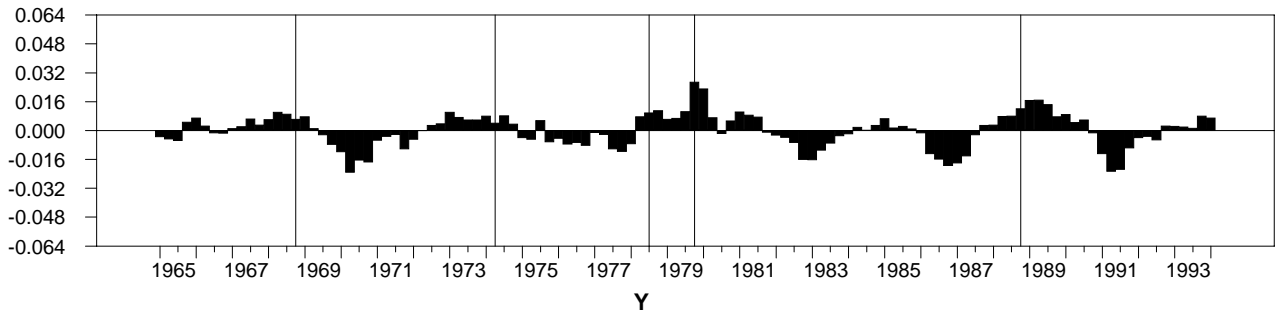
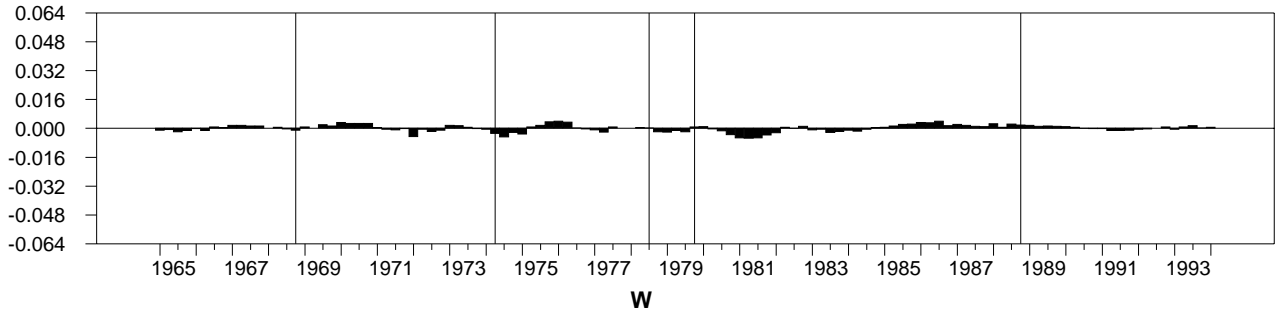
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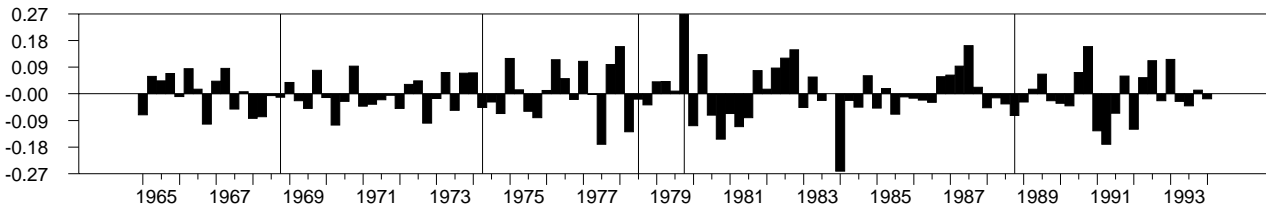
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