Safe Assets*

Robert J. Barro, Jesús Fernández-Villaverde, Oren Levintal, Andrew Mollerus,
Harvard University, University of Pennsylvania,
Interdisciplinary Center (IDC) Herzliya, Columbia University

June 2020

Abstract

To analyze the process of safe-asset creation, we build a Lucas-tree model with heterogeneity in risk aversion among agents and rare macroeconomic disasters. Safe assets take the form of private bond issues from low-risk-aversion to high-risk-aversion agents. We derive the equilibrium values of the ratio of safe-to-total assets, the shares of each agent in equity ownership and wealth, and the rates of return on safe and risky assets. Our model matches the observed risk-free rate of 1.0% per year, the unlevered equity premium of 4.2%, and the safe-asset share of 37% (once we adjust for human capital) with a plausible calibration of risk-aversion parameters. A disaster shock leads to an extended period in which the share of wealth held by the low-risk-averse agent and the risk-free rate are low but rising, and the ratio of safe-to-total assets is high but falling. Ricardian equivalence holds in our model: added government bonds have no effect on rates of return and the net quantity of safe assets. Surprisingly, the crowding-out coefficient for private bonds with respect to public bonds is not 0 or -1, but around -0.5, a value found in some existing empirical studies.

* This version: June 2020. We appreciate comments from Andrew Abel, Ben Broadbent, Ricardo Caballero, Emmanuel Farhi, Xavier Gabaix, Oded Galor, Nicola Gennaioli, Matteo Maggiori, Ricardo Perez-Truglia, Andrei Shleifer, Jon Steinsson, Mike Woodford, Luigi Zingales, Marco Del Negro, Alex Cukierman, Stefan Lewellen, participants of seminars in Columbia, Brown, Wharton, McGill, IDC and Banco de España, and participants at the conference on “Safe Assets” at Columbia University in February 2015.
1 Introduction

In a Lucas-tree world (Lucas, 1978), the aggregate risk reflects the uncertainty in the GDP process, which equals the fruit that drops from the tree. This process may include rare macroeconomic disasters, which correspond to sharp and possibly permanent drops in the number or productivity of the trees. Examples of rare disasters include financial crises, political upheavals, armed conflicts, natural catastrophes, and health crises such as the 2020 COVID-19 pandemic.

A safe asset in this environment is one whose real value is insulated from shocks, including rare disasters. If the GDP process is given, safe assets cannot mitigate overall risk, but can only redistribute this risk across agents. In a representative-agent setting, the redistribution of aggregate risk cannot occur, and the economy’s equilibrium quantity of safe assets will be nil. While it is possible to construct safe assets by issuing risk-free bonds, by creating a financial structure with risk-free tranches, by entering into a variety of insurance contracts, and so on, the creation of any of these safe assets always goes along with the symmetric expansion in the riskiness of levered claims on the underlying asset, i.e., the Lucas tree. In equilibrium, the representative agent ends up holding the overall risk, and this total magnitude is unaffected by financial arrangements. Hence, a meaningful analysis of safe assets requires heterogeneity across agents.

Differences in the degree of risk aversion are a natural form of heterogeneity for a study of safe assets. Our analysis exploits this idea by building a simple setup with two types of agents. Agents of type 1 have a relatively low risk aversion and agents of type 2 have a relatively high risk aversion. Specifically, if an agent $i$ has a constant coefficient of relative risk aversion $\gamma_i$, we assume that $0 < \gamma_1 \leq \gamma_2$, so that agent 1 is at least as willing as agent 2 to absorb risk.

In our model, the desire to redistribute risk across agents is the source of safe private assets. In equilibrium, agent 1, with relatively low risk aversion, issues private safe bonds (or equivalent claims)
that are held by agent 2 with relatively high risk aversion. Correspondingly, agent 1 owns a disproportionate share of risky assets, which are equity claims on the Lucas tree. The quantity of safe assets equals the magnitude of the bonds issued by agents of type 1 and held by agents of type 2. The equilibrium amount of these assets depends on the level and differences in risk aversion across the agents, the characteristics of the stochastic process (including rare disasters) that drive GDP, and some other parameters.

In this environment, a pure claim on the Lucas tree corresponds to unlevered equity. A match with the empirically observed high equity premium requires the expected rate of return on this equity to be substantially higher than the risk-free rate, which equals the rate of return on non-contingent, private bonds. Previous analyses with rare-disaster models, summarized in Barro and Ursúa (2012), found that the replication of this high equity premium requires, first, a coefficient of relative risk aversion, $\gamma$, around 3-4 (for a representative agent) and, second, the presence of fat-tailed uncertainty, such as a non-negligible potential for drops in GDP in the short run by more than 10%. Thus, our model incorporates these two features.

However, with a CRRA utility, a coefficient of relative risk aversion, $\gamma$, of 3-4 implies an intertemporal elasticity of substitution (IES) of $1/3$-$1/4$, which seems unrealistically low.¹ Specifically, the high $\gamma$ needed to generate the observed equity premium precludes the case of log utility in the sense of IES=1. More generally, in a standard utility formulation, it is impossible for all agents to have the same IES along with coefficients of relative risk aversion that differ across agents.

We tackle this problem by using Epstein-Zin/Weil (henceforth EZW) recursive preferences (Epstein and Zin, 1989, and Weil, 1990), which distinguish between relative risk aversion and the intertemporal elasticity of substitution (IES). Typically, this benefit from EZW preferences comes at

---

¹ For example, the well-identified estimation in Gruber (2013) estimates an IES of around 2 and van Binsbergen et al. (2012), using a rich DSGE model, estimate the IES to be 1.7. Bansal and Yaron (2004) and Barro (2009) argue that an IES below 1 produces puzzling patterns in the relation of growth rates and uncertainty to ratios of stock prices to earnings.
the cost of analytical complexity when compared with time-separable power utility. Surprisingly, with heterogeneity in risk aversion, the EZW specification allows for a simpler analysis than with a CRRA utility function because we can maintain reasonable values of the IES that are the same for all agents while introducing \( \gamma_i \) coefficients that are high and different across \( i \). The equality of the IES across agents simplifies many expressions. For parsimony, we also keep the rate of time preference, \( \rho \), the same for all \( i \).

Since a basic model with permanent differences in relative risk aversion is non-stationary (in the long run, the wealth share of the agent with lowest risk aversion approaches 1), we assume that agents die off and are continually replaced by new agents who are randomly assigned one of the two possible coefficients of relative risk aversion. The economy then has an interior steady state in which the mean wealth shares of each type of agent are between 0 and 1.

To assess the model empirically, we measure the quantity of safe assets in 34 OECD countries. Our measure follows the methodology proposed by Gorton, Lewellen, and Metrick (2012; henceforth GLM) for the United States. Using the financial balance sheets of OECD countries, we estimate that the (weighted) average safe-asset ratio (safe assets to total assets) in these countries in 2015 was 37%. Further, we show that this ratio of safe assets is relatively stable over time (GLM have already documented this last fact, but just for the United States).

We show that, under a plausible calibration of risk-aversion parameters, our two-agent model with rare disasters matches the observed risk-free rate of 1.0% per year, the unlevered equity premium of 4.2%, and the safe-asset share of 37% (once we adjust for human capital). To our knowledge, our paper is the first to match all of these three observations simultaneously without calibrating risk aversion at unrealistically high values. Our model, despite its simplicity, illustrates how safe-asset creation works quantitatively and how it fits with observations of asset prices.
Motivated by this success, we carry out two further experiments within our model—the realization of a macroeconomic disaster and the experience of tranquility (no disasters) for 40 years—and document how consumption and wealth shares evolve over time in these situations. A disaster shock leads to an extended period in which the share of wealth held by the low-risk-averse agent and the risk-free rate are low but rising, and the ratio of safe-to-total assets is high but falling. Thus, our model displays high persistence in the income and wealth effects of shocks.

Another result comes when we introduce public debt into the model. Added government bonds create more safe assets while simultaneously creating corresponding “safe liabilities” in the form of the present value of taxes. In the baseline setting, where the government and the private sector are equally good at creating safe assets, Ricardian equivalence holds, in the sense that changes in the quantity of government bonds do not affect rates of return and the net quantity of safe assets. Surprisingly, the model predicts that an increase of 1 unit in government bonds crowds out around 0.5 units of private bonds. This prediction accords with some existing empirical evidence.

The equilibrium we find requires an enforcement mechanism for repayments of safe claims. More concretely, agents of type 1 must make payments of principal and interest to agents of type 2 even in bad states of the world, such as realizations of macroeconomic disasters. Repayment mechanisms may involve collateral, liquidity, and contractual features related to the legal system. However, these mechanisms are not the subject of our analysis, which focuses on the underlying supply of and demand for safe private assets. Potentially complementary research that emphasizes liquidity, collateral, and asymmetric information includes Holmström and Tirole (1998) and Gorton and Ordoñez (2013).

Our paper is broadly related to the growing literature on safe assets sparked by the global financial crisis of 2008. This literature studies the scarcity of safe assets (Gourinchas and Jeanne, 2012, Caballero, Farhi, and Gourinchas, 2017, and Caballero and Farhi, 2018), the competition among types

More specifically, our paper connects with the literature on asset-pricing models with two types of agents distinguished by their coefficients of relative risk aversion. Among the most influential papers in this field, we can highlight Dumas (1989), Wang (1996), Chan and Kogan (2002), Gărleanu and Pedersen (2011), Longstaff and Wang (2012), Gennaioli, Shleifer, and Vishny (2012), Caballero and Farhi (2018), Coen-Pirani (2005), Brumm et al. (2015), Gărleanu and Panageas (2015), and Drechsler, Savov, and Schnabl (2018). Except for the last four references, these analyses assume time-separable power utility, augmented in Chan and Kogan (2002) to include external habits.² The main focus of this literature is on asset-pricing effects from heterogeneous risk aversion. Our paper contributes to this literature by studying, in addition to the standard asset-pricing effects, how heterogeneous risk aversion affects the quantity of safe assets, as measured by the safe-asset share.

Coen-Pirani (2005), Brumm et al. (2015), and Drechsler et al. (2018) study two-agent economies with EZW preferences, where agents differ only in their risk aversion, as in our model, but they also introduce constraints on the trade of assets. Coen-Pirani (2005) considers a margin requirement and finds that the model converges asymptotically to a representative-agent economy, despite the occasionally binding constraint. Brumm et al. (2015) study a disaster model similar to ours but impose constraints on the sale of equity and bonds. In the absence of realized disasters, their model converges asymptotically to an equilibrium with limited stock-market participation, where all equity

²In Wang (1996), one agent has log utility and the other has square-root utility. In Gărleanu and Pedersen (2011), one agent has log utility and the other has a coefficient of relative risk aversion greater than one. In the main analysis of Longstaff and Wang (2012), one agent has log utility and the other has squared utility. Gennaioli, Shleifer, and Vishny (2012) assume that one agent is risk neutral and the other has infinite risk aversion. Caballero and Farhi (2018) use an analogous setup.
is held by the low-risk-averse agent, and the two agents trade only in bonds. By comparison, in our model, agents trade freely in equity and bonds, so that quantities and prices in those markets are determined by agents’ desire to share risk, rather than by exogenously imposed constraints on asset trading. Drechsler et al. (2018) introduce into the baseline two-agent model a liquidity requirement on the private issuance of safe assets (e.g., bank deposits). Consequently, the ratio of safe-to-total assets in their model is extremely sensitive to the liquidity cost, a result that deviates from observed data. Gârleanu and Panageas (2015) also allow for two agents with EZW utility. However, they focus on cases in which each agent’s IES is close to the reciprocal of its coefficient of relative risk aversion. This specification means that risk-aversion coefficients well above one are constrained to be associated with IES values well below one. Finally, Hall (2016) uses the heterogeneous risk-aversion framework developed in a working paper version of our study, Barro and Mollerus (2014), to analyze the evolution of safe real interest rates.

