Job Search Models

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Motivation

- Trade in the labor market is a decentralized economic activity:
  1. It takes time and effort.
  2. It is uncoordinated.

- Central points:
  1. Matching arrangements.
  2. Productivity opportunities constantly arise and disappear.
Empirical Observations

- Huge amount of labor turnover.
- Pioneers in this research: Davis and Haltiwanger.
- Micro data:
  2. Job opening and labor turnover survey (JOLTS): 16,000 establishments, monthly.
  4. Longitudinal employer household dynamics (LEHD): matched data.
Basic Accounting Identity

- For each period $t$ and level of aggregation $i$:

$$\text{Net Employment Change}_{ti} = \left( \text{Hires}_{ti} - \text{Separations}_{ti} \right)$$

$$= \left( \text{Creation}_{ti} - \text{Destruction}_{ti} \right)$$

- Difficult to distinguish between voluntary and involuntary separations.
Four Models of Random Matching

- Pissarides (1985).
Setup

- Pissarides (1985)

- Continuous time.

- Constant and exogenous interest rate $r$: stationary world.

- No capital (we will change this later).
Workers

- Continuum of measure $L$ of worker. A law of large numbers hold in the economy.

- Workers are identical.

- Linear preferences (risk neutrality).

- Thus, worker maximizes total discounted income:

$$
\int_0^\infty e^{-rt} y(t) \, dt
$$

where $r$ is the interest rate and $y(t)$ is income per period.
Firms

- Endogenous number of small firms:
  1. One firm = one job.
  2. Competitive producers of the final output at price $p$.

- Free entry into production:
  1. Perfectly elastic supply of firm operators.
  2. Zero-profit condition.

- Vacancy cost $c > 0$ per unit of time.
Matching Function I

- $L$ workers, $u$ unemployment rate, and $v$ vacancy rate.
- How do we determine how many matches do we have?
- Define matching function:
  \[ f_L = m(u_L, v_L) \]

  where $f$ is the rate of jobs created.

- Increasing in both argument, concave, and constant returns to scale.
- Why CRS?
  1. Argument against decreasing returns to scale: submarkets.
  2. But possibly increasing returns to scale (we will come back to this).

- Then
  \[ f = m(u, v). \]
Matching Function II

- All matches are random.


- Empirical evidence:

\[ f_t = e^{\varepsilon_t} u_t^{0.72} v_t^{0.28} \]

- \( \varepsilon_t \) is the sum of:

  1. High frequency noise.

  2. Very low frequency movement (for example, demographics).
What if Increasing Returns to Scale?

- Multiple equilibria:
  1. High activity equilibrium.
  2. Low activity equilibrium.


- In any case, a matching function implies externalities and opens door to inefficiencies.
Properties of Matching Function I

- Define vacancy unemployment ratio (or market tightness) as:
  \[ \theta = \frac{v}{u} \]

- Then:
  \[ q(\theta) = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right) \]

- We can show:
  1. \( q'(\theta) \leq 0 \).
  2. \( \frac{q'(\theta)}{q(\theta)} \theta \in [-1, 0] \).
Properties of Matching Function II

Since \( \frac{f}{v} = \frac{m(u,v)}{v} = q(\theta) \), we have:

1. \( q(\theta) \) is the (Poisson) rate at which vacant jobs become filled.
2. Mean duration of a vacancy is \( \frac{1}{q(\theta)} \).

Since \( \frac{f}{u} = \frac{m(u,v)}{u} = \theta q(\theta) \), we have:

1. \( \theta q(\theta) \) is the (Poisson) rate at which unemployed workers find a job.
2. Mean duration of unemployment is \( \frac{1}{\theta q(\theta)} \).
Externalities

- Note that \( q(\theta) \) and \( \theta q(\theta) \) depend on market tightness.

- This is called a search or congestion externality.

- Think about a party where you take 5 friends.

- Prices and wages do not play a direct role for the rates.

- Competitive versus search equilibria.
Job Creation and Job Destruction

- Job creation: a firm and a worker match and they agree on a wage.

- Job creation in a period: \( fL = u\theta q(\theta)L \).

- Job creation rate: \( \frac{u\theta q(\theta)}{1-u} \).

- Job destruction: exogenous at (Poisson) rate \( \lambda \).

- Job destruction in a period: \( \lambda(1-u)L \).

