

# Was Malthus Right?

## Economic Growth and Population Dynamics\*

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### Abstract

This paper studies the relationship between population dynamics and economic growth. Prior to the Industrial Revolution increases in total output were roughly matched by increases in population. In contrast, during the last 150 years, increments in per capita income have coexisted with slow population growth. Why are income and population growth no longer positively correlated? This paper presents a new answer, based on the role of *capital-specific technological change*, that provides a unifying account of lower population growth and sustained economic growth. An overlapping generations model with *capital-skill complementarity* and endogenous fertility, mortality and education is constructed and parametrized to match English data. The key finding is that the observed fall in the relative price of capital accounts for more than 60% of the fall in fertility and over 50% of the increase in income per capita in England occurred during the demographic transition. Additional experiments show that neutral technological change or the reduction in mortality cannot account for the fall in fertility.

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# 1. Introduction

This paper studies the relationship between population dynamics and economic growth. It makes two contributions. First, it presents a new propagation channel from technological change into population growth: the combination of capital-specific technological change and capital-skill complementarity. Second, it quantitatively evaluates to what extent theory can account for the economic growth and fertility change observed in England during the key years of the demographic transition.

The key motivation for the paper comes from the historical evidence on economic growth and population dynamics. For centuries, the world experienced important technological developments: the discovery of agriculture or the invention of writing and science are just some remarkable examples. Despite these advances, existing evidence suggests that increases in total output were roughly matched by increases in population. In contrast, during the last 150 years, the pattern is quite different: production has been growing steadily while population growth has slowed down, dramatically raising income per capita. Why is modern economic growth so different from other periods? Why are income and population growth no longer correlated? In particular, why have developed countries experience the *demographic transition*, the movement from a high mortality-high fertility regime to a low mortality-low fertility scenario? and how is this demographic transition related with economic growth?

Explaining the relationship between income and population is one of the oldest challenges in economics. Malthus (1803) developed a powerful model that links better technology with constant living standards. According to his model, technological change allows a higher total output. The increase in total output causes higher population through higher fertility and lower mortality. However, in the presence of a fixed input as land, this higher population translates into lower marginal productivities that decrease per capita income back to the stationary level previous to the technological advance. Malthus' model is quite successful at accounting for the main facts that prevailed until the nineteenth century, but it fails to explain the coexistence of growth in per capita income and low fertility. This problem is pervasive in neoclassical theory. Fertility choice models need to include children as an argument in preferences to explain why parents use time and resources for childbearing. However this inclusion usually makes children normal goods with positive rent-elasticity, i.e. fertility should increase as income increases. This suggests a puzzle: while theory predicts a positive relationship between income and fertility, a negative correlation is observed both cross sectionally and over time.

The seminal idea for breaking this Malthusian knot was developed in Becker (1960) and Becker and Lewis (1973). Those papers argued for the existence of a trade-off between quantity and quality of children. They also suggested that, if the income-elasticity of quality

is sufficiently high, quantity will go down with higher income, explaining the observed negative relation between fertility and income levels. The interest in this mechanism was revived with the presentation of an operational dynastic model of fertility in Barro and Becker (1989) and Becker and Barro (1988).

Building on this initial work, Becker, Murphy and Tamura (1990), Lucas (1998), Jones (2001) and specially in an important contribution, Galor and Weil (2000) present models that try to capture the historical evolution of population and output. These models emphasize how modern economic growth can be made consistent with the demographic transition if the process of economic growth raises the return to human capital investment. Facing this higher return on their children's human capital, parents will move away from having a lot of poorly educated children towards having fewer, highly educated ones. However these papers are either silent on the source of the higher human capital returns or present very reduced form characterizations of the process that make difficult to map observables into theory. Also, with the partial exception of Jones (2001), there has been no previous attempt to quantitatively compare theory and data and evaluate whether we can account for measurements with a carefully parametrized model.

This paper tries to fill these two voids. First, it presents an operational and empirically based mechanism to relate economic growth and increasing returns to human capital. Second, it builds a dynamic general equilibrium model and compares its price and quantities predictions with data.

The two empirical observations used to develop an alternative theory of increments in the returns to capital are the trend in the relative price of capital and the elasticities of substitution among different inputs. The first observation, a long-run, declining, trend in the relative price of capital have been extensively documented (Collins and Williamson (1999) and Greenwood *et al.* (1997) among others). A simple interpretation of this fall involves a two-sector model where technological progress is higher in the sector that produces the capital good than in the sector that produces the consumption good. This difference is often known as *capital-specific technological change*. The second observation is the high complementarity of physical capital and skilled labor and the substitutability of physical capital and unskilled labor observed in the data (Krusell *et al.* (2000)). This feature of technology is also called *capital-skill complementarity*.

How might the combination of *capital-specific technological change* and *capital-skill complementarity* lead to growth in per capita income and the fall in fertility? More productive capital (or equivalently, cheaper capital) raises the skill-premium attached with higher human capital when physical capital and skilled labor are complementary inputs and physical capital and unskilled labor are substitutes. The increment in the skill premium induces parents to choose more education instead of more children causing the fall in fertility.

This intuition qualitatively shows how an increase in the return to human capital investment can reduce fertility. To quantitatively analyze this mechanism, this paper develops a version of the neoclassical growth model in which parents choose consumption, work effort, savings, fertility, and education for their children. Mortality is endogenously treated as depending on age-specific factors and consumption. The model is calibrated to match English data prior to the demographic transition. Two main results appear: when neutral technological change is introduced in the model, fertility increases as predicted by Malthus and the evidence previous to 1800. However, when the observed fall in the relative price of capital is fed through capital specific technological change, the model reproduces more than 60% of the fertility drop and more than 50% of per capita income increase in England. That suggests that standard neoclassical theory can quantitatively account for the dynamics of economic growth and population change.

Evidence for England is consistent with the hypothesis. Around 1870, the relative price of capital began to decline (Collins and Williamson (1999)) and the skill-premium increased substantially despite large unskilled migration to England offshoots (Anderson (2001) and Williamson (1997)). In addition, there is a strong negative correlation between education achievements and the fall in fertility across cohorts. Also, an important literature in development economics initiated by Schultz (1975) emphasizes how technological change increases the returns to the ability to analyze and adapt new production procedures. To the extent that formal schooling raises these abilities, we should observe technological change associated with higher schooling and lower fertility. Rosenzweig (1990) presents striking evidence of how the Green Revolution in India triggered both higher schooling and lower fertility.

A further theoretical finding involves the effects of a reduction in mortality. Contrary to other previous results in the literature (Meltzer (1995) or Eckstein *et al.* (1999)), a reduction in mortality increases fertility. The reason is simple: the increase in utility of having children with longer life expectancies dominates the incentives for additional investment in human capital justified by the same longer life.

The rest of the paper is organized as follows. Section 2 reviews the empirical evidence on the demographic transition with an special treatment of the English case. Section 3 presents the model and section 4 parametrizes it. Section 5 discusses the different experiments and section 6 reports sensitivity analysis. Section 7 concludes. An appendix explains data and computational issues in detail.

## 2. Some basic facts

The main fact of population dynamics and economic growth this paper is concerned about is the change from a high mortality-high fertility regime to a low mortality-low fertility scenario

at the beginning of the modern epoch of economic growth. Since the history of income levels is well known (Maddison (1991)), this section concentrates in reviewing the empirical evidence on the demographic transition, both from a general view and from an English perspective.

## 2.1. Population Dynamics and the Demographic Transition

The demographic transition is the change from a regime of high fertility-high mortality, predominant for centuries, to the present situation (at least in developed countries) of low fertility-low mortality. This change is characterized by an intermediate period where mortality is falling but not yet fertility, generating a large surge in population. This pattern is well documented in literature (see Chesnais (1992))<sup>1</sup>.