After discussing empirical patterns in safe assets in Section 2, Section 3 works out a model that derives equilibrium holdings of equity claims and private bonds by the two types of agents, differentiated by their degrees of relative risk aversion. For heuristic purposes, we begin with an approximate solution to a tractable case where the IES for both types of agents equals one. We show subsequently with numerical techniques that the results extend to settings with general EZW preferences. Section 4 carries out quantitative analyses based on different specifications of the underlying parameters, including coefficients of relative risk aversion and the characteristics of the macro-disaster process. Section 5 distinguishes further between the gross quantity of private bonds and the net quantity corresponding to loans among agents and studies the role of government bonds. Section 6 relates the model’s predictions on the quantity of safe assets to our empirical measurements in Section 2. Section 7 concludes with suggestions for future research.
2 The Safe-Asset Share in OECD Countries

We estimate the safe-asset share (safe assets to total assets) in 34 OECD countries following the methodology of GLM, who undertook this exercise for the United States. Safe assets are defined as the sum of debt liabilities\(^3\) issued by the government, the central bank, and the financial sector.\(^4\) Total assets are defined as the sum of all liabilities issued by all sectors in the economy.\(^5\) The data are from the financial balance sheets of OECD countries (SNA 2008). These data are roughly equivalent to the U.S. flow of funds used by GLM, but cover many countries, though at a lower level of detail.

Table 1 presents our estimates for 2015. The values range from 22\% (Chile) to 66\% (Greece). Among the largest economies, the United States has the lowest safe-asset share of 30\%,\(^6\) while Japan has the highest share of 54\%. The weighted average by country assets of the safe-asset share is 37\%. This estimate can be interpreted as the aggregate safe-asset share in our sample of OECD countries. Later, we will show that our model can replicate this safe-asset share.

GLM also found that the safe-asset share in the United States was remarkably stable from 1952 to 2010, despite the dramatic changes in U.S. financial markets over this period. Figure 1 shows a similar finding for other countries for the period 1995-2017 for which data are available. The figure plots the safe-asset share for the United States and a group of non-U.S. countries. The safe-asset share moves around a long-run level of 30\% for the United States and 43\% for non-U.S. countries. The safe-asset share is stable over time and has not changed significantly even after the global financial crisis of 2008.

---

\(^3\) Debt liabilities include currency and deposits, debt securities, loans, and money-market-fund shares.

\(^4\) GLM exclude intra-government loans (e.g., government bonds held by federal retirement programs) and 25\% of long-term bonds issued by financial corporations (e.g., MBS, ABS). Due to data limitations, we are not able to make similar adjustments in our OECD sample.

\(^5\) The sectors are general government, the financial sector, non-financial corporations, households, and the rest of the world.

\(^6\) GLM estimate the safe-asset ratio at 31\%-33\% for the United States. Despite using data from a different year (2015), we replicate their results by estimating a safe-asset share of 30\%. For other countries, we find an average ratio of 43\%. 
The stability of the safe-asset share can also be observed in the cross section. We find substantial differences between countries in the size of their financial markets, but a much smaller difference in their safe-asset shares. For instance, the ratio of total assets to GDP in Mexico is 3.9, the smallest in our sample. Ireland has the largest ratio of total assets to GDP of 26.9. Nevertheless, the safe-asset share in both countries is similar: 0.30 in Mexico and 0.26 in Ireland. More generally, the cross-country correlation between the safe-asset share and the total assets to GDP ratio is close to zero, suggesting that the safe-asset share is not driven by the size of the financial sector. Indeed, for the long time series for the U.S. economy reported by GLM, the safe-asset share has not changed despite the jump in the ratio of total assets to GDP from 4 in 1952 to 10 in 2010.

The stock of safe assets consists of claims issued by the public sector (government and central bank) and the private sector (financial institutions). Figure 2 presents these two types of safe assets in the United States (panel A) and in non-U.S. countries (panel B). In both cases, there is a clear crowding-out effect. When public safe assets increase, private safe assets decline. This phenomenon is strongly evident after the global financial crisis. The U.S. public safe-asset share increased between 2007 and 2015 from 7% to 12%, while the private share declined from 23% to 18%. In non-U.S. countries, the public safe-asset share increased over the same period from 10% to 14% and the private safe-asset share declined from 33% to 29%.

Using U.S. data, GLM and Krishnamurthy and Vissing-Jorgensen (2013) estimate the crowding-out coefficient at around -0.5. We later show that a theoretical benchmark for the crowding-out coefficient in a Ricardian equivalence economy is -0.5, which is consistent with these studies. In our sample, regressions of private safe assets on public safe assets yield a coefficient of -0.70 (S.E. 0.09) for the United States and -0.77 (S.E. 0.19) for non-U.S. countries. While these coefficients are larger than those estimated by GLM and Krishnamurthy and Vissing-Jorgensen (2013), given our data limitations and the large standard errors, we consider them to be in line with the previous evidence.
Our empirical findings suggest that the safe-asset share depends on deep parameters that are relatively robust over time and across countries. In the next section, we build a model that exhibits this property. We calibrate the model to match the observed equity premium and show that it can generate a safe-asset share with a magnitude consistent with the estimates in Table 1 and without resorting to extreme values of risk aversion. Moreover, in line with the empirical findings, we show how the safe-asset share in our model is fairly insensitive to most model parameters, and hence unlikely to change much over time.

3 Baseline Model

We work with an economy with two types of agents (one more risk averse than the other), a Lucas tree, and two assets (an unlevered equity claim on the Lucas tree and a non-contingent, one-period private bond). The model is set up, for convenience, in discrete time, but we think of the period as short. Parts of the structure of our model parallel Longstaff and Wang (2012).

3.1 Agents

The representative agent $i$, for $i \in \{1, 2\}$, has EZW preferences of the form:

$$U_{it} = \left\{ \left( \frac{\rho}{1+\rho} \right) C_{it}^{1-\theta} + \left( \frac{1}{1+\rho} \right) \left[ E_t(U_{i,t+1}^{1-\gamma}) \right]^{(1-\theta)/(1-\gamma)} \right\}^{1/(1-\theta)} .$$

The coefficients controlling relative risk aversion satisfy $0 < \gamma_1 \leq \gamma_2$; that is, agent 1 has relatively low risk aversion. However, the IES, $1/\theta > 0$, and the rate of time preference, $\rho > 0$, are the same for the two agents. This assumption makes the mechanism behind our results transparent. For much of our analysis, we simplify the derivations by assuming $\theta = 1$ (log utility).

3.2 The Lucas Tree

The economy is endowed with a single Lucas tree, with size normalized to one, that generates real GDP of $Y_t$ in period $t$. The two agents consume this GDP:
\[ C_{1,t} + C_{2,t} = Y_t. \] (2)

Ownership of the tree is given by \( K_{1,t} \) and \( K_{2,t} \):

\[ K_{1,t} + K_{2,t} = 1. \] (3)

We use a convention whereby \( K_{i,t} \) applies at the end of period \( t \), after the payment of the dividend, \( K_{i,t-1} Y_t \), to agent \( i \) in period \( t \). This timing convention is of little consequence since the length of the period is short. The price of the tree in period \( t \) in units of consumables is \( P_t \).

The stochastic process that generates \( Y_t \) follows specifications similar to those in previous rare-disaster models, except for the omission of a normally distributed business-cycle shock, which is quantitatively unimportant for our argument. There is a constant probability \( p \) of a disaster per period. More concretely, with probability \( 1 - p \), real GDP grows over one period by the factor \( 1 + g \), where \( g \geq 0 \) is constant. With probability \( p \), a disaster occurs and real GDP grows over one period by the factor \( (1 + g) \cdot (1 - b) \), where \( b > 0 \) is the size of a disaster. When the length of the period is short, the expected growth rate per year of GDP, denoted \( g^* \), is given by:

\[ g^* \approx g - pb, \] (4)

where \( g \) and \( p \) are measured in annual units.

In this simple setting, disasters last for only one “period” and have a single size. The analysis can be extended to allow for a time-invariant size distribution of disasters, as in Barro and Ursúa (2012). With more complexity, we could allow disasters to have stochastic duration and cumulative size and to be followed by a tendency to recovery with above-normal growth rates (Nakamura et al., 2013 and Barro and Jin, 2016).\(^7\) Other feasible extensions include time variation in \( p \) (as in Gabaix, 2012) and \( g \) (as in Bansal and Yaron, 2004).

---

\(^7\) The recovery tendency lowers the effective size, \( b \), of a disaster. Therefore, for some purposes, we could allow for recoveries within the present framework by adjusting \( b \).
As we will describe below, the baseline calibration sets \( p = 0.04 \) per year. This probability corresponds to the empirical frequency of disasters -defined as short-term declines in real per capita GDP of at least 10%- in a long-term panel of countries. A drop of 10% is also in line with the early estimates of per capita GDP reductions in countries that imposed aggressive COVID-19 lockdowns during the first half of 2020, such as Italy and Spain. The effective disaster size -i.e., the single value in the representative-agent economy that generates an equity premium corresponding roughly to the full-size distribution of disasters- is \( b = 0.32 \). The growth-rate parameter, corresponding to the non-disaster mean growth rate of real per capita GDP or consumption, is \( g = 0.025 \) per year.

### 3.3 Assets

Agents can trade in two assets. The first asset is an unlevered equity claim, \( K_{t,t} \), on the tree. Individual agents can go short on this claim. We also consider a non-contingent, one-period private bond, \( B_{t,t} \). The quantity \( B_{i,t} \) is negative for a borrower (issuer of a bond) and positive for a lender (holder of a bond). Since we work with a closed economy, the total quantity of these private bonds, when added up across the two agents, is always zero:

\[
B_{1,t} + B_{2,t} = 0. \quad (5)
\]

In the baseline case with a single disaster size and no normally distributed shock, the two forms of assets are sufficient to generate a complete-markets solution. In the more general case, a wider array of assets would be needed to span the possible outcomes. Also, we will ignore possibilities of default and neglect transaction costs associated with payments of interest and principal and collecting on loans. In the cases we consider in this paper, debtors always have enough assets in equilibrium to make the prescribed principal and interest payments on bonds. Bonds pay the risk-free interest rate for period \( t \), denoted \( r_t^f \). The amount of principal and interest received or paid on bonds by agent \( i \) in period \( t \) is \((1 + r_t^f)B_{i,t-1}\).
Each agent’s budget constraint for period $t$ is:

$$C_{i,t} + P_t K_{i,t} + B_{i,t} = (Y_t + P_t) K_{i,t-1} + (1 + r_t^f) B_{i,t-1}.$$  \hfill (6)

The choice for period $t$ of $C_{i,t}$ and the portfolio allocation, $(K_{i,t}, B_{i,t})$, occur when $Y_t$, $P_t$, and $r_t^f$ are known, but $Y_{t+1}$ and $P_{t+1}$ are not.

