THE
ECONOMIC
JOURNAL

The Economic Journal, 1–26 https://doi.org/10.1093/ej/ueac017

SAFE ASSETS*

Robert J. Barro, Jesús Fernández-Villaverde, Oren Levintal and Andrew Mollerus

This paper investigates the quantity of safe assets. First, we estimate that the average safe-asset ratio (ratio of safe to total assets) in 34 OECD countries was 37% in 2015. Further, we document that this ratio is relatively stable over time. Second, we build a heterogeneous-agent model with rare disasters and risk aversion coefficients that accounts for (i) the average level of the safe-asset ratio; (ii) the stability of this ratio over time; (iii) the observed risk-free rate of around 1.0% per year; and (iv) the empirical unlevered equity premium of about 4.2%. The model also replicates the observed highly concentrated distributions of wealth and equity. Finally, Ricardian equivalence holds in our model: issuing additional government bonds has no effect on rates of return and the net quantity of safe assets. Surprisingly, the crowding-out coefficient for private bonds with respect to public bonds is around −0.5, a value found in empirical studies.

A safe asset is one whose real value is insulated from shocks. Since the global financial crisis of 2008, researchers have been concerned about the scarcity of these safe assets (Caballero *et al.*, 2017; Caballero and Farhi, 2018), the competition among different types of them (Krishnamurthy and Vissing-Jorgensen, 2013; 2015; He *et al.*, 2016; 2019) and the effects from changes in their supply (Krishnamurthy and Vissing-Jorgensen, 2012; Benigno and Nisticò, 2017; Lenel, 2018; Infante, 2020). For a general review of this literature, see Gorton (2017).

Motivated by these concerns, this paper investigates the quantity of safe assets. This task is important because policies taken since the financial crisis (e.g., QE programmes) target the quantities of safe assets and, yet, the literature does not provide a quantitative model of safe-asset creation. We address this issue by, first, measuring the safe-asset ratio (ratio of safe to total assets) in 34 OECD countries and, second, providing a model that can match the observed safe-asset ratio along with key asset-pricing moments.

Our measure of the safe-asset ratio follows the methodology proposed by Gorton *et al.* (2012) for the United States. Using the financial balance sheets of OECD countries, we find that the weighted average safe-asset ratio in these countries was 37% in 2015. Further, we document that this ratio is relatively stable over time (Gorton *et al.*, 2012, have already documented this last fact for the United States).

Our theory comprises a heterogeneous-agent model with rare disasters and risk aversions calibrated at realistic levels. This quantitative model accounts for (i) the average level of the safe-asset ratio; (ii) the stability of this ratio over time; (iii) the observed risk-free rate of 1.0%

* Corresponding author: Jesús Fernández-Villaverde, Economics, University of Pennsylvania, Philadelphia, PA 19104-6297, USA. Email: jesusfv@econ.upenn.edu

This paper was received on 8 July 2020 and accepted on 14 February 2022. The Editor was Francesco Lippi.

The data and codes for this paper are available on the Journal repository. They were checked for their ability to reproduce the results presented in the paper. The replication package for this paper is available at the following address: https://zenodo.org/record/5979683.

We appreciate comments from the editor, two referees, Andrew Abel, Ben Broadbent, Ricardo Caballero, Emmanuel Farhi, Xavier Gabaix, Oded Galor, Nicola Gennaioli, Matteo Maggiori, Ricardo Perez-Truglia, Andrei Shleifer, Jon Steinsson, Mike Woodford, Luigi Zingales, Marco Del Negro, Alex Cukierman, Stefan Lewellen, participants in seminars at Columbia, Brown, Wharton, McGill, IDC, and Banco de España, and at the conference on “Safe Assets” at Columbia University in February 2015.

per year; and (iv) the empirical unlevered equity premium of 4.2%. Thus, we can match assets' quantities and prices, a novel result in the literature.

Two additional results regarding untargeted moments are noteworthy. First, the model replicates highly concentrated wealth and equity distributions. Specifically, the top 5% of agents own 60% of aggregate wealth and 90% of aggregate equity, values close to the shares measured in the United States. Second, the model generates a negative correlation between public and private safe assets with a crowding-out coefficient around −0.5, in line with estimated coefficients documented in previous US studies and in our OECD sample. Interestingly, this crowding-out coefficient is compatible with the version of Ricardian equivalence that holds in our model: issuing additional government bonds does not affect rates of return or the net quantity of safe assets.

We build our model around two key components. First, we have heterogeneous agents to allow risk transfer among agents and, thus, have a meaningful analysis of safe-asset creation. In comparison, a representative-agent setting does not allow for the redistribution of aggregate risk and the economy's quantity of safe assets is nil. Among the many possible forms of agent heterogeneity, differences in the degree of risk aversion are a natural choice for the study of safe-asset creation because this heterogeneity highlights the incentives to transfer risks among agents. We introduce two types of agents, one with a lower risk aversion than the other, so that an agent of type 1 is more willing than an agent of type 2 to absorb risk.[1] At the same time, our use of recursive preferences (Epstein and Zin, 1989; Weil, 1990) lets us keep reasonably high and homogeneous values of the intertemporal elasticity of substitution (IES).[2]

Second, we work with a Lucas-tree world with rare macroeconomic disasters (Lucas, 1978; Barro, 2009). For analytic tractability, we assume that the fruit of the tree is divided between a share that corresponds to labour income (i.e., human capital) and a share that corresponds to equity (i.e., financial capital). In our environment, rare disasters correspond to sharp and possibly permanent drops in the number or productivity of the trees. Examples of rare disasters include financial crises, political upheavals, armed conflicts, natural catastrophes, and pandemics. Rare disasters are key for our analysis because they create a sufficiently strong incentive to share risk among types of agents while keeping reasonable levels of risk aversion and enforcing a consumption process that matches important aspects of the data.

In our model, agents of type 1 issue private safe bonds that are held by agents of type 2. Correspondingly, agents of type 1 own a disproportionate share of risky assets, which in our model correspond to unlevered equity claims on the Lucas tree. In this setting, the equilibrium of our calibrated model can match the observed risk-free rate and equity premium, the safe-asset ratio (and its stability), the highly concentrated wealth and equity distributions, and the negative correlation between public and private safe assets. Also, the model displays high persistence in the income and wealth effects of shocks.

Our paper connects with the literature on asset-pricing models with two types of agents distinguished by their relative risk aversions. Among many others, we highlight Dumas (1989), Wang (1996), Chan and Kogan (2002), Coen-Pirani (2005), Gârleanu and Pedersen (2011), Longstaff and Wang (2012), Brumm *et al.* (2015), Gârleanu and Panageas (2015), Caballero and Farhi (2018), Drechsler *et al.* (2018), and Gomez (2019). Except for the last five references,

[1] To make the model stationary, we assume that agents change their types randomly over their (infinite) lifetime. Type changes can be interpreted as transitions between generations within the same dynasty.

[2] For example, Gruber (2013) estimates an IES of around 2, while Van Binsbergen *et al.* (2012) find it to be 1.7. Bansal and Yaron (2004) and Barro (2009) argue that an IES below 1 produces puzzling patterns in the relationship of growth rates and uncertainty to ratios of stock prices to earnings.

these analyses assume time-separable power utility, augmented in Chan and Kogan (2002) to include external habits. The focus of this literature is on asset-pricing effects from heterogeneous risk aversion. Our paper expands on this literature by studying how heterogeneous risk aversion affects the quantity of safe assets.

Coen-Pirani (2005), Brumm *et al.* (2015), and Drechsler *et al.* (2018) study two-agent economies with recursive preferences, where agents differ only in their risk aversion, but they also introduce constraints on the trade of assets. Coen-Pirani (2005) considers a margin requirement and finds that the model converges asymptotically to a representative-agent economy, despite the occasionally binding constraint. Brumm *et al.* (2015) study a disaster model, but impose constraints on the sale of equity and bonds. By comparison, in our model, agents trade freely in equity and bonds, so that quantities and prices in these markets are determined by agents' desire to share risk, rather than by exogenously imposed constraints on asset trading. Drechsler *et al.* (2018) introduce into the baseline two-agent model a liquidity requirement on the private issuance of safe assets (e.g., bank deposits). Consequently, the ratio of safe to total assets in their model is extremely sensitive to the liquidity cost, a result that deviates from observed data. Gârleanu and Panageas (2015) and Gomez (2019) allow for two agents with recursive utility, but they focus on cases in which each agent's IES is close to the reciprocal of its coefficient of relative risk aversion (i.e., approximate CRRA).

An alternative route to generating high-risk premia with rare disasters is heterogeneity in beliefs across agents. The first paper to point this out was Chen *et al.* (2012), where high risk premia result from the optimistic beliefs of a small group of agents. We are also close to Collin-Dufresne *et al.* (2017), who generate high risk premia through differences in beliefs induced by learning about the mean growth rate of consumption and the probability of rare disasters across generations. However, neither of these two papers investigates the quantity of safe assets, the main goal of our paper.

The rest of the paper is organised as follows. Section 1 documents empirical patterns for safe assets. Section 2 presents our heterogeneous-agent model. For heuristic purposes, we begin with the tractable case where the IES for both types of agents equals one. Section 3 shows that the results extend to settings with a general IES, including variable disaster size and defaultable long-term bonds. Section 4 distinguishes between gross and net quantities of private bonds and studies the role of government bonds. Section 5 relates the model's predictions to our empirical measurements in Section 1. Section 6 concludes.

1. The Safe-Asset Ratio in OECD Countries

We estimate the safe-asset ratio in 34 OECD countries following the methodology of Gorton *et al.* (2012), who undertook this exercise for the United States. Safe assets are defined as the sum of debt liabilities issued by three sectors: (i) government; (ii) central bank; and (iii) financial sector. We include in this measure four types of debt liabilities: (i) currency and deposits; (ii) debt securities; (iii) loans; and (iv) money-market-fund shares.[3] Total assets are defined as the sum of all liabilities issued by all sectors in the economy: general government, the financial sector,

[3] Gorton *et al.* (2012) exclude intra-government loans (e.g., government bonds held by federal retirement programmes) and 15% of long-term bonds issued by financial corporations (e.g., MBS, ABS). Detailed breakdowns into intra-government holdings and asset-backed securities are not available for the OECD data, which follow the classification of SNA2008 (see European Commission *et al.*, 2009, pp.226–7). Hence, we are not able to make similar adjustments in our OECD sample.