- Job destruction rate: \( \frac{\lambda(1-u)}{1-u} \).
Evolution of Unemployment

- Evolution of unemployment:

\[ \dot{u} = \lambda (1 - u) - u\theta q(\theta) \]

- In steady state:

\[ \lambda (1 - u) = u\theta q(\theta) \]

or

\[ u = \frac{\lambda}{\lambda + \theta q(\theta)} \]

- This relation is a downward-slopping and convex to the origin curve: the Beveridge Curve.
Beveridge Curve
2000:Q4 - 2010:Q2*

Source: BLS, Job Openings and Labor Turnover Survey and Current Population Survey

* Q2:2010 is average of Apr & May
Labor Contracts and Firm’s Value Functions

- Wage $w$.
- Hours fixed and normalized to 1.
- Either part can break the contract at any time without cost.
- $J$ is the value function of an occupied job.
- $V$ is the value function of a vacant job.
- Then, in a stationary equilibrium:
  \[ rV = -c + q(\theta)(J - V) \]
  \[ rJ = p - w - \lambda J \]
- Note $J = \frac{p - w}{r + \lambda}$ and $J' = -\frac{1}{r + \lambda}$.
Job Creation Condition

- Because of free entry

\[ V = 0 \]
\[ J = \frac{c}{q(\theta)}. \]

- Then:

\[ p - w - (r + \lambda) J = 0 \implies \]
\[ p - w - (r + \lambda) \frac{c}{q(\theta)} = 0 \]

- This equation is known as the job creation condition.
- Interpretation.
Workers I

- Value of not working: $z$.
- Includes leisure, UI, home production.
- Because of linearity of preferences, we can ignore extra income.
- $U$ is the value function of unemployed worker.
- $W$ is the value function of employed worker.
- Then:

$$rU = z + \theta q(\theta) (W - U)$$
$$rW = w + \lambda (U - W)$$

- Note $W = \frac{w}{r + \lambda} + \frac{\lambda}{r + \lambda} U$ and $W' = \frac{1}{r + \lambda}$. 
Workers II

- With some algebra:

\[(r + \theta q(\theta)) U - \theta q(\theta) W = z\]
\[-\lambda U + (r + \lambda) W = w\]

and

\[
U = \frac{(r + \lambda) z + \theta q(\theta) w}{(r + \theta q(\theta))(r + \lambda) - \lambda \theta q(\theta)} = \frac{\lambda z + \theta q(\theta) w + rz}{r^2 + r \theta q(\theta) + \lambda r}
\]
\[
W = \frac{(r + \theta q(\theta)) w + \lambda z}{(r + \theta q(\theta))(r + \lambda) - \lambda \theta q(\theta)} = \frac{\lambda z + \theta q(\theta) w + rw}{r^2 + r \theta q(\theta) + \lambda r}
\]

- Clearly, for \(r > 0\), \(W > U\) if and only if \(w > z\).

- Note that if \(r = 0\), \(W = U\).
Wage Determination I

- We can solve Nash Bargaining Solution:

\[ w = \arg \max (W - U)\beta (J - V)^{1-\beta} \]

- First order conditions:

\[ \beta \frac{W'}{W - U} = - (1 - \beta) \frac{J'}{J - V} \]

- Since \( W' = -J' = \frac{1}{r+\lambda} \) and \( V = 0 \):

\[ W = U + \beta \left( W - U + J \right)_{\text{surplus of the relation}} = U + \beta S \]
Wage Determination II

Also

\[ W - U = \frac{\beta}{1 - \beta} J = \frac{\beta}{1 - \beta} c (\theta) \]

Since \( J = \frac{p - w}{r + \lambda} \) and \( W = \frac{w}{r + \lambda} + \frac{\lambda}{r + \lambda} U \)

\[ \frac{w}{r + \lambda} - \frac{r}{r + \lambda} U = \beta \left( \frac{w}{r + \lambda} - \frac{r}{r + \lambda} U + \frac{p - w}{r + \lambda} \right) \Rightarrow \]

\[ w = rU + \beta (p - rU) \]

Interpretation.
Now, note:

\[
    w = rU + \beta (p - rU) \Rightarrow \\
    w = (1 - \beta) rU + \beta p \Rightarrow \\
    w = (1 - \beta) (z + \theta q (\theta) (W - U)) + \beta p \Rightarrow \\
    w = (1 - \beta) \left( z + \theta q (\theta) \frac{\beta}{1 - \beta} \frac{c}{q (\theta)} \right) + \beta p \Rightarrow \\
    w = (1 - \beta) z + \beta (p + \theta c)
\]