Let $R_{t+1}$ represent the gross return on an asset between periods $t$ and $t+1$. This return equals $(Y_{t+1} + P_{t+1})/P_t$ for equity and $(1 + r_{t+1}^f)$ for bonds. Each agent seeks to maximize expected utility, given in equation (1), subject to the budget constraint in equation (6) and its initial assets. The first-order optimization conditions for each agent can be expressed by means of a perturbation argument for periods $t$ and $t+1$ as:

$$E_t \left( U_{l,t+1}^{1-\gamma_l} \right)^{\gamma_l / (1-\gamma_l)} = \frac{1}{1+\rho} E_t \left[ U_{l,t+1}^{\theta-\gamma_l} \cdot \left( \frac{C_{l,t+1}}{C_{l,t}} \right)^{-\theta} \cdot R_{t+1} \right].$$  \hfill (7)

The results simplify in straightforward ways under log utility, $\theta = 1$. When $R_{t+1}$ equals the risk-free return, $1 + r_{t+1}^f$, and $\theta = 1$, equation (7) implies:

$$1 + r_{t+1}^f = (1 + \rho) / E_t \left[ \frac{C_{l,t}}{C_{l,t+1}} \cdot \left( \frac{U_{l,t+1}^{1-\gamma_l}}{E_t \left( U_{l,t+1}^{1-\gamma_l} \right)} \right) \right].$$  \hfill (8)

Thus, a key implication of the first-order conditions is that, in equilibrium, the right-hand side of equation (8) must be the same for each agent; i.e., it needs to hold for both $\gamma_1$ and $\gamma_2$. This correspondence implies that the prospective paths of uncertain consumption and utility levels for the two agents must accord with the differences in the coefficients of relative risk aversion.

When $\theta = 1$, each agent’s consumption in period $t$ is the fraction $\rho/(1+\rho)$ of that agent’s resources for period $t$:

$$C_{i,t} = \frac{\rho}{1+\rho} \cdot [(Y_t + P_t) \cdot K_{i,t-1} + (1 + r_t^f) \cdot B_{i,t-1}].$$  \hfill (9)

Giovannini and Weil (1989, Appendix B) derive this condition for a representative-agent economy. Coen-Pirani (2005) shows that (9) also holds in a two-agent model with heterogeneous risk aversion.
Equations (6), (8), and (9) jointly determine agent i’s choices of consumption, \( C_{i,t} \), and portfolio allocation, \( (K_{i,t}, B_{i,t}) \). Adding up equation (9) for the two agents and using the conditions from equations (2), (3), and (5)—total consumption equals GDP, equity holdings add to one, and bond holdings add to zero—leads to:

\[ P_t = Y_t / \rho. \]  

(10)

Thus, under log utility, the equity price and, hence, the value of total assets are independent of parameters related to the expected growth rate, the uncertainty of the Lucas tree’s yield, and the degree of risk aversion. This result implies that the expected rate of return on equity, \( r^e \), equals the dividend yield, \( \rho \), plus the expected rate of capital gain, which equals \( g^* \), the expected growth rate of GDP and consumption:

\[ r^e \approx \rho + g^* \approx \rho + g - pb, \]  

(11)

where \( g^* \) is given in equation (4). Since \( r^e \) is independent of uncertainty parameters, the effects of these parameters on the equity premium work through the risk-free rate, \( r^f \), rather than \( r^e \).  

3.4 Market Equilibrium

Agent i’s wealth at the end of period t-1 is \( W_{i,t-1} = P_{t-1}K_{i,t-1} + B_{i,t-1} \), so that agent 1’s wealth share at the end of period t-1 is:

\[ \frac{W_{1,t-1}}{W_{t-1}} = \frac{K_{1,t-1}}{P_{t-1}} + \frac{B_{1,t-1}}{P_{t-1}}. \]

Notice that total wealth, \( W_{t-1} \), equals the equity price, \( P_{t-1} \).

To determine the equilibrium, we require an initial value for agent 1’s wealth share. For example, this share might start at 0.5 in period 0 with zero debt issue \( (K_{1,0} = .5, B_{1,0} = 0) \). Heuristically, if \( \gamma_1 < \gamma_2 \), there is an incentive in this initial position for agent 1 to issue risk-free bonds,

---

\(^8\) We know from previous analyses of this i.i.d. setting with a representative agent, such as Barro (2009), that the equity premium is independent of the parameter \( \theta \). Thus, setting \( \theta = 1 \) does not affect the model’s implications for the equity premium.
so that $B_{1,1} < 0$ in period 1, and these bonds will be held by agent 2, so that $B_{2,1} > 0$. That is, agent 1 borrows from agent 2 on a safe basis. Correspondingly, agent 1 uses its bond issue to increase its share of equity, so that $K_{1,1} > .5$ and $K_{2,1} < .5$. In a richer model with endogenous capital, this process of safe credit creation could affect the equilibrium amount and composition of investment.

The pattern of bond and equity positions shifts risk from the high-risk-aversion agent 2 to the low-risk-aversion agent 1. However, the process does not entail complete risk shifting. Rather, enough bond issuance occurs so that the resulting stochastic paths of future consumption for each agent make the right-hand side of equation (8) the same for each agent $i$.

We can use equations (8) and (9), along with the agents’ budget constraints and utility functions, to solve numerically the equilibrium values of the safe interest rate, each agent’s consumption and utility, and each agent’s allocation of assets between equity and bonds, as functions of the agents’ previous portfolios and current GDP.\(^9\) The realization for $Y_t$ (disaster or no disaster in the present case) then determines each agent’s portfolio at the end of period $t$. These portfolios together with the realization of $Y_{t+1}$ pin down the equilibrium values for period $t + 1$, and so on.

Using the budget constraint from equation (6) and the conditions for consumption and the price of the Lucas tree in equations (9) and (10), we can show that agent 1’s wealth share at the end of period $t$ is related to asset holdings from the end of period $t - 1$ in accordance with:

$$\frac{w_{1,t}}{w_t} = K_{1,t-1} + \frac{\rho(1+r_f^{1})B_{1,t-1}}{(1+\rho)y_t}. \quad (12)$$

We can also demonstrate that the change in agent 1’s wealth share from $t - 1$ to $t$ is:

$$\frac{w_{1,t}}{w_t} - \frac{w_{1,t-1}}{w_{t-1}} \approx \frac{\rho B_{1,t-1}}{y_{t-1}} (r_f^{t} - \rho - g_t), \quad (13)$$

where $g_t \equiv (Y_t/y_{t-1} - 1)$ is the stochastic growth rate of GDP. The approximation is exact for infinitely short periods.

\(^9\) We carry out this analysis numerically on a detrended version of the model using a quarterly calibration. See Section 4.
Since the risk-free rate, $r^f$, will be less than $\rho + g^*$, which equals the expected rate of return on equity, $r^e$, the expectation of the right-hand side of equation (13) is positive if $B_{1,t-1} < 0$. The expected change in agent 1’s wealth share is positive whenever agent 1 (the low-risk-aversion agent) is borrowing in a risk-free manner from agent 2. The reason that agent 1’s wealth share tends to rise over time is that this agent’s wealth is relatively concentrated in risky equity, which has a higher expected rate of return than risk-free bonds (even after factoring in the occasional macroeconomic disasters, which tend to reduce agent 1’s wealth share). Consequently, we find numerically that agent 1’s wealth share asymptotically approaches one, and the ratio of risk-free bonds, $B_{1,t}$, to total assets or GDP asymptotically approaches zero. In effect, there is a selection effect whereby wealth is concentrated asymptotically in the agent with lower risk aversion. Hence, the model behaves in the long run like a representative-agent economy with a coefficient of relative risk aversion equal to $\gamma_1$.

The non-stationarity of the initial form of the model makes it unsatisfactory for studying the determination of wealth shares and quantities of safe assets. Previous models with this problem are Dumas (1989) and Longstaff and Wang (2012, p. 3208). To go further, we must modify the model to achieve stationarity. In particular, we need to keep the expected wealth share of agent 1 asymptotically bounded away from one.

### 3.5 Replacement and Stationarity

A natural way to achieve stationarity is to have agents die, with replacement by new agents who may be type 1 or type 2, that is, who have a relative risk aversion of $\gamma_1$ or $\gamma_2$. Since type-1 agents tend to have above-average wealth, this process redistributes wealth back to type-2 agents.\textsuperscript{10,11}

\textsuperscript{10} Chan and Kogan (2002) generate stationarity by having each agent’s coefficient of relative risk aversion, $\gamma_i$, be an increasing function of that agent’s wealth share. However, richer agents having higher coefficients of relative risk aversion seems hard to reconcile with other observations. In any event, the type of model postulated by these authors functions in the steady state as a representative-agent model with a single coefficient of relative risk aversion.

\textsuperscript{11} Some forms of tax/transfer systems provide an alternative mechanism to induce a stationary steady-state equilibrium. We explored this approach and found that a graduated income tax rate system can support an interior steady state in equilibrium, whereas a pure flat tax rate does not. Results are available upon request.
Specifically, we assume that a percentage $\nu$ of the agents of each type die off per period (which is also the mortality probability of each agent within its type) and that they are immediately replaced by a newborn. These new agents are type 1 with probability $\mu$ and type 2 with probability $1 - \mu$. These probabilities are independent of the characteristics of the parents. The case $\mu = 0.5$ corresponds to equal numbers of agents of each type. The replacement agents inherit the assets of their predecessors and, thus, the representative agent within each type has a new wealth that reflects the transfers from dead agents. We interpret these intergenerational transfers as unintended bequests, as we assume that agents derive zero utility from bequests.\(^{12}\)

Following Gârleanu and Panageas (2015), we adjust the preferences in equation (1) for mortality risk by replacing the discount factor $1/(1 + \rho)$ with the adjusted factor $(1 - \nu)/(1 + \rho)$:

$$U_{i,t} = \left\{ \left( \frac{\nu + \mu}{1 + \rho} C_t^{\frac{1}{1-\nu}} + \left( \frac{1 - \nu}{1 + \rho} E_t [U_{i,t+1}^{1 - \gamma_i} (1-\theta)/(1-\gamma_i)] \right)^{(1-\theta)/(1-\gamma_i)} \right\}^{1/(1-\theta)}. \quad (14)$$

This specification assumes that utility from bequests is zero, aggregate shocks are realized before the survival shock, and aversion toward mortality risk equals $\theta$ (see the online appendix D of Gârleanu and Panageas [2015] for more details and the interpretation of aversion toward mortality risk).\(^{13}\)