Table 1. *The Safe-Asset Ratio—OECD Countries (2015).*

	Safe assets / Total assets	Govt debt / Total assets	Central bank debt / Total assets	Financial debt / Total assets	Total assets / GDP
Austria	0.44	0.13	0.03	0.27	8.2
Belgium	0.37	0.11	0.01	0.24	12.2
Brazil	0.40	0.13	N/A	0.27	6.5
Canada	0.29	0.08	0.01	0.20	11.4
Chile	0.22	0.03	0.03	0.16	7.0
Czech Republic	0.40	0.10	0.07	0.22	4.8
Denmark	0.31	0.04	0.02	0.26	13.4
Estonia	0.27	0.03	0.05	0.19	6.0
Finland	0.34	0.08	0.03	0.24	8.5
France	0.37	0.09	0.02	0.26	12.5
Germany	0.46	0.10	0.04	0.32	7.6
Greece	0.66	0.27	0.12	0.26	7.0
Hungary	0.34	0.13	0.04	0.17	6.7
Iceland	0.25	0.05	0.03	0.17	12.9
Ireland	0.26	0.03	0.01	0.22	26.9
Israel	0.36	0.13	0.05	0.19	7.2
Italy	0.52	0.20	0.04	0.28	7.9
Japan	0.54	0.18	0.06	0.31	12.9
Korea	0.36	0.06	0.03	0.27	8.5
Latvia	0.43	0.09	0.09	0.26	4.9
Lithuania	0.39	0.16	0.07	0.16	3.9
Mexico	0.30	0.13	0.04	0.13	3.9
Netherlands	0.35	0.04	0.01	0.29	20.5
Norway	0.29	0.04	0.01	0.24	8.8
Poland	0.37	0.14	0.04	0.19	4.3
Portugal	0.46	0.17	0.05	0.24	10.3
Slovak Republic	0.43	0.18	N/A	0.25	4.2
Slovenia	0.43	0.21	0.04	0.18	5.2
Spain	0.48	0.14	0.04	0.30	9.6
Sweden	0.27	0.04	0.01	0.22	12.4
Switzerland	0.36	0.02	0.06	0.27	14.2
Turkey	0.34	0.09	0.04	0.21	3.9
United Kingdom	0.40	0.07	N/A	0.33	15.2
United States	0.30	0.10	0.02	0.18	10.4
Mean	0.37	0.11	0.04	0.23	9.4
Mean (weighted)	0.37	0.10	0.03	0.24	11.4
Mean (weighted*)	0.37	0.10	0.03	0.24	11.6
Min.	0.22	0.02	0.01	0.13	3.9
Max.	0.66	0.27	0.12	0.33	26.9

Notes: Column 1 presents the safe-asset ratio, defined as the ratio of safe to total assets. Columns 2–4 show the components of the safe-asset ratio, and column 5 has the ratio of total assets to GDP. The weighted average is based on country values of total assets. Data source is financial balance sheets of OECD countries (SNA 2008).
* Asset-weighted means over a subsample of 16 countries with bond return data, reported in Table 2.

non-financial corporations, households, and the rest of the world. The data are from the financial balance sheets of OECD countries (SNA 2008). These data are roughly equivalent to the US flow of funds used by Gorton *et al.* (2012), but cover many countries, though at a lower level of detail. Our definition of the safe-asset ratio is fixed across countries and time.

Table 1 presents our cross-country estimates in 2015 of safe-asset ratios and Table 2 shows the composition of these ratios. The safe-asset ratios range from 22% (Chile) to 66% (Greece). Among the largest economies, the United States has the lowest safe-asset ratio of 30%, while Japan has the highest ratio, 54%.[4] To be consistent with the model definition of safe assets

[4] Gorton *et al.* (2012) estimate the safe-asset ratio at 31%–33% for the United States. Our estimate of the US safe-asset ratio of 30% is close to Gorton *et al.* (2012), suggesting that our data limitations described in Note 3 are not substantial.

Table 2. *Composition of Safe Liabilities Across Sectors (2015).*

	Government		Central bank		Financial sector			Real returns	
	Bonds	Deposits	Bonds	Deposits	Bonds	Deposits	Money market funds	Bills	Bonds
Austria	0.80	0.20	0.00	1.00	0.22	0.78	0.00	–	–
Belgium	0.79	0.21	0.00	1.00	0.14	0.86	0.00	1.21	3.01
Brazil	0.76	0.24	N/A	N/A	0.23	0.77	0.00	–	–
Canada	0.95	0.05	0.00	1.00	0.31	0.69	0.00	–	3.92
Chile	0.97	0.03	0.46	0.54	0.23	0.69	0.08	–	–
Czech Republic	0.87	0.13	0.00	1.00	0.11	0.89	0.00	–	–
Denmark	0.74	0.26	0.22	0.78	0.49	0.51	0.00	3.08	3.58
Estonia	0.06	0.94	0.00	1.00	0.01	0.99	0.00	–	–
Finland	0.78	0.22	0.00	1.00	0.24	0.75	0.01	0.64	3.22
France	0.82	0.18	0.00	1.00	0.21	0.75	0.04	−0.47	1.54
Germany	0.74	0.26	0.00	1.00	0.18	0.82	0.00	1.51	3.15
Greece	0.18	0.82	0.00	1.00	0.15	0.85	0.00	–	–
Hungary	0.85	0.15	0.00	1.00	0.08	0.89	0.03	–	–
Iceland	0.61	0.39	0.00	1.00	0.37	0.63	0.00	–	–
Ireland	0.65	0.35	0.00	1.00	0.33	0.67	0.00	–	–
Israel	0.97	0.03	0.30	0.70	0.21	0.77	0.02	–	–
Italy	0.82	0.18	0.00	1.00	0.23	0.77	0.00	1.20	2.53
Japan	0.87	0.13	0.00	1.00	0.13	0.87	0.00	0.68	2.54
Korea	0.91	0.09	0.43	0.57	0.22	0.75	0.03	–	–
Latvia	0.59	0.41	0.00	1.00	0.03	0.97	0.00	–	–
Lithuania	0.69	0.31	0.00	1.00	0.00	1.00	0.00	–	–
Mexico	0.89	0.11	0.02	0.98	0.11	0.89	0.00	–	–
Netherlands	0.71	0.29	0.00	1.00	0.33	0.67	0.00	1.37	2.71
Norway	0.51	0.49	0.00	1.00	0.36	0.62	0.01	1.10	2.55
Poland	0.74	0.26	0.23	0.77	0.04	0.96	0.00	–	–
Portugal	0.49	0.51	0.00	1.00	0.18	0.82	0.00	−0.01	2.23
Slovak Republic	0.62	0.38	N/A	N/A	0.06	0.94	0.00	–	–
Slovenia	0.77	0.23	0.00	1.00	0.03	0.97	0.00	–	–
Spain	0.74	0.26	0.00	1.00	0.24	0.69	0.07	−0.04	1.41
Sweden	0.76	0.24	0.27	0.73	0.42	0.57	0.02	1.77	3.25
Switzerland	0.66	0.34	0.00	1.00	0.08	0.92	0.00	0.89	2.41
Turkey	0.85	0.15	0.00	1.00	0.06	0.94	0.00	–	–
United Kingdom	0.87	0.13	N/A	N/A	0.19	0.81	0.00	1.16	2.29
United States	1.00	0.00	0.00	1.00	0.40	0.52	0.08	2.23	2.85

Notes: The table presents the composition of debt liabilities issued by the safe sectors (government, central bank and financial sector) in each country. The figures are shares of total debt liabilities issued by the respective sectors (i.e., columns within each sector add to one). Bonds include short- and long-term debt securities. Deposits include currency, transferable deposits, other deposits, short-term loans and long-term loans (i.e., loans to the safe sectors). Money market funds are shares in money-market funds (on the liability side of the balance sheet). The last two columns report real returns on bills and government bonds from Jordà *et al.* (2019) for the available countries, where data for Canada are from Barro and Ursúa (2008).

(i.e., fixed return with zero default probability), we later focus on a subsample of countries with particularly safe government bonds. In particular, Table 2 also reports bond and bill returns for 16 countries, estimated by Barro and Ursúa (2008) and Jordà *et al.* (2019), for which samples of more than 100 years are available. For these 16 countries, the average bond return is low, varying from 1.4% (Spain) to 3.92% (Canada), close to the US bond return of 2.85%. Hence, we evaluate the government bonds in these 16 countries as safe and calibrate the model to match the average safe-asset ratio in this subsample, which is 37%.

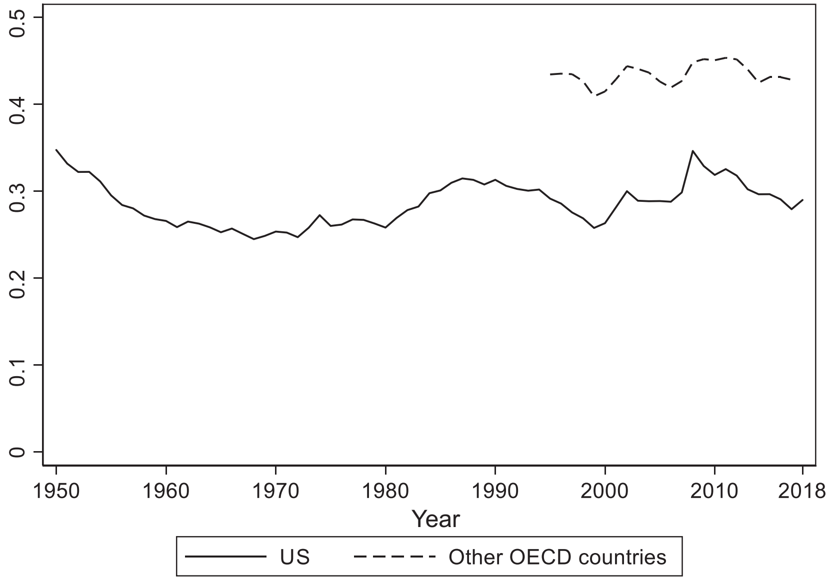

Fig. 1. *The Safe-Asset Ratio in OECD Countries. Safe-asset ratio (safe assets/total assets) for the United States and for the following group of countries (for which we have full data from 1995): Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Japan, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden and United Kingdom.*

Gorton *et al.* (2012) found that the US safe-asset ratio was remarkably stable from 1952 to 2010, despite the dramatic changes in US financial markets over this period. Figure 1 shows a similar finding for other countries for the period 1995–2017 for which data are available. The figure plots the safe-asset ratio for the United States and other countries. This ratio moves around a long-run level of 30% for the United States and 43% for the other countries. The safe-asset ratio is stable over time and in the cross section and did not change significantly even after the global financial crisis of 2008.

While there are large differences across countries in the size of their financial markets, the differences in their safe-asset ratios are much smaller. For instance, Mexico has the smallest ratio of total assets to GDP in the sample, 3.9, while Ireland has the largest, 26.9. Nevertheless, the safe-asset ratio in Mexico is 0.30, close to the 0.26 in Ireland. More generally, the cross-country correlation between the safe-asset ratio and the ratio of total assets to GDP is close to zero, suggesting that the safe-asset ratio is not driven by the size of the financial sector. As we mentioned before, for the long time series for the US economy reported by Gorton *et al.* (2012), the safe-asset ratio has not varied much despite the jump in the ratio of total assets to GDP from 4 in 1952 to 10 in 2010.

The stock of safe assets consists of claims issued by the public sector (government and central bank) and the private sector (financial institutions). Figure 2 shows both types of safe assets in the United States (panel A) and other countries (panel B). Figure 2 shows a crowding-out effect. When public safe assets increase, private safe assets decline. This phenomenon is salient after the

