DSGE Models with Uncertainty Shocks

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Two class of DSGE models

1. Representative agent models with time-varying volatility.

2. Heterogenous agent models with time-varying volatility.

- Complementary test beds for our empirical investigations.

A simple RBC model with stochastic volatility

- Let me start with a canonical RBC model.
- This model is unlikely to be of much use in real research.
- But, it helps fixing ideas and notation.
- In fact, all the important issues that we will deal with already appear in this model.

Environment

- Representative agent with utility function:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

- The household's budget constraint is given by:

$$c_t + i_t + \frac{b_{t+1}}{R_t} = w_t l_t + r_t k_t + b_t$$

- Capital is accumulated according to the law of motion:

$$k_{t+1} = (1 - \delta) k_t + i_t$$

- The final good is produced by a competitive firm with a technology:

$$y_t = e^{z_t} A k_t^{\alpha} l_t^{1-\alpha}$$

- Thus, the economy must satisfy the aggregate resource constraint:

$$y_t = c_t + i_t$$

Stochastic volatility

- Productivity, z_t, follows an autoregressive process (structural shock):

$$z_t = \rho z_{t-1} + \sigma_t \varepsilon_t,$$

where

$$\varepsilon_t \sim \mathcal{N}(0, 1).$$

- And σ_t evolves over time as an autoregressive process (volatility shock):

$$\log \sigma_t = (1 - \rho_\sigma) \log \sigma + \rho_\sigma \log \sigma_{t-1} + \left(1 - \rho_\sigma^2\right)^{\frac{1}{2}} \eta u_t,$$

where

$$u_t \sim \mathcal{N}(0, 1).$$

FOCs

- These are the equilibrium conditions:

$$u_1(c_t, l_t) = \mathbb{E}_t u_1(c_{t+1}, l_{t+1}) \beta (1 + r_{t+1} - \delta)$$
$$u_2(c_t, l_t) = u_1(c_t, l_t) w_t$$
$$w_t = (1-\alpha) e^{z_t} k_t^{\alpha} l_t^{-\alpha}$$
$$r_t = \alpha e^{z_t} k_t^{\alpha-1} l_t^{1-\alpha}$$

- Next we write the decision rules of the agents in the economy.

- Different approaches.

A perturbation solution

- We have controls as a function of $(\log k_t, z_{t-1}, \log \sigma_{t-1}, \varepsilon_t, u_t, \Lambda)$:

$$\begin{aligned}\log c_t &= c\left(\log k_t, z_{t-1}, \log \sigma_{t-1}, \varepsilon_t, u_t, \Lambda\right)\\ \log l_t &= l\left(\log k_t, z_{t-1}, \log \sigma_{t-1}, \varepsilon_t, u_t, \Lambda\right)\\ \log k_{t+1} &= k\left(\log k_t, z_{t-1}, \log \sigma_{t-1}, \varepsilon_t, u_t, \Lambda\right)\end{aligned}$$

- We will find the Taylor expansions to these functions.

- Alternative solution methods?

Example

- Greenwood-Hercowitz-Huffman (GHH) preferences:

$$u\left(c_t, l_t\right) = \log\left(c_t - \psi \frac{l_t^{1+\zeta}}{1+\zeta}\right).$$

- No wealth effect.

- We set $\beta = 0.99$ to get an annual interest rate of around 4 percent, we set $\zeta = 0.5$ to get a Frisch elasticity of 2, and $\psi = 3.4641$ to get average labor supply to be $1/3$ of available time. We set $\alpha = 1/3$ to match labor income share in national income, $A = 0.9823$ to normalize $y = 1$, and $\delta = 0.025$.

- The stochastic process parameters, we set $\rho = 0.95$ and $\log \sigma = \log(0.007)$, and $\rho_\sigma = 0.95$ and $\eta = 0.1$.