Mortality rates began to fall during the middle of the eighteenth century in England, France, North America and Scandinavian countries and continued during the nineteenth century, including more and more European countries. In the twentieth century the decline has extended worldwide and mortality is still falling.

For decades, the study of mortality relied extensively on the use of parish records of burials (Gooner (1913)). A main conclusion was that the first decline in mortality during the eighteenth century corresponded basically to the elimination of traditional mortality crises (Goubert (1960)). However, when national data were assembled (Wrigley and Schofield (1981) or Dupâquier (1989)), it was discovered that about 90% of the reduction in mortality was due to a fall in normal causes of death<sup>2</sup>. Existing evidence supports now the idea that famines were at a lower order of magnitude in comparison with chronic malnutrition as a death cause.

Following this evidence, McKeown (1976) proposed the increment of average caloric consumption and the as the most likely candidates to explain the initial decline in mortality. To support this view, Fogel and Floud (1994) produced a series of *per capita* caloric intake from the estimates of English food production that showed a raise from 1802 calories at the beginning of the eighteenth century to 2346 a century later. But this emphasis on consumption over medical improvements created a huge controversy. Lee (1981) showed little correlation between the short-term variations in death rates and wheat price. Razzel (1974) pointed out how mortality rates of nobility rapidly fell after 1725 although there is no sign of change in the diet of the peerage. Preston and Haines (1991) documented the development of scientific knowledge of causes of death and the subsequent improvement in public health, especially after the quick diffusion of the smallpox vaccine, discovered by Jenner in 1796.

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<sup>1</sup>Guillard (1855), the inventor of the word ‘demography’, already noticed the close relationship between levels (and trends) of mortality and fertility. The concept of demographic transition was developed by Thompson (1929).

<sup>2</sup>Famines were local and with low correlation, so they smoothed out in the aggregate. Wrigley and Schofield (1991) show that between 1550 and 1750 mortality crisis accounted for less than 6% of deaths in England.

After a momentary pause in the middle decades of the nineteenth century<sup>3</sup>, mortality rates reassumed their downward trend in Western countries, accelerating after the discovery by Pasteur in 1884 of the microbic origin of infectious diseases. Immediately after the first world war, most non-European countries experienced a sudden decline in mortality (most significantly China and India) that reached Africa in the fifties, as the emergence of antibiotics and modern vaccines led to an efficient treatment of contagious diseases. Nowadays, mortality is still falling worldwide, slowly in rich countries, and rapidly in poor ones.

The history of fertility is simpler: the fall in fertility occurred later in time and it was, except for the *baby boom* after the second world war, rather monotone and rapid. Thanks to the data collected by the *Princeton European Fertility Project*, undertaken in 1963, the statistical picture of the process is clear. The first decline in fertility appeared in France around 1760 but a general phenomenon of fertility decline does not appear in most rich countries until 1875, well over a century after the initial fall in mortality. Not only did fertility not decrease on the eve of industrial revolution but there is evidence that in English reproduction rates increased substantially in the last decades of the eighteenth century and beginning of the nineteenth century (Wrigley and Schofield (1981)).

The fall in fertility was rather simultaneous within Europe, moving from Scandinavia to northern Europe and then to the center and south of Europe and other European stock populations (United States, Australia, Argentina). Most of this fall was the consequence of a reduction in marital fertility and not in a increase of marriage age or an increase in celibacy. Most rich countries later experienced a rise in the number of births, commonly known as the ‘baby boom’, after the second world war. For instance, in the United States, the reproduction rate went from 2.19 in 1940 to 3.58 in 1957. A short-run movement in fertility may have occurred in the last decades as fertility rates declined below replacement levels in several countries and now seem to be raising again, although the data are inconclusive.

The decline of fertility in poor countries does not appear until after the second world war, first in the middle fifties in small, densely populated societies as Cyprus or Singapore and in the sixties in middle income countries such as Brazil or Egypt. China and India began the fall around 1970. Today, extremely high fertility rates are only found in specific cases as Afghanistan, in the Arab middle east and subsaharian Africa.

The picture of the demographic transition is then as follows. For centuries high mortality coexist with high fertility. Then, mortality begins to fall in the seventeenth century and keeps this trend until today. Fertility stays high until the end of nineteenth century and then drops very quickly, reaching a new combination of low mortality and low fertility.

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<sup>3</sup>The industrial revolution, overpopulation (Sandberg and Steckel (1988)) or the growth and crowding of inner cities (Haines and Anderson (1988)) are possible explanations of this detour.

## 2.2. The English Case

Three reasons suggest concentrating on the English case. First, England is nearly a textbook example of the demographic transition. Second, it was the first country to experience the industrial revolution and the effects of modern growth. Third, English historical statistics are both abundant and well regarded by historians<sup>4</sup> (Mitchell (1988) and Wrigley *et al.* (1997)).

The English fall in mortality began in the middle decades of the seventeenth century and continued until the beginning of the twentieth century, with a small detour in the middle decades of the nineteenth century (see Figure 2.2). The changes in fertility happened well after this initial decline in mortality. In fact, the first decades of the nineteenth century witnessed a rise in the rate of births. The main fall in fertility was concentrated from 1875 to 1916. In 1876, the Crude Birth Rate (births per one thousand inhabitants in a year, death rate is defined analogously as deaths per thousand inhabitants) in England was 35.48 (slightly above historical levels) while in 1916 it was 20.18: a fall of 43% in only 40 years. Interestingly enough is the homogeneity within regions: most English counties experienced the fall in fertility in the very same years, without outliers.

The study of this fall in fertility has not yet found satisfactory explanations. In the words of Dudley Baines (1994): “The main conclusion about the reason for the fall in fertility is that we know too little about it”. Indeed, several of the most popular alternative explanations are not supported by the data:

1. Changes in marriage patterns. If women married later or less, fertility would fall. However, from 1851-1914, the share of women marrying (83-88%) and their average age when first married (25-26 years) stayed at normal historical levels (Baines (1994)).
2. Labor participation of women. This explanation points to the higher opportunity cost of raising children when women work in the formal sector or when their wage is higher. Census data show that in 1871, the labor force participation rate of women was 41% of all women, while in 1911, it was 36%, a fall in participation rate of 5%: during the crucial years of the fall in fertility, female labor supply went down.
3. Urbanization. If children are less productive in cities than in farms, a shift of population to cities will reduce fertility. Again, the evidence is wanting: by 1841 (well *before* the fall in fertility) 61% of English population already lived in cities (Cairncross (1953)). In addition, fertility was not lower in cities: Teitelbaum (1984) shows how the Crude Birth Rate in London was higher than England as a whole in the period 1840-1910 (32.27 *vs.*

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<sup>4</sup>A problem with English data is the difficulty in separating statistics from England, Scotland and Wales. Whenever this separation has not been possible it is noted explicitly.

32.25). Also the homogeneity of the fall in fertility across counties indicates the lack of relevance of urbanization.

4. The movement from farming to industry. This explanation is similar to the previous one: children are less valuable in factories than in farms. A test of this hypothesis is the evolution of the composition of the labor force. The results are not favorable. In 1871, when fertility began to fall rapidly, only 15% of Great Britain's labor force worked in agriculture. In 1911, around the end of the fall, this figure had only moved to 7.8% (Clark (1957)). In a similar fashion, Teitelbaum (1984) finds that the proportion of population in manufacturing activities and the fall in marital fertility are negatively related for a cross-section of English counties.
5. Fall in infant mortality. If parents have a target number of children, they need high fertility rates when infant mortality is high. Data, however, show how the fall in infant mortality was *after*, not before, the fall in fertility. Indeed, as described in detail in the appendix, infant mortality fails to granger-cause fertility while fertility granger-causes infant mortality.

