A Model of Financial Intermediation

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A Model with Financial Intermediation

- Previous models have a very streamlined financial intermediation structure.

- Many of the events of the 2007-2010 recession were about breakdowns in intermediation.

- Kiyotaki and Gertler (2011) incorporate a richer financial intermediation sector.

- In particular, we will deal with liquidity.

- Different concepts of liquidity.

- Help us to think about unconventional monetary policy.
Figure 1: Selected Corporate Bond Spreads

NOTE: The black line depicts the average credit spread for our sample of 5,269 senior unsecured corporate bonds; the red line depicts the average credit spread associated with very long maturity corporate bonds issued by firms with low to medium probability of default (see text for details); and the blue line depicts the standard Baa credit spread, measured relative to the 10-year Treasury yield. The shaded vertical bars denote NBER-dated recessions.
Figure 2: Scatter chart of \( \{ (\Delta A_t, \Delta E_t) \} \) and \( \{ (\Delta A_t, \Delta D_t) \} \) for changes in assets, equity and debt of US investment bank sector consisting of Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley between 1994Q1 and 2011Q2 (Source: SEC 10Q filings).
Representative Household

- Preferences:
  \[
  \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log(c_t - \gamma c_{t-1}) - \chi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right)
  \]

- Household can save in:
  1. Deposits at the financial intermediary, \(d_t\), that pay an uncontingent nominal gross interest rate \(R_t\).
  2. Public debt, \(d_{gt}\), that pay an uncontingent nominal gross interest rate \(R_t\).
  3. Arrow securities (net zero supply in equilibrium).

- The budget constraint is then:
  \[
  c_t + d_{h,t} = w_t l_t + R_{t-1} d_{h,t-1} + T_t + F_t
  \]
  where \(d_{h,t} = d_t + d_{gt}\). 
Representative Household

- Continuum of members of measure one with perfect consumption insurance within the family.

- A fraction $1 - f$ are workers and $f$ are bankers.

- Workers work and send wages back to the family.

- Bankers run a bank that sends (non-negative) dividends back to the family.

- In each period, a fraction $(1 - \sigma)$ of bankers become workers and a fraction $(1 - \sigma) \frac{f}{1-f}$ of workers become bankers. Why?
Optimality Conditions

- The first-order conditions for the household are:

\[
\frac{1}{c_t - \gamma c_{t-1}} - \beta \mathbb{E}_t \frac{\gamma}{c_{t+1} - \gamma c_t} = \lambda_t
\]

\[
\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_t
\]

\[
\chi l_t^\varphi = \lambda_t w_t
\]

- Asset pricing kernel:

\[
SDF_t = \beta \frac{\lambda_t}{\lambda_{t-1}}
\]

and standard non-arbitrage conditions.
Island model: continuum of islands of measure 1.

In each island, there is a firm that produces the final good with capital (not mobile) and labor (mobile across islands) and a Cobb-Douglas production function.

Then, by equating the capital-labor ratio across islands, aggregate output is:

$$y_t = A_t k_t^\alpha l_t^{1-\alpha}$$

where $A_t$ is a random variable.

Wages satisfy:

$$w_t = (1 - \alpha) \frac{y_t}{l_t}$$

Profits per unit of capital:

$$z_t = \frac{y_t - w_t l_t}{k_t}$$
Liquidity Risk

- Each period, investment opportunities arrive randomly to a fraction $\pi^i$ of islands.

- There is no opportunity in $\pi^n = 1 - \pi^i$.

- Investment opportunities are i.i.d. across time and islands.

- Only firms in islands with investment opportunities can accumulate capital.

Then:

$$k_{t+1} = \psi_{t+1} \left[ \pi^i (1 - \delta) k_t + i_t \right] + \psi_{t+1} \pi^n (1 - \delta) k_t$$

$$= \psi_{t+1} \left[ (1 - \delta) k_t + i_t \right]$$

where $\psi_{t+1}$ is a shock to productivity of capital.
Capital Good Producers

- Adjustment costs in investment:

\[ f \left( \frac{i_t}{i_{t-1}} \right) \]

with \( f(1) = f'(1) = 0 \) and \( f'' \left( \frac{i_t}{i_{t-1}} \right) > 0 \).

- Relative price of capital: \( q_t^i \).