The last condition is known as the Wage Equation.
Steady State

- Three equations:

\[ w = (1 - \beta) z + \beta \theta c + \beta p \]

\[ p - w - (r + \lambda) \frac{c}{q(\theta)} = 0 \]

\[ u = \frac{\lambda}{\lambda + \theta q(\theta)} \]

- Combine the first two conditions:

\[ (1 - \beta) (p - z) - \frac{r + \lambda + \beta \theta q(\theta)}{q(\theta)} c = 0 \]

\[ u = \frac{\lambda}{\lambda + \theta q(\theta)} \]

that we can plot in the Beveridge Diagram.
Comparative Statics

- Raise $z$: higher unemployment because less surplus to firms. Relation with unemployment insurance.

- Changes in matching function.

- Changes in Nash parameter.

- Dynamics?
Can the equilibrium achieve social efficiency despite search externalities?

Social Planner:

$$\max_{u, \theta} \int_{0}^{\infty} e^{-rt} \left( p(1 - u) + zu - c\theta u \right) dt$$

s.t. $u = \frac{\lambda}{\lambda + \theta q(\theta)}$

The social planner faces the same matching frictions than the agents.

First order conditions of the Hamiltonian:

$$-e^{-rt} (p - z + c\theta) + \mu (\lambda + \theta q(\theta)) - \dot{\mu} = 0$$

$$-e^{-rt} cu + \mu u q(\theta) (1 - \eta(\theta)) = 0$$

where $\mu$ is the multiplier and $\eta(\theta)$ is (minus) the elasticity of $q(\theta)$. 
Efficiency II

- From the second equation:

\[ \mu = e^{-rt} \frac{cu}{uq(\theta) (1 - \eta(\theta))} \]

- Now:

\[ e^{-rt} cu = \mu uq(\theta) (1 - \eta(\theta)) \]

\[ -rt + \log cu = \log \mu + \log uq(\theta) (1 - \eta(\theta)) \]

and taking time derivatives:

\[ -r = \frac{\dot{\mu}}{\mu} \Rightarrow -\dot{\mu} = r\mu \]

and

\[ -e^{-rt} (p - z + c\theta) + \mu (\lambda + \theta q(\theta)) - \dot{\mu} = 0 \Rightarrow \]

\[ -e^{-rt} (p - z + c\theta) + \mu (r + \lambda + \theta q(\theta)) = 0 \]
Thus we get:

\[-e^{-rt}(p - z + c\theta) + e^{-rt} \frac{cu(r + \lambda + \theta q(\theta))}{uq(\theta)(1 - \eta(\theta))} = 0 \Rightarrow\]

\[(1 - \eta(\theta))(p - z) - \frac{r + \lambda + \eta(\theta)\theta q(\theta)}{q(\theta)} c = 0\]

Remember that the market job creation condition:

\[(1 - \beta)(p - z) - \frac{r + \lambda + \beta\theta q(\theta)}{q(\theta)} c = 0\]

Both conditions are equal if, and only if, \(\eta(\theta) = \beta\).
Hosios’ Rule

- Imagine that matching function is \( m = Au^{\eta} v^{1-\eta} \).

- Then \( \eta(\theta) = \eta \).

- We have that efficiency is satisfied if \( \eta = \beta \).

- This result is known as the Hosios Rule (Hosios, 1990):
  1. If \( \eta > \beta \) equilibrium unemployment is below its social optimum.
  2. If \( \eta < \beta \) equilibrium unemployment is above its social optimum.

- Intuition: externalities equal to share of surplus.
Introducing Capital

- Production function $f(k)$ per worker with depreciation rate $\delta$.

- Arbitrage condition in capital market $f'(k) = (r + \delta)$.