The solution I

- Consumption:

$$
\begin{aligned}
\widehat{c}_t = {} & 0.055115\widehat{k}_t + 0.576907 z_{t-1} + 0.004251\varepsilon_t \\
& -0.000830\widehat{k}_t^2 + 0.036281\widehat{k}_t z_{t-1} + 0.000267\widehat{k}_t \varepsilon_t \\
& +0.315513 z_{t-1}^2 + 0.004650 z_{t-1}\varepsilon_t \\
& +0.000017\varepsilon_t^2 + 0.004251\varepsilon_t u_t + 0.004038\varepsilon_t \widehat{\sigma}_{t-1} \\
& +0.000013
\end{aligned}
$$

The solution II

- Labor:

$$
\begin{aligned}
\widehat{l}_t &= 0.014040\widehat{k}_t + 0.253333 z_{t-1} + 0.001867\varepsilon_t \\
&\quad -0.000444\widehat{k}_t^2 + 0.010671\widehat{k}_t z_{t-1} + 0.000079\widehat{k}_t\varepsilon_t \\
&\quad +0.096267 z_{t-1}^2 + 0.001419 z_{t-1}\varepsilon_t \\
&\quad +0.000005\varepsilon_t^2 + 0.001867\varepsilon_t u_t + 0.001773\varepsilon_t\widehat{\sigma}_{t-1}
\end{aligned}
$$

The solution III

- Capital:

$$
\begin{aligned}
\widehat{k}_{t+1} = \; & 0.983067\widehat{k}_t + 0.563093z_{t-1} + 0.004149\varepsilon_t \\
& -0.0005\widehat{k}_t^2 + 0.035747\widehat{k}_t z_{t-1} + 0.000263\widehat{k}_t\varepsilon_t \\
& +0.3342873z_{t-1}^2 + 0.004926z_{t-1}\varepsilon_t \\
& +0.000018\varepsilon_t^2 + 0.004149\varepsilon_t u_t + 0.003942\varepsilon_t\widehat{\sigma}_{t-1} \\
& -0.000013
\end{aligned}
$$

A new solution I

- Log-CRRA $u(c_t, l_t) = \log c_t - \psi \frac{l_t^{1+\zeta}}{1+\zeta}$. Calibration $\psi = 4.5$ so $l = 1/3$.
- The new policy functions for consumption:

$$\begin{aligned}
\widehat{c}_t = \; & 0.043421\widehat{k}_t + 0.199865z_{t-1} + 0.001473\varepsilon_t \\
& -0.000810\widehat{k}_t^2 + 0.005249\widehat{k}_t z_{t-1} + 0.000039\widehat{k}_t\varepsilon_t \\
& +0.053136z_{t-1}^2 + 0.000783z_{t-1}\varepsilon_t \\
& +0.000003\varepsilon_t^2 + 0.001473\varepsilon_t u_t + 0.001399\varepsilon_t\widehat{\sigma}_{t-1} \\
& -0.000003
\end{aligned}$$

A new solution II

- Labor:

$$
\begin{aligned}
\widehat{l}_t = & -0.008735\widehat{k}_t + 0.148498 z_{t-1} + 0.001094\varepsilon_t \\
& +0.000449\widehat{k}_t^2 - 0.000676\widehat{k}_t z_{t-1} - 0.000005\widehat{k}_t \varepsilon_t \\
& +0.018944 z_{t-1}^2 + 0.000279 z_{t-1}\varepsilon_t \\
& +0.000001\varepsilon_t^2 + 0.001094\varepsilon_t u_t + 0.001039\varepsilon_t \widehat{\sigma}_{t-1} \\
& +0.000002
\end{aligned}
$$

A new solution III

- Capital:

$$
\begin{aligned}
\widehat{k}_{t+1} = \; & 0.949211\widehat{k}_t + 0.730465 z_{t-1} + 0.005382\varepsilon_t \\
& -0.000214\widehat{k}_t^2 + 0.017585\widehat{k}_t z_{t-1} + 0.000130\widehat{k}_t\varepsilon_t \\
& +0.351353 z_{t-1}^2 + 0.005178 z_{t-1}\varepsilon_t \\
& +0.000019\varepsilon_t^2 + 0.005382\varepsilon_t u_t + 0.005113\varepsilon_t\widehat{\sigma}_{t-1} \\
& +0.000006
\end{aligned}
$$

- Consequences of the wealth effect.
- In particular, switch on the sign of the precautionary behavior for consumption and capital.

Other sources of SV

- We introduced SV in the process for productivity Fernández-Villaverde and Rubio-Ramírez (2007).

- But there is nothing specific in productivity.

- We could have SV in:

  1. Spread of the interest rates at which a small open economy borrows (Fernández-Villaverde *et al.*, 2009).
  2. Other financial spreads.
  3. Taxes and/or monetary policy (Fernández-Villaverde *et al.*, 2015).
  4. Preference shocks.

- Key idea: empirical relevance.