- Capital good producers:

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left( q_t^i - \left( 1 + f \left( \frac{i_t}{i_{t-1}} \right) \right) i_t \right)
\]

- Optimality condition:

\[
q_t^i = 1 + f \left( \frac{i_t}{i_{t-1}} \right) + \frac{i_t}{i_{t-1}} f' \left( \frac{i_t}{i_{t-1}} \right) - \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{i_{t+1}}{i_t} \right)^2 f' \left( \frac{i_{t+1}}{i_t} \right)
\]
Aggregate Resource Constraint

- Government consumption $g_t$.

- Then:

$$y_t = c_t + \left(1 + f \left(\frac{i_t}{i_{t-1}}\right)\right) i_t + g_t$$

- We will also have, later on, a wide set of policies, that will imply a relatively involved government budget constraint.

- I will skip details because it is mere accounting.
Banks are born with a small initial transfer from the family.

Initial equity is increased with retained earnings.

Dividends are only distributed when the bank dies.

Banks are attached to a particular island, which in this period may be

\[ h = \{i, n\} . \]

Thus, ex post, they may not be able to lend⇒wholesale market.
Banks move across islands over time to equate expected rate of return:

1. Before moving, they sell their loans.

2. This allows us to forget about distributions: ratio of total financial intermediary net worth to total capital is the same in each island.

Discussion: specificity in bank relational capital?
Balance Sheet

- Net worth: \( n_t^h \).

- Besides equity, banks raise funds in a national financial market:
  
  1. Retail market: from the households, \( d_t \) at cost \( R_t \). Before investment shock is realized.
  
  2. Wholesale market: from each other, \( b_t^h \) at cost \( R_{bt} \). After investment shock is realized.

- Then, bank lend to non-financial firms in their island \( s_t^h \). No enforcement problem (we can think about \( s_t^h \) as equity).
Balance sheet constraint (where $q_t^h$ is the price of a loan):

$$q_t^h s_t^h = n_t^h + b_t^h + d_t$$

Evolution of net worth:

$$n_t^h = \left[ z_t + (1 - \delta) q_t^h \right] \psi_t s_{t-1}^h - R_{t-1} d_{t-1} - R_{bt-1} b_{t-1}^h$$

Objective function of bank:

$$V_t = \mathbb{E}_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \beta^i \frac{\lambda_{t+i}}{\lambda_t} n_{t+i}^h$$
Value function: maximized objective function:

\[
V_t \left( s_t^h, b_t^h, d_t \right) = \max V_t \\
= \mathbb{E}_t \beta^t \frac{\lambda_{t+i}}{\lambda_t} \sum_h \pi^h \left\{ + \sigma \max_{d_{t+1}} \max_{s_t^h, b_t^h} V_t \left( s_{t+1}^h, b_{t+1}^h, d_{t+1} \right) \right\}
\]
We need some financial friction to make the intermediation problem interesting.

Simply costly-enforcement mechanism.

Diversion of funds to family:

\[ \theta \left( q_t^h s_t^h - \omega b_t^h \right) \]

default, and close down.

Interpretation of \( \omega \).
Three cases:

1. $\omega = 1$ (frictionless interbank market). The interbank and loan rates are the same. They are bigger than the deposit rate if banks are constrained (only one aggregate constrain holds).

2. $\omega = 0$ (symmetric frictions). The interbank and deposit rate are the same. The returns on loans if banks on non-investing islands are constrained.

3. $\omega \in (0, 1)$. The interbank rate lies between the return on loans and the deposit rate.
Then, the incentive constraint (IC) is:

\[ V_t \left( s_t^h, b_t^h, d_t \right) \geq \theta \left( q_t^h s_t^h - \omega b_t^h \right) \]

The Lagrangian associated with the IC is \( \lambda_t^h \) and:

\[ \lambda_t^h = \pi^i \lambda_t^i + \pi^n \lambda_t^n \]

Problem:

\[
\max V_t \left( s_t^h, b_t^h, d_t \right) + \lambda_t^h \left( V_t \left( s_t^h, b_t^h, d_t \right) - \theta \left( q_t^h s_t^h - \omega b_t^h \right) \right) \\
= \max \left( 1 + \lambda_t^h \right) V_t \left( s_t^h, b_t^h, d_t \right) - \lambda_t^h \theta \left( q_t^h s_t^h - \omega b_t^h \right)
\]
Guess of Value Functions

- We guess that value function is linear in states:

\[ V \left( s_t^h, b_t^h, d_t \right) = \nu_{st} s_t^h - \nu_{bt} b_t^h - \nu_{dt} d_t \]

- Interpretation of coefficients:

1. \( \nu_{st} \): marginal value of assets.
2. \( \nu_{bt} \): marginal cost of interbank borrowing.
3. \( \nu_{dt} \): marginal cost of deposits.