This section has documented the main facts of the demographic transition. While mortality have been falling steadily, fertility did not fall until the end of the nineteenth century. This difference increased population growth. Simultaneously income per capita has jumped by a factor of thirty. Can we quantitatively account for these observations? In the next section we develop a simple overlapping model to undertake that task.

### 3. The Environment

#### 3.1. Households

In any period  $t$  there is a continuum of households with measure  $\Gamma_t$ . Each household lives up to four periods: childhood, young adulthood, middle adulthood and old adulthood. Survival from one period to another is stochastic -agents can die earlier in life-. Let  $s_j^t$  be the conditional probability that a household born at date  $t$  and alive at  $j - 1$  will be alive at date  $j$ . The case  $s_t^t$  can be understood as the probability of surviving the gestation, birth and first year of life. These probabilities are given by:

$$\begin{aligned}
 s_t^t &= g(c_t^{t-1}, m_{0,t}) \\
 s_{t+1}^t &= g(c_t^t, m_{1,t}) \\
 s_{t+2}^t &= g(c_{t+1}^t, m_{2,t+1}) \\
 s_{t+3}^t &= g(c_{t+2}^{t+2}, m_{3,t+2})
 \end{aligned}$$

and  $s_{t+4}^t = 0$  where  $g$  is a function of the consumption  $c_j^t$  at date  $j$  of the household born in  $t$  (except  $s_t^t$  that depends on the consumption of the parents<sup>5</sup>), and an age-specific and possibly time variant parameter  $m_{ij}$  that accounts for different mortality risk over the life cycle. The function  $g$  is increasing, concave in both arguments and satisfies appropriate boundary conditions,  $g(0, m_{i,j}) = \lim_{m_{i,t} \rightarrow 0} g(c_j^t, m_{i,j}) = 0$  and  $\lim_{c_j^t \rightarrow \infty} g(c_j^t, m_{i,j}) = \lim_{m_{i,j} \rightarrow \infty} g_q(c_j^t, m_{i,j}) = 1$ , for all  $c_j^t, m_{i,j} > 0$ . The unconditional survival probability of a household born in  $t$  for date  $j$  are then defined as  $s_{jt} = \prod_{q=0}^j g_q(\cdot)$ .

Utility in each period  $j$  for a person born in  $t$  is given by a concave, increasing function  $u(c_j^t)$  such that  $\lim_{c \rightarrow 0} u'(c) = \infty$ . These preferences admit a Von Neumann-Morgenstern representation where the relevant probabilities are given by the survival probabilities outlined above. The discount factor between two periods is  $\beta \in (0, 1)$ .

Fertility decisions are taken at the beginning of the young adulthood period, over a continuous variable  $n_t$  of children such that  $n_t \in [0, \bar{n}]$ . Applying an appropriate law of large numbers (Uhlig (1996)), the share of surviving children realized in each particular household equates the population survival probabilities .

Altruism induces households to have children: to their own utility they add the lifetime utility of their children multiplied by a function  $b(n_t)$  such that  $b(n_t) \geq 0$ ,  $b'(n_t) > 0$ ,  $b''(n_t) \leq 0$  and  $b(\bar{n}) < 1$  (see Barro and Becker (1988) and Becker and Barro (1989)). This function encompasses both a pure time discount factor and a degree of selfishness. Given this form of the altruism, if the utility function is concave in the resources invested in each child, egalitarian treatment for children will appear (Alvarez (1994)).

However, children are costly to produce, both in terms of time and resources. Each child requires an amount of resources  $(\kappa_1 + \kappa_2 s_t^t) c_{t+1}^t$ , that depends on parameters  $\kappa_1$  and  $\kappa_2$  and the consumption of its parents. Each child has a fixed cost  $\kappa_1$  and another cost  $\kappa_2$  if it survives the first period of life. The multiplicative character of consumption reflects indivisibilities inside the household. Parents' time is needed to educate children. The educational effort will be translated into a higher ability by the children to supply skilled labor services. We will call an index of this ability human capital. The relation between parents' effort in units of time per child,  $e_{t+1}$ , and the human capital of a child is governed by a continuously differentiable, strictly increasing and concave function  $\varphi(\cdot) : [0, \infty) \rightarrow [0, \infty)$ :

$$h_{t+1} = \varphi(e_{t+1}) \tag{1}$$

such that  $\varphi(0) = 0$  and  $\lim_{e_{t+1} \rightarrow \infty} \varphi(e_{t+1}) < \infty$ . This function can be inverted to generate a time-cost function  $e_{t+1} = \bar{\varphi}(h_{t+1})$ . Using the time constraint of the household, we get  $s_{1t} n_t$

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<sup>5</sup>Medical studies show that a mother's ability to nourish her own child is established during her own previous life. See Barker (1994).

$\leq \frac{1}{e_{t+1}}$  and  $n_t \leq \min(\bar{n}, \frac{1}{s_{1t}e_{t+1}})$ .

The supply of market labor time by the household,  $l_j^t \in [0, 1]$ , is given by  $l_{t+1}^t = (1 - e_{t+1}s_{1t}n_t)$  and  $l_{t+2}^t = 1$ . Each unit of time provides one efficiency unit of unskilled labor services and  $h_t$  efficiency units of skilled labor services. Unskilled labor receives a wage  $w_j^u$  per efficiency unit and skilled labor  $w_j^s$ . Total labor income is then  $(w_j^s h_t + w_j^u) l_j^t$ .

In addition, households born at time  $t$  are allowed to buy a positive quantity  $a_j^t$  of a non-contingent bond at time  $j$  (since individuals do not make decisions during childhood,  $j$  only takes values in  $\{t+1, t+2, t+3\}$ ). If they die with positive positions, unintended bequest are redistributed in a lump-sum fashion among the rest of adults in the economy<sup>6</sup>.

The value function  $V_t(\cdot)$  of an individual born in  $t$  can then be written as:

$$V_t(\cdot) = \max_{c_{t+1}^t, c_{t+2}^t, c_{t+3}^t, e_{t+1}, n_t \leq \bar{n}} u(c_{t+1}^t) + \beta E u(c_{t+2}^t) + \beta^2 E u(c_{t+3}^t) + b(n_t) E V_{t+1}(\cdot) \quad (2)$$

$$\begin{aligned} s.t. \quad (1 + \kappa_1 + \kappa_2 s_{1t}^t) c_{t+1}^t + a_{t+1}^t &= (w_{t+1}^s h_t + w_{t+1}^u) l_{t+1}^t + tr_{t+1} \\ c_{t+2}^t + a_{t+2}^t &= w_{t+2}^s h_t + w_{t+2}^u + (1 + r_{t+2}) a_{t+1}^t + tr_{t+2} \\ c_{t+3}^t &= (1 + r_{t+3}) a_{t+2}^t + q_{t+3} \\ l_{t+1}^t &= (1 - e_{t+1} s_{1t} n_t), l_j^t \in [0, 1] \end{aligned}$$

where  $\{w_t^s, w_t^u, r_t\}$  are wages and the interest rate,  $tr_t$  is the lump-sum transfer and the expectations are taken with respect to the survival probabilities given the consumption choice<sup>7</sup>. Note that this problem is not convex because of the term  $e_{t+1}s_{1t}n_t$  in the budget constraint.

To keep accounting clear, define  $v \in \mathcal{V} \equiv \{1, 2, 3, 4\}$  to indicate the age of each generation,  $A \equiv (a \mid a \geq 0)$  the set of possible values for assets and  $H \equiv (h \mid h \geq 0)$  the set of possible values for human capital. Let  $\Omega \equiv \mathcal{V} \times A \times H$  and  $\Psi_t(v, da, dh)$  the measure over the population. Then  $\Gamma_t = \int d\Psi_t(v, da, dh) = \Psi_t(\Omega)$  where, again assuming that an appropriate law of large numbers holds, the distribution collapses at just four atom points, eliminating the burden of keeping track of the continuous measure, an infinite-dimensional object.