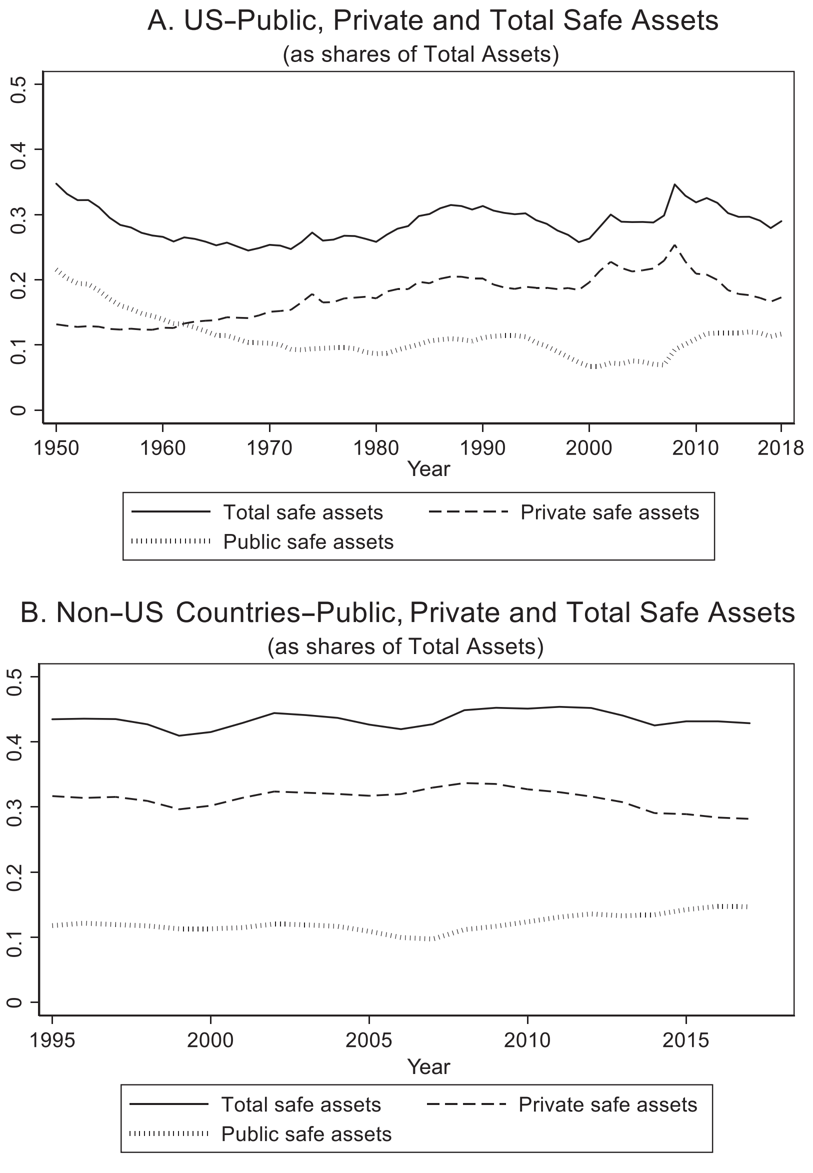

Fig. 2. *Public and Private Safe Assets (as shares of total assets). Public safe assets are defined as debt liabilities of the government and central bank. Private safe assets are debt liabilities of financial institutions. All presented as ratios to total assets. Non-US countries are listed in the note to Figure 1.*

global financial crisis. The US public safe-asset ratio (ratio of public safe assets to total assets) increased between 2007 and 2015 from 7% to 12%, while the private safe-asset ratio (ratio of private safe assets to total assets) declined from 23% to 18%. In non-US countries, the public safe-asset ratio increased over the same period from 10% to 14% and the private safe-asset ratio declined from 33% to 29%.

Table 3. *Regressions of Private Safe Assets on Public Safe Assets.*

		Coefficient	SE	Sample	No. obs.
1.	United States	−0.70	0.09	1950–2018	69
2.	Non-US countries	−0.77	0.19	1995–2017	23
3.	OECD	−0.70	0.24	1995–2017	23
4.	OECD—balanced panel with fixed effects	−0.31	0.04	1995–2017	575
5.	OECD—unbalanced panel with fixed effects	−0.34	0.03	1950–2018	642

Notes: The table presents results for the following regression: private debt $= \alpha + \beta^*$ public debt. Private debt is the ratio of financial corporations' debt to total safe debt. Public debt is the ratio of government and central bank debt to total safe debt. The first row reports regression for US time-series data shown in Figure 2, panel A. The second row reports a similar regression for aggregate time-series data of non-US countries shown in Figure 2 panel B. The third row reports a similar regression for aggregate data of all OECD countries. The fourth row reports a balanced-panel regression with fixed effects. The fifth row reports an unbalanced-panel regression with fixed effects.

Using US data, Gorton *et al.* (2012) and Krishnamurthy and Vissing-Jorgensen (2013) estimate the crowding-out coefficient at around −0.5. We show later that a theoretical benchmark for the crowding-out coefficient in an economy that satisfies Ricardian equivalence is consistent with these studies. In our sample, regressions of private safe assets on public safe assets yield a coefficient of −0.70 (SE 0.09) for the United States and −0.77 (SE 0.19) for non-US countries (Table 3).[5] While these coefficients are larger than those estimated by Gorton *et al.* (2012) and Krishnamurthy and Vissing-Jorgensen (2013), given our data limitations and the large standard errors, we consider them to be in line with the previous evidence.

Our findings suggest that the safe-asset ratio depends on deep parameters, which are relatively robust over time and across countries. Next, we build a model that matches the observed equity premium with realistic risk-aversion coefficients and generates a safe-asset ratio with a magnitude consistent with Table 1. In line with the empirical findings, our model's safe-asset ratio is fairly insensitive to most model parameters and, hence, unlikely to vary much over time.

2. Baseline Model

We work with an economy with two types of agents (one more risk averse than the other), a Lucas tree and two assets (an unlevered equity claim on the Lucas tree and a non-contingent, one-period private bond). The model is set up, for convenience, in discrete time, but we think of the period as short. Parts of the structure of our model parallel Longstaff and Wang (2012).

2.1. *Agents*

The total population is normalised to 1, where a fraction μ is of type 1 and a fraction $1-\mu$ is of type 2. The representative agent of type i, for $i \in \{1, 2\}$, has preferences of the form:

$$U_{i,t} = \left\{ \left(\frac{\rho}{1+\rho}\right) C_{i,t}^{1-\theta} + \left(\frac{1}{1+\rho}\right) \left[E_t \left(U_{i,t+1}^{1-\gamma_i} \right) \right]^{(1-\theta)/(1-\gamma_i)} \right\}^{1/(1-\theta)}. \tag{1}$$

The coefficients controlling relative risk aversion, γ_i, satisfy $0 < \gamma_1 \leq \gamma_2$; that is, agent 1 has relatively low risk aversion. However, the IES, $1/\theta > 0$, and the rate of time preference,

[5] Panel regressions with fixed effects yield a smaller coefficient of −0.3 (Table 3, rows 4–5). Since these regressions do not account for cross-border holdings of safe assets (which may be significant for small open economies), the coefficients may be biased downward. We do not claim causality in our regressions. Instead, we argue that our model generates equilibrium behaviour regarding changes in safe assets that is entirely consistent with these regressions.

$\rho > 0$, are the same for the two agents. This assumption makes the mechanism behind our results transparent. For much of our analysis, we simplify the derivations by assuming $\theta = 1$ (log utility).

2.2. *The Lucas Tree*

The economy is endowed with a single Lucas tree, with size normalised to one, that generates real GDP of Y_t in period t. The tree owners receive αY_t as dividend income and the remaining $(1 - \alpha)Y_t$ is paid to all agents equally as labour income. By market clearing:

$$C_{1,t} + C_{2,t} = Y_t. \tag{2}$$

Ownership of the tree is shared between $K_{1,t}$ and $K_{2,t}$:

$$K_{1,t} + K_{2,t} = 1. \tag{3}$$

In (2)–(3), $C_{i,t}$ and $K_{i,t}$ denote aggregate consumption and equity, respectively, of type i agents. We use a convention whereby $K_{i,t}$ applies at the end of period t, after the payment of the dividend, $K_{i,t-1}\alpha Y_t$, to agent i. The price of the tree in period t in units of consumables is P_t, of which $(1 - \alpha)P_t$ corresponds to the valuation of human capital and αP_t is the market value of financial capital.

The stochastic process that generates Y_t follows specifications like those in previous rare-disaster models, except for the omission of a normally distributed business-cycle shock, which is quantitatively unimportant for our argument once we have rare disasters. There is a constant probability p per period of a disaster. With probability $1 - p$, real GDP grows over one period by the factor $1 + g$, where $g \geq 0$ is constant. With probability p, a disaster occurs, and real GDP grows over one period by the factor $(1 + g) \cdot (1 - b)$, where $b > 0$ is the size of a disaster. When the length of the period is short, the expected growth rate per year of GDP, denoted g^*, is:

$$g^* \approx g - pb, \tag{4}$$

where g and p are measured in annual units.

In the baseline setting, disasters last for only one period and have a single size (Barro, 2009). In Subsection 3.3 we examine the case of a time-invariant size distribution of disasters, as in Barro (2009), and show that our main results hold. With more complexity, we could allow disasters with stochastic duration followed by a tendency for recovery with above-normal growth rates (Nakamura *et al.*, 2013; Barro and Jin, 2021).[6] Other feasible extensions include time variation in p (as in Gabaix, 2012) and g (as in Bansal and Yaron, 2004).

As described later, the baseline calibration sets $p = 0.04$ per year. This probability corresponds to the empirical frequency of disasters—defined as short-term declines in real per capita GDP of at least 10%—in a long-term panel of countries. A drop of 10% is also in line with the early estimates of per capita GDP reductions in countries that imposed aggressive COVID-19 lockdowns during the first half of 2020, such as Italy and Spain. The effective disaster size—i.e., the single value in the representative-agent economy that generates an equity premium corresponding roughly to the full-size distribution of disasters—is $b = 0.32$. The growth-rate parameter, corresponding to the non-disaster mean growth rate of real per capita GDP or consumption, is $g = 0.025$ per year.

[6] The recovery tendency lowers the effective size, b, of a disaster. For some purposes, we could allow for recoveries within the present framework by adjusting b.

2.3. *Assets*

Agents can trade in two assets. The first asset is an unlevered equity claim, $K_{i,t}$, on the tree. Individual agents can go short on this claim. We also consider a non-contingent, one-period private bond, $B_{i,t}$. The bond pays the risk-free interest rate for period t, denoted r_t^f. The amount of principal and interest received or paid on bonds by agent i in period $t+1$ is $(1+r_{t+1}^f)B_{i,t}$. The quantity $B_{i,t}$ is negative for a borrower (issuer of a bond) and positive for a lender (holder of a bond). Since we work with a closed economy, the total quantity of these private bonds, when added up across the two agents, is zero:

$$B_{1,t} + B_{2,t} = 0, \tag{5}$$

where $B_{i,t}$ denotes aggregate bond holdings of type i agents. We ignore other motives to issue safe assets, such as tax incentives, or their use as collateral, medium of exchange, etc.

The baseline model assumes a fixed disaster size. In Subsection 3.3 we extend the model to the case of variable disaster size while maintaining the restriction on two asset types only (equity and bonds). We will also explore the possibility of default and consider the case of long-term bonds. We will find that extensions to variable disaster size, defaultable bonds, and long-term durations do not significantly change our quantitative results.

Each agent's budget constraint for period t is:

$$C_{i,t} + \alpha P_t K_{i,t} + B_{i,t} = \mu_i \ (1-\alpha) Y_t + (\alpha Y_t + \alpha P_t) K_{i,t-1} + \left(1 + r_t^f\right) B_{i,t-1}, \tag{6}$$

where $\mu_i(1-\alpha)Y_t$ denotes labour income obtained by type i agents, with population shares $\mu_1 = \mu$ and $\mu_2 = (1-\mu)$. Dividend income of type i agents is $\alpha Y_t K_{i,t-1}$ and the stock price is αP_t. The choice for period t of $C_{i,t}$, $K_{i,t}$ and $B_{i,t}$ occurs when Y_t, P_t and r_{t+1}^f are known, but Y_{t+1} and P_{t+1} are not.

Let R_{t+1} represent the gross return on an asset between periods t and $t+1$. This return equals $(Y_{t+1} + P_{t+1})/P_t$ for equity and $(1 + r_{t+1}^f)$ for bonds. Each agent seeks to maximise expected utility, given in (1), subject to the budget constraint in (6) and its initial assets. The first-order condition for each agent is:

$$\left[E_t\left(U_{i,t+1}^{1-\gamma_i}\right)\right]^{\left(\frac{\theta-\gamma_i}{1-\gamma_i}\right)} = \left(\frac{1}{1+\rho}\right) E_t\left[{U_{i,t+1}}^{\theta-\gamma_i} \cdot \left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\theta} \cdot R_{t+1}\right]. \tag{7}$$

The results simplify in straightforward ways under log utility, $\theta = 1$. When R_{t+1} equals the risk-free return, $1 + r_{t+1}^f$, and $\theta = 1$, (7) implies:

$$1 + r_{t+1}^f = (1+\rho)/E_t\left[\left(\frac{C_{i,t}}{C_{i,t+1}}\right)\left(\frac{U_{i,t+1}^{1-\gamma_i}}{E_t\left(U_{i,t+1}^{1-\gamma_i}\right)}\right)\right]. \tag{8}$$

In equilibrium, the right-hand side of (8) must be the same for both γ_1 and γ_2. Thus, the prospective paths of uncertain consumption and utility levels for the two agents must accord with the differences in the coefficients of relative risk aversion.