- We have four equations:

  $$f'(k) = (r + \delta)$$
  $$w = (1 - \beta)z + \beta \theta c + \beta p(f(k) - (r + \delta)k)$$
  $$p(f(k) - (r + \delta)k) - w - (r + \lambda) \frac{c}{q(\theta)} = 0$$
  $$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$
Setup

- Similar to previous model but we endogeneize job destruction.
- Productivity of a job $px$ where $x$ is the idiosyncratic component.
- New $x$'s arrive with Poisson rate $\lambda$.
- Distribution is $G(\cdot)$.
- Distribution is memoryless and with bounded support $[0, 1]$.
- Initial draw is $x = 1$. Why?
Policy Function of the Firm

- Value function for a job is $J(x)$.
- Then:
  - If $J(x) \geq 0$, the job is kept.
  - If $J(x) < 0$, the job is destroyed.
- There is an $R$ such that $J(R) = 0$.
- This $R$ is the reservation productivity.
Flows into Unemployment

- A law of large numbers hold for the economy.

- Job destruction: \( \lambda G(R)(1 - u) \).

- Unemployment evolves:

  \[
  \dot{u} = \lambda G(R)(1 - u) - u\theta q(\theta)
  \]

- In steady state:

  \[
  u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}
  \]
Value Functions

- Value functions for the firm:

\[
\begin{align*}
    rV &= -c + q(\theta) (J(1) - V) \\
    rJ(x) &= px - w(x) + \lambda \int_{R}^{1} J(s) dG(s) - \lambda J(x)
\end{align*}
\]

- Value functions for the worker:

\[
\begin{align*}
    rU &= z + \theta q(\theta) (W(1) - U) \\
    rW(x) &= w(x) + \lambda \int_{R}^{1} W(s) dG(s) + \lambda G(R) U - \lambda W(x)
\end{align*}
\]

Because of free entry, \( V = 0 \) and \( J(1) = \frac{c}{q(\theta)} \).
Also, by Nash bargaining:

\[
W(x) - U = \beta (W(x) - U + J(x))
\]
Equilibrium Equations

\[ u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)} \]

\[ J(R) = 0 \]

\[ J(1) = \frac{c}{q(\theta)} \]

\[ W(x) - U = \beta (W(x) - U + J(x)) \]
Solving the Model I

- First, repeating the same steps than in the Pissarides model:

\[ w(x) = (1 - \beta)z + \beta(px + \theta c) \]

- Second:

\[ W(R) - U = \beta (W(R) - U + J(R)) = \beta (W(R) - U) \Rightarrow W(R) = U \]

- Third:

\[ rJ(x) = px - (1 - \beta)z - \beta(px + \theta c) + \lambda \int_{R}^{1} J(s) \, dG(s) - \lambda J(x) \Rightarrow \\
(r + \lambda) J(x) = (1 - \beta)px - (1 - \beta)z - \beta\theta c + \lambda \int_{R}^{1} J(s) \, dG(s) \]
Solving the Model II

- At $x = R$

$$ (r + \lambda) J(R) = (1 - \beta) pR - (1 - \beta) z - \beta c + \lambda \int_{R}^{1} J(s) dG(s) = 0 $$

- Thus:

$$ (r + \lambda) J(x) = (1 - \beta) p(x - R) \Rightarrow $$
$$ (r + \lambda) J(1) = (1 - \beta) p(1 - R) \Rightarrow $$
$$ (r + \lambda) \frac{c}{q(\theta)} = (1 - \beta) p(1 - R) \Rightarrow $$

$$ (1 - \beta) p \frac{1 - R}{r + \lambda} = \frac{c}{q(\theta)} $$
Solving the Model III

- Note that

\[(r + \lambda) J(x) = (1 - \beta) p(x - R) \Rightarrow J(x) = \frac{(1 - \beta)}{r + \lambda} p(x - R)\]

- Then

\[(r + \lambda) J(x) = (1 - \beta) (px - z) - \beta \theta c + \lambda \int_{R}^{1} J(s) dG(s) \Rightarrow\]

\[(r + \lambda) J(x) = (1 - \beta) (px - z) - \beta \theta c + \frac{\lambda (1 - \beta) p}{r + \lambda} \int_{R}^{1} (s - R) dG(s)\]
Evaluate the previous expression at \( x = R \) and using the fact that \( J(R) = 0 \)

\[
(r + \lambda) J(R) = 0 = (1 - \beta) (pR - z) - \beta \theta c + \frac{\lambda (1 - \beta) p}{r + \lambda} \int_{R}^{1} (s - R) \, dG(s) \]

\[
\Rightarrow \quad R - \frac{z}{p} - \frac{\beta}{1 - \beta} \theta c + \frac{\lambda}{r + \lambda} \int_{R}^{1} (s - R) \, dG(s) = 0
\]
Solving the Model V

- We have two equations on two unknowns, $R$ and $\theta$:

$$
(1 - \beta) \frac{p}{r + \lambda} \frac{1 - R}{r + \lambda} = \frac{c}{q(\theta)}
$$

$$
R - \frac{z}{p} - \frac{\beta}{1 - \beta} \theta c + \frac{\lambda}{r + \lambda} \int_R^1 (s - R) dG(s) = 0
$$

- The first expression is known as the job creation condition.