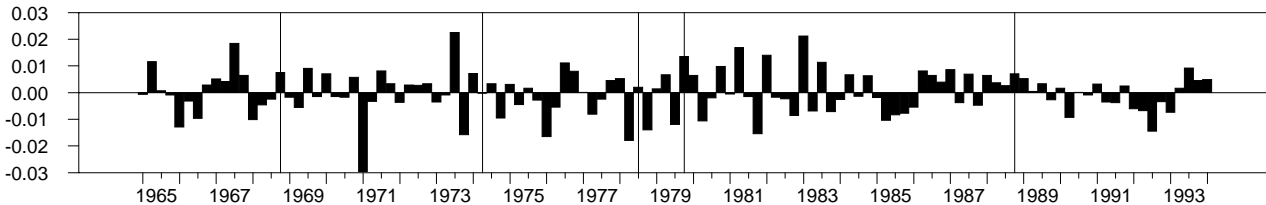
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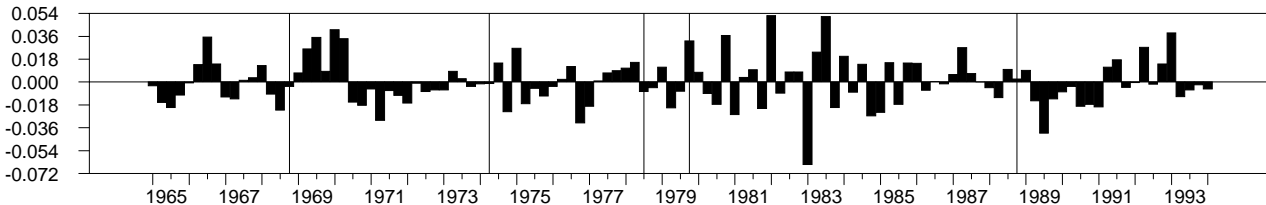
# Time Series of Structural Disturbances: M2 Model