A convenient feature of the model is that the agent’s optimization problem can be normalized by her beginning-of-period resources (Coen-Pirani, 2005). Defining resources for period $t$ by:

\[^{12}\text{Alternatively, as in Blanchard (1985), agents may leave no bequests and hold all their assets as annuities, on which the returns factor in the probability of dying. This setting is used by Gârleanu and Panageas (2015). Our analysis ignores a market for annuities. Empirically, this assumption is not too restrictive. According to the investment firm LIMRA, the total size of the annuities market in 2017 was roughly 1\% of GDP.}\]

\[^{13}\text{Gârleanu and Panageas (2015) distinguish between aversion toward aggregate risk, captured by parameter $\gamma_i$, and aversion toward mortality risk, captured by a different parameter $\chi_i$. Accordingly, they modify the standard EZW preferences in equation (1) to:} \]

$$U_{i,t} = \left\{ \left( \frac{\nu + \mu}{1 + \rho} C_t^{\frac{1}{1-\nu}} + \left( \frac{1 - \nu}{1 + \rho} E_t \left( (1 - \varepsilon)U_{i,t+1}^{1 - \chi_i} + \varepsilon V_{i,t+1}^{1 - \chi_i} \right) (1-\theta)/(1-\gamma_i) \right)^{(1-\theta)/(1-\gamma_i)} \right\}^{1/(1-\theta)},$$

\[^{13}\text{where $\varepsilon$ is an idiosyncratic mortality shock, which takes the value 1 with probability $\nu$ and zero otherwise, and $V_{i,t+1}$ denotes utility from a bequest. The outer expectation is taken conditional on information known in period $t$ only, whereas the inner expectation is conditional also on the aggregate shock $Y_{i,t+1}$, which is realized before the mortality shock $\varepsilon$. Assuming $V_{i,t+1} = 0$ and $\chi_i = \theta$, these preferences simplify to (14), after adjusting the constant $\frac{\rho}{1 + \rho}$ to $\frac{\rho + \nu}{1 + \rho}$.}\]
the normalized utility function is:

\[
u_{i,t} = \left\{ \left( \frac{\rho + v}{1 + \rho} \right) c_{i,t}^{1-\theta} + \left( \frac{1-v}{1 + \rho} \right) (1 - c_{i,t})^{1-\theta} \right\}^{1/(1-\theta)} \left( R_{i,t+1} + 1 u_{i,t+1} (1 - \gamma_i) \right)^{1/(1-\theta)},
\]

where \( c_{i,t} \) and \( u_{i,t} \) denote consumption and utility as ratios to \( A_{i,t} \), and \( R_{i,t+1} \) is the total return on the agent’s portfolio. If the portfolio share of equity is \( x_{i,t} = P_t K_{i,t} / (P_t K_{i,t} + B_{i,t}) \), then \( R_{i,t+1} \) is:

\[
R_{i,t+1} = x_{i,t} \cdot \left( \frac{R_{i,t+1} + 1}{P_t} \right) + (1 - x_{i,t}) \cdot (1 + r_t^f) + (1 - x_{i,t}) \cdot (1 + r_t^f).
\]

The agents maximize utility in equation (16) by choosing the ratio of consumption to period t’s resources, \( c_{i,t} \), and the equity share, \( x_{i,t} \). Since these decisions are independent of resources, \( A_{i,t} \), the optimal consumption ratios and equity shares (\( c_{i,t}, x_{i,t} \)) are identical for all agents with the same \( \gamma_i \).

This property allows us to aggregate easily across agents with the same \( \gamma_i \). The value of assets held as equity and bonds at the end of period t is related to the choices of \( c_{i,t} \) and \( x_{i,t} \) and to resources, \( A_{i,t} \) (given in equation [15]), in accordance with:

\[
P_t K_{i,t} = x_{i,t} \cdot (1 - c_{i,t}) \cdot A_{i,t},
\]

\[
B_{i,t} = (1 - x_{i,t}) \cdot (1 - c_{i,t}) \cdot A_{i,t}.
\]

We now modify the analysis to allow for the transfer of assets across types that occurs when newborn agents are of different types from their parents. The analysis is modified only by changing the expression for resources in equation (15). The revised expression is:

\[
A_{i,t} = (Y_t + P_t) \cdot [K_{i,t-1} - \nu \cdot (K_{i,t-1} - \mu_i)] + (1 - \nu) \cdot (1 + r_t^f) \cdot B_{i,t-1},
\]

where \( \mu_i \) is the probability that a newborn agent is of type i. In our case, \( \mu_1 = \mu \) and \( \mu_2 = 1 - \mu \).

Equation (20) says that the resources of type-i agents are reduced by the transfers when the equity share of type-i agents, \( K_{i,t-1} \), exceeds the “population share,” \( \mu_i \), of those agents. Equations (18)-(20) fully characterize the laws of motion of the portfolios of the two types of agents.
For the case of log utility, $\theta = 1$, the expression for the change in the wealth share of agents of type 1 is modified from equation (13) to:

$$
\frac{W_{1,t}}{w_t} - \frac{W_{1,t-1}}{w_{t-1}} \approx (\rho + \nu) \cdot \frac{B_{1,t-1}}{Y_{t-1}} \cdot (r^f_t - \rho - g_t) + \nu \cdot (\mu - \frac{W_{1,t-1}}{w_{t-1}}).
$$

(21)

If $\nu = 0$, as before, the expectation of the right-hand side of equation (20) is positive if $B_{1,t-1} < 0$. As agent 1’s wealth share approaches one, $B_{1,t-1}/Y_{t-1}$ asymptotically approaches zero and equation (21) implies that the expectation of the change in agent 1’s wealth share asymptotically approaches zero. Another property of the equilibrium with $\nu = 0$ is that $K_{1,t-1}$ asymptotically approaches one.

If $\nu > 0$, when $K_{1,t-1}$ is close to one and $B_{1,t-1}/Y_{t-1}$ is negligible, the term on the far right of equation (21) is negative and dominates in magnitude the first term on the right. It follows that the expected change in agent 1’s wealth share reaches zero before $K_{1,t-1}$ gets close to one and $B_{1,t-1}/Y_{t-1}$ becomes negligible. For this reason, the economy tends to approach a stochastic steady state in which the mean wealth shares for each type of agent are between zero and one. We will next compute these (steady-state) mean wealth shares as well as the means of safe assets (expressed relative to total assets or GDP) and risk-free rates, $r^f$.

### 4 Quantitative Analysis of the Stationary Model

We now extend our analysis and consider cases beyond $\theta = 1$ (with the IES still the same for both agents). To do so, we solve the model numerically using the Taylor projection algorithm proposed by Levintal (2018). This method has been shown to work well in models with rare disasters by Fernández-Villaverde and Levintal (2018). The algorithm approximates the solution by polynomials that zero the Taylor series of the residual function (up to a finite order) at a given point in the state space. By Taylor’s theorem, the residual function is approximately zero around that point; hence, the solution is locally accurate in this neighborhood. We make the approximation over the long-
run domain of the model and simulate the model for 2,000 years. See Levintal (2018) and Fernández-Villaverde and Levintal (2018) and the online computational appendix for further details.

Aside from the coefficients of relative risk aversion, $\gamma_1$ and $\gamma_2$, the baseline parameter values, listed in the note to Table 2, are $\rho = 0.02$ per year (rate of time preference), $\nu = 0.02$ per year (replacement rate), $g = 0.025$ per year (growth-rate parameter), $p = 0.04$ per year (disaster probability), and $b = 0.32$ (effective disaster size). These values accord with the empirical analysis in Barro and Ursúa (2012). These parameter values imply from equation (4) that the expected growth rate is

$$g^* = g - p \cdot b = 0.0122 \text{ per year.}$$

(22)

We begin with a gross replacement rate (death and birth rate) of $\nu = 0.02$ per year, which corresponds approximately to average U.S. adult mortality rates. The baseline case assumes $\theta = 1$ (log utility) and $\mu = 0.5$ (the population shares of the two types are equal). Later, we will carry out sensitivity analyses for the parameters $\nu, \mu, \theta,$ and $p$.

### 4.1 A Representative Agent

Table 2 considers the case with a representative agent, $\gamma_1 = \gamma_2 = \gamma$, for a range of values of $\gamma$. In this situation, if we start with agent 1’s wealth share at 0.5, $B_{i,t}$ and $K_{i,t}$ stay constant over time at 0 and 0.5, respectively, irrespective of the realizations of $Y_t$. Because of log utility, the expected rate of return on equity, $r^e$, is fixed at $\rho + \nu + g^*$, where $\rho + \nu = 0.04$ per year and $g^* = 0.0122$, so that $r^e = 0.052$ per year.

A higher $\gamma$ lowers the risk-free rate, $r^f$, and thereby raises the equity premium. Specifically, Table 2 shows that $r^f$ varies from 0.046 at $\gamma = 1$ to $-0.055$ at $\gamma = 6$. In the present model, which lacks risk-free and costless storage of final product, there is nothing special about a risk-free rate of zero. An unlevered equity premium between 0.03 and 0.06 (corresponding to historical data) requires
\( \gamma \) to be between 3 and 4.5. For a given \( \gamma \), \( r^f \) is fixed over time, regardless of the realizations of \( Y_t \). This risk-free rate is a shadow rate in the sense that no risk-free borrowing and lending occur in equilibrium. That is, no net safe assets are created in this representative-agent environment.

### 4.2 Heterogeneity in Risk Aversion

We move now to the case where agents are heterogeneous in their risk aversion. Table 3 reports our results. Since the gross replacement rate is \( v = 0.02 \) per year and \( \mu = 0.5 \), an agent of type \( i \) is replaced by an agent of the other type at the rate \( 0.5 \cdot v = 0.01 \) per year.

Table 3 shows combinations of \( \gamma_1 \) and \( \gamma_2 \) that generate a mean steady-state risk-free rate of \( r^f = 0.010 \) and a mean steady-state unlevered equity premium of \( r^e - r^f = 0.042 \). That is, these combinations of \( \gamma_1 \) and \( \gamma_2 \) roughly match the empirical averages of the risk-free rate and the equity premium. The table shows the corresponding means of a set of variables: agent 1’s share of risky assets, \( K_1 \), and wealth, \( W/W \), and the ratio of the amount of safe assets, \( |B_1| \), to wealth and GDP. Because economy-wide assets equal annual GDP times 25, \( 1/(\rho + v) \), in this model, the amount of safe assets expressed relative to annual GDP is 25 times the ratio to total assets. These total assets correspond to the capitalization of the entire flow of GDP, effectively including human capital as well as physical capital. We come back to this point in Section 6.

The first row of Table 3 shows that, if \( \gamma_1 = \gamma_2 \), the value of \( \gamma_1 \) and \( \gamma_2 \) needed to generate a mean steady-state \( r^f \) of 0.010 is 3.86 (see Table 2). Columns 1 and 2 of Table 3 show that values of \( \gamma_1 \) below 3.86 require higher values of \( \gamma_2 \). For example, \( \gamma_1 = 3.6 \) matches up with \( \gamma_2 = 4.25 \), \( \gamma_1 = 3.4 \) with \( \gamma_2 = 4.9 \), \( \gamma_1 = 3.2 \) with \( \gamma_2 = 7.0 \), and \( \gamma_1 = 3.1 \) with \( \gamma_2 \) above 10. For still lower values of \( \gamma_1 \), the required value of \( \gamma_2 \) explodes. For \( \gamma_2 = 10 \), the risk-free rate is still slightly larger than 0.010. To get \( r^f = 0.010 \), \( \gamma_2 \) should be larger than 10, but it is hard to compute the risk-free rate with enough accuracy in this region.
In column 5 of Table 3, the mean of the share of risky assets held by agent 1, $K_1$, equals 0.50 when $\gamma_1 = \gamma_2$, then rises toward 1.0 as $\gamma_1$ falls and $\gamma_2$ rises. When $\gamma_1 = 3.1$ and $\gamma_2 = 10$, the mean of $K_1$ is 0.93.

In column 6, the mean of the wealth share, $W_1/W$, starts at 0.50 when $\gamma_1 = \gamma_2$, then rises as $\gamma_1$ falls and $\gamma_2$ rises. This wealth share equals 0.77 when $\gamma_1 = 3.1$ and $\gamma_2 = 10$. Equity ownership is much more unequally distributed than overall wealth.