Application: small open economies I

- Now, I summarize the results in Fernández-Villaverde *et al.* (2009).
- Changes in the volatility of the spread have a effect on real variables.
- These effects appear when the level of the real interest rate remains constant.
- We use the evidence of SV in the spreads for Argentina.
- Into an small open economy model calibrated to match Argentina.

Application: small open economies II

- No theory of volatility changes. It as an exogenous process.

- The findings of Uribe and Yue (2006) and Longstaff *et al.* (2007) justify our strategy. The evidence in both papers is strongly supportive of the view that a substantial component of changes in volatility is exogenous to the country.

- Literature on financial contagion is to understand phenomena that distinctively look like exogenous shocks to small open economies (Kaminsky *et al.*, 2003).

- How to think about the more general case.

The model I

- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-v}}{1-v} - \omega \frac{H_t^{1+\eta}}{1+\eta} \right).$$

- The household's budget constraint is given by:

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t H_t - R_t K_t + C_t + I_t + \frac{\Phi_D}{2} \left(D_{t+1} - D \right)^2$$

- The real interest rate

$$r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$$

The model II

- Both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow $AR(1)$ processes:

$$\begin{aligned}\varepsilon_{tb,t} &= \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}} u_{tb,t} \\ \varepsilon_{r,t} &= \rho_{r}\varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t}\end{aligned}$$

- The standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ also follow:

$$\begin{aligned}\sigma_{tb,t} &= \left(1-\rho_{\sigma_{tb}}\right)\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t} \\ \sigma_{r,t} &= \left(1-\rho_{\sigma_{r}}\right)\sigma_{r} + \rho_{\sigma_{r}}\sigma_{r,t-1} + \eta_{r} u_{\sigma_{r},t}\end{aligned}$$

The model III

- The stock of capital evolves according to

$$K_{t+1} = (1-\delta)K_t + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$

- Firms rent capital and labor from households to produce output in a competitive environment according to the technology

$$Y_t = K_t^{\alpha}\left(e^{X_t} H_t\right)^{1-\alpha}$$

where

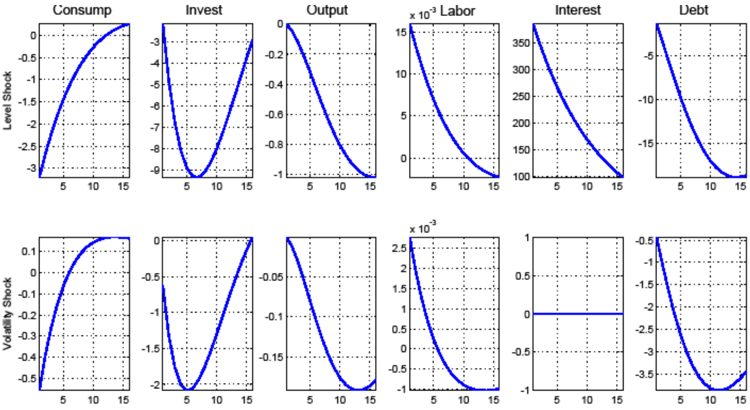
$$X_t = \rho_x X_{t-1} + e^{\sigma_x} u_{x,t}$$

and $u_{x,t} \sim \mathcal{N}(0,1)$.

Solving and calibrating the model

- We solve the model by relying on perturbation methods.
- We need to obtain a *third* approximation of the policy functions.
- We first estimate the process for the spread using EMBI+ data and a Bayesian approach.
- We pick target some moments of the Argentinian economy.
- Our calibration must target the moments of interest generated by the ergodic distributions and not the moments of the deterministic steady state, since those last ones are not representative of the stochastic dynamics.

Impulse response functions



A model with heterogeneous agents

- Bloom et al. (20012)
- Large number of heterogeneous firms that employ capital and labor to produce a single final good.
- Large number of identical households with measure one.
- Loading heterogeneity in the firms' side.
- But this is not required.
- We could think about important sources of heterogeneity from the side of the household.

Firms

- Production function:

$$y_{j,t} = A_t z_{j,t} k_{j,t}^{\alpha} n_{j,t}^{\upsilon}, \ \alpha + \upsilon < 1$$

- Processes:

$$\log(A_t) = \rho^A \log(A_{t-1}) + \sigma_{t-1}^A \varepsilon_t$$

  and

$$\log(z_{j,t}) = \rho^Z \log(z_{j,t-1}) + \sigma_{t-1}^Z \varepsilon_{j,t}$$

- Variance of innovations, σ_{t-1}^A and σ_{t-1}^Z, move over time according to two-state Markov chains.