Now, we can define the initial size of the generation  $t$ ,  $N_t \equiv \int n_t d\Psi_t(2, \cdot)$ , population,  $Pop_t \equiv s_{0t}N_t + s_{1t-1}N_{t-1} + s_{2t-2}N_{t-2} + s_{3t-3}N_{t-3} \equiv \Psi_t(\Omega)$ , deaths  $D_t \equiv (1 - s_{0t})N_t + (1 - s_{1t-1})N_{t-1} + (1 - s_{2t-2})N_{t-2} + (1 - s_{3t-3})N_{t-3} + N_{t-4}$ , the skilled and unskilled labor

<sup>6</sup>We make this assumption by necessity. Allowing intergenerational transfers generates a non-trivial distribution of agents within each cohort that makes the model intractable. Using a model similar to ours, Hendricks (2001) presents computational evidence that suggests this assumption is relatively unimportant.

<sup>7</sup>Since probabilities depend on previous periods consumption, the household has control over her “modified” discount factor. This feature does not generate problems of dynamic inconsistency because the marginal rate of substitution between two periods is independent of the moment in time when it is evaluated.

supply in units of efficiency:

$$S_t \equiv s_{1t-1} \int l_t^{t-1} h_{t-1} d\Psi_t(2, \cdot) + s_{2t-2} \int h_{t-2} d\Psi_t(3, \cdot) \quad (3)$$

$$U_t \equiv s_{1t-1} \int l_t^{t-1} d\Psi_t(2, \cdot) + s_{2t-2} N_{t-2} \quad (4)$$

the reproduction rate,  $CBR_t \equiv \frac{N_t}{Pop_t}$ , and the death rate,  $CDR_t \equiv \frac{D_t}{Pop_t}$ .

### 3.2. Firms

There are two sectors in the economy: one produces the consumption good and the other produces the investment good. Output is given by a common constant returns to scale, strictly quasi-concave production function with a nested C.E.S. form:

$$C_t = A_t \left[ ((B_t K_{ct})^\omega + S_{ct}^\omega)^\frac{\rho}{\omega} + U_{ct}^\rho \right]^\frac{1}{\rho} \quad (5)$$

$$X_t = q_t A_t \left[ ((B_t K_{kt})^\omega + S_{kt}^\omega)^\frac{\rho}{\omega} + U_{kt}^\rho \right]^\frac{1}{\rho} \quad (6)$$

where  $A_t$  is the general productivity level,  $q_t$  the technology factor of capital production,  $B_t$  is the productivity level of capital,  $K_{it}$  the physical capital services,  $S_{it}$  is the amount of skilled labor services and  $U_{it}$  the amount of unskilled labor services used by the sector  $i$  during period  $t$ . All the technology parameters  $z_t$ ,  $q_t$  and  $B_t$  are assumed to be exogenous, deterministic, processes. To maintain strict quasi-concavity we restrict  $\rho, \omega \in (-\infty, 1)$ . Also we assume  $\rho > \omega$ : the elasticity of substitution between unskilled labor and a composite of capital and skilled labor is higher than that between capital and skilled labor.

This parameter reflects two facts. First, capital is in general complementary with skilled labor. Second, there is a fairly high degree of substitution between the two kinds of labor. The notion (and the micro evidence) that capital investment (or *capital-specific technological change*) benefits primarily skilled workers goes at least as far back as Rosen (1968) and Griliches (1969) and it is summarized by Hammermesh (1993). Most of the estimates show evidence consistent with the assumed restriction (Rosen (1968), Griliches (1969), Fallon and Layard (1975) and Brown and Christensen (1981) among others). In addition, this approach has been shown to be useful in explaining observed data. For instance, similar technologies have been used by Krusell *et al.* (2000) and Heckman, Lochner and Taber (1998) to account for the evolution of the skill premium in the United States.

The properties of the technologies guarantee that, under perfect competition in factor markets, profit-maximizing firms will equalized factor ratios across sectors and  $1/q_t$  will be the equilibrium relative price of capital in terms of our numeraire, the consumption good.

Then total output is given by:

$$Y_t = C_t + \frac{X_t}{q_t} = A_t \left[ ((B_t K_t)^\omega + S_t^\omega)^{\frac{\rho}{\omega}} + U_t^\rho \right]^{\frac{1}{\rho}} \quad (7)$$

where inputs are now defined as the sum of inputs in both sectors. Finally, the law of motion for capital is given by  $K_{t+1} = (1 - \delta) K_t + X_t$  where  $\delta$  is the depreciation rate.

To simplify computation, changes in relative prices can be mapped into changes in the capital productivity level as shown by Greenwood, Hercowitz and Krusell (1997). To do this, consider a transformed model where the capital services provided by the physical capital stock change depending on when the stock was accumulated. Call  $\widetilde{K}_t$  the amount of services provided by the current stock of capital. The production function is now:

$$Y_t = A_t \left[ \left( (B_t \widetilde{K}_t)^\omega + S_t^\omega \right)^{\frac{\rho}{\omega}} + U_t^\rho \right]^{\frac{1}{\rho}}$$

and the stock of services provided evolves according to  $\widetilde{K}_{t+1} = (1 - \delta) \widetilde{K}_t + q_t X_t$ .

Clearly  $\widetilde{K}_t$  is a function of the whole history of investment decisions and technology levels. To make this value operative define the following function  $I_t \equiv \frac{\widetilde{K}_t}{K_t}$ . Then, if we let  $B_t = \widetilde{B}_t I_t$ , the transformed production function can be written as:

$$Y_t = A_t \left[ ((B_t I_t K_t)^\omega + S_t^\omega)^{\frac{\rho}{\omega}} + U_t^\rho \right]^{\frac{1}{\rho}} \quad (8)$$

Now, assuming that the economy is in a steady-state such that  $X_t = \delta K_t$  and normalizing  $q_j = 1$  for  $j \leq t$  but  $q_{t+1} \neq 1$ , it is the case that  $B_t = \widetilde{B}_t$ ,  $K_{t+1} = K_t = \widetilde{K}_t$  and, after some algebra:

$$I_{t+1} = \frac{\widetilde{K}_{t+1}}{K_t} = 1 + \delta (q_{t+1} - 1) \quad (9)$$

If we then set  $B_{t+1} = I_{t+1} B_t$ , the period  $t + 1$  production function is given by:

$$Y_{t+1} = A_{t+1} \left[ ((B_{t+1} K_{t+1})^\omega + S_{t+1}^\omega)^{\frac{\rho}{\omega}} + U_{t+1}^\rho \right]^{\frac{1}{\rho}} \quad (10)$$

where only physical capital matters and a fall in its relative prices is equivalent to a raise in its productivity.

Using (10), and for some given endowments, the log skill premium  $\pi_t$  to human capital is  $\log \pi_t = \alpha_1 \log U_t + \alpha_2 \log S_t + \alpha_3 \log ((B_t K_t)^\omega + S_t^\omega)$  where  $\alpha_1 = 1 - \rho$ ,  $\alpha_2 = \omega - 1$  and  $\alpha_3 = \frac{\rho}{\omega} - 1$ . This equation shows the three forces that drive the skill premium. First, the endowment of unskilled labor services,  $\log U_t$ . Since  $\alpha_1 > 0$  this term always increases the ratio. Second is the endowment of skilled labor services,  $\log S_t$ . As  $\alpha_2 < 0$  this term

decreases the skill premium. Finally, the capital-skill complementarity,  $\log((B_t K_t)^\omega + S_t^\omega)$ . Since  $\rho > \omega$ , the sign of  $\alpha_3$  is ambiguous but not the total effect<sup>8</sup>: the higher the aggregate amount of capital or its productivity, the higher the skill premium.