Column 7 shows that $|B_1|/W$, the mean of the ratio of the magnitude of safe-to-total assets, rises from 0 when $\gamma_1 = \gamma_2$ to 3.4% when $\gamma_1 = 3.6$ ($\gamma_2 = 4.25$), 7.0% when $\gamma_1 = 3.4$ ($\gamma_2 = 4.9$), 12.6% when $\gamma_1 = 3.2$ ($\gamma_2 = 7.0$), and 15.9% when $\gamma_1 = 3.1$ ($\gamma_2 = 10$). For subsequent purposes, we are interested in the model’s predictions about the size of safe assets. Thus, an important result is that the predicted quantity of safe assets remains below 16% of economy-wide assets as long as $\gamma_2$ is less than 10, which is a high degree of relative risk aversion. In column 8, the corresponding ratio to GDP is 4.0.

Table 4 redoes the analysis for alternative settings of four of the parameters: the reciprocal of the IES, $\theta$, is allowed to be 0.5 or 2.0, rather than 1.0; the gross replacement rate, $\nu$, is 0.05 per year, rather than 0.02; the disaster probability, $p$, is 0.02 per year, rather than 0.04; and the population share of type 1 agents, $\mu$, is 0.25, rather than 0.5. In each case, the table shows steady-state values of $r^e$ and $r^f$ and the other variables for three of the combinations of $(\gamma_1, \gamma_2)$ considered in Table 3. The other parameters are held fixed at the values assumed in Table 3.

Shifts in the reciprocal of the IES, $\theta$, have only moderate effects on the steady state. Consider the case where $\gamma_1 = 3.4$ and $\gamma_2 = 4.9$. With $\theta = 1$ (in Table 3), the rates of return are $r^e = 0.052$ and $r^f = 0.010$. Table 4 shows that these rates of return change to 0.053 and 0.013, respectively, when $\theta = 0.5$ and to 0.049 and 0.007, respectively, when $\theta = 2$. Correspondingly, the means of $K_1$ and $W_1/W$ were 0.693 and 0.623, respectively, in Table 3. These values change in Table 4 to 0.752 and
0.690 when \( \theta = 0.5 \) and to 0.655 and 0.579 when \( \theta = 2 \). The mean of \(|B_1|/W\) was 0.070 in Table 3 and changes in Table 4 to 0.062 when \( \theta = 0.5 \) and 0.076 when \( \theta = 2 \). The mean of \(|B_1|/Y\) was 1.74 in Table 3 and changes in Table 4 to 1.50 when \( \theta = 0.5 \) and 2.05 when \( \theta = 2 \). Most importantly, the magnitude of safe assets shows little sensitivity to the assumed IES. Or, to put it another way, the results with log utility, \( \theta = 1 \), provide a reasonable approximation when we depart from the log case.

An increase in the replacement rate, \( \nu \), means that the high-risk-aversion type, agent 2, counts more for the new steady state. Therefore, the rise in \( \nu \) to 0.05 in the middle of Table 4 lowers the steady-state shares of agent 1 in equity and wealth. For the case where \( \gamma_1 = 3.4 \) and \( \gamma_2 = 4.9 \), the values of \( K_1 \) and \( W_1/W \) go from 0.693 and 0.623, respectively, in Table 3 to 0.628 and 0.554 in Table 4. The rise in \( \nu \) increases expected returns on all assets, because agents are less patient when mortality risk is higher. The change in \( \nu \) has only a minor effect on the ratio of safe-to-total assets, \(|B_1|/W\), which goes from 0.070 in Table 3 to 0.074 in Table 4. The ratio of safe assets to GDP, \(|B_1|/Y\), falls more significantly from 1.74 in Table 3 to 1.04 in Table 4 due to the fall in the ratio of total assets to GDP, given by \( P_t/Y_t = 1/(\rho + \nu) \). Hence, our results do not change greatly when \( \nu \) is 0.05, rather than 0.02, except for the ratio of safe assets to GDP, which falls sharply.

The next part of Table 4 sets the disaster probability, \( p \), at 0.02 per year, instead of 0.04. The decrease in \( p \) makes the steady-state risk-free rate, \( r^f \), substantially higher, around 0.036, rather than 0.010. At the same time, the equity premium becomes too low in Table 4, when compared with the empirical averages. Thus, as in previous research, the model does not accord with regularities on mean rates of return unless the disaster risk is sufficiently high. A similar conclusion arises if the disaster size, \( b \), is lowered substantially below its initially assumed value of 0.32.

The last part of Table 4 shows the effects from setting the population share, \( \mu \), of low-risk-aversion agents to 0.25, rather than 0.50. This shift effectively lowers the supply of safe assets (from
agents of type 1) compared to the demand (from agents of type 2) and results in a drop in the risk-free rate, $r^f$, and a rise in the equity premium.

4.3 Dynamics

The dynamics of the economy can be described in terms of the evolution of the share of type 1 agents in total wealth, $W_1/W$. Disaster shocks and long periods free of disasters affect this wealth share and have persistent influences on the risk-free interest rate, $r^f$, the ratio of safe-to-total assets, and other variables. We consider first the dynamic effects from a disaster and then examine the consequences from an extended period free of disasters. In these examples, the parameter values are those shown in the notes to Table 2.

4.3.1 Aftermath of a Disaster

Figure 3 shows the dynamics of the economy starting from a steady state and assuming the realization of a disaster of size $b = 0.32$ in period 1. The results correspond to the parameter combination $\gamma_1 = 3.3$ and $\gamma_2 = 5.6$ in Table 3. The paths of variables in Figure 3 assume no further disasters. The variables considered over 10 years are agent 1’s wealth share, $W_1/W$, the risk-free interest rate, $r^f$, agent 1’s share of total equity, $K_1$, and the ratio of safe-to-total assets, $|B_1|/W$.

Because of agent 1’s relatively high concentration in risky assets, its wealth share, $W_1/W$, falls with the disaster from 0.670 to 0.626. The share rises thereafter (in the absence of further disasters) but remains below the steady-state value even after 10 years, when the share reaches 0.654. Another way to look at this pattern is that relatively low inequality of wealth and consumption persists for a long time after a disaster shock. However, the recovery toward the steady state is accompanied by rising inequality. These patterns also appear in agent 1’s share of equity, $K_1$. This share falls on impact from its steady-state value of 0.767 to 0.731, then rises to 0.755 after 10 years.
For the risk-free rate, $r^f$, we can view the disaster shock and consequent shift in relative wealth toward agent 2 as raising the demand for safe bonds (from agent 2) compared to the supply (from agent 1). In response to this shift in excess demand, $r^f$ falls on impact from its mean value of 0.0100 to 0.0083. That is, the disaster leads to a low risk-free interest rate. In the recovery period, $r^f$ rises but remains below its steady-state value. After 10 years, $r^f$ reaches 0.0094.

The enhanced wealth share of agent 2 is accompanied on impact by a rise in the ratio of the magnitude of safe-to-total assets, $|B_1|/W$. This ratio increases initially from its steady-state value of 0.097 to 0.106. Thus, safe assets are comparatively large immediately after a disaster. The ratio then falls gradually and reaches 0.101 after 10 years.

To summarize, disasters generate low but rising wealth and consumption inequality, low but increasing risk-free real interest rates, and high but declining ratios of safe-to-total assets. In particular, low inequality and risk-free interest rates and high safe-asset ratios are all symptoms of a gradual recovery from a severe adverse shock to the economy.

An essential feature of the disaster shock is that it disproportionately affects agent 1, with low risk aversion, and, thus, shifts the wealth share initially toward agent 2, with high risk aversion. This pattern arises because the shock affects the value of equity, which is for the most part held by agent 1. Hart and Zingales (2014) argue that this kind of pattern characterizes some macro-financial shocks, such as the bursting of the Internet boom in 2000. They point out, however, that other shocks -notably the Great Recession of 2007-2009- feature an erosion on the value of assets that were previously viewed as nearly safe. In the 2007-2009 case, this pattern applied particularly to claims associated with real estate, whose safety had been greatly exaggerated.

In our model, we could analyze the Hart and Zingales case by allowing for an unexpected (i.e., zero-probability event) decline in the value of the existing “safe” assets, which are the private bonds.
In this case, agent 1’s wealth share would initially shift discretely above its steady-state value. The subsequent dynamic corresponds to that described in our next example.

4.3.2 Forty Years of Tranquility

Figure 4 assumes that, starting from the steady state, the economy has a long period with no disasters (“40 years of tranquility”). This situation accords broadly with the U.S. experience from the 1950s up to the Great Recession of 2007-2009. The parametric assumptions for Figure 4 are the same as those for Figure 3 (corresponding to values shown in the notes to Table 2).

In Figure 4, agent 1’s wealth share rises gradually above its steady-state value of 0.670. Conditional on no disasters, this ratio rises after 40 years to 0.711—and would asymptotically approach a higher value, 0.723, if no disaster ever occurred. The value 0.723 is a quasi-steady-state wealth share (we will refer to this quasi-steady-state as “steady state,” with quotes, below and in the figures) in that it applies asymptotically conditional on the realization of no disasters (although agents still consider, while making their decisions, the possibility that disasters can hit the economy). In contrast, the lower mean wealth share of 0.670 is defined inclusive of the occasional occurrence of disasters.

The dynamic path of the wealth share in Figure 4 shows that sustained tranquility is accompanied by rising inequality, with growing wealth and consumption shares of agent 1. The dynamic also features a rising risk-free rate, which increases above its steady-state value of 0.010 and eventually approaches 0.0121. The ratio of safe-to-total assets falls from its steady-state value of 0.10 and gradually approaches 0.087.

In the paths shown in Figure 3, agent 1’s wealth share would never rise above the “steady-state” value of 0.723. However, a shock mentioned before—where the value of safe assets declines sharply because this safety had been exaggerated—could put agent 1’s wealth share above 0.723. In

---

14 A countervailing force in 2007-2009 is that large financial institutions, including Lehman Brothers, experienced sharp losses in the value of assets linked to real estate. This aspect of the shock tends to lower agent 1’s wealth share and, thus, works like the disaster realization that was already analyzed.
that case, the post-shock dynamic paths (conditional on no further disasters) would feature a gradually declining wealth share of agent 1, with this share asymptotically approaching from above the value 0.723. Correspondingly, the risk-free rate would rise initially above its “steady-state” value and then fall gradually, whereas the ratio of safe to total assets would fall initially below its “steady-state” value and then rise gradually.

5 Gross versus Net Lending and Ricardian Equivalence

The bond holdings, $B_1$, shown in Table 3 correspond to net safe lending from high-risk-aversion agent 2 to low-risk-aversion agent 1. There is a sense, however, in which gross bond issuance is not pinned down, because the model would admit unlimited borrowing and lending within types. That is, agent 1 could effectively issue an arbitrary amount of bonds to itself, and analogously for agent 2.

If the model were augmented to include an infinitesimal amount of transaction costs for bond issuance or collection of interest and principal, then borrowing and lending within types would not occur in equilibrium. In this case, the quantity of bonds, $B_1$, shown in Table 3 would be the unique equilibrium for the gross amount outstanding.