When $\theta = 1$, each agent's consumption in period t is the fraction $\rho/(1+\rho)$ of that agent's resources (including human capital):

$$C_{i,t} = \frac{\rho}{1+\rho} \cdot \left[(Y_t + P_t) \cdot \left((1-\alpha)\mu_i + \alpha K_{i,t-1}\right) + \left(1 + r_t^f\right) \cdot B_{i,t-1}\right]. \tag{9}$$

Giovannini and Weil (1989, app. B) derive this condition for a representative-agent economy. Coen-Pirani (2005) shows that (9) also holds in a two-agent model with heterogeneous risk aversion.

Equations (6), (8) and (9) jointly determine agent i's choices of consumption, $C_{i,t}$, and portfolio allocation, $(K_{i,t}, B_{i,t})$. Adding up (9) for the two agents and using the market clearing conditions from (2), (3) and (5) lead to:

$$P_t = Y_t/\rho. \tag{10}$$

Hence, under log utility, the equity price and the value of total assets are independent of parameters related to the expected growth rate, the uncertainty of the Lucas tree's yield and the degree of risk aversion. Thus, the expected rate of return on equity, r^e, equals the dividend yield, ρ, plus the expected rate of capital gain, which equals g^*, the expected growth rate of GDP and consumption:

$$r^e \approx \rho + g^* \approx \rho + g - pb,$$

where g^* is given in (4). Since r^e is independent of uncertainty parameters, the effects of these parameters on the equity premium work through the risk-free rate, r^f, rather than r^e.[7]

2.4. *Market Equilibrium*

Agent i's wealth at the end of period $t-1$ is $W_{i,t-1} = \alpha P_{t-1} K_{i,t-1} + B_{i,t-1}$, so that agent 1's wealth share at the end of period $t-1$ is $\frac{W_{1,t-1}}{W_{t-1}} = K_{1,t-1} + \frac{B_{1,t-1}}{\alpha P_{t-1}}$. Notice that total wealth, W_{t-1}, equals the market value of equity, αP_{t-1}.

To determine the equilibrium, we require an initial value for agent 1's wealth share. For example, this share might start at the population share (μ) in period 0 with zero debt issue ($K_{1,0} = \mu$, $B_{1,0} = 0$). Heuristically, if $\gamma_1 < \gamma_2$, there is an incentive in this initial position for agent 1 to issue risk-free bonds, so that $B_{1,1} < 0$ in period 1, and these bonds will be held by agent 2, so that $B_{2,1} > 0$. That is, agent 1 borrows from agent 2 on a safe basis. Correspondingly, agent 1 uses its bond issue to increase its share of equity, so that $K_{1,1} > \mu$ and $K_{2,1} < 1 - \mu$. In a richer model with endogenous capital, this process of safe credit creation could affect the equilibrium amount and composition of investment.

The pattern of bond and equity positions shifts risk from the high-risk-aversion agent 2 to the low-risk-aversion agent 1. However, the process does not entail complete risk shifting. Rather, enough bond issuance occurs so that the resulting stochastic paths of future consumption for each agent make the right-hand side of (8) the same for each agent i.

We can use (8) and (9), along with the agents' budget constraints and utility functions, to solve numerically the equilibrium values of the safe interest rate, each agent's consumption and utility, and each agent's allocation of assets between equity and bonds, as functions of the agents' previous portfolios and current GDP.[8] The realisation of Y_t (disaster or no disaster in the present case) then determines each agent's portfolio at the end of period t. These portfolios together with the realisation of Y_{t+1} pin down the equilibrium values for period $t+1$, and so on.

[7] We know from previous analyses of this independent and identically distributed (i.i.d.) setting with a representative agent, such as Barro (2009), that the equity premium is independent of the parameter θ. Thus, setting $\theta = 1$ does not affect the model's implications for the equity premium.

[8] Section 3 carries out this analysis numerically on a detrended version of the model using a quarterly calibration.

Using (6), (9) and (10), we can show for the case $\theta = 1$ that agent 1's wealth share at the end of period t is related to asset holdings from the end of period $t - 1$ in accordance with:

$$\frac{W_{1,t}}{W_t} = K_{1,t-1} + \frac{\rho\left(1 + r_t^f\right) B_{1,t-1}}{(1+\rho)\,\alpha Y_t}.$$

If $g_t \equiv (Y_t/Y_{t-1} - 1)$ is the GDP growth rate, the change in agent 1's wealth share from $t - 1$ to t is:

$$\frac{W_{1,t}}{W_t} - \frac{W_{1,t-1}}{W_{t-1}} \approx \frac{\rho B_{1,t-1}}{\alpha Y_{t-1}}\left(r_t^f - \rho - g_t\right). \tag{11}$$

The approximation is exact for infinitely short periods.

Since the risk-free rate, r^f, will be less than $\rho + g^*$, which equals the expected rate of return on equity, r^e, the expectation of the right-hand side of (11) is positive if $B_{1,t-1} < 0$. The expected change in agent 1's wealth share is positive whenever agent 1 (the low-risk-aversion agent) is borrowing in a risk-free manner from agent 2. We find numerically that agent 1's wealth share asymptotically approaches one, and the ratio of risk-free bonds, $B_{1,t}$, to total assets or GDP asymptotically approaches zero. The reason that agent 1's wealth share tends to rise over time is that this agent's wealth is relatively concentrated in risky equity, which has a higher expected rate of return than risk-free bonds (even after factoring in the occasional macroeconomic disasters, which tend to reduce agent 1's wealth share).[9] Hence, the model behaves in the long run like a representative-agent economy with a coefficient of relative risk aversion equal to γ_1.

The non-stationarity of the initial form of the model makes it unsatisfactory for studying the determination of wealth shares and quantities of safe assets. Previous models with this problem are Dumas (1989) and Longstaff and Wang (2012, p.3208). To go further, we must modify the model to achieve stationarity and keep the expected wealth share of agent 1 asymptotically bounded away from one.

2.5. *Type Changes and Stationarity*

A natural way to achieve stationarity is to have random type changes. Since type 1 agents tend to have above-average wealth, this process redistributes wealth back to type 2 agents.[10] Specifically, assume that a percentage υ of the agents change type every period. The probability of changing from type 1 to type 2 is $\nu(1-\mu)$ and the probability of changing from type 2 to type 1 is $\nu\mu$. Thus, the representative agent within each type has a new wealth that reflects the wealth transfers from agents of the other type.

To allow for the type change, let i and j denote the agent type in period t and $t+1$, respectively, where $i, j \in \{1, 2\}$. Then, agents' preferences are given by:

$$U_{i,t} = \left\{\left(\frac{\rho}{1+\rho}\right) C_{i,t}^{1-\theta} + \left(\frac{1}{1+\rho}\right)\left[E_t\left(U_{j,t+1}^{1-\gamma_i}\right)\right]^{(1-\theta)/(1-\gamma_i)}\right\}^{1/(1-\theta)},$$

where the expectation operator is taken with respect to the distribution of future output and type changes. Namely, agents internalise these type changes in their optimising decisions.

[9] This result holds numerically for our calibrated parameters. Gârleanu and Panageas (2015) provide a full analytical solution of the wealth-share process in a closely related model.

[10] Some forms of tax/transfer systems provide an alternative mechanism to induce a stationary steady-state equilibrium. We explored this approach and found that a graduated income tax rate system can support an interior steady state in equilibrium, whereas a pure flat tax rate does not.

A convenient feature of the model is that the agent's optimisation problem can be normalised by the beginning-of-period resources (Coen-Pirani, 2005). Defining resources, including the market value of human capital, for period t by:

$$A_{i,t} = (Y_t + P_t) \cdot \left((1-\alpha)\,\mu_i + \alpha K_{i,t-1}\right) + \left(1 + r_t^f\right) \cdot B_{i,t-1}, \tag{12}$$

the normalised utility function is:

$$u_{i,t} = \left\{ \left(\frac{\rho}{1+\rho}\right) c_{i,t}^{1-\theta} + \left(\frac{1}{1+\rho}\right) \left(1 - c_{i,t}\right)^{1-\theta} \left[E_t \left[\left(R_{i,t+1} u_{j,t+1}\right)^{1-\gamma_i} \right] \right]^{(1-\theta)/(1-\gamma_i)} \right\}^{1/(1-\theta)}, \tag{13}$$

where $c_{i,t}$ and $u_{i,t}$ denote consumption and utility as ratios to $A_{i,t}$ and $R_{i,t+1}$ is the total return on the agent's portfolio. If the portfolio share of equity and human capital is $x_{i,t} = P_t\,((1-\alpha)\mu_i + \alpha K_{i,t-1})/(P_t((1-\alpha)\mu_i + \alpha K_{i,t-1}) + B_{i,t})$, then $R_{i,t+1}$ is:

$$R_{i,t+1} = x_{i,t} \cdot \left(\frac{Y_{t+1} + P_{t+1}}{P_t}\right) + \left(1 - x_{i,t}\right) \cdot \left(1 + r_{t+1}^f\right).$$

The agents maximise utility in (13) by choosing the ratio of consumption to period t's resources, $c_{i,t}$, and the equity share, $x_{i,t}$. Given that these decisions are independent of $A_{i,t}$, the optimal consumption ratios and equity shares $(c_{i,t}, x_{i,t})$ are identical for all agents of the same type and we can aggregate easily across agent types. The value of assets held as equity and bonds at the end of period t is related to the choices of $c_{i,t}$ and $x_{i,t}$ and to resources, $A_{i,t}$, given in (12), in accordance with:

$$P_t \left((1-\alpha)\,\mu_i + \alpha K_{i,t}\right) = x_{i,t} \cdot (1 - c_{it}) \cdot A_{i,t} \tag{14}$$

$$B_{i,t} = \left(1 - x_{i,t}\right) \cdot \left(1 - c_{i,t}\right) \cdot A_{i,t}. \tag{15}$$

We allow for the transfer of assets across types that occurs when agents change type by changing the expression for resources in (12) to:

$$\begin{aligned} A_{i,t} = (Y_t + P_t) &\cdot \left[(1-\alpha)\,\mu_i + \alpha K_{i,t-1} - \alpha\nu \cdot \left(K_{i,t-1} - \mu_i\right)\right] \\ &+ (1-\nu) \cdot \left(1 + r_t^f\right) \cdot B_{i,t-1}, \end{aligned} \tag{16}$$

where μ_i is the population share of type i. In our case, $\mu_1 = \mu$ and $\mu_2 = 1 - \mu$. Equation (16) says that the resources of type i agents are reduced by the transfers when the equity share of type i agents, $K_{i,t-1}$, exceeds the 'population share', μ_i. Equations (14)–(16) determine the laws of motion of the portfolios of the two types of agents.

For the case of log utility, $\theta = 1$, the expression for the change in the wealth share of agents of type 1 is modified from (11) to:

$$\frac{W_{1,t}}{W_t} - \frac{W_{1,t-1}}{W_{t-1}} \approx \rho \cdot \frac{B_{1,t-1}}{\alpha Y_{t-1}} \cdot \left(r_t^f - \rho - g_t\right) + \nu \cdot \left(\mu - \frac{W_{1,t-1}}{W_{t-1}}\right). \tag{17}$$

If $\upsilon = 0$, as before, the expectation of the right-hand side of (16) is positive if $B_{1,t-1} < 0$. As agent 1's wealth share approaches one, $B_{1,t-1}/Y_{t-1}$ asymptotically approaches zero and (17) implies that the expectation of the change in agent 1's wealth share asymptotically approaches zero. Another property of the equilibrium with $\upsilon = 0$ is that $K_{1,t-1}$ asymptotically approaches one.