### 3.3. Equilibrium

Now we are ready to define an equilibrium for the economy as:

**Definition 1.** A *Recursive Competitive Equilibrium* is a value function  $V_t(h_t; \Psi_t(\cdot))$  and a set of policy functions  $\{c_j^t(h_t; \Psi_t(\cdot)), l_j^t(h_t; \Psi_t(\cdot)), a_j^t(h_t; \Psi_t(\cdot)), n_t(h_t; \Psi_t(\cdot))\}$  for each generation of households, and functions for allocations  $\{Y_t(\Psi_t(\cdot)), C_t(\Psi_t(\cdot)), X_t(\Psi_t(\cdot))\}$ , aggregate inputs,  $\{K_t(\Psi_t(\cdot)), U_t(\Psi_t(\cdot)), S_t(\Psi_t(\cdot))\}$ , their rental prices  $\{r_t(\Psi_t(\cdot)), w_t^u(\Psi_t(\cdot)), w_t^s(\Psi_t(\cdot))\}$ , the relative price of capital  $q_t$  and a law of motion  $\Phi_t(\Psi_t(\cdot))$  for the measure of the population measure  $\{\Psi_t(\cdot)\}$  for all  $t$  such that 1) households solve their recursive problem (2), 2) prices for inputs are the marginal productivities, 3) the lump-sum transfers are equal to the aggregate involuntary bequests, 4) markets clear and 5) the law of motion  $\Phi_t(\Psi_t(\cdot))$  is consistent with individual behavior,  $\Psi_{t+1}(\cdot) = \Phi_t(\Psi_t(\cdot))$ .

## 4. Parametrizing the Model

This section discusses the data used and how functional forms and the parameters of the model are chosen. A basic consideration will be to pick standard functional forms and accepted parameter values to the maximum possible extent to facilitate comparison with other studies.

### 4.1. Data

Parameters are selected to match the English Crude Birth and Crude Death Rates for the period 1541-1800, reserving the end of the sample to evaluate the model. To assure consistency in the moment estimation undertaken below, data stationarity needs to be checked, especially when even a casual inspection of both series (see Figure 2.2) suggest potential problems. Formally this problem is addressed by testing for the presence of unit roots. After an easy rejection of the presence of a linear trend (to avoid spurious detrending), the null of stationarity against the alternative of a unit root without a linear trend is tested following the procedure in Kwiatkowski *et al.* (1992). The results of the test strongly support the null hypothesis of stationarity and, with the non-significativity of the time trend, the existence of constant first moments<sup>9</sup>.

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<sup>8</sup>If  $\omega < 0$ , then  $\alpha_3 < 0$  but the log decreases with higher capital, skilled labor or productivity of capital. If  $\omega > 0$ , then  $\alpha_3 > 0$  and the log increases.

<sup>9</sup>See the appendix for further discussion.

## 4.2. Functional Forms

First, a CRRA utility function  $\frac{c^{1-\sigma}-1}{1-\sigma}$  is assumed. The altruism function, following Becker, Murphy and Tamura (1990), has a constant-elasticity form  $\gamma n^\varepsilon$ . The human capital investment function is given by  $h_{t+1} = e_t^\alpha$  (originally from Ben-Porath (1967)) where  $\alpha < 1$ . Survival probabilities are described by hazard functions  $\lambda(\tau, x)$  where  $\tau$  denotes the time and  $x$  is a vector of covariates. From the hazard function we can find the conditional density of durations  $f(\tau, x) = \lambda(\tau, x) \exp[-\int_0^\tau \lambda(u, x) du]$ . The vector of covariates  $x \equiv \{m_{i,j}, c_j^t\}$  includes the consumption level  $c_j^t$  and some age-specific factor  $m_{i,j}$ . If the covariates enter in a multiplicative fashion and the hazard is constant within each period of life,  $\lambda(\cdot, \cdot)$  can be written as  $e^{-m_{i,t}c_j^t}$  and the survival function takes the simple form  $1 - e^{-m_{i,t}c_j^t}$  for  $i = \{0, 1, 2, 3\}$ , assuring that the survival probability belongs to the unit interval and is increasing in both arguments.

## 4.3. Calibration

Now the parameters of the model are calibrated. With respect to preference parameters, the values for  $\beta$  and  $\sigma$  are selected to follow the standard choices in the literature. The discount factor is set equal to 0.97 on a yearly basis, or, following the interpretation of a period as 20 years, 0.54. The risk aversion is chosen to be one. For simplicity  $\gamma$  is set equal to  $\beta$ . For the elasticity  $\varepsilon$ , a value of 0.4 is chosen to match the observed fertility rate (number of children per woman). Section 6 discusses the effect of these choices in the quantitative behavior of the model, including a discussion of their relation with the endogenous mortality probabilities.

For the technology parameters, the final output parameters ( $A, B$ ) are chosen to match two observations, the capital structures-output ratio (slightly over 1) and the average real interest rate in the period 1870-1914 (around 3%). Appropriate numbers are 2.3 and 0.4 (note that the second number basically plays the role of a share parameter). With respect to  $\rho$  and  $\omega$ , estimates of the elasticity of substitution between unskilled labor and capital are between 0.5 and 3 and the values for the elasticity of substitution between skilled labor and capital are definitively below 1.2 and some close to 0. Since unfortunately, there are no estimations of these parameters for England during the time period considered in the paper, we use Krusell *et al.* (2000) suggestions of  $\rho = 0.33$  and  $\omega = -0.67$ . Following Greenwood, Hercowitz and Krusell (1997) we set  $\delta$  equal to 0.93 (0.125 annually). The parameter in the human capital production function  $\alpha$  is chosen to be 0.6, a middle point in the range of estimates reported in Browning, Hansen and Heckman (1999). The cost of children comes from the Oxford-scale, used by the OCDE and EUROSTAT to impute consumption in households (Van Praag and Warnaar (1997)). This scale puts the first adult at one, the second at 0.7 and each child below 18 at 0.5. Then the cost of two children is given by  $(0.5 \cdot 2)/1.7 \simeq 0.6$ . As an upper

bound two thirds of these expenditures can be assumed to be education related and since in the model each descendant is equivalent to two in the data, the total cost is given by  $0.2^{10}$ . Dividing this cost evenly by years of life,  $\kappa_2$  is equal to 0.19 and  $\kappa_1 = 0.01$ . Table 4.1 summarizes the previous discussion.

Table 4.1: Calibration of the Model

Preferences Parameters: $\beta = 0.54, \sigma = 1, \gamma = 0.54, \varepsilon = 0.4$
Technology Parameters: $\kappa_1 = 0.01, \kappa_2 = 0.19, A = 2.3, B = 0.4, \delta = 0.93$
$\rho = 0.33, \omega = -0.67, \alpha = 0.6$

#### 4.4. A Simulated Method of Moments

The remaining parameters in the model that need to be determined are the age-specific factors. In this section, three tasks are described: how to understand the fluctuations in the data, how to express all the age specific factors as functions of just one of them and how to estimate this one parameter.