- Periods of low and high macro and micro uncertainty.

Investment and hours

- Law of motion for capital:

$$k_{j,t+1} = (1 - \delta_k)\, k_{j,t} + k_{i,t}$$

- Irreversible investment: resale of capital occurs at a price that is only a share $(1 - S)$ of its purchase price.

- Hours

$$n_{j,t} = (1 - \delta_n)\, n_{j,t-1} + s_{j,t}$$

- Fixed cost of adjusting hours F^L as a share H of the annual wage bill per worker.

Recursive formulation I

- Value function problem of the firm:

$$V\left(k, n_{-1}, z; A, \sigma^{Z}, \sigma^{A}, \Phi\right)$$
$$= \max_{i,n} \left\{ \begin{array}{c} y - w\left(A, \sigma^{Z}, \sigma^{A}, \Phi\right) n - i \\ -AC^{k}\left(k, k'\right) - AC^{n}\left(n_{-1}, n\right) \\ +\mathbb{E} m V\left(k', n, z'; A', \sigma^{Z\prime}, \sigma^{A\prime}, \Phi'\right) \end{array} \right\}$$
$$\text{s.t. } \Phi' = \Gamma\left(A, \sigma^{Z}, \sigma^{A}, \Phi\right)$$

- Pricing kernel:

$$m = \Psi\left(A, \sigma^{Z}\sigma^{A}, \Phi, A', \sigma^{Z\prime}, \sigma^{A\prime}, \Phi'\right)$$

Recursive formulation II

- Value function problem of the household:

$$W\left(A, \sigma^{Z}, \sigma^{A}, \Phi\right) = \max\left\{\log c_t - \phi\frac{n^{1+\xi}}{1+\xi} + \beta\mathbb{E}W\left(A', \sigma^{Z\prime}, \sigma^{A\prime}, \Phi'\right)\right\}$$

- Market clearing conditions.
- Standard recursive competitive equilibrium solution.
- We will discuss in the next lecture how to solve the model.