- The second expression is known as the job destruction condition.

- Together with $u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$ and $w(x) = (1 - \beta) z + \beta (px + \theta c)$, we complete the characterization of the equilibrium.
Efficiency

- **Social Welfare:**

\[
\max_{u, \theta} \int_0^{\infty} e^{-rt} (y + zu - c\theta u) \, dt
\]

\[
s.t. \quad u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}
\]

where \(y\) is the average product per person in the labor market.

- **The evolution of \(y\) is given by:**

\[
\dot{y} = p\theta q(\theta) u + \lambda (1 - u) \int_R^1 psdG(s) - \lambda y
\]

- **Again, Hosios’ rule.**
Motivation


- Wage dispersion: different wages for the same work.

- Violates the law of one price.

- What is same work? Observable and unobservable heterogeneity.

- Evidence of wage dispersion: Mincerian regression

\[ w_i = X_i' \beta + \varepsilon_i \]

- Typical Mincerian regression accounts for 25-30% of variation in the data.
Theoretical Challenge

- Remember Diamond’s paradox: elasticity of labor supply was zero for the firm.

- Not all the deviations from a competitive setting deliver wage dispersion.

- Wage dispersion you get from Mortensen-Pissarides is very small (Krusell, Hornstein, Violante, 2007).

- Main mechanism to generate wage dispersion: on-the-job search.
Environment

- Unit measure of identical workers.

- Unit measure of identical firms.

- Each worker is unemployed (state 0) or employed (state 1).

- Poisson arrival rate of new offers $\lambda$. Same for workers and unemployed agents.

- Offers come from an equilibrium distribution $F$. 
Previous Assumptions that We Keep

- No recall of offers.
- Job-worker matches are destroyed at rate $\delta$.
- Value of not working: $z$.
- Discount rate $r$.
- Vacancy cost $c$. 
Value Functions for Workers

- Utility of unemployed agent:
  \[ rV_0 = z + \lambda \left[ \int \max \{ V_0, V_1(w') \} \, dF(w') - V_0 \right] \]

- Utility of worker employed at wage \( w \):
  \[ rV_1(w) = w + \lambda \left[ \int [\max \{ V_1(w), V_1(w') \} - V_1(w)] \, dF(w') \right] + \delta [V_0 - V_1(w)] \]

- As before, there is a reservation wage \( w_R \) such that \( V_0 = V_1(w_R) \).

- Clearly, \( w_R = z \).
Firms Problem

- \( G(w) \): distribution of workers.


- The profit for a firm:

\[
\pi(p, w) = \frac{[u + (1 - u) G(w)]}{r + \delta + \lambda (1 - F(w))} (p - w)
\]

- Firm sets wages \( w \) to maximize \( \pi(p, w) \). No symmetric pure strategy equilibrium.

- Firms will never post \( w \) lower than \( z \).
Unemployment

- Steady state unemployment:

\[ \lambda (1 - F(z)) u = \delta (1 - u) \]

- Then:

\[ u = \frac{\delta}{\delta + \lambda [1 - F(z)]} = \frac{\delta}{\delta + \lambda} \]

where we have used the fact that no firm will post wage lower than \( z \) and that \( F \) will not have mass points (equilibrium property that we have not shown yet).

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Distribution of Workers

- Workers gaining less than $w$:
  \[ E(w) = (1 - u) G(w) \]

- Then:
  \[ \dot{E}(w) = \lambda F(w) u - (\delta + \lambda [1 - F(w)]) E(w) \]

- In steady state:
  \[ E(w) = \frac{\lambda F(w)}{\delta + \lambda [1 - F(w)]} u \Rightarrow \]
  \[ G(w) = \frac{E(w)}{1 - u} = \frac{\delta F(w)}{\delta + \lambda [1 - F(w)]} \]
Equilibrium objects: \( u, F(w), \lambda, G(w) \).