Time series averages of birth and death rates are going to be used to match the steady state implication of a version of the model that abstracts from changes in age-specific factors. That abstraction explains the sources of fluctuations in the data by changes in the age-specific factors unobservable by the econometrician, reducing the search of values to those that reproduce first moments of the data. Formally, let  $Q \equiv \{Q_t : \Delta \rightarrow \mathfrak{R}^2, t = 1, 2, \dots\}$  be the observed two dimensional stochastic process (fertility and mortality rates). This process is defined on a complete probability space  $(\Delta, \mathfrak{S}, P_0)$  where  $\Delta = \mathfrak{R}^{2\infty} \equiv \times_{t=1}^{\infty} \mathfrak{R}^2$  and  $\mathfrak{S} = B^{2\infty} \equiv B(\mathfrak{R}^{2\infty})$  is the Borel  $\sigma$ -algebra generated by the measurable finite-dimensional product cylinders. Call  $P_0$  the data generation process and a sample of size  $n$  is described by  $P_0(B) \equiv P_0[X^n \in B]$  where  $B^{2n} \equiv B(\mathfrak{R}^{2n})$  is the Borel  $\sigma$ -algebra generated by the open sets of  $\mathfrak{R}^{2n} \equiv \times_{t=1}^n \mathfrak{R}^2$ . If, as suggested by the failure to reject stationarity, we assume that  $P_0$  is invariant over time, constant values for the age-specific factors  $m = \{m_0, m_1, m_2, m_3\}$  can be found such that

$$E_0 \begin{pmatrix} CBR(m_t) \\ CDR(m_t) \end{pmatrix} = \begin{pmatrix} CBR(m) \\ CDR(m) \end{pmatrix} \quad (11)$$

This problem is clearly overidentified as four parameters can be adjusted to match two moments. However, given the functional assumption for the hazard function,  $m_0$  and  $m_2$  can be expressed as a linear function of  $m_1$ . In addition, if  $c_{t+1}^t \simeq v c_{t+1}^{t-1}$  is assumed, where  $v$  is some constant (as it is approximately true in the computations), it is also the case that  $m_3$  is

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<sup>10</sup> An underestimation of the cost of children makes the reduction in fertility smaller when higher technology is introduced, biasing down the effects of capital-specific technological change the paper emphasizes.

a linear function of  $m_1$ . Then only  $m_1$  must be estimated independently and the relation between  $m_1$  and the other factors can be determined using the observed life tables.

To estimate  $m_1$  the procedure matches the first moments of the model with the first moments of the data using a modified version of the *Simulated Method of Moments* that exploits the restriction on the value of the expectation to find a suitable value for the age-specific factor.

In particular, let  $\chi(\cdot, \cdot) : \mathfrak{R}_+^n \times \Theta \rightarrow \mathfrak{R}^2$  be a simulated  $C^1$  function where  $\Theta \subseteq \mathfrak{R}$  is a compact set such that there exists a unique parameter  $m_{10} \in \text{int}(\Theta)$  that solves the equation:

$$E_0[\chi(\xi, m_1)] = E_0\left(\begin{array}{c} CBR(m_{1t}) - CBR(m_1) \\ CDR(m_{1t}) - CDR(m_1) \end{array}\right) = \mathbf{0} \quad (12)$$

where  $\xi$  is the vector of calibrated parameters and  $\mathbf{0}$  a  $2 \times 1$  vector of zeros. A natural sample analog is the estimating function  $\vartheta : \mathfrak{R}^T \times \mathfrak{R}_+ \rightarrow \mathfrak{R}^2$ :

$$\vartheta(\xi, m_1) = \left( \begin{array}{c} CBR(m_1) - \overline{CBR} \\ CDR(m_1) - \overline{CDR} \end{array} \right) \quad (13)$$

where  $\overline{CBR}$  and  $\overline{CDR}$  are sample means. The main computational burden comes from the fact that neither  $CBR(m_1)$  nor  $CDR(m_1)$  have an analytical solution and they need to be found by simulation.

The covariance matrix is given by  $S = \lim_{t \rightarrow \infty} T * E\{\vartheta(\xi, m_1)\vartheta(\xi, m_1)'\}$ . Since this matrix does not depend on  $m_1$ , we can use a Newey-West estimator  $\tilde{S} = \widehat{\Gamma}_0 + \sum_{v=1}^q \left[1 - \frac{v}{q+1}\right] (\widehat{\Gamma}_v + \widehat{\Gamma}_v')$  where  $\widehat{\Gamma}_v = \frac{1}{T} \sum_{t=v+1}^T \{(\psi_t - \bar{\psi})(\psi_{t-v} - \bar{\psi})\}$  and  $\psi_t' = (CBR_t, CDR_t)$ .

Then we write the estimation problem as:

$$\widetilde{m}_1 = \arg \min_{\widetilde{m}_1 \in \mathfrak{R}_+} \vartheta(\xi, m_1) \tilde{S}^{-1} \vartheta(\xi, m_1)' \quad (14)$$

Under the regularity condition stated in Pakes and Pollard (1989), this estimator is both consistent and asymptotically normally distributed,  $\sqrt{T}(\widetilde{m}_1 - m_1) \sim \mathcal{N}(0, V)$ , where  $V$  is estimated by  $\widehat{V} = \left( \frac{\partial \vartheta(\xi, m_1)}{\partial m_1} \Big|_{\widetilde{m}_1 - m_1} \tilde{S}^{-1} \frac{\partial \vartheta(\xi, m_1)}{\partial m_1} \Big|_{\widetilde{m}_1 - m_1} \right)^{-1}$  and the derivative is computed numerically.

## 5. Quantitative Results

This section presents the results for the benchmark model and the outcome of three experiments proposed to explore the influence of different factors in the demographic transition.

### 5.1. The Steady State

After the calibration,  $\widetilde{m}_1$  is estimated following the procedure outlined in the previous section. The algorithm, described in greater detail in the computational appendix, begins with a guess of  $\widetilde{m}_1$  and finds the associated prices  $\{r, w^u, w^s\}$  that clear markets in the steady state. The estimating function is then evaluated and a new  $\widetilde{m}_1$  is tried. The process continues until a global minimum of the quadratic form is found.

For the chosen calibration, the estimated value of  $\widetilde{m}_1$  is 0.45 with asymptotic variance of 0.001. The remaining values ( $\widetilde{m}_0 = 0.49$ ,  $\widetilde{m}_2 = 0.40$ ,  $\widetilde{m}_3 = 0.20$ ) follow the  $u$ -pattern of survival probabilities observed in the data. With these estimations different dimensions of the model can be compared with the empirical evidence. First, the simulated moments (Crude Birth and Death Rates) are shown in Table 5.1 where the numbers in parenthesis are sample standard deviations.

Table 5.1: Simulated *vs.* Observed Moments

Moments	Observed	Steady-State
CBR	32.4 (2.7)	32.5
CDR	26.9 (3.2)	22.3

The model is able to match fertility quite closely and mortality rates in a range around one and half standard deviations<sup>11</sup>. Mortality rates seem difficult to match due to the very stylized demographic structure of the model. However, even if the aggregate rate is not as high as desired, the behavior of mortality over the life-cycle closely match observed rates (as seen in table 5.2) and in the model-generated life expectancy (39 years *versus* 38 years measured in the data for the period 1640-1800) suggest that a lower aggregate death rate does not substantially affect individual choices<sup>12</sup>.

Table 5.2: Mortality Probabilities

	Observed	Steady-State
P( <i>alive at 1</i> )	0.83	0.84
P( <i>alive at 21 1</i> )	0.80	0.82
P( <i>alive at 41 21</i> )	0.76	0.78
P( <i>alive at 61 41</i> )	0.58	0.58

An important final test of the performance of the model is the comparison between the

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<sup>11</sup>An standard  $\chi^2$  test rejects the overidentifying restriction at conventional levels.

<sup>12</sup>In particular, it seems that the main reason for the low mortality is that deaths are concentrated only at the end of each period. That makes the population structure move in jumps instead of following a smooth curve and decreases death rates for a given fertility rate.

observed and the simulated age distribution of the population (Table 5.3). The fit in the population age distribution is close, with a Kullback-Liebler Information Criterion of 0.6. This result is remarkable since the model was not estimated to replicate that age distribution. Going through different age groups, the only important difference is in the first bin as the share of children is too high (42% in the data in comparison with 47.1% in the model). Indeed, if we multiply the relative share of the other three generations by 1.1 (to compensate for the bigger first generation) the new fit is quite close. The reason for this divergence is that child mortality is disproportionately concentrated in the first years of life while in the model children survive the whole first period. As discuss in section 6, this feature does not seem to affect the performance of the model.