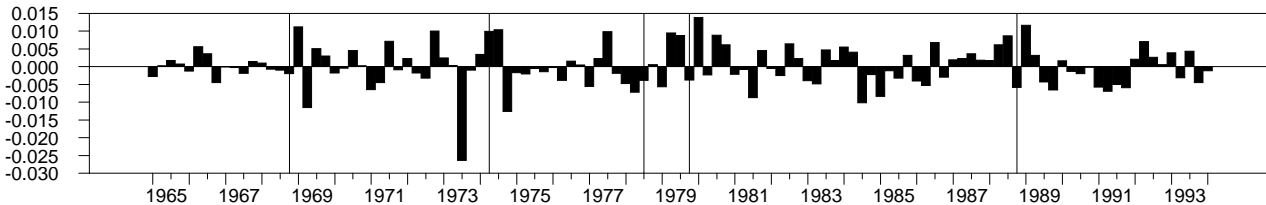
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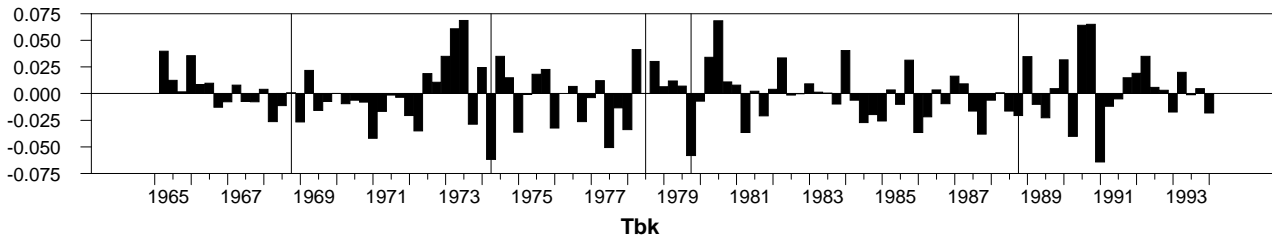
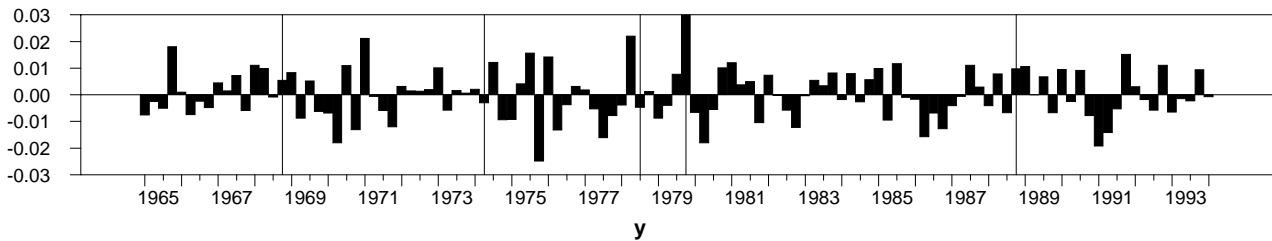
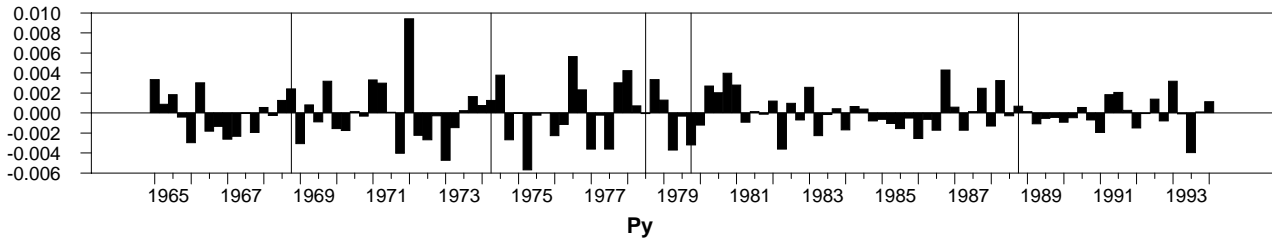
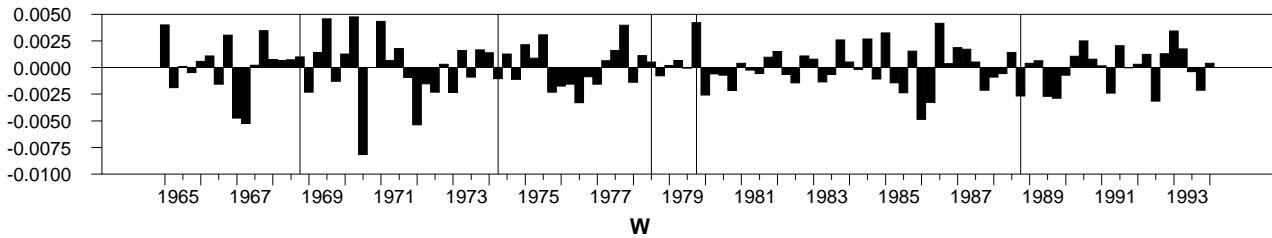