Table 4. *Distribution of Wealth and Risky Assets (%).*

	Bottom 90%	90–95%	Top 5%	Top 1%
Net worth	23.5	11.5	64.9	37.2
All equity claims	11.4	9.6	79.0	50.0
Stocks	7.7	7.3	84.9	50.7
Stock funds	7.3	7.5	85.2	51.5
Business	6.0	5.1	88.8	64.6
Indirect stock holdings*	25.0	19.5	55.5	23.7
Real estate (net equity)*	41.0	12.3	46.6	21.6
Jewellery, art, etc.	29.6	9.3	61.1	37.0
Vehicles	74.9	8.3	16.8	6.3

Notes: The table presents net worth share and risky asset shares of the bottom and top percentiles of the net worth distribution.
* Indirect stock holdings through investment funds, IRAs/Keoghs and other funds, as defined by Bhutta *et al.* (2020).
** Residential and non-residential holdings net of debt.
Source. Survey of Consumer Finances, 2019.

If $\upsilon > 0$, when $K_{1,t-1}$ is close to one and $B_{1,t-1}/Y_{t-1}$ is negligible, the term on the far right of (17) is negative and dominates in magnitude the first term on the right. It follows that the expected change in agent 1's wealth share reaches zero before $K_{1,t-1}$ gets close to one and $B_{1,t-1}/Y_{t-1}$ becomes negligible. For this reason, the economy tends to approach a stochastic steady state in which the mean wealth shares for each type of agent are between zero and one. We will next compute these (steady state) mean wealth shares as well as the means of safe assets (expressed relative to total assets or GDP) and risk-free rates, r^f.

3. Quantitative Analysis

We now extend our analysis and consider cases beyond $\theta = 1$, with the IES still the same for both agents. To do so, we solve the model numerically using the Taylor projection algorithm proposed by Levintal (2018). This method has been shown to work well in models with rare disasters by Fernández-Villaverde and Levintal (2018). The algorithm approximates the solution by polynomials that zero the Taylor series of the residual function (up to a finite order) at a given point in the state space. By Taylor's theorem, the residual function is approximately zero around that point; hence, the solution is locally accurate in this neighboorhood. We make the approximation over the long-run domain of the model and simulate the model for 2,000 years. See Levintal (2018) and Fernández-Villaverde and Levintal (2018) and the computational Online Appendix for further details.

Aside from the coefficients of relative risk aversion, γ_1 and γ_2, the baseline parameter values, listed in the note to Table 5 are $\rho = 0.04$ per year (rate of time preference), $g = 0.025$ per year (growth-rate parameter), $p = 0.04$ per year (disaster probability) and $b = 0.32$ (effective disaster size). These values accord with the empirical analysis in Barro and Ursúa (2012). These parameter values imply from (4) that the expected growth rate is

$$g^* = g - p \cdot b = 0.0122 \, \text{per year}.$$

The population share of low-risk-aversion agents is calibrated at $\mu = 0.05$. These agents correspond to the shareholders of the financial system that issue safe liabilities. According to the Survey of Consumer Finances (SCF), Table 4, the top 5% of the population owns almost the entirety of stocks (85% of direct stock holdings, 85% of stock funds and 89% of businesses).

Thus, the data suggest that the top 5% of households own most of the financial sector. Our calibration is in line with previous studies of heterogeneous-agent models that interpret the low-risk-aversion agents as a relatively small group. For instance, Kekre and Lenel (2021) calibrate the population share of low-risk-aversion agents at 4%, Gârleanu and Panageas (2015) at 1% and Gomez (2019) at 5%.[11]

We calibrate the type-change rate at $\nu = 0.02$ per year, which corresponds approximately to the average US adult mortality rate. Accordingly, type changes are interpreted implicitly as transitions between generations within the same dynasty. The baseline case assumes $\theta = 1$ (log utility). Later, we carry out sensitivity analyses for the parameters ν, μ, θ and p.

3.1. *A Representative Agent*

Consider the case of a representative agent, $\gamma_1 = \gamma_2 = \gamma$. In this situation, if we start with agent 1's wealth share at $\mu = 0.05$, the equilibrium quantities of bonds, $B_{1,t}$ and $B_{2,t}$, are zero and the ownership of equity is evenly distributed, $K_{1,t} = 0.05$, $K_{2,t} = 0.95$, over time, irrespective of the realisations of Y_t. Because of log utility, the expected rate of return on equity, r^e, is fixed at $\rho + g^*$, where $\rho = 0.04$ per year and $g^* = 0.0122$, so that $r^e = 0.052$ per year, which is independent of γ.

The price of equity is $P = Y/\rho = 25 \cdot Y$. In this representative-agent case with infinitely short periods, the equilibrium risk-free rate can be written in closed form, if $\gamma \neq 1$, as:

$$r^f = \rho + \theta g + p\left(\frac{\theta - 1}{\gamma - 1}\right) - p(1-b)^{-\gamma} + p\left(\frac{\gamma - \theta}{\gamma - 1}\right)(1-b)^{1-\gamma}.$$

If $\theta = 1$, as γ approaches 1, r^f approaches $\rho + g - pb/(1-b)$.

A higher γ lowers the risk-free rate, r^f, and thereby raises the equity premium. For instance, r^f varies from 0.046 at $\gamma = 1$ to -0.055 at $\gamma = 6$. In our model, which lacks risk-free and costless storage of output, there is nothing special about a negative risk-free rate. An unlevered equity premium between 0.03 and 0.06 (corresponding to historical data) requires γ to be between 3 and 4.5. For a given γ, r^f is fixed over time, regardless of the realisations of Y_t. Recall, however, that this risk-free rate is a shadow rate because no risk-free borrowing and lending occur in equilibrium. That is, no net safe assets are created in this representative-agent environment.

3.2. *Heterogeneity in Risk Aversion*

We move now to the case where agents are heterogeneous in their risk aversion. Table 5 reports our main results. Since the type-change rate is $\nu = 0.02$ per year and $\mu = 0.05$, an agent with high risk aversion (type 2) changes to low risk aversion (type 1) at the rate $\mu \cdot \nu = 0.001$ per year. We also assume that the labour-income share is $\alpha = 2/3$, roughly the average in US data, while the other parameters are the same as in the previous subsection.

Table 5 shows combinations of γ_1 and γ_2 that generate a mean steady-state risk-free rate of $r^f = 0.010$ and a mean steady-state unlevered equity premium of $r^e - r^f = 0.042$. That is, these combinations of γ_1 and γ_2 match the empirical averages of the risk-free rate and the equity

[11] We follow previous papers in sharing labour income equally among all agents. This improves the comparability of our findings with existing results. In any case, changing how labour income is distributed has a minor quantitative effect for the purposes of this paper.

Table 5. *Steady-State Equity Ownership, Wealth Share, and Safe Assets Alternative values of γ_1 and γ_2 that generate $r^e = 0.052$ and $r^f = 0.010$.*

(1) γ_1	(2) γ_2	(3) r^e	(4) r^f	(5) σ^e	(6) σ^f	(7) K_1	(8) W_1/W	(9) $\lvert B_1\rvert/W$	(10) $\lvert B_1\rvert/Y$
3.85	3.85	0.052	0.010	0.0646	0.0000	0.050	0.050	0.000	0.00
3.30	3.89	0.052	0.010	0.0646	0.0000	0.122	0.096	0.026	0.22
2.90	3.98	0.052	0.010	0.0646	0.0002	0.251	0.179	0.072	0.60
2.80	4.02	0.052	0.010	0.0646	0.0004	0.308	0.216	0.092	0.77
2.70	4.07	0.052	0.010	0.0646	0.0005	0.382	0.263	0.119	0.99
2.60	4.15	0.052	0.010	0.0646	0.0008	0.487	0.330	0.157	1.31
2.50	4.29	0.052	0.010	0.0646	0.0012	0.649	0.433	0.216	1.80
2.40	4.54	0.052	0.010	0.0646	0.0017	0.905	0.593	0.312	2.60
2.30	5.50	0.052	0.010	0.0646	0.0032	1.607	1.035	0.571	4.76

Notes: This analysis assumes that the population share of type 1 agents is $\mu = 0.05$, the IES is $\theta = 1$ (log utility), the growth-rate parameter is $g = 0.025$ per year, the rate of time preference is $\rho = 0.04$ per year, the gross type change rate is $\nu = 0.02$ per year, the disaster probability is $p = 0.04$ per year (corresponding in the historical data to contractions in per capita GDP of at least 10%), the disaster size is $b = 0.32$, and the labour-income share is $\alpha = 2/3$. The coefficients of relative risk aversion for the two agents, γ_1 and γ_2, are values that generate a steady-state rate of return on equity, r^e, of 0.052 and a risk-free interest rate, r^f, of 0.010. Volatilities of returns on equity and risk-free rate are denoted σ^e and σ^f, respectively. Columns 7–10 show the (steady-state) means of agent 1's share of equity ownership, K_1, and financial assets, W_1/W, and the ratio of the magnitude of safe assets, B_1, to total financial assets and GDP.

premium.[12] Table 5 documents the corresponding means of a set of variables: agent 1's share of risky assets, K_1, and wealth, W_1/W, and the ratio of the amount of safe assets, $|B_1|$, to wealth and GDP.

The first row of Table 5 shows that, if $\gamma_1 = \gamma_2$, the value of γ_1 and γ_2 needed to generate a mean steady state r^f of 0.010 is 3.85. The next rows show that values of γ_1 below 3.85 require higher values of γ_2. For example, $\gamma_1 = 3.3$ matches up with $\gamma_2 = 3.89$, $\gamma_1 = 2.9$ with $\gamma_2 = 3.98$, $\gamma_1 = 2.6$ with $\gamma_2 = 4.15$, and $\gamma_1 = 2.3$ with $\gamma_2 = 5.5$.

In column 7, the mean of the share of risky assets held by agent 1, K_1, equals 0.05 when $\gamma_1 = \gamma_2$, then increases as γ_1 falls and γ_2 rises. When $\gamma_1 = 2.4$ and $\gamma_2 = 4.54$, the mean of K_1 is 0.91. For $\gamma_1 = 2.3$ and $\gamma_2 = 5.5$, we get $K_1 = 1.6$ so type 2 agents are short on equity (their overall wealth, including the market value of human capital, is still positive). However, as seen in Table 4, this region where equity is shorted does not accord with observed distributions of wealth and risky assets.

In column 8, the mean of the wealth share, W_1/W, starts at 0.05 when $\gamma_1 = \gamma_2$, then rises as γ_1 falls and γ_2 rises. This wealth share equals 0.593 when $\gamma_1 = 2.4$ and $\gamma_2 = 4.54$. As in the data, equity ownership is much more unequally distributed in our model than overall wealth.

Column 9 shows that $|B_1|/W$, the mean of the ratio of safe assets to total assets, rises from 0 when $\gamma_1 = \gamma_2$ to 9.2% when $\gamma_1 = 2.8$ ($\gamma_2 = 4.02$) to 57.1% when $\gamma_1 = 2.3$ ($\gamma_2 = 5.5$). In column 10, the corresponding ratio of safe assets to GDP grows from 0 to 4.8.

Table 6 redoes the analysis for alternative settings of four of the parameters: the reciprocal of the IES, θ, is allowed to be 0.5 or 2.0, rather than 1.0; the gross type-change rate, υ, is 0.03

[12] The table also reports the volatility of equity return, σ^e, and the risk-free rate, σ^f. For the case of unit IES ($\theta = 1$) reported in Table 5, the price-dividend ratio is fixed. Hence, the volatility of the equity return is equal to the volatility of output growth, which is substantially lower than the observed stock-market volatility. For instance, Barro and Ursúa (2012) estimate stock-return volatility and output-growth volatility at 32% and 6.0%, respectively. To increase the model stock-market volatility, Wachter (2013) studies variable disaster probability and Barro and Jin (2021) combine disaster risk with long-run risks. The volatility of the 5-year yield on US Treasury Inflation-Protected Securities (TIPS) is estimated by Gârleanu and Panageas (2015) and Gomez (2019) at 0.7–0.9%, which is slightly higher than the risk-free rate volatility generated by the model.