Table 5.3: Population Structures

Structure	Observed	Steady-State
1-20/t	42.0	47.1
21-40/t-1	30.2	27.8
41-60/t-2	19.7	16.8
+60/t-3	8.1	7.3

The general assessment is that the model performance is highly satisfactory: despite its simplicity, it generates a population with very similar moments and with an age structure close to the data. This suggests that the model is a good laboratory for the task of specifying changes in the environment and generating reliable answers to these changes.

Now we will proceed with the computational experiments. First we will test the if higher income leads parents to some kind of trade-off between quantity and education of children. Then the role of a fall in the relative price of capital is explored. Finally, the last experiment assesses the role of the fall in mortality in fertility levels.

## 5.2. A change in total productivity

This experiment is designed to test the hypothesis that higher income caused the demographic transition through higher productivity. This hypothesis is based on Becker (1960) and an important subsequent literature that have argued in favor of the existence of a quantity-quality trade-off between children and education. If this trade-off holds, as income rises, we should observe that parents substitute in favor of fewer, higher quality, children. This move from quantity to quality would explain why income and fertility are negatively correlated both cross sectionally and over time.

The design of this experiment is as follows. We increase the value of  $A_t$  by 10% in the production function  $A_t \left[ ((B_t K_t)^\omega + S_t^\omega)^{\frac{\rho}{\omega}} + U_t^\rho \right]^{\frac{1}{\rho}}$  for  $t > \bar{t}$ . This change is neutral in the

sense that, for a given supply of endowments, all the input prices will rise exactly 10%. So, even if along the equilibrium path relative prices can change, there is no a priori reason why this should increase or reduce skilled *versus* unskilled relative prices. Also this change is perfectly forecasted by agents.

The outcome of this experiment (which holds for a wide set of parameter values when sensitivity analysis is performed) is very different from the quality-quantity trade-off: parents prefer to have more children. Fertility increases from 33.1 to 35.6 and the death rate falls to 22.1 thanks to the higher level of consumption associated with the higher level of income. Human capital accumulation stays roughly constant.

The transitional dynamics for this experiment (see Figure 5.1) are rather smooth, and completed after few periods<sup>13</sup>. Fertility grows with a small plateau without overshooting and mortality falls in a soft path. This pattern of relatively rapid convergence to the new steady state is common to different experiments. It is also important to notice how in the transition several waves are observed. This kind of behavior is common in demographic models where echoes from past decisions propagate for a number of generations.

The intuition behind these results is simple: in addition to the pure income effect (children are normal goods), as income rises for any level of human capital, higher investment in skills yields a smaller marginal utility, making the household's choice of more children relatively cheaper and, as a consequence, increasing the quantity of children produced in equilibrium.

It is interesting to see why the quantity-quality trade-off fails in our model. This trade off usually works through two mechanisms: a high income-elasticity of quality and the opportunity cost of time. The first effect works as follows: if preferences are representable by a utility function with high income-elasticity of quality, the demand for quality will raise so much that, even if the income-elasticity of quantity is also positive, fertility may fall. This can happen if the slope of the value function with respect to human capital increase monotonically with an increment in income, *i.e.* households with high human capital improve their situation relatively more than low human capital households after a change in productivity. However, the increment in productivity studied in the experiment is neutral *ceteris paribus*. In our model general equilibrium effects make skilled labor profit more than unskilled labor from the technology change (the skilled wage rises a 5.7% while the unskilled wage rises 4.5%) but not enough to force the fertility choice to low levels. With respect to the second effect, the increased opportunity cost of time invested in children does not seem to have an important quantitative effect: the income effect dominates for the chosen parametrization and in the

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<sup>13</sup>The simulation plots begin before the actual change occurs. The fact that the shock is anticipated induces changes in the choices of forward-looking agents before it hits the economy. In all cases the effects more than three periods before the shock are minimal. Perfect forecast is introduced for convenience. Results are nearly invariant if technological change is a measure zero shock.

sensitivity analysis<sup>14</sup>.

Finally note that the results of the experiment can explain the increment in fertility observed in England in the first decades of the nineteenth century and the fall in mortality. The initial phases of the industrial revolution raised income. This higher consumption decreased mortality and led parents to increase the number of children. The fact that England experienced that increment in fertility basically non existing in other European countries can be then simply linked to the fact that England entered first the process of modern economic growth. The results also helps to understand the lack of relation between income per capita and fertility fall in Europe. As already mention in section 2, a surprising fact of demographic transitions in European countries was their simultaneity. If we follow the dating of the *Princeton European Fertility Project* (based on an index of marital fertility), the fertility transition begins in Belgium in 1881, Germany in 1888, England and Wales in 1892 and the Netherlands in 1897 (Coale and Treadway (1986)). Even more, the fall in Hungary (one of the least developed areas of Europe at the time) is at the same time as England (Alter (1992))<sup>15</sup>. A similar result holds within countries: there is no relationship between the fall in fertility and the relative income level of different regions.

This fact is puzzling. If income level do not affect fertility why do we see fertility drops associated with modern economic growth? And why do we see a negative relation between income and fertility in a cross-section of countries? Maybe our attention should move to the sources of this higher income.

### 5.3. The Effect of Capital-Specific Technological Change

The first experiment suggests that the key to generate a fall in fertility lies in moving up the slope of the value function with respect to human capital. A natural way to obtain this result is to increase the skill premium through the effects of *capital-skill complementarity*.

The design of this experiment is as follows. We increase the value of the capital productivity,  $B$ , by 60% in the production function  $A_t \left[ ((B_t K_t)^\omega + S_t^\omega)^{\frac{\rho}{\omega}} + U_t^\rho \right]^{\frac{1}{\rho}}$  for  $t > \bar{t}$  in an anticipated and permanent way. Using the results in section 3, this number is chosen to match the observed fall in the relative price of capital in England during the demographic transition (1875-1920).

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<sup>14</sup>A third mechanism, proposed by Becker, Murphy and Tamura (1990), is the presence of increasing returns in human capital: when human capital is abundant, the rate of return on human capital is higher and then parents move from quantity into education. This increasing returns generate two stable steady-states, one with low fertility and one with high. However, as pointed out by Browning, Hansen and Heckman (1998), it seems there is weak empirical evidence in favor of these increasing returns in human capital investment.

<sup>15</sup>More recently Guinnane, Okun and Trusel (1994) have argued that this dating procedure can lead to misleading results but even with their timing the fall in fertility is not correlated with income levels.

The evidence on the relative price of capital shows that this price fall by a substantial amount between 1875 and 1920. Different measures are reported extensively in Collins and Williamson (1999) for the United Kingdom<sup>16</sup>. In terms of the final goods (that include also capital prices and then tend to underestimate the fall), the relative price of capital fall from 100 in 1875-1879 to 86 in 1920-1925. However, for the purposes of our model, the relevant price is the price of equipment. The evidence suggests (Krusell *et al.* (2000)) that the capital-skill complementarity exist between equipment and skill labor and it is not very important with respect to structures. The relative price of equipment fall from a level of 100 in 1875-1879 to 62 in 1920-1925<sup>17</sup>. If we interpret the capital in the production function as structures, we can make  $q = 1.62$  ( $1/0.619$ ) and, using the depreciation value for 40 years, obtain  $B_{t+1} = I_{t+1}B_t \simeq 1.60B_t$ .