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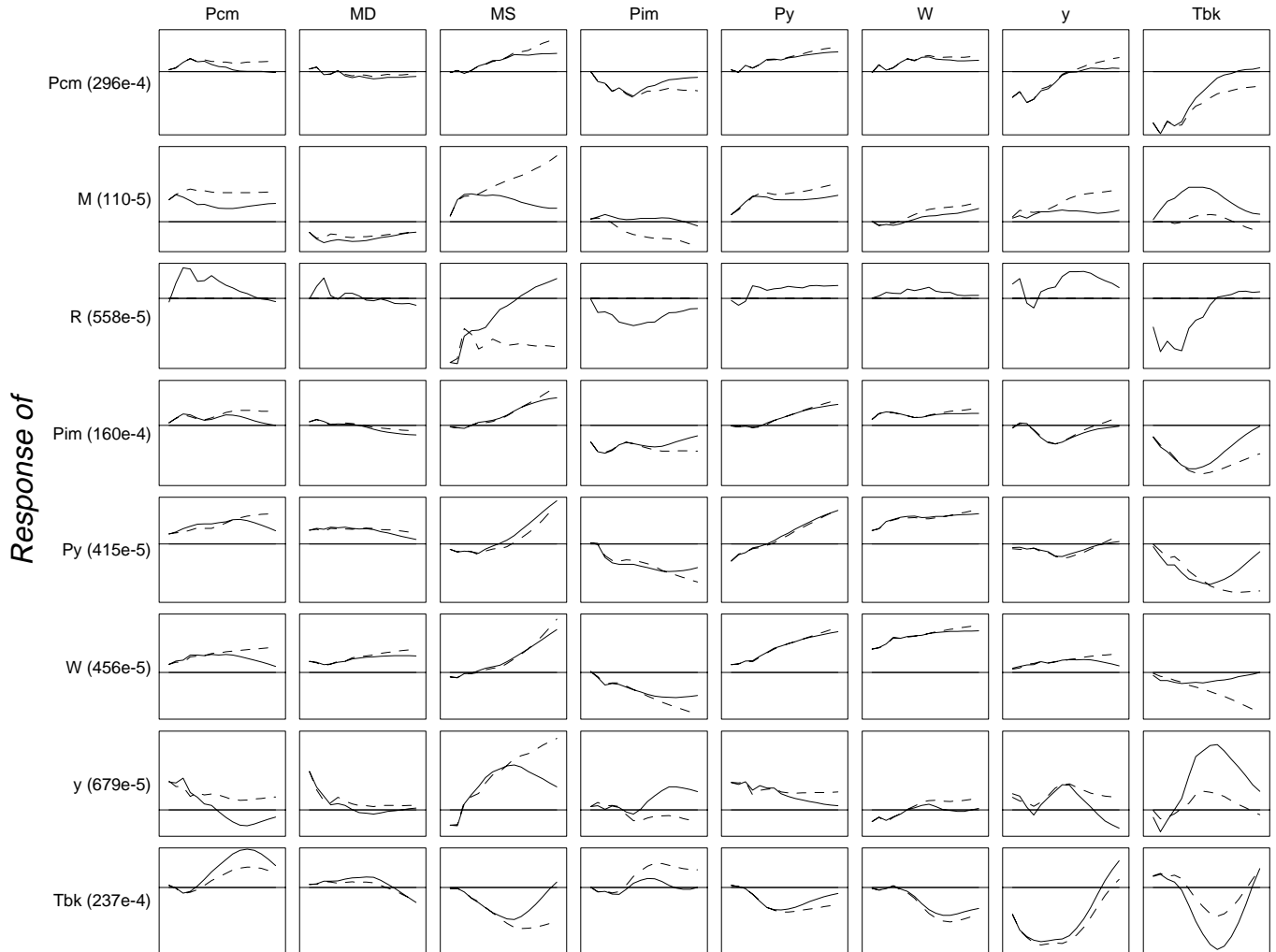