Table 6. *Alternative Parameter Values.*

(1) γ_1	(2) γ_2	(3) r^e	(4) r^f	(5) σ^e	(6) σ^f	(7) K_1	(8) W_1/W	(9) $\|B_1\|/W$	(10) $\|B_1\|/Y$
$\theta = 0.5$									
2.40	4.54	0.053	0.016	0.0657	0.0013	1.295	0.923	0.373	3.01
$\theta = 2.0$									
2.40	4.54	0.047	0.004	0.0626	0.0023	0.681	0.415	0.266	2.51
$\nu = 0.03$ per year									
2.40	4.54	0.052	0.005	0.0646	0.0013	0.578	0.348	0.229	1.91
$p = 0.02$ per year									
2.40	4.54	0.058	0.033	0.0458	0.0005	0.416	0.236	0.180	1.50
$\mu = 0.1$									
2.40	4.54	0.052	0.013	0.0646	0.0015	1.016	0.662	0.354	2.95

Notes: These results use the parameter values from Table 5, except for the change in the indicated parameter value. The first three lines use the reciprocal of the IES $\theta = 0.5$ (instead of 1.0), the next three use $\theta = 2.0$, the next three use the gross type-change rate $\nu = 0.03$ per year (instead of 0.02), the next three use the disaster probability $p = 0.02$ per year (instead of 0.04), and the last three use the population share $\mu = 0.1$ for the low-risk-aversion type (instead of 0.05).

per year, rather than 0.02; the disaster probability, p, is 0.02 per year, rather than 0.04; and the population share of type 1 agents, μ, is 0.10, rather than 0.05. In each case, the table shows steady-state values of r^e and r^f and the other variables for the case $\gamma_1 = 2.40$, $\gamma_2 = 4.54$, which will be our preferred calibration, as discussed in Subsection 5.1. The other parameters are held fixed at the values assumed in Table 5.

Shifts in the reciprocal of the IES, θ, have moderate effects on the steady-state safe-asset ratio. With $\theta = 1$ (in Table 5), the safe-asset ratio, $|B_1|/W$, for the case $\gamma_1 = 2.4$ ($\gamma_2 = 4.54$) was 0.312, which roughly accords with the data (see Section 5). When $\theta = 0.5$, the safe-asset ratio rises to 0.373, and when $\theta = 2$, it declines to 0.266. These effects are moderate given the large impact on the wealth and equity distributions (the wealth share and the equity share of type 1 agents almost double when θ drops from 2 to 0.5). The effect on rates of return is also moderate. For $\theta = 1$, the mean returns are $r^e = 0.052$ and $r^f = 0.010$. These rates of return change to 0.053 and 0.016, respectively, when $\theta = 0.5$ and to 0.047 and 0.004, respectively, when $\theta = 2$. Return volatilities also change little.

An increase in the type-change rate, ν, means that heterogeneity in risk aversion is lower as agents internalise these type changes in their optimal decisions. Consider a rise in ν from 0.02 to 0.03. The results in the middle of Table 6 show that agent 1's steady-state shares in equity and wealth drop. When $\gamma_1 = 2.4$ and $\gamma_2 = 4.54$, the values of K_1 and W_1/W go from 0.905 and 0.593, respectively, in Table 5 to 0.578 and 0.348 in Table 6. The safe-asset ratio, $|B_1|/W$, changes from 0.312 to 0.229.

The next part of Table 6 halves the disaster probability, p, to 0.02 per year. A lower p makes the steady-state risk-free rate, r^f, substantially higher, around 0.033, rather than 0.010, and stock-market volatility, σ^e, substantially lower (0.046 rather than 0.065). At the same time, the equity premium becomes too low when compared with empirical averages. As in previous research, the model does not match the mean rates of return unless the disaster risk is sufficiently high. A similar conclusion arises if the disaster size, b, is lowered substantially below its initially assumed value of 0.32.

Table 7. *Variable Disaster Size, Long-Term Bonds and Default.*

(1) γ_1	(2) γ_2	(3) r^e	(4) r^f	(5) σ^e	(6) σ^f	(7) K_1	(8) W_1/W	(9) $\|B_1\|/W$	(10) $\|B_1\|/Y$
Variable disaster size									
1.46	4.13	0.052	0.010	0.050	0.0034	0.913	0.604	0.309	2.57
Default probability									
1.46	4.13	0.052	0.018	0.050	0.0257	0.824	0.533	0.291	2.43
Long-term bonds, no default									
1.46	4.13	0.052	0.004	0.050	0.0059	0.847	0.590	0.257	2.15
Long-term bonds with default									
1.46	4.13	0.052	0.013	0.050	0.0256	0.771	0.520	0.251	2.09

Notes: This table introduces a distribution of disaster size following the estimates of Barro and Ursúa (2012). The first row recalibrates the parameters $g = 0.021$, $\gamma_1 = 1.46$, $\gamma_1 = 4.13$ to match asset returns and the ratio of safe to total assets (roughly 31%). The second row assumes the same parameter values but introduces a 40% probability of default on bonds in disaster periods with a default size of 20%. The third and fourth rows assume that bonds pay an infinite stream of geometrically declining coupons, with an average duration of 5 years. Full model specifications are provided in the Online Appendix.

The last part of Table 6 shows the effects from setting the population share, μ, of low-risk-aversion agents to 0.10, rather than 0.05. This shift increases the supply of safe assets (from agents of type 1) compared to the demand (from agents of type 2) and results in a rise in the risk-free rate, r^f, and a drop in the equity premium. The safe-asset ratio changes moderately. For the case $\gamma_1 = 2.4$ and $\gamma_2 = 4.54$, this ratio moves from 0.312 to 0.354.

3.3. *Variable Disaster Size, Default Probability, and Long-Term Bonds*

Our baseline model assumes a fixed disaster size b. In this section, we extend the model to the case of variable disaster size. Specifically, we assume that the disaster size b is an i.i.d. stochastic variable and calibrate its distribution based on data from Barro and Ursúa (2012). We discretise the disaster-size distribution into six disaster-size values: 14%, 24%, 33%, 43%, 55% and 65%, with corresponding frequencies: 60%, 20%, 9%, 7%, 2% and 2%. The mean disaster size is 21.6% and the standard deviation is 12.2%, so the distribution is highly dispersed. We maintain the restriction that agents can issue only equity and bonds. To keep the expected growth rate as in the baseline model, we recalibrate the annual growth rate in normal periods at $g = 0.021$.

Table 7 reports the results. The first row recalibrates the risk-aversion parameters to match an equity premium $r^e - r^f = 0.042$ and a safe-asset ratio $|B_1|/W = 0.31$. The new parameters that match these targets are $\gamma_1 = 1.46$ and $\gamma_2 = 4.13$. In the baseline version, we obtained the same equity premium of 4.2% and safe-asset ratio of 31% for $\gamma_1 = 2.40$ and $\gamma_2 = 4.54$ (see Table 5, row 8). Namely, the introduction of variable disaster size changed the calibrated γ_i's by a small amount. More interestingly, the untargeted moments K_1 and W_1/W (equity and wealth share of type 1 agents) in the variable-disaster-size version are 91% and 60%, respectively, very close to their values in the baseline version (90% and 59%, respectively, see Table 5 row 8). Hence, our main results are robust to the introduction of variable disaster size.

The remaining rows in Table 7 examine the sensitivity of the model to bond default and long-term bonds. First, we introduce a probability that bonds may default in disaster periods, but not in normal periods. We assume that the default probability in disaster periods is 40% and the size of default is 20%.[13] The remaining parameters are the same as in the variable-disaster-size version. Table 7, row 2, shows that the introduction of default probability increases the average bond return from 1.0% to 1.8%. The safe-asset ratio changes slightly from 31% to 29%, the wealth share of type 1 agents decreases from 60% to 53% and the equity share of type 1 agents falls from 91% to 82%.

Second, we relax the assumption that bonds mature after one period. Instead, we assume that bonds pay an infinite stream of coupons $1, \tau, \tau^2, \ldots$, where $1/(1-\tau)$ is a measure of average duration ($\tau \in [0, 1]$).[14] Table 7, row 3, presents the results for a duration of 5 years. We find that the bond return decreases from 1.0% to 0.4% and the safe-asset ratio declines from 31% to 26%.[15] When we combine long-term bonds with default probability, the safe-asset ratio is 25%, compared to 31% in the baseline variable-disaster-size model. Overall, the extension of the model to defaultable long-term bonds generates relatively small changes in the quantitative results of the model.

3.4. *Dynamics*

The dynamics of the economy can be described in terms of the evolution of type 1 agents' share in total wealth, W_1/W. Disaster shocks affect this wealth share and have persistent influences on variables such as the risk-free interest rate, r^f, and the ratio of safe to total assets.

Figure 3 shows the dynamics of the economy starting from a steady state and assuming the realisation of a disaster of size $b = 0.32$ in period 1 when $\gamma_1 = 2.4$ and $\gamma_2 = 4.54$. The paths of variables in Figure 3 assume no further disasters. We plot, over 10 years, agent 1's wealth share, W_1/W, the risk-free interest rate, r^f, agent 1's share of total equity, K_1, and the safe-asset ratio, $|B_1|/W$.

Because of agent 1's relatively high concentration in risky assets, its wealth share, W_1/W, falls with the disaster from 0.59 to 0.45. The share rises thereafter (in the absence of further disasters), but remains below the steady-state value even after 10 years when the share reaches 0.52. In other words: a relatively low inequality of wealth and consumption persists for a long time after a disaster shock. Simultaneously, the recovery towards the steady state is accompanied by rising inequality. These patterns also appear in agent 1's share of equity, K_1. This share falls on impact from its steady-state value of 0.91 to 0.72, then rises to 0.82 after 10 years.

The disaster shock and consequent shift in relative wealth towards agent 2 raises the demand for safe bonds (from agent 2) compared to the supply (from agent 1). In response to this shift in excess demand, r^f falls on impact from its mean value of 0.0100 to 0.0075. In the recovery period, r^f rises, but remains below its steady-state value. After 10 years, r^f reaches 0.0088. The decline in agent 1's wealth share is accompanied on impact by a fall in the safe-asset ratio, $|B_1|/W$, from its steady-state value of 0.31 to 0.27. The ratio then rises gradually and reaches 0.30 after 10 years. In line with our previous arguments, the variations in the safe-asset ratio are relatively moderate, despite the large changes in the wealth distribution caused by the disaster shock. The stability of the safe-asset ratio can also be demonstrated by simulating the

[13] Specifically, if the bond price is q_t, the bond return next period is $1/q_t - 1$ in normal periods, and $(1 - \delta_{t+1})/q_t - 1$ in disaster periods, where δ_{t+1} is an i.i.d. shock that equals 0.2 with probability 0.4 and zero otherwise. We model default as a reduced payment by the issuer of the bonds to the holder (i.e., a form of partial risk-sharing).

[14] The return on the long-term bond is $(1 + \tau q_{t+1})/q_t - 1$, where q_t is the bond price in period t.

[15] Long-term bond returns are negatively correlated with output growth. Hence, they pay a lower return than short-term bonds. In a disaster, the wealth share of type 1 agents falls and the safe return rises. Consequently, long-term bond prices increase, creating a negative correlation with GDP growth.