Simple yet boring arguments show that \( F(w) \) does not have mass points and has connected support.

First, by free entry:

\[
\pi(p, z) = \frac{\delta}{\delta + \lambda} \frac{p - z}{r + \delta + \lambda} = c
\]

which we solve for \( \lambda \).

Hence, we also know \( u = \frac{\delta}{\delta + \lambda} \).
Second, by the equality of profits and with some substitutions:

\[
\pi(p, w) = \frac{\delta}{\delta + \lambda} + \frac{\lambda}{\delta + \lambda} \frac{\delta F(w)}{\delta + \lambda [1 - F(w)]} (p - w)
\]

\[
= \frac{\delta}{\delta + \lambda [1 - F(w)]} \frac{p - w}{r + \delta + \lambda (1 - F(w))}
\]

\[
= \frac{\delta}{\delta + \lambda} \frac{p - z}{r + \delta + \lambda}
\]

Previous equality is a quadratic equation on \( F(w) \).

To simplify the solution, set \( r = 0 \). Then:

\[
F(w) = \frac{\delta + \lambda}{\delta} \left[ 1 - \left( \frac{p - w}{p - z} \right)^{0.5} \right]
\]
Now, we get:

\[ G(w) = \frac{\delta}{\lambda} \left[ \left( \frac{p-w}{p-z} \right)^{0.5} - 1 \right] \]

Highest wage is \( F(w^{\text{max}}) = 1 \)

\[ w^{\text{max}} = \left( 1 - \frac{\delta}{\delta + \lambda} \right)^2 p + \left( \frac{\delta}{\delta + \lambda} \right)^2 z \]

Empirical content.

Modifications to fit the data.
Competitive Search


- A market maker chooses a number of markets $m$ and determines the wage $w_i$ in each submarket.

- Workers and firms are free to move between markets.

- Two alternative interpretations:
  1. Clubs charging an entry fee. Competition drives fees to zero.
  2. Wage posting by firms.
Workers

- Value functions:
  \[ rU_i = z + \theta_i q(\theta_i) (W_i - U_i) \]
  \[ rW_i = w_i + \lambda (U_i - W_i) \]

- Then:
  \[ W_i = \frac{1}{r + \lambda} w_i + \frac{\lambda}{r + \lambda} U_i \]
  \[ rU_i = z + \theta_i q(\theta_i) \left( \frac{w_i - rU_i}{r + \lambda} \right) \]

- Workers will pick the highest \( U_i \).
- In equilibrium, all submarkets should deliver the same \( U_i \). Hence:
  \[ \theta_i q(\theta_i) = \frac{rU - z}{w_i - rU} (r + \lambda) \]

- Negative relation between wage and labor market tightness.
- If \( w_i < rU \), the market will not attract workers and it will close.
Firms

- Value Functions:

\[ rV_i = -c + q(\theta_i) (J_i - V_i) \]
\[ rJ_i = p - w_i - \lambda J_i \]

- Thus:

\[ rV_i = -c + q(\theta_i) \left( \frac{p - w_i}{r + \lambda} - V_i \right) \]

- Each firm solves

\[ rV_i = \max_{w_i, \theta_i} \left( -c + q(\theta_i) \left( \frac{p - w_i}{r + \lambda} - V_i \right) \right) \]
\[ s.t. \ rU_i = z + \theta_i q(\theta_i) \left( \frac{w_i - rU}{r + \lambda} \right) \]
Equilibrium

- Impose equilibrium condition $V_i = 0$ and solve the dual:

$$rU_i = \max_{w_i, \theta_i} \left( z + \theta_i q(\theta_i) \frac{w_i - rU}{r + \lambda} \right)$$

$$\text{s.t. } c = q(\theta_i) \frac{p - w_i}{r + \lambda}$$

- Plugging the value of $w_i$ from the constraint into the objective function:

$$rU_i = \max_{\theta_i} \left( z - c\theta_i + \theta_i q(\theta_i) \frac{p - rU}{r + \lambda} \right)$$

- Solution:

$$c = q(\theta_i) \frac{p - rU}{r + \lambda} + \theta_i q'(\theta_i) \frac{p - rU}{r + \lambda}$$

that is unique if $\theta_i q(\theta_i)$ is concave.