If transaction costs associated with bonds are substantial, the quantity of net bond issuance and the risk-free rate might differ significantly from the values shown in Table 3. Correspondingly, the risk-free rate received by lenders (agent 2) would deviate from that paid by borrowers (agent 1). For example, if transaction costs were prohibitive, the results would correspond to those shown in Table 2. The quantity of net bond issuance would be 0, and the share of capital held by each agent type would be 0.5 (assuming $\mu = 0.5$). As an example, if $\gamma_1 = 3.0$, the shadow risk-free rate for agent1 would be 0.025 (from Table 2) and if $\gamma_2 = 5.0$, the shadow risk-free rate for agent 2 would be −0.019 (again from Table 2). That is, agent 1 would be willing to pay a rate of 0.025 per year at the margin on risk-
free borrowing, whereas agent 2 would be willing to accept a rate of $-0.019$ per year at the margin on risk-free lending. However, no issuance of safe debt occurs because of the prohibitive transaction costs.

Suppose now that the government issues one-period bonds with characteristics corresponding to those of private bonds. The real interest rate on government bonds held from $t$ to $t+1$ must then be $r^f$, the same as that on private bonds. The simplest way to introduce public debt is for the government to make a lump-sum transfer of bonds in year $t$ in the aggregate quantity $B_t^g$. This distribution is assumed to go 50-50 to agents 1 and 2 (corresponding to the assumed population share $\mu = 0.5$). The aggregate principal and interest, $(1 + r^f_{t+1})B_t^g$, is paid out to government bondholders in period $t+1$. This payout is financed by lump-sum taxes, levied equally in period $t+1$ on agents 1 and 2.

The representative-agent version of this model, where $\gamma_1 = \gamma_2 = \gamma$, exhibits full Ricardian equivalence. That is, the representative agent willingly holds additional government bonds with no changes in equilibrium rates of return, including the risk-free rate, $r^f$. This result differs from those in Blanchard (1985) and Gârleanu and Panageas (2015), where new agents receive no bequests and, therefore, arrive with below-average assets (although they have labor income). In their settings, a rise in government bonds tends to generate an increase in $r^f$. In contrast, in our model, new agents start life with a bequest (unintended) that corresponds to the assets of an agent who has just died. For this reason, a change in the quantity of government bonds does not change equilibrium rates of return.

In our model, where $\gamma_1 < \gamma_2$, government bond issuance does not affect the rates of return or the amount of net borrowing and lending between agents 1 and 2. That is, the equilibrium features Ricardian equivalence with respect to net quantities of safe assets and the various rates of return.

Consider now how the added government bonds end up being held by agents 1 and 2. One possibility, assumed in the upper part of Table 5, is that each type holds the 50% of the government bonds that they initially received (corresponding to $\mu = 0.5$). These quantities balance the present
value of the (certain) tax liabilities imposed on each type. The quantity of net private borrowing and lending, corresponding to $B_{1,t}$, is then the same as before.

The problem with this proposed equilibrium is that agent 1 is simultaneously holding government bonds and issuing private bonds. Since the two types of bonds are assumed to be indistinguishable, we can think of them as trading on a single bond market with a single rate of return. Hence, in the upper part of Table 5, agent 1 would be operating simultaneously on both sides of this bond market. As before, if there are infinitesimal transaction costs for bond issuance or collection of interest and principal, this equilibrium would be ruled out. Starting from the configuration in the upper part of Table 5, agent 1 would be motivated to sell its government bonds and use the proceeds to retire private bonds.

In the full equilibrium, shown in the lower part of Table 5, the magnitude of the reduction in private bonds equals the amount of government bonds received by agent 1. Since we have assumed that this amount was one-half of the total government bond issue, it follows that the magnitude of the reduction in private bonds expressed as a ratio to government bonds issued equals one-half. That is, the crowding-out coefficient for private bonds with respect to government bonds is minus one-half. More generally, this coefficient equals minus the share of the government bond issue that goes to agents of type 1 -the type issuing the private bonds.$^{15}$

When compared to the equilibrium prior to the government bond issue, the only difference in the lower part of Table 5 is that some of the borrowing and lending between agents 1 and 2 is purely private, while some works through the government as intermediary (collecting taxes from agent 1 and

---

$^{15}$ This generalization of our one-half result was pointed out to us by Xavier Gabaix. He noted that “our basic finding was independent of the disaster theme and would come from pretty much any reason to hold debt.” He then observed: “If there is a fraction $f$ of lenders and $1 - f$ of borrowers, then the crowding-out coefficient is $\frac{d(Gross\ private\ debt)}{d(Govt\ debt)} = -(1 - f)$. ” The result that we stress, where $f = \mu = 0.5$, is not general but is likely to be a good approximation because private lenders and borrowers must be balanced in terms of dollars lent and borrowed even if not in terms of numbers of persons or wealth. Abel (2017) used the working paper version of our analysis, Barro and Mollerus (2014), to generalize our results on crowding-out along the lines sketched by Gabaix.
using the proceeds to pay principal and interest on half of the government bonds held by agent 2). When viewed this way, the finding of Ricardian equivalence is not surprising: it corresponds to the assumption that the private sector and the government are equally good at arranging loans between agents 1 and 2.

The surprising part of our result is that the crowding-out coefficient for private bonds with respect to public bonds is -0.5, not 0.0 or -1.0. The one-half result came from a model with a number of simplifying assumptions; notably, there were just two agent types characterized by their coefficients of relative risk aversion, $\gamma_1$, and the incidence of the present value of taxes net of transfers associated with the government bond issue was the same for each type. However, the crowding-out coefficient around 0.5 does not depend on these assumptions holding precisely. For example, the restriction to two types is unimportant.\textsuperscript{16} The assumption that matters most is that there is little relation across types between $\gamma_1$ and the share of taxes net of transfers applying to the type. For example, in our baseline case, the share of taxes net of transfers is one-half for each type.

The model’s predicted crowding-out coefficient is related to the study by Krishnamurthy and Vissing-Jorgensen (2013, p. 1), who argue “that government debt ...should crowd out the net supply of privately issued short-term debt.” They test this hypothesis on U.S. data for 1914-2011 and find (Table 4, Panel A) that an increase in the quantity of net U.S. government debt has a significantly negative effect on the net short-term debt created by the private financial sector. Remarkably, their estimated coefficient was close to −0.5, the value predicted by our baseline model.\textsuperscript{17} Similarly, Table

\textsuperscript{16} Suppose, for example, that there are four types of agents, where $\gamma_1 < \gamma_2 < \gamma_3 < \gamma_4$. Suppose further that the initial equilibrium involves private bond holdings of $B_1 = -100$, $B_2 = -50$, $B_3 = 50$, and $B_4 = 100$. Assume that the government issues 4 units of bonds, with the present value of taxes rising by 1 unit for each type. In this case, the two private borrowers go, in equilibrium, to $B_1 = -99$ and $B_2 = -49$, thereby preserving their positions for bond holdings net of tax liabilities (of 1 each) at -100 and -50, respectively. The two private lenders go, in equilibrium, to overall bond positions (inclusive of government bonds) of $B_3 = 51$ and $B_4 = 101$, thereby preserving their positions for bond holdings net of tax liabilities at 50 and 100, respectively. The additional 4 units of government bonds crowd out the total of private bonds by 2 units.

\textsuperscript{17} Krishnamurthy and Vissing-Jorgensen (2013, p. 23) say: “These results suggest that a one-dollar increase in Treasury supply reduces the net short-term debt issued by the financial sector by 50 cents.” In their theory (Section 3), they derive
1 in GLM reports a crowding-out coefficient close to −0.5 for a broad concept of private financial-sector liabilities (their “high estimate”) in the United States for 1952-2011.\textsuperscript{18}

Ricardian equivalence would not hold in our model if the government is superior to the private sector in the technology of creating safe assets.\textsuperscript{19} In particular, the government might be able to commit better than private agents to honoring payments of principal and interest on its bonds and can also use the coercive power of the tax system to ensure the financing of these payments. On the other hand, a private lending arrangement requires only that agent 1 makes principal and interest payments in period t + 1 to agent 2, whereas the public setup entails the government collecting taxes in period t + 1 from agent 1 and then using the proceeds to pay off agent 2. Once the distorting influences from taxation are considered, it is not obvious that the public process entails lower “transaction costs” overall.\textsuperscript{20}

An additional consideration is that the expansion of public debt and the associated taxation are poorly targeted. In our baseline case described by the lower part of Table 5, 50% of the added government bonds –held by agent 2– match the added present value of tax liabilities for this type and, therefore, do not serve to shift risk toward agent 1. Only the remaining 50% of government bonds corresponds to this shifting of risk. In contrast, all private bonds issued by agent 1 and held by agent 2 are associated with risk shifting.

To highlight a case where the issuance of public debt is important, suppose that the private sector’s technology for creating safe assets is so poorly developed that no issuance of private bonds

\textsuperscript{18} GLM, p. 1, say: “These results suggest that financial liabilities and government liabilities may be substitutes.”

\textsuperscript{19} Caballero and Farhi (2018, p. 3) make this assumption, although they do not clarify the elements that underlie the government’s superior technology: “Public debt … plays a central role … as typically the government owns a disproportionate share of the capacity to create safe assets while the private sector owns too many risky assets. … The key concept then is that of fiscal capacity: How much public debt can the government credibly pledge to honor should a major macroeconomic shock take place in the future?” They also do not consider that public debt issuance creates additional “safe liabilities” in the form of taxes that match the added safe assets in a present-value sense.

\textsuperscript{20} Even if the interest rate on government bonds is lower than that on private bonds, the overall transaction costs -including the distorting effects from taxation- associated with the public process might exceed those for the private process.
occurs in the initial equilibrium (where no government bonds exist). In this case, analyzed at the beginning of this section, agents 1 and 2 are effectively autonomous, and the equilibrium for each type is the one that would apply in the corresponding representative-agent economy. The risk-free interest rate for type-1 agents can then diverge substantially from that for type-2 agents.

In this environment, the government’s issuance of bonds can substitute for the private lending that would have occurred if the private sector had possessed the technology to create safe assets. In this setting, Ricardian equivalence fails, and the government’s debt issue moves the economy toward a more efficient outcome, where risk is shifted from type-2 to type-1 agents, and the risk-free interest rates of the two types converge. This observation can partly explain why, historically, the issuance of public debt has been linked with the development of deeper and more efficient financial markets.

We can assess how much public debt is required to get the economy into the equilibrium of our baseline model with private debt. The answer -related to the crowding-out coefficient of one-half discussed before- is that the required quantity of public debt is twice the level of private bonds that arose in the initial setting. Moreover, if public debt expands beyond this quantity, it has no further effect on equilibrium. That is, Ricardian equivalence holds in this range at the margin even though private bonds are assumed to be absent.

6 The Quantity of Safe Assets

In our model, the quantity of safe assets corresponds to the shifting of risk from agent 2 to agent 1. Table 3 shows that, for reasonable parameter values, the mean for the ratio of safe-to-total assets ranges up to 15%.  

21 We focus on the model’s predictions about the ratio of safe-to-total assets, rather than the ratio of safe assets to annual GDP. The latter ratio depends on the ratio of total assets to annual GDP, which equals $1/\phi$ in the baseline model with log utility. This last ratio equals 25 when $\rho = 0.02$ per year and $\nu = 0.02$ per year but is sensitive to the choices of $\rho$ and $\nu$.  