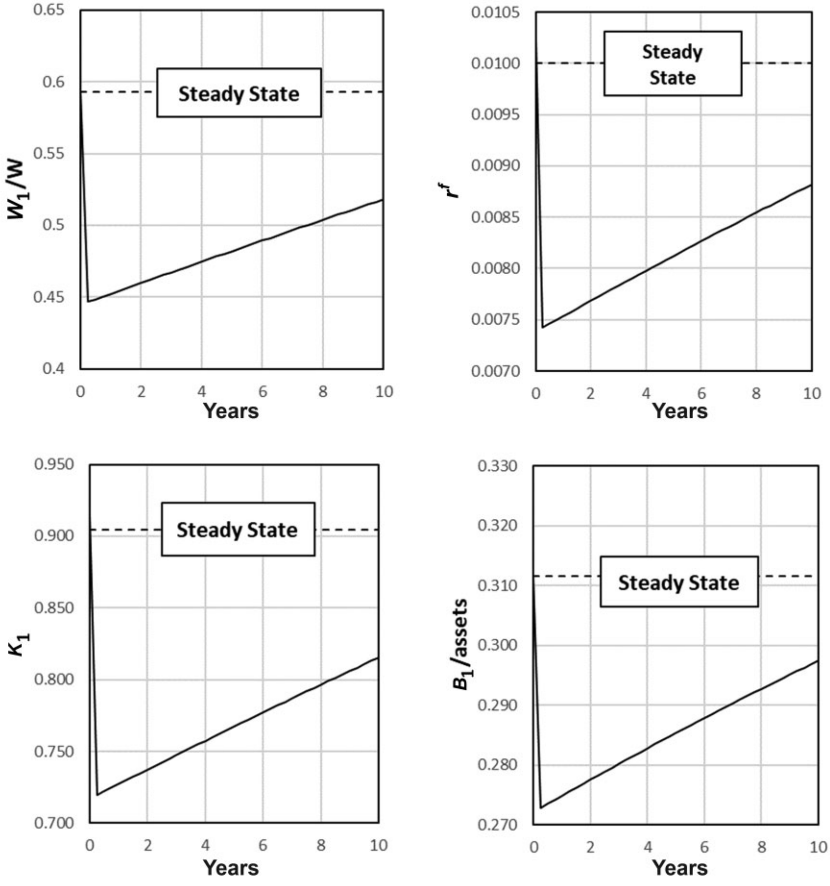

Fig. 3. *Dynamic Paths Following a Disaster. These results correspond to the case where* $\gamma_1 = 2.4$ *and* $\gamma_2 = 4.54$ *in Table 6. The simulated paths start from the steady-state value of* W_1/W*, 0.593, then assume that a disaster of proportionate size 0.32 materialises in period 1. Subsequently, no further disasters occur. The panels show the dynamic paths after period 1 for agent 1's wealth share* W_1/W*, the risk-free interest rate,* r^f*, agent 1's share of total equity,* K_1*, and the ratio of the magnitude of safe assets,* B_1*, to total assets.*

model for a long period with no realised disasters, which is consistent with the stable US safe-asset ratio post-World War II shown in Figure 2 (see the working paper version, Barro *et al.*, 2017).

4. Gross versus Net Lending and Ricardian Equivalence

The bond holdings, B_1, shown in Table 5, correspond to net safe lending from high-risk-aversion agent 2 to low-risk-aversion agent 1. However, gross bond issuance is not pinned down because the model would admit unlimited borrowing and lending within types. That is, agent 1 could issue an arbitrary amount of bonds to itself, and analogously for agent 2. If the model were augmented to include an infinitesimal amount of transaction costs for bond issuance or collection of interest and principal, borrowing and lending within types would not occur in equilibrium. In this case, the quantity of bonds, B_1, in Table 5 would be the unique equilibrium for the gross amount outstanding.

If transaction costs associated with bonds are substantial, the quantity of net bond issuance and the risk-free rate might differ significantly from the values in Table 5. Correspondingly, the risk-free rate received by lenders (agent 2) would deviate from that paid by borrowers (agent 1). For example, if transaction costs were prohibitive, the quantity of net bond issuance would be 0, and the share of capital held by each agent type would be equal to its population share. If $\gamma_1 = 2.4$ and $\gamma_2 = 4.5$, we can calculate that agent 1 would be willing to pay a rate of 0.033 per year at the margin on risk-free borrowing, whereas agent 2 would be willing to accept a rate of -0.005 per year at the margin on risk-free lending. However, no issuance of safe debt occurs because of the prohibitive transaction costs.

Suppose now that the government issues one-period bonds with characteristics corresponding to those of private bonds. The real interest rate on government bonds held from t to $t+1$ must then be r^f, the same as that on private bonds. The simplest way to introduce public debt is for the government to make a lump-sum transfer of bonds B_t^g. The aggregate principal and interest, $(1 + r_{t+1}^f)B_t^g$, is paid out to government bondholders in period $t+1$. This payout is financed by lump-sum taxes, levied in period $t+1$. We assume that future tax payments may be distributed unequally, based on some measure of inequality (e.g., income, consumption or wealth distribution). Without loss of generality, suppose that type 1 agents pay next period ω taxes and type 2 pay $1-\omega$. The government transfers the bonds to the agents according to their tax shares: type 1 agents get ω bonds and type 2 get $1-\omega$. Given the future tax liabilities, the net present value of the bond transfer is zero for each agent. Hence, the wealth distribution is not affected by the bond transfer.[16]

The representative-agent version of this model, where $\gamma_1 = \gamma_2 = \gamma$, exhibits full Ricardian equivalence. That is, the representative agent willingly holds additional government bonds with no changes in equilibrium rates of return, including the risk-free rate, r^f. This result differs from those in Blanchard (1985) and Gârleanu and Panageas (2015), where new agents receive no bequests and, therefore, arrive with below-average assets (although they have labour income). In their settings, a rise in government bonds tends to generate an increase in r^f. In contrast, when we set $\gamma_1 < \gamma_2$ in our model, bond issuance by the government does not affect the rates of return or the amount of net borrowing and lending between agents 1 and 2. That is, the equilibrium features Ricardian equivalence with respect to net quantities of safe assets and the various rates of return.

Consider now how the added government bonds end up being held by agents 1 and 2. One possibility is that each type holds the government bonds they initially received (corresponding to ω and $1-\omega$ for types 1 and 2, respectively). These quantities balance the present value of the (certain) tax liabilities imposed on each type. The quantity of net private borrowing and lending, corresponding to $B_{1,t}$, is then the same as before.

In this proposed equilibrium, agent 1 is simultaneously holding government bonds and issuing private bonds. Since both types of bonds are assumed to be indistinguishable, this is equivalent to trading on a single bond market with a single rate of return. Hence, agent 1 would be operating simultaneously on both sides of this bond market. As before, if there are infinitesimal transaction costs for bond issuance or collection of interest and principal, this equilibrium would be ruled

[16] If the government transfer changes the wealth distribution, it induces distributional effects, and Ricardian equivalence no longer holds. Our analysis focuses on cases where Ricardian equivalence holds, and yet public bonds crowd out private bonds.

out. Agent 1 would be motivated to sell its government bonds and use the proceeds to retire private bonds.

In the full equilibrium, the magnitude of the reduction in private bonds equals the present value of future taxes paid by agent 1. Since we have assumed that this amount was ω of the total government bond issue, the reduction in private bonds expressed as a ratio to government bonds issued is ω as well. That is, the crowding-out coefficient for private bonds with respect to government bonds is $-\omega$.[17]

When compared to the equilibrium prior to the government bond issue, the only difference is that some of the borrowing and lending between agents 1 and 2 is purely private, while some works through the government as intermediary (collecting taxes from agent 1 and using the proceeds to pay principal and interest on the government bonds held by agent 2). When viewed this way, the finding of Ricardian equivalence is not surprising: it corresponds to the assumption that the private sector and the government are equally good at arranging loans between agents 1 and 2.

The surprising part of our result is that the crowding-out coefficient for private bonds with respect to public bonds is $-\omega$, not 0.0 or -1.0. In the data, the tax share ω of type 1 agents is likely to be related to the income or wealth distribution. In the United States, the top 5% of the income distribution paid 0.424 of all federal taxes in 2017 (Congressional Budget Office, 'The Distribution of Household Income 2017'). In our benchmark calibration, the wealth share of type 1 agents is 0.593, the consumption share is 0.231, and the income share (not including capital gains) is 0.326. Hence, ω should be in the range of these numbers, perhaps around 0.5.[18]

5. The Quantity of Safe Assets

In our model, the quantity of safe assets corresponds to the shifting of risk from agent 2 to agent 1. Using data to match the model's predictions for the quantity of safe assets is challenging because it is unclear how to measure empirically the amounts of these assets.[19] In Section 1, we defined safe assets as total debt liabilities of the government and the private financial sector. Our evidence suggested a weighted average for the safe-asset ratio of 37%, with a range from 22% to 66%. Also, this ratio was stable over time. How does our model account for these observations?

5.1. *Level*

We calibrate the model to match the equity premium found by Barro and Ursúa (2012) and the safe-asset ratio measured in Section 1. The weighted average of the safe-asset ratio of 37% reported in Table 1 (for the subsample of 16 safe countries) includes a ratio of 13% of total assets attributable to the public sector (government and central bank debt). We think of our baseline model as predicting the safe-asset ratio in the absence of public-sector debt. We adjust the observed 37% value by subtracting the public-sector ratio (13%) net of the estimated crowding-out of private safe assets, assuming as in the previous section a crowding-out coefficient of 0.5.

[17] Abel (2017) used an earlier version of our analysis, Barro and Mollerus (2014), to generalise our results on crowding-out.

[18] The earlier version of this paper, Barro *et al.* (2017), discusses the effects of transaction costs in private borrowing, distortionary taxes and different allocation of government bonds to the agents.

[19] We focus on the model's predictions about the ratio of safe to total assets, rather than the ratio of safe assets to annual GDP. The latter ratio depends on the ratio of total assets to annual GDP, which equals $1/\rho$ in the baseline model with log utility (e.g., 25 when $\rho = 0.04$). However, in a more general model, this ratio would differ from $1/\rho$.

We then estimate that the private safe-asset ratio (absent public-sector debt) would be around 31%. To match an equity premium of 4.2% and a safe-asset ratio of 31%, we choose from Table 5 the plausible values of risk aversion of $\gamma_1 = 2.40$ and $\gamma_2 = 4.54$. Under these values, our model matches successfully the two targeted moments.

What is the model's performance in predicting untargeted moments? To answer that question, we compare the portfolio of the low-risk-aversion agents in our model to the top 5% of the wealth distribution, which we have used to calibrate the population share of the low-risk-aversion agents. The wealth and equity shares of type 1 agents in our calibrated model are 0.59 and 0.91, respectively (Table 5, row 8). These numbers match reasonably well with the portfolios of the top 5% reported in the SCF (Table 4). In the data, this group's net-worth share is 64.9%, only slightly larger than our model prediction of 59%. Also, in the data, this group holds 79% of equity claims, close to our model prediction of 90%.

We need, however, to discuss three important adjustments to the measure of safe assets that go beyond the scope of our simple environment. First, Gorton *et al.* (2012) include government liabilities as safe assets, but do not include any portion of capitalised future taxes as 'safe liabilities', even at the margin. Although tax liabilities cannot be directly traded, it is also true that these liabilities—and how they vary along with changes in the quantity of government bonds—affect economic analyses of public debt. To the extent that agents factor in future taxes, the gross public debt would overstate a meaningful measure of safe assets. A richer model that includes a government could analyse this issue.

Second, the Gorton *et al.* (2012) measure does not compute a net figure for liabilities of financial institutions; that is, there is no deduction for safe assets held by these institutions. For instance, in 2007–2008, Lehman Brothers issued bonds and commercial paper but also held US government securities and liabilities of other financial firms. Our model could be extended to account for this kind of borrowing and lending within types. These patterns might arise because of idiosyncratic shocks that affect individual agents within types still defined by coefficients of relative risk aversion. On this ground, Gorton *et al.*'s (2012) measured liabilities of government and financial institutions would overstate the net quantity of safe assets.