- Then:

\[ \left( 1 + \bar{\lambda}_t^h \right) \left( \nu_{st} s_t^h - \nu_{bt} b_t^h - \nu_{dt} d_t \right) - \bar{\lambda}_t^h \theta \left( q_t^h s_t^h - \omega b_t^h \right) \]
Optimality Conditions

- Remember that, from the balance sheet constraint:

\[ b_t^h = q_t^h s_t^h - n_t^h - d_t \]

- The FOC are (note the bank takes \( n_t^h \) as given and use chain rule to take derivatives of \( b_t^h \)):

\[ d_t : \left( 1 + \lambda_t^h \right) (\nu_{bt} - \nu_{dt}) = \lambda_t^h \theta \omega \]

\[ s_t^h : \left( 1 + \lambda_t^h \right) \left( \frac{\nu_{st}}{q_t^h} - \nu_{bt} \right) = \lambda_t^h \theta (1 - \omega) \]

\[ \lambda_t^h : \nu_{dt} n_t^h \geq \left( \theta - \left( \frac{\nu_{st}}{q_t^h} - \nu_{dt} \right) \right) q_t^h s_t^h - (\theta \omega - (\nu_{bt} - \nu_{dt})) b_t^h \]
Interpretation:

1. Marginal cost of interbank borrowing is higher than cost of deposits iff \( \tilde{\lambda}_t^h > 0 \) and \( \omega > 0 \).

2. Marginal value of assets is higher than marginal cost of interbank borrowing if \( \lambda_t^h > 0 \) and \( \omega < 1 \).

3. Balance sheet effect: equity in bank must be sufficiently high in relation with assets and interbank borrowing.
Case A: Frictionless Wholesale Financial Market I

- $\omega = 1$.
- Arbitrage across asset markets:

$$q^b_t = q^l_t = q_t$$

- Marginal value of asset must be equal to the marginal cost of borrowing on the interbank market:

$$\frac{\nu_{st}}{q_t} = \nu_{bt}$$

- The incentive constraint is simply:

$$q_t s_t - b_t = \phi_t n_t$$

where

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}$$
Case A: Frictionless Wholesale Financial Market II

- With a bit of work (which I skip), and by matching coefficients

\[
\mu_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1} \left( R_{t+1}^k - R_{t+1} \right)
\]

\[
\nu_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1} R_{t+1}
\]

where

\[
\Omega_{t+1} = 1 - \sigma + \sigma (\nu_{t+1} + \phi_{t+1} \mu_{t+1})
\]

\[
R_{t+1}^k = \psi_t \frac{z_{t+1} + (1 - \delta) q_{t+1}}{q_t}
\]

- Aggregating:

\[
q_t s_t = \phi_t n_t
\]

- In this economy, a crisis increases the excess returns for banks of all types.
Case B: Symmetric Frictions I

- $\omega = 0$.

- Deposits and interbank loans become perfect substitutes:

  $$\nu_t = \nu_{bt}$$

- Thus, in general, there will be differences in prices of assets across islands:

  $$q^n_t > q^i_t$$

  and

  $$\mu^i_t > \mu^n_t \geq 0$$
Case B: Symmetric Frictions II

- The leverage ratio:

\[
\frac{q_t^i s_t^i}{n_t^i} = \phi_t^i = \frac{\nu_t}{\theta - \mu_t^i}
\]

\[
\frac{q_t^n s_t^n}{n_t^n} \leq \phi_t^n = \frac{\nu_t}{\theta - \mu_t^n}
\]

\[
\left( \frac{q_t^n s_t^n}{n_t^n} - \phi_t^n \right) \mu_t^n = 0
\]

- With a bit of work (which I skip), and by matching coefficients

\[
\mu_t = \beta^t E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} \left( R_{t+1}^{hh'} - R_{t+1} \right)
\]

\[
\nu_t = \beta^t E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} R_{t+1}
\]
Case B: Symmetric Frictions III

where

\[ \Omega_{t+1}^h = 1 - \sigma + \sigma \left( \nu_{t+1} + \phi_{t+1}^h \mu_{t+1}^h \right) \]

\[ R_{t+1}^{hh'} = \psi_t \frac{z_{t+1} + (1 - \delta) q_{t+1}^{h'}}{q_t^h} \]

Note that now we need to index also by the type of the island in next period and integrate over it.