31
Using data to match the model’s predictions for the quantity of safe assets is challenging because it is unclear how to measure empirically the amounts of these assets. In Section 2, we followed GLM and defined safe assets as total debt liabilities of the government and the private financial sector. Our evidence suggested a weighted average of safe assets of 37%, with a range from 22% to 66%. Also, this safe-asset share was stable over time. How does our model account for these two observations?

6.1 Level

Our model predicts, under the reasonable values of risk aversion of $\gamma_1 = 3.2$ and $\gamma_2 = 7.0$, a 12.6% safe-asset ratio in the steady state (row 5, Table 3). In our theory, total assets are equal to the discounted value of GDP (which, because we do not have investment, is itself equal to aggregate consumption). This model-consistent definition of total assets includes human capital as well as physical capital. Since income from physical capital constitutes, in developed economies, around one-third of GDP, 12.6% of total assets in our model corresponds to around 38% of physical assets (12.6% divided by one-third), a finding that accords with the GLM measure.

Recall that we reported in Section 2 an observed 37% safe-asset ratio following GLM’s methodology. Therefore, our computed 38% safe-asset ratio matches surprisingly well the measured 37% ratio, especially given the measurement issues involved in the latter number.

We need, however, to discuss three important adjustments to the measure of safe assets that go beyond the scope of our simple environment. First, GLM include government liabilities as safe assets, but do not include any portion of capitalized future taxes as “safe liabilities,” even at the margin. Although tax liabilities cannot be directly traded, it is also true that these liabilities -and how they vary along with changes in the quantity of government bonds- affect economic analyses of public debt. To the extent that future taxes are factored in by agents, the gross public debt would overstate a meaningful measure of safe assets. A richer model that includes a government could analyze this issue.
Second, the GLM measure does not compute a net figure for liabilities of financial institutions; that is, there is no deduction for safe assets held by these institutions. For instance, in 2007-2008, Lehman Brothers issued bonds and commercial paper, but also held U.S. government securities and liabilities of other financial firms. Our model could be extended to account for this kind of borrowing and lending within types. These patterns might arise because of idiosyncratic shocks that affect individual agents within types still defined by coefficients of relative risk aversion. On this ground, GLM’s measured liabilities of government and financial institutions would overstate the net quantity of safe assets.

Third, an array of financial arrangements (including structured finance, stock options, and insurance contracts) can be used to convert risky assets into relatively safe assets. On this ground, the measured liabilities of governments and financial institutions might understate the quantity of safe assets.

The net effects from these adjustments to the measured quantity of safe assets are ambiguous.

6.2 Stability

In comparison with the previous discussion, the observed stability of the ratio of safe-to-total assets accords with the model without further considerations. The results in Table 3 indicate that large changes in the mean of the ratio of safe-to-total assets might arise from changes in the gap between the risk aversion coefficients of the high- and low-risk-aversion types, that is, $\gamma_1 - \gamma_2$. However, if this gap were roughly constant (and the parameters indicated in the note to Table 2 were fixed), then the mean of the ratio of safe-to-total assets would be stable.

A comparison of the results in Table 4 with those in Table 3 indicates that the one-time variations considered in a set of other parameters (the IES, $1/\theta$, the gross replacement rate, $\nu$, the disaster probability, $p$, and the share of type-1 agents, $\mu$) do not have large effects on the mean of the steady-state ratio of safe-to-total assets for given values of $\gamma_1$ and $\gamma_2$. Thus, variations over time in
these other parameters are unlikely to be sources of instability in the mean of the steady-state ratio of safe-to-total assets.

Finally, the results in Figures 2 and 3 show that the ratio of safe-to-total assets does not vary greatly along a dynamic path that is approaching its steady-state values. For instance, in Figure 3, the ratio of safe-to-total assets varies only from 10.6% to 9.7% over a period of 10 years.

7 Conclusions

We constructed a model with heterogeneity in risk aversion to study the determination of the equilibrium quantity of safe assets. The model achieves tractability and transparency by assuming two types of agents with EZW preferences. The agents differ by coefficients of relative risk aversion but have the same IES and rate of time preference. In the baseline model, each agent has log utility, in the sense of IES=1, but we have also considered other values of the IES.

We focused on a stationary version of the model in which agents die off and are replaced by new agents who may have low or high risk aversion. In the baseline setting, Ricardian equivalence holds in that the quantity of government bonds does not affect rates of return or the net quantity of safe assets. The predicted crowding-out coefficient for private bonds with respect to government bonds is around -0.5, in line with some existing empirical evidence.

We generated quantitative implications for the quantity of safe assets by calibrating the model with sufficient disaster risk to get the model’s predictions into the right ballpark for the average equity premium and risk-free rate. Our model matches the safe-asset share ratio of 37% once we adjust for human capital. To the best of our understanding, our paper is the first to simultaneously replicate the observed risk-free rate, equity premium, and safe-asset share with plausible risk-aversion coefficients.

The basic structure of the model with heterogeneity in coefficients of relative risk aversion can be applied to other economic problems. For example, the framework can incorporate credit-market
imperfections, including the necessity for enforcement mechanisms to ensure repayment of private debts. This extension is related to issues concerning collateral, liquidity, and asymmetric information. This type of extension would be important for assessing implications for the magnitude and composition of investment.
References


<table>
<thead>
<tr>
<th>Safe Assets Total Assets</th>
<th>Gov Debt Total Assets</th>
<th>Central Bank Debt Total Assets</th>
<th>Financial Debt Total Assets</th>
<th>Total Assets</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.44</td>
<td>0.13</td>
<td>0.03</td>
<td>0.27</td>
<td>8.20</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.37</td>
<td>0.11</td>
<td>0.01</td>
<td>0.24</td>
<td>12.20</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.40</td>
<td>0.13</td>
<td>NA</td>
<td>0.27</td>
<td>6.50</td>
</tr>
<tr>
<td>Canada</td>
<td>0.29</td>
<td>0.08</td>
<td>0.01</td>
<td>0.20</td>
<td>11.40</td>
</tr>
<tr>
<td>Chile</td>
<td>0.22</td>
<td>0.03</td>
<td>0.03</td>
<td>0.16</td>
<td>7.00</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.40</td>
<td>0.10</td>
<td>0.07</td>
<td>0.22</td>
<td>4.80</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.31</td>
<td>0.04</td>
<td>0.02</td>
<td>0.26</td>
<td>13.40</td>
</tr>
<tr>
<td>Estonia</td>
<td>0.27</td>
<td>0.03</td>
<td>0.05</td>
<td>0.19</td>
<td>6.00</td>
</tr>
<tr>
<td>Finland</td>
<td>0.34</td>
<td>0.08</td>
<td>0.03</td>
<td>0.24</td>
<td>8.50</td>
</tr>
<tr>
<td>France</td>
<td>0.37</td>
<td>0.09</td>
<td>0.02</td>
<td>0.26</td>
<td>12.50</td>
</tr>
<tr>
<td>Germany</td>
<td>0.46</td>
<td>0.10</td>
<td>0.04</td>
<td>0.32</td>
<td>7.60</td>
</tr>
<tr>
<td>Greece</td>
<td>0.66</td>
<td>0.27</td>
<td>0.12</td>
<td>0.26</td>
<td>7.00</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.34</td>
<td>0.13</td>
<td>0.04</td>
<td>0.17</td>
<td>6.70</td>
</tr>
<tr>
<td>Iceland</td>
<td>0.25</td>
<td>0.05</td>
<td>0.03</td>
<td>0.17</td>
<td>12.90</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.26</td>
<td>0.03</td>
<td>0.01</td>
<td>0.22</td>
<td>26.90</td>
</tr>
<tr>
<td>Israel</td>
<td>0.36</td>
<td>0.13</td>
<td>0.05</td>
<td>0.19</td>
<td>7.20</td>
</tr>
<tr>
<td>Italy</td>
<td>0.52</td>
<td>0.20</td>
<td>0.04</td>
<td>0.28</td>
<td>7.90</td>
</tr>
<tr>
<td>Japan</td>
<td>0.54</td>
<td>0.18</td>
<td>0.06</td>
<td>0.31</td>
<td>12.90</td>
</tr>
<tr>
<td>Korea</td>
<td>0.36</td>
<td>0.06</td>
<td>0.03</td>
<td>0.27</td>
<td>8.50</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.43</td>
<td>0.09</td>
<td>0.09</td>
<td>0.26</td>
<td>4.90</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.39</td>
<td>0.16</td>
<td>0.07</td>
<td>0.16</td>
<td>3.90</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.30</td>
<td>0.13</td>
<td>0.04</td>
<td>0.13</td>
<td>3.90</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.35</td>
<td>0.04</td>
<td>0.01</td>
<td>0.29</td>
<td>20.50</td>
</tr>
<tr>
<td>Norway</td>
<td>0.29</td>
<td>0.04</td>
<td>0.01</td>
<td>0.24</td>
<td>8.80</td>
</tr>
<tr>
<td>Poland</td>
<td>0.37</td>
<td>0.14</td>
<td>0.04</td>
<td>0.19</td>
<td>4.30</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.46</td>
<td>0.17</td>
<td>0.05</td>
<td>0.24</td>
<td>10.30</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>0.43</td>
<td>0.18</td>
<td>NA</td>
<td>0.25</td>
<td>4.20</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.43</td>
<td>0.21</td>
<td>0.04</td>
<td>0.18</td>
<td>5.20</td>
</tr>
<tr>
<td>Spain</td>
<td>0.48</td>
<td>0.14</td>
<td>0.04</td>
<td>0.30</td>
<td>9.60</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.27</td>
<td>0.04</td>
<td>0.01</td>
<td>0.22</td>
<td>12.40</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.36</td>
<td>0.02</td>
<td>0.06</td>
<td>0.27</td>
<td>14.20</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.34</td>
<td>0.09</td>
<td>0.04</td>
<td>0.21</td>
<td>3.90</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.40</td>
<td>0.07</td>
<td>NA</td>
<td>0.33</td>
<td>15.20</td>
</tr>
<tr>
<td>United States</td>
<td>0.30</td>
<td>0.10</td>
<td>0.02</td>
<td>0.18</td>
<td>10.40</td>
</tr>
<tr>
<td>mean</td>
<td>0.37</td>
<td>0.11</td>
<td>0.04</td>
<td>0.23</td>
<td>9.40</td>
</tr>
<tr>
<td>mean (weighted)</td>
<td>0.37</td>
<td>0.10</td>
<td>0.03</td>
<td>0.24</td>
<td>11.40</td>
</tr>
<tr>
<td>min</td>
<td>0.22</td>
<td>0.02</td>
<td>0.01</td>
<td>0.13</td>
<td>3.90</td>
</tr>
<tr>
<td>max</td>
<td>0.66</td>
<td>0.27</td>
<td>0.12</td>
<td>0.33</td>
<td>26.90</td>
</tr>
</tbody>
</table>

Note: Column 1 presents the safe-asset share, defined by the ratio of safe assets to total assets. Columns 2-4 present the components of the safe-asset share, and column 5 presents the ratio of total assets to GDP. Data source is financial balance sheets of OECD countries (SNA 2008).
Table 2
Representative-Agent Economy
(Single Coefficient of Relative Risk Aversion)