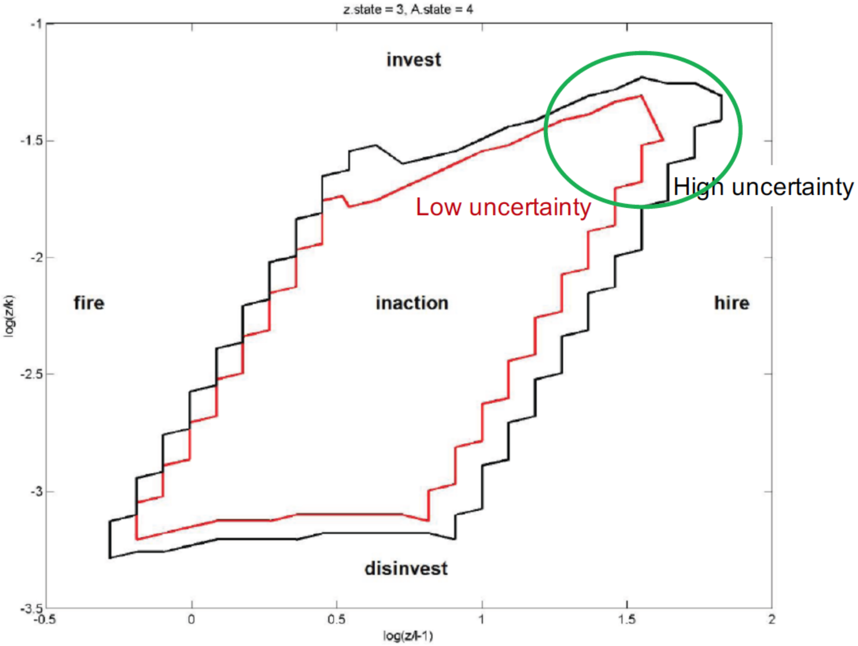
Two effects

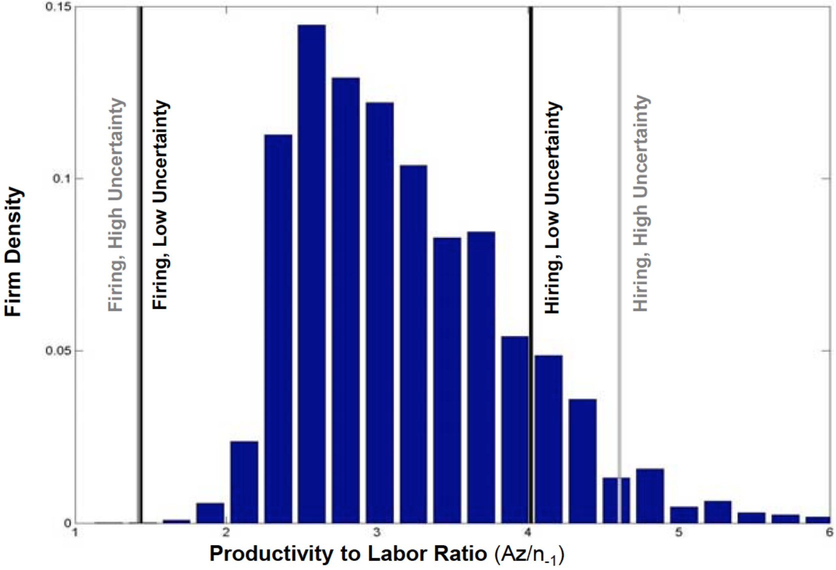
- Delay effect: higher uncertainty leads firms to postpone decisions

$$\frac{\partial I}{\partial \sigma^i} < 0$$

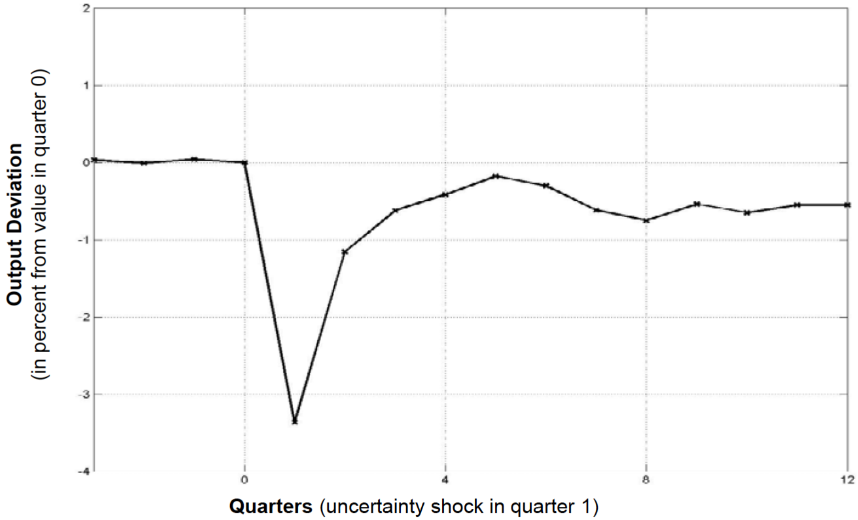
- Caution effect: higher uncertainty reduces firms response to other changes, like prices or TFP

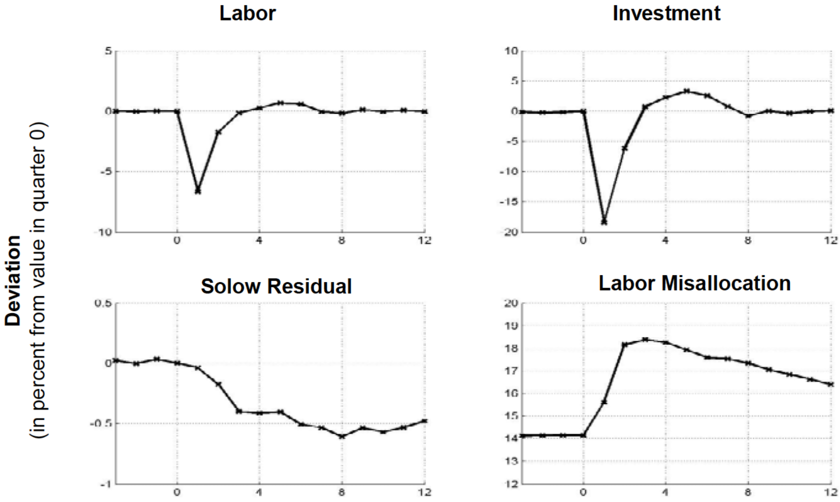
$$\frac{\partial I^2}{\partial A \partial \sigma^i} < 0$$