Third, an array of financial arrangements (including structured finance, stock options, and insurance contracts) can be used to convert risky assets into relatively safe assets. Thus, the measured liabilities of governments and financial institutions might understate the quantity of safe assets.

The net effects from these adjustments to the measured quantity of safe assets are ambiguous. Quantification of the net effects requires empirical work beyond the scope of the present paper.

5.2. *Stability*

The observed stability of the safe-asset ratio accords well with the model. The results in Table 5 indicate that large changes in the mean of the safe-asset ratio might arise from changes in the gap between the risk-aversion coefficients of the high- and low-risk-aversion types, that is, $\gamma_1 - \gamma_2$. However, if this gap was roughly constant (and the other parameters of the model were fixed), then the mean of the safe-asset ratio would be stable.

Comparing Table 6 with Table 5 indicates that variations in a set of other parameters (the IES, $1/\theta$, the gross replacement rate, ν, and the share of type 1 agents, μ) do not have large effects on the steady-state safe-asset ratio. Thus, variations over time in these other parameters are unlikely to be major sources of variation in safe-asset ratios. In contrast, permanent changes in the disaster probability, p, generate substantial effects on the equity premium and the safe-asset ratio. In practice, however, changes in the disaster probability (estimated from options prices) are short-lived (Barro and Liao, 2021) and, hence, would not generate sustained variations in safe-asset ratios.

Finally, the results in Figure 3 show that the safe-asset ratio does not vary greatly along a dynamic path that is approaching its steady state. For instance, in Figure 3, the safe-asset ratio varies only from 31% to 27% over a period of 10 years.

5.3. *Cross-Sectional Differences*

Table 1 shows that the safe-asset ratio in our sample of OECD countries ranges from 22 to 66%. An alternative interpretation of Table 5 is that our model can account for these differences through relatively small variations in risk-aversion heterogeneity across countries. For example, to go from the ratio of 22% of Chile to the ratio of 31% of Denmark, we only need to move from $\gamma_1 = 2.50$ and $\gamma_2 = 4.29$ to $\gamma_1 = 2.40$ and $\gamma_2 = 4.54$.

In addition, we can also account for the cross-sectional differences in the safe-asset ratio in our sample through variations in aggregate risk or public debt, in the latter case due to the crowding-out effect of $-\omega$. For instance, compare Japan and the United States. With a benchmark value of $\omega = 0.5$, the difference in public debt (24% in Japan vs. 12% in the United States) can account for one-quarter of the difference in the safe asset ratio (54% in Japan vs. 30% in United States). We leave for future research a thorough investigation of the differences in the cross-sectional distribution of safe assets.

6. Conclusions

We constructed a model with heterogeneity in risk aversion to study the quantity of safe assets. Despite its simplicity, the model can quantitatively account for the observed risk-free rate, equity premium, safe-asset ratio (and its stability), concentrated wealth and equity distributions, and the negative correlation between public and private safe assets, all while using plausible risk-aversion coefficients. Furthermore, in the baseline setting, Ricardian equivalence holds in that the quantity of government bonds does not affect rates of return or the net quantity of safe assets.

The basic structure of the model with heterogeneity in coefficients of relative risk aversion can be applied to other economic problems. For example, the framework can incorporate credit-market imperfections, including the necessity for enforcement mechanisms to ensure repayment of private debts. This extension relates to issues concerning collateral, liquidity and asymmetric information. This type of extension would be important for assessing implications for the magnitude and composition of investment.

Harvard University, USA
University of Pennsylvania, USA
Reichman University, Israel
Columbia University, USA

Additional Supporting Information may be found in the online version of this article:

Online Appendix
Replication Package

References

Abel, A.B. (2017). 'Crowding out in Ricardian economies', *Journal of Monetary Economics*, vol. 87, pp. 52–66.

Bansal, R. and Yaron, A. (2004). 'Risks for the long run: A potential resolution of asset pricing puzzles', *Journal of Finance*, vol. 59(4), pp. 1481–509.

Barro, R.J. (2009). 'Rare disasters, asset prices, and welfare costs', *American Economic Review*, vol. 99(1), 243–64.

Barro, R.J., Fernández-Villaverde, J., Levintal, O. and Mollerus, A. (2017). 'Safe assets', Working paper no. 20652, NBER.

Barro, R.J. and Jin, T. (2021). 'Rare events and long-run risks', *Review of Economic Dynamics*, vol. 39, pp. 1–25.

Barro, R.J. and Liao, G.Y. (2021). 'Rare disaster probability and options pricing', *Journal of Financial Economics*, vol. 139(3), pp. 750–69.

Barro, R.J. and Mollerus, A. (2014). 'Safe assets', Ms., Harvard University.

Barro, R.J. and Ursúa, J.F. (2008). 'Macroeconomic crises since 1870', *Brookings Papers on Economic Activity*, vol. 2008(1), pp. 255–350.

Barro, R.J. and Ursúa, J.F. (2012). 'Rare macroeconomic disasters', *Annual Review of Economics*, vol. 4(1), pp. 83–109.

Benigno, P. and Nisticò, S. (2017). 'Safe assets, liquidity, and monetary policy', *American Economic Journal: Macroeconomics*, vol. 9(2), pp. 182–227.

Bhutta, N., Bricker, J., Chang, A.C., Dettling, L.J., Goodman, S., Hsu, J.W., Moore, K.B., Reber, S., Volz, A.H. and Windle, R. (2020). 'Changes in US family finances from 2016 to 2019: Evidence from the Survey of Consumer Finances', *Federal Reserve Bulletin*, vol. 106(5), pp. 1–42.

Blanchard, O.J. (1985). 'Debt, deficits, and finite horizons', *Journal of Political Economy*, vol. 93(2), pp. 223–47.

Brumm, J., Grill, M., Kubler, F. and Schmedders, K. (2015). 'Collateral requirements and asset prices', *International Economic Review*, vol. 56(1), pp. 1–25.

Caballero, R.J. and Farhi, E. (2018). 'The safety trap', *Review of Economic Studies*, vol. 85(1), pp. 223–74.

Caballero, R.J., Farhi, E. and Gourinchas, P. (2017). 'The safe assets shortage conundrum', *Journal of Economic Perspectives*, vol. 31(3), pp. 29–46.

Chan, Y.L. and Kogan, L. (2002). 'Catching up with the Joneses: Heterogeneous preferences and the dynamics of asset prices', *Journal of Political Economy*, vol. 110(6), pp. 1255–85.

Chen, H., Joslin, S. and Tran, N.K. (2012). 'Rare disasters and risk sharing with heterogeneous beliefs', *Review of Financial Studies*, vol. 25(7), pp. 2189–224.

Coen-Pirani, D. (2005). 'Margin requirements and equilibrium asset prices', *Journal of Monetary Economics*, vol. 52(2), pp. 449–75.

Collin-Dufresne, P., Johannes, M. and Lochstoer, L.A. (2017). 'Asset pricing when this "time is different"', *Review of Financial Studies*, vol. 2, pp. 505–38.

Drechsler, I., Savov, A. and Schnabl, P. (2018). 'A model of monetary policy and risk premia', *Journal of Finance*, vol. 73(1), pp. 317–73.

Dumas, B. (1989). 'Two-person dynamic equilibrium in the capital market', *Review of Financial Studies*, vol. 2, pp. 157–88.

Epstein, L.G. and Zin, S.E. (1989). 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework', *Econometrica*, vol. 57(4), pp. 937–69.

European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations, and World Bank. (2009). *System of National Accounts 2008*, New York: Authors.

Fernández-Villaverde, J. and Levintal, O. (2018). 'Solution methods for models with rare disasters', *Quantitative Economics*, vol. 9(2), pp. 903–44.

Gabaix, X. (2012). 'Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance', *Quarterly Journal of Economics*, vol. 127(2), pp. 645–700.

Gârleanu, N. and Panageas, S. (2015). 'Young, old, conservative and bold: The implications of heterogeneity and finite lives for asset pricing', *Journal of Political Economy*, vol. 123(3), pp. 670–85.

Gârleanu, N. and Pedersen, L. (2011). 'Asset prices and deviations from the law of one price', *Review of Financial Studies*, vol. 24(6), pp. 1980–2022.

Giovannini, A. and Weil, P. (1989). 'Risk aversion and intertemporal substitution in the capital asset pricing model', Working paper no. 2824, NBER.

Gomez, M. (2019). 'Asset prices and wealth inequality', Ms., Columbia University.

Gorton, G. (2017). 'The history and economics of safe assets', *Annual Review of Economics*, vol. 9, pp. 547–86.

Gorton, G., Lewellen, S. and Metrick, A. (2012). 'The safe-asset share', *American Economic Review*, vol. 102(3), pp. 101–6.

Gruber, J. (2013). 'A tax-based estimate of the elasticity of intertemporal substitution', *The Quarterly Journal of Finance*, vol. 3(1), pp. 1–20.

He, Z., Krishnamurthy, A. and Milbradt, K. (2016). 'What makes US government bonds safe assets?', *American Economic Review*, vol. 106(5), pp. 519–23.

He, Z., Krishnamurthy, A. and Milbradt, K. (2019). 'A model of safe asset determination', *American Economic Review*, vol. 109(4), pp. 1230–62.

Infante, S. (2020). 'Private money creation with safe assets and term premia', *Journal of Financial Economics*, vol. 136(3), pp. 828–56.

Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M. and Taylor, A.M. (2019). 'The rate of return on everything, 1870–2015', *Quarterly Journal of Economics*, vol. 134(3), pp. 1225–98.
Kekre, R. and Lenel, M. (2021). 'Monetary policy, redistribution and risk premia', Working paper no. 28869, NBER.
Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). 'The aggregate demand for treasury debt', *Journal of Political Economy*, vol. 120(2), pp. 233–67.
Krishnamurthy, A. and Vissing-Jorgensen, A. (2013). 'Short-term debt and financial crises: What we can learn from US treasury supply', Ms., Northwestern University.
Krishnamurthy, A. and Vissing-Jorgensen, A. (2015). 'The impact of treasury supply on financial sector lending and stability', *Journal of Financial Economics*, vol. 118(3), pp. 571–600.
Lenel, M. (2018). 'Safe assets, collateralized lending and monetary policy', Ms., Stanford University.
Levintal, O. (2018). 'Taylor projection: A new solution method for dynamic general equilibrium models', *International Economic Review*, vol. 59(3), pp. 1345–73.
Longstaff, F.A. and Wang, J. (2012). 'Asset pricing and the credit market', *Review of Financial Studies*, vol. 25(11), pp. 3169–215.
Lucas, R.E., Jr. (1978). 'Asset prices in an exchange economy', *Econometrica*, vol. 46(6), pp. 1429–45.
Nakamura, E., Steinsson, J., Barro, R.J. and Ursúa, J.F. (2013). 'Crises and recoveries in an empirical model of consumption disasters', *American Economic Journal: Macroeconomics*, vol. 5(3), pp. 35–74.
Van Binsbergen, J.H., Fernández-Villaverde, J., Koijen, R.S.J. and Rubio-Ramírez, J. (2012). 'The term structure of interest rates in a DSGE model with recursive preferences', *Journal of Monetary Economics*, vol. 59(7), 634–48.
Wachter, J.A. (2013). 'Can time-varying risk of rare disasters explain aggregate stock market volatility?' *Journal of Finance*, vol. 68(3), pp. 987–1035.
Wang, J. (1996). 'The term structure of interest rates in a pure exchange economy with heterogeneous investors', *Journal of Financial Economics*, vol. 41(1), pp. 75–110.
Weil, P. (1990). 'Nonexpected utility in macroeconomics', *Quarterly Journal of Economics*, vol. 105(1), pp. 29–42.