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# Time Series of Structural Disturbances: M2 Model

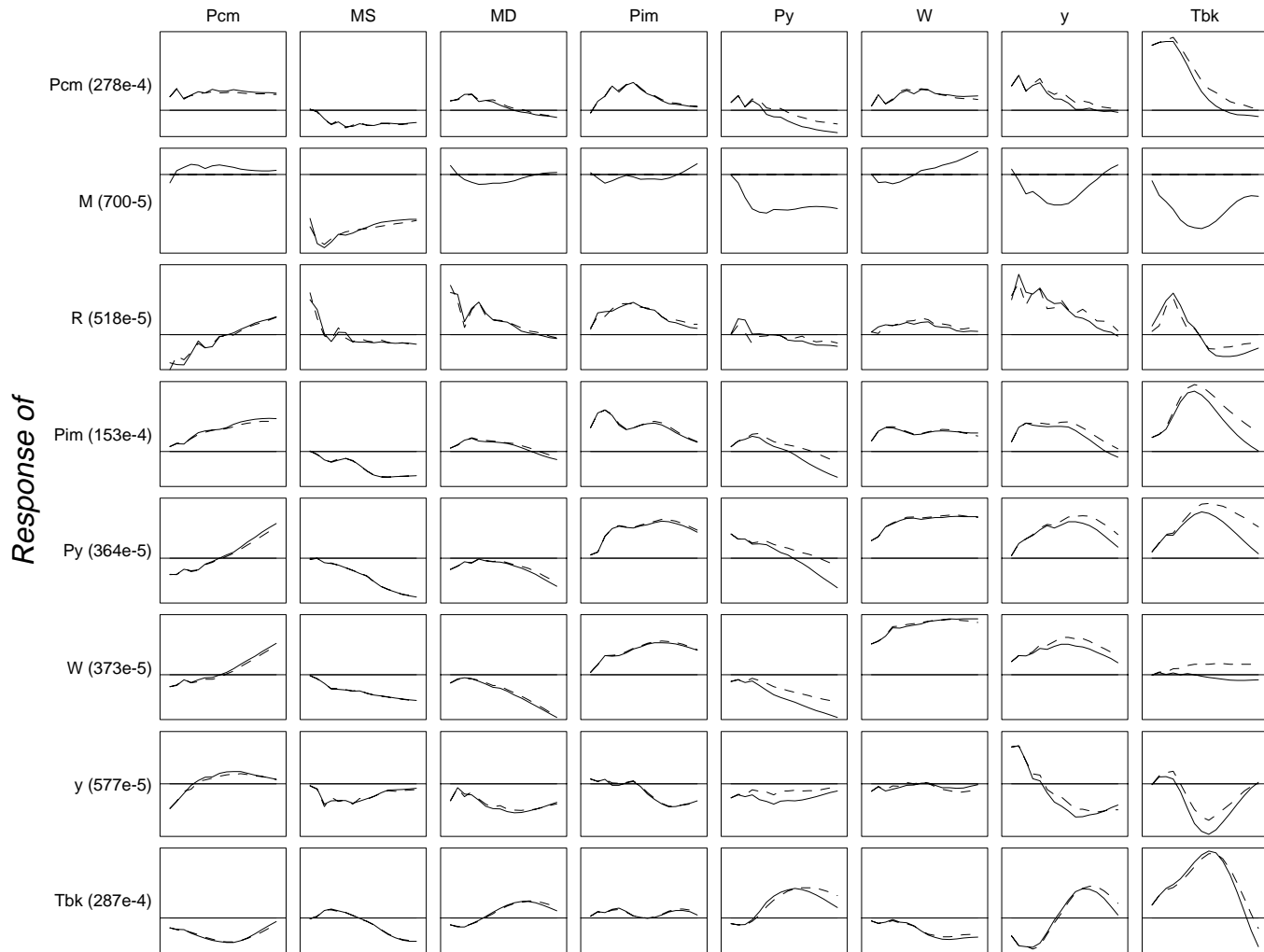


# Fixed R Policy: M2 Model





# Fixed M Policy: M2 Model



## Assessment and Criticisms of SVARs

- SVARs offer an attractive approach to estimation.
- They promise to coax interesting patterns from the data that will prevail across a set of incompletely specified dynamic economic model with a minimum of identifying assumptions.
- Moreover, SVAR are easy to estimate.
- SVAR have contributed to the understanding of aggregate fluctuations, to clarify the importance of different economic shocks, and to generate fruitful debates among macroeconomists.

## Criticisms of SVARs I

- Economic shocks recovered from a SVAR do not resemble the shocks measured by other mechanisms, like market expectations (Rudebusch, 1998).
- Shocks recovered from a SVAR may reflect variables omitted from the model. If these omitted variables correlate with the included variables, the estimated economic shocks will be biased.
- Example→ “price puzzle”: early SVARs found that inflation increased after a contractionary monetary policy shock.
- Sims (1992): the Fed looks forward when it sets the federal fund rate. Solution?

## Criticisms of SVARs II

- Many SVAR exercises, even simple ones, are sensitive to the identification restrictions, Stock and Watson (2001).
- Many of the identification schemes are the product of an specification search in which researchers look for “reasonable” answers. If an identification scheme matches the conventional wisdom is called successful, if it does not is called a puzzle or, even worse, a failure (Uhlig, 2005).