<table>
<thead>
<tr>
<th>( \gamma_1 = \gamma_2 = \gamma )</th>
<th>( r^e )</th>
<th>( r^f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.052</td>
<td>0.046</td>
</tr>
<tr>
<td>1.5</td>
<td>0.052</td>
<td>0.042</td>
</tr>
<tr>
<td>2</td>
<td>0.052</td>
<td>0.038</td>
</tr>
<tr>
<td>2.5</td>
<td>0.052</td>
<td>0.032</td>
</tr>
<tr>
<td>3</td>
<td>0.052</td>
<td>0.025</td>
</tr>
<tr>
<td>3.5</td>
<td>0.052</td>
<td>0.017</td>
</tr>
<tr>
<td>4</td>
<td>0.052</td>
<td>0.007</td>
</tr>
<tr>
<td>4.5</td>
<td>0.052</td>
<td>-0.005</td>
</tr>
<tr>
<td>5</td>
<td>0.052</td>
<td>-0.019</td>
</tr>
<tr>
<td>5.5</td>
<td>0.052</td>
<td>-0.035</td>
</tr>
<tr>
<td>6</td>
<td>0.052</td>
<td>-0.055</td>
</tr>
</tbody>
</table>

Note: This analysis assumes that the population share of type-1 agents is \( \mu = 0.5 \) and the IES is \( \theta = 1 \) (log utility). When the coefficients of relative risk aversion are the same for the two types of agents, \( \gamma_1 = \gamma_2 = \gamma \), the equilibrium quantities of bonds, \( B_1 \) and \( B_2 \), are zero, and the ownership of equity is evenly distributed, \( K_1 = K_2 = 0.5 \). The table shows the equilibrium risk-free rate, \( r^f \), for each value of \( \gamma \). The calculations assume that the growth-rate parameter is \( g = 0.025 \) per year, the rate of time preference is \( \rho = 0.02 \) per year, the gross replacement rate is \( \nu = 0.02 \) per year, the disaster probability is \( p = 0.04 \) per year (corresponding in the historical data to contractions in per capita GDP of at least 10%), and the disaster size is \( b = 0.32 \).

The expected growth rate is \( g^* = g - p \cdot b = 0.0122 \) per year. The expected rate of return on equity, given \( \theta = 1 \), is \( r^e = \rho + \nu + g^* = 0.052 \) per year, which is independent of \( \gamma \). The price of equity is \( P = Y / (\rho + \nu) = 25 \cdot Y \). In this representative-agent case, the equilibrium risk-free rate can be written in closed form, if \( \gamma \neq 1 \), as:

\[
r^f = \rho + \theta g + p \left( \frac{\theta - 1}{\theta - 1} \right) - p (1 - b)^{-\gamma} + p \left( \frac{\gamma - \theta}{\gamma - 1} \right) (1 - b)^{1-\gamma}.
\]

If \( \theta = 1 \), as \( \gamma \) approaches 1, \( r^f \) approaches \( \rho + g - pb/(1 - b) \).
Table 3
Steady-State Equity Ownership, Wealth Share, and Safe Assets

Alternative values of $\gamma_1$ and $\gamma_2$ that generate $r^e = 0.052$ and $r^f = 0.010$

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
<td>$r^e$</td>
<td>$r^f$</td>
<td>$K_1$</td>
<td>$W_1/W$</td>
<td>$</td>
<td>B_1</td>
</tr>
<tr>
<td>3.86</td>
<td>3.86</td>
<td>0.052</td>
<td>0.010</td>
<td>0.500</td>
<td>0.500</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.052</td>
<td>0.010</td>
<td>0.593</td>
<td>0.560</td>
<td>0.034</td>
<td>0.84</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.052</td>
<td>0.010</td>
<td>0.693</td>
<td>0.623</td>
<td>0.070</td>
<td>1.74</td>
</tr>
<tr>
<td>3.30</td>
<td>5.60</td>
<td>0.052</td>
<td>0.010</td>
<td>0.762</td>
<td>0.667</td>
<td>0.095</td>
<td>2.37</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.052*</td>
<td>0.010</td>
<td>0.846</td>
<td>0.720</td>
<td>0.126</td>
<td>3.14</td>
</tr>
<tr>
<td>3.10*</td>
<td>10.00*</td>
<td>0.052*</td>
<td>0.011*</td>
<td>0.930*</td>
<td>0.771*</td>
<td>0.159*</td>
<td>3.96*</td>
</tr>
</tbody>
</table>

Note: The coefficients of relative risk aversion for the two agents, $\gamma_1$ and $\gamma_2$, are values that generate a steady-state rate of return on equity, $r^e$, of 0.052 and a risk-free interest rate, $r^f$, of 0.010. The other parameters are the same as those indicated in the notes to Table 1. The other columns show the (steady-state) means of agent 1’s share of equity ownership, $K_1$, and total assets, $W_1/W$, and the ratio of the magnitude of safe assets, $B_1$, to total assets and GDP.

* Figures in the last row of the table correspond to a risk-free rate of 0.011. Reducing the risk-free rate to 0.010 requires an increase in $\gamma_2$ above 10, but our numerical solutions in this region were insufficiently accurate.
Table 4
Steady-State Equity Ownership, Wealth Share, and Safe Assets

Alternative Parameter Values

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
<td>$r^e$</td>
<td>$r^f$</td>
<td>$K_1$</td>
<td>$W_1/W$</td>
<td>$</td>
<td>B_1</td>
</tr>
<tr>
<td>$\theta = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.054</td>
<td>0.012</td>
<td>0.635</td>
<td>0.603</td>
<td>0.033</td>
<td>0.78</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.053</td>
<td>0.013</td>
<td>0.752</td>
<td>0.690</td>
<td>0.062</td>
<td>1.50</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.053</td>
<td>0.014</td>
<td>0.880</td>
<td>0.784</td>
<td>0.096</td>
<td>2.35</td>
</tr>
<tr>
<td>$\theta = 2.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.049</td>
<td>0.007</td>
<td>0.570</td>
<td>0.536</td>
<td>0.034</td>
<td>0.92</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.049</td>
<td>0.007</td>
<td>0.655</td>
<td>0.579</td>
<td>0.076</td>
<td>2.05</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.049</td>
<td>0.009</td>
<td>0.816</td>
<td>0.657</td>
<td>0.160</td>
<td>4.30</td>
</tr>
<tr>
<td>$\nu = 0.05$ per year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.083</td>
<td>0.040</td>
<td>0.558</td>
<td>0.524</td>
<td>0.034</td>
<td>0.48</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.083</td>
<td>0.038</td>
<td>0.628</td>
<td>0.554</td>
<td>0.074</td>
<td>1.04</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.083</td>
<td>0.034</td>
<td>0.771</td>
<td>0.621</td>
<td>0.150</td>
<td>2.12</td>
</tr>
<tr>
<td>$p = 0.02$ per year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.059</td>
<td>0.037</td>
<td>0.564</td>
<td>0.530</td>
<td>0.034</td>
<td>0.84</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.059</td>
<td>0.036</td>
<td>0.640</td>
<td>0.567</td>
<td>0.073</td>
<td>1.82</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.059</td>
<td>0.035</td>
<td>0.789</td>
<td>0.644</td>
<td>0.145</td>
<td>3.61</td>
</tr>
<tr>
<td>$\mu = 0.25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.60</td>
<td>4.25</td>
<td>0.052</td>
<td>0.006</td>
<td>0.338</td>
<td>0.308</td>
<td>0.030</td>
<td>0.75</td>
</tr>
<tr>
<td>3.40</td>
<td>4.90</td>
<td>0.052</td>
<td>0.003</td>
<td>0.487</td>
<td>0.410</td>
<td>0.076</td>
<td>1.90</td>
</tr>
<tr>
<td>3.20</td>
<td>7.00</td>
<td>0.052</td>
<td>0.003</td>
<td>0.774</td>
<td>0.605</td>
<td>0.169</td>
<td>4.21</td>
</tr>
</tbody>
</table>

Note: These results use the parameter values shown in the notes to Table 1 and used in Table 2, except for the change in the indicated parameter value. The first three lines use the reciprocal of the IES $\theta = 0.5$ (instead of 1.0), the next three use $\theta = 2.0$, the next three use the gross replacement rate $\nu = 0.05$ per year (instead of 0.02), the next three use the disaster probability $p = 0.02$ per year (instead of 0.04), and the last three use the population share $\mu = 0.25$ for the low-risk-aversion type (instead of 0.5).
Table 5

Changes in Safe Assets When the Government Issues Bonds

<table>
<thead>
<tr>
<th>Changes in:</th>
<th>Agent 1</th>
<th>Agent 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1: Government bonds up by 100, held 50-50</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private bond holdings, B</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Government bond holdings, B^g</td>
<td>+50</td>
<td>+50</td>
<td>+100</td>
</tr>
<tr>
<td>Taxes (present value)</td>
<td>+50</td>
<td>+50</td>
<td>+100</td>
</tr>
<tr>
<td>Net safe assets in model</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Net safe assets as measured by GLM</td>
<td>+50</td>
<td>+50</td>
<td>+100</td>
</tr>
</tbody>
</table>

| Changes in:                               |         |         |       |
| **Case 2: Government bonds up by 100, all held by agent 2** |         |         |       |
| Private bond holdings, B                 | +50*    | -50     | 0     |
| Government bond holdings, B^g             | 0       | +100    | +100  |
| Taxes (present value)                     | +50     | +50     | +100  |
| Net safe assets in model                  | 0       | 0       | 0     |
| Net safe assets as measured by GLM        | 0       | +50     | +50   |

Note: In all cases, the government issues 100 bonds, B^g, and transfers these bonds 50-50 to agents 1 and 2. The present value of taxes rises by 100, divided 50-50 between agents 1 and 2. In case 1, the added government bonds are held 50-50 by agents 1 and 2. In case 2, all of the added government bonds are held by agent 2.

*Borrowing by agent 1 goes down by 50.
The figure shows the safe-asset share (safe assets/total assets) for the U.S. and for the following group of countries (for which we have full data from 1995): Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Japan, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, and United Kingdom.
Figure 2

Public and Private Safe Assets (as shares of total assets)

A. U.S.

B. Non-U.S. Countries

Note: Public safe assets are defined as debt liabilities of the government and central bank. Private safe assets are debt liabilities of financial institutions. All presented as shares of total assets. Non-U.S. countries are listed in the note to Figure 1.
Note: These results correspond to the case where $\gamma_1 = 3.3$ and $\gamma_2 = 5.6$ in Table 2. The simulated paths start from the steady state value of $W_1/W$, 0.670, then assume that a disaster of proportionate size 0.32 materializes in period 1. Subsequently, no further disasters occur. The panels show the dynamic paths after period 1 for agent 1’s wealth share, $W_1/W$, the risk-free interest rate, $r^f$, agent 1’s share of total equity, $K_1$, and the ratio of the magnitude of safe assets, $B_1$, to total assets.
Figure 4

Dynamic Paths for 40 Years of Tranquility

Note: These results correspond to the case where $\gamma_1 = 3.3$ and $\gamma_2 = 5.6$ in Table 2. The simulated paths start from the steady-state value of $W_1/W$, 0.670, then assume that no disasters occur over the next 40 years. The panels show the dynamic paths after period 1 for agent 1’s wealth share, $W_1/W$, the risk-free interest rate, $r^f$, agent 1’s share of total equity, $K_1$, and the ratio of the magnitude of safe assets, $B_1$, to total assets. The lines marked as “steady states” are values that would be approached asymptotically conditional on disasters never happening.