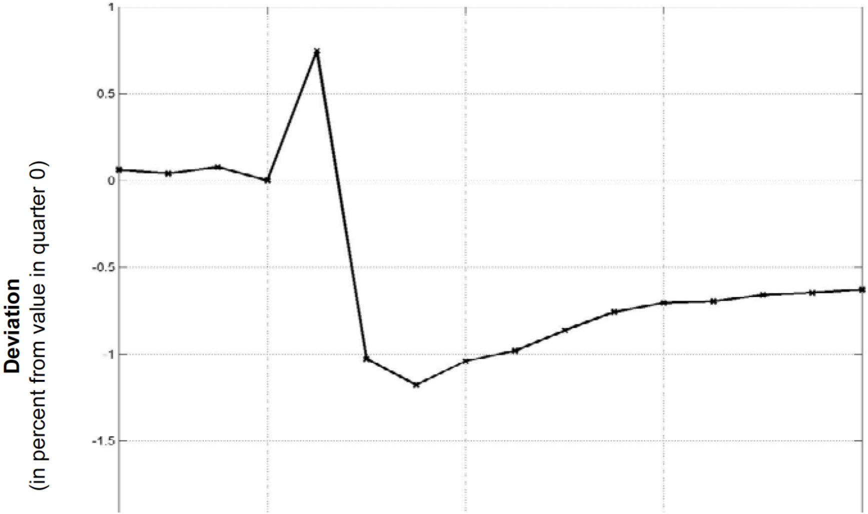

z.state = 3, A.state = 4
invest
fire
inaction
hire
disinvest
Low uncertainty
High uncertainty
log(z/k)
log(z/l-1)
-1
-1.5
-2
-2.5
-3
-3.5
-0.5
0
0.5
1
1.5
2

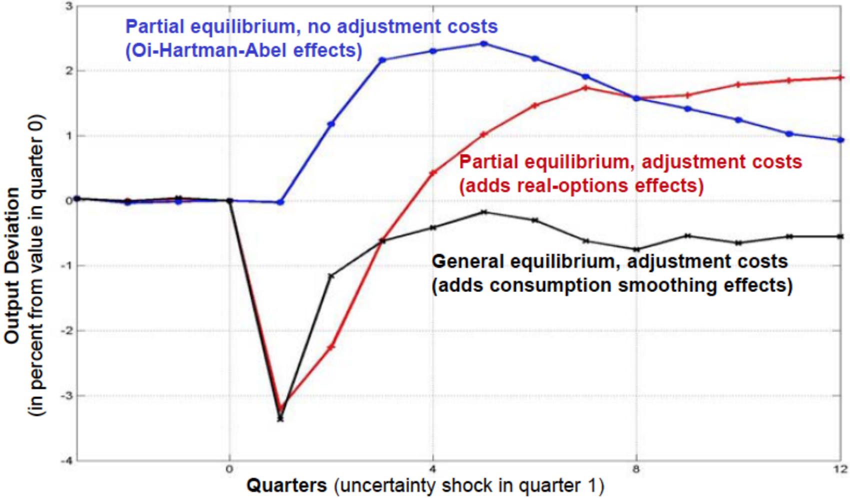


Firm Density
Productivity to Labor Ratio (Az/n-1)
Firing, High Uncertainty
Firing, Low Uncertainty
Hiring, Low Uncertainty
Hiring, High Uncertainty
0.15
0.1
0.05
0
1
2
3
4
5
6


Output Deviation
(in percent from value in quarter 0)
2
1
0
-1
-2
-3
-4
0
4
8
12
Quarters (uncertainty shock in quarter 1)


Labor
Investment
Solow Residual
Labor Misallocation
Deviation
(in percent from value in quarter 0)
5
0
-5
-10
0
4
8
12
10
5
0
-5
-10
-15
-20
0.5
0
-0.5
-1
20
19
18
17
16
15
14
13
12


Deviation
(in percent from value in quarter 0)
1
0.5
0
-0.5
-1
-1.5


Partial equilibrium, no adjustment costs
(Oi-Hartman-Abel effects)
Partial equilibrium, adjustment costs
(adds real-options effects)
General equilibrium, adjustment costs
(adds consumption smoothing effects)
Output Deviation
(in percent from value in quarter 0)
Quarters (uncertainty shock in quarter 1)
3
2
1
0
-1
-2
-3
-4
0
4
8
12