Aggregating:

\[ q_t^i s_t^i = \phi_t^i n_t^i \]

\[ q_t^n s_t^n \leq \phi_t^n n_t^n \]

\[ (q_t^n s_t^n - \phi_t^n n_t^n) \mu_t^n = 0 \]
Case B: Symmetric Frictions IV

- In this economy, a crisis increases the excess returns for banks of all types.

- Also:

\[
\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} R_{kt+1}^{ih'} > \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} R_{kt+1}^{nh'} \\
\geq \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} R_{bt+1} \\
= \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1}^{h'} R_{t+1}
\]
Aggregation

- Total bank net worth:
  \[ n_t^h = n_{ot}^h + n_{yt}^h \]

- Total net worth of old banks:
  \[ n_{ot}^h = \sigma \pi^h \left\{ \left[ z_t + (1 - \delta) q_t^h \right] \psi_t s_{t-1} - R_{t-1} d_{t-1} \right\} \]
  where we have net out the interbank loans.

- Total net worth of new banks:
  \[ n_{yt}^h = \xi \left[ z_t + (1 - \delta) q_t^h \right] \psi_t s_{t-1} \]

- Aggregate balance sheet constraint:
  \[ d_t = \sum_{h=i,n} \left( q_t^h s_t^h - n_t^h \right) \]
Market Clearing

- Market for securities:
  \[ s^i_t = i_t + (1 - \delta) \pi^i k_t \]
  \[ s^n_t = (1 - \delta) \pi^n k_t \]

- Labor market
  \[ \chi l^\theta_t = \lambda_t w_t \]

- Debt market:
  \[ d_{ht} = d_t + d_g \]
Policy Experiments

- Unconventional monetary policy:
  1. Lending facilities.
  2. Liquidity facilities.
  3. Equity injections.

- Classic discussion from *Sargent and Wallace (1983)*: real bills doctrine.

- How do we decide between these different policies?

- Effect on government budget position.

Sir John Houblon.  
Governor.

Sir John Somers.  
Lord Keeper.

Mr. Michael Godfrey  
Deputy Governor.
Lending Facilities

- The central bank lends directly to banks that are constrained.

- Central bank is not constrained by its balance sheet (this is more subtle than it seems, but let us assume it for a moment).

- But additional cost $\tau$ of underwriting a loan (monitoring, politics...).

- The central bank does not subsidize loans...

- ...but, by increasing funds available, it has an impact on equilibrium prices and allocations.

- New equilibrium condition:

$$q_t^h s_t^h = q_t^h \left( s_{pt}^h + s_{gt}^h \right)$$
Liquidity Facilities

- Central bank discounts loans from the interbank lending market.

- Banks can divert less funds from the central bank than from the regular interbank market:
  \[
  \theta (1 - \omega_g)
  \]
  with \( \omega_g > 0 \).

- Then:
  \[
  q^h_t s^h_t = n^h_t + b^h_t + m^h_t + d_t
  \]

- Penalty rate for discount: difference in the default.
Equity Injections

- Treasury transfers wealth to banks.

- Government takes direct ownership position.

- Then:

\[ q_t^h s_t^h = n_t^h + n_t^g + b_t^h + d_t \]

- Government must pay a premium.

### Households

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.500</td>
<td>Habit parameter</td>
</tr>
<tr>
<td>$\chi$</td>
<td>5.584</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.100</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
</tbody>
</table>

### Financial intermediaries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^t$</td>
<td>0.250</td>
<td>Probability of new investment opportunities</td>
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<tr>
<td>$\theta$</td>
<td>0.383</td>
<td>Fraction of assets divertable: perfect interbank market</td>
</tr>
<tr>
<td></td>
<td>0.129</td>
<td>Fraction of assets divertable: imperfect interbank market</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.003</td>
<td>Transfer to entering bankers: perfect interbank market</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>Transfer to entering bankers: imperfect interbank market</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.972</td>
<td>Survival rate of the bankers</td>
</tr>
</tbody>
</table>

### Intermediate good firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
<td>Effective capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Steady-state depreciation rate</td>
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### Capital producing firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_f'/f$</td>
<td>1.500</td>
<td>Inverse elasticity of net investment to the price of capital</td>
</tr>
</tbody>
</table>

### Government

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C/Y$</td>
<td>0.200</td>
<td>Steady-state proportion of government expenditures</td>
</tr>
</tbody>
</table>
Issues Ahead

- More detailed structure of bank capital.
- Different wholesale markets.
- Heterogeneity.
- Non-linearities.
- Optimal policy.