Once this change in the productivity of capital is fed into the model, the transition and the new steady-state are computed. The experiment delivers an important fall in fertility. Crude Birth Rate falls 9 points, to 25.0, in comparison with the observed fall of 14 points between the mean 1541-1796 and 1921-1929. Mortality raises, counterfactually, to 19.2, because of age compositional effects. However this does not seem a drawback of the model since age-specific factors variations or pure income level changes can generate the fall in mortality. At the same time, as a response to the new situation, human capital accumulation raises more than a 40% and per capita income rises slightly over 50%. The results of the experiment prove how neoclassical theory can quantitatively account for the simultaneous fall in fertility and increment in per capita income using observables: prices and capital-skill complementary.

The transitional dynamics are shown in Figure 5.2. Again a smooth path appears. Fertility begins to decline before the actual change and the change is basically finish in only two generations. Sticking to our interpretation of a period in the model (20 years) that corresponds basically to the period 1875-1920 where most of the fall in fertility is observed.

The intuition behind the new steady state is now that the accumulation of capital drives down the relative price of unskilled labor. The increased skill premium transforms the strategy of many, low-educated children into a much less interesting option: parents move away from quantity into quality and fertility falls. The experiment indicates that modern economic growth indeed caused the demographic transition but not through increments in income, as usually suggested, but through changes in relative prices.

The results also outline the following explanation of the demographic transition: beginning around the last decades of the eighteenth century improved health practices and higher income

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<sup>16</sup>This evidence is also available for other western countries. In all of them, it is observed a substantial fall in the relative price of capital more or less at the same time of a big drop in fertility. Also note that the evidence suggest that before 1870 the relative price of capital was relatively constant.

<sup>17</sup>Other measures reported in Collins and Williamson (1999) is the user cost of capital (interest rate plus depreciation less change in prices of capital), which declined 50% between 1880-1884 and 1910-1914.

increased fertility and decreased mortality in England. As a consequence, the next hundred years experienced a sharp raise in population. Later, at the end of nineteenth century, as the relative price of capital began to fall, the return to investment in human capital rose, inducing parents to move from quantity into quality of children, and causing a big drop in fertility.

To further test this theory we can check whether the set of responses predicted are observed in the data: is there an increase in human capital accumulation? Is there a rise in the relative prices of skilled labor?

### 5.3.1. Changes in education,

A good proxy for human capital accumulation is given by the years of formal education. Using this measure, the correlation between education achievements and the fall in fertility is striking for the English case. Figure 5.3 graphs fertility rate against the average years of schooling of males in England and Wales by Birth Cohort. A simple linear regression of these two variables gives an  $R^2$  of 0.86. The increase is even more significant if we remember that school attendance was not compulsory until 1890 (and even then with exceptions) and until 1891 tuition was not free (Teitelbaum (1984)). The number of schooling years increased by around 50%, closed to the prediction of the model (40%).

### 5.3.2. changes in relative prices and ....

Observed changes in relative prices are difficult to quantify in a setting equivalent to the model. In the fifty years before the first world war, England was a large exporter of both capital and persons. For labor, the Net Immigration Rate between 1870 and 1910 was -2.25 per thousand, reducing labor force by 11 per cent at the end of this period as compared to the hypothetical situation without migration (Williamson (1997)). Most of this emigration was of unskilled workers. A similar pattern of strong exports affects capital. Only from 1911 to 1914 the United Kingdom lent a 10% of its GNP abroad and on the eve of the first world war, the stock of Britain's overseas investments amounted to 173% of total national output (Eichengreen (1994)). These exports of capital tend to reduce the skill premium. Despite this unfavorable bias, the ratio of unskilled labor wage to GDP per worker hour fall a 0.9% a year on average between 1870 and 1913, with a total fall of 31%. As noted by Williamson (1997) if labor share is constant<sup>18</sup>, this ratio indicates the movement of skilled *vs.* unskilled wages<sup>19</sup>. In the model skill premium only needs to grow a 15% in the new steady state to

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<sup>18</sup>To show this is the case we take data from Table 6.1 in Matthews, Feinstein and Odling-Smee (1982). Labor share is built as labor income plus farm profits over GNP less income from abroad. In 1873 this labor share is 64.35% and in 1913 63.83%.

<sup>19</sup>Similarly an index of skilled manual wages rose a 10% more than an index of total wages.

generate the drop in fertility. Data strongly suggests that the evidence of the relative prices changes required by theory exists.

### 5.3.3. ... changes in the composition of migration

The model deals with a closed economy without inputs mobility. As previously noted this feature misses important aspects of the English experience. We can think of a simple extension of the theory to deal with these issues. Households would be given the choice of “opting out” from markets and get some uncertain, external endowment if they pay some fixed cost. A simple interpretation of this opting out is emigration: households can leave the country and move to another one with higher average, but uncertain, wages. If the distribution of this endowment rises with the technological change in the home country and relative prices move against unskilled labor, we should observe a higher number of unskilled household choosing this possibility. Again the evidence clearly support the prediction of the theory. In her classic study, Erickson (1972) showed how the character of English emigration was very different in the last decades of the nineteenth century from what it had been in the middle of the century. The main characteristic of the emigration of the middle decades of the nineteenth century was a balanced cross-section of the English society. In last decades it was strongly tilted towards unskilled workers. Even in 1886 and 1887, when a cyclical downturn raised unemployment rate among skilled workers substantially, the emigration rate of skilled workers was very low.

### 5.4. A change in mortality

Galor and Weil (1999), Kalemli-Ozcan, Ryder and Weil (2000) and Meltzer (1995) among others have linked changes in life expectancy and fertility. If mortality falls, the distribution of years when an investment in human capital pays off shifts to the right. These papers argue that, as human capital becomes a more attractive investment, parents substitute away from quantity of children into education and fertility falls.

However, even abstracting from relative price changes induced by general equilibrium, the total effects of a reduction in mortality are ambiguous. A lower mortality also means a higher value of children (longer lives imply higher utilities) and consequently, a higher demand for children. In addition, a reduced mortality also increases the return on the parents’ human capital and, as they become richer, their relative demand for children will increase if children are a normal good<sup>20</sup>.

What are the quantitative effects of a reduction of mortality in the model? To answer this question we conduct the following experiment. Given the transition function  $1 - e^{-m_i,t^c t}$ ,

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<sup>20</sup>In the absence of inter vivos transfers households will also like to accumulate more physical capital as the probability of living until retirement has raised. This saving need increases the opportunity cost of children.

the age-specific factors  $m$  are increased by a 10% to match the fall in mortality observed in England during the nineteenth century. As before this change is perfectly foreseen.

The results of this experiment are strong: a lower mortality increases fertility by 1.4 points and lowers the death rate by 0.8 points while the average level of human capital remains nearly constant. The transition path associated with this change is shown in Figure 5.4. Fertility begins to increase after the shock and quickly overshoots the new steady state level, decreasing then over the next few periods. Mortality falls quickly and most of the adjustment is achieved after two periods.

The reason behind the results is simple: the effect of higher utility compensates any incentive to increase investment in human capital. The elasticity of the children's value function with respect to changes in human capital is reduced by a fall in mortality because the new value function is both monotonically higher and flatter along the human capital axis. The higher level is explained by the longer life expectancy while the lower slope is due to the higher effect of a lower mortality on low human capital households. The derivative of the survival function is decreasing in  $m_{1t}c_t$  and thus, for those households where consumption is lower, the marginal effect on survival probabilities is bigger and higher the increment in utility<sup>21</sup>. In contrast, the altruism function exhibits constant elasticity and it is not affected by the change in mortality. The optimality condition relating these two elasticities and fertility implies, then, a reduction in human capital accumulation and an increase in fertility. To see this, we can think in terms of a simpler model with a linear human capital production function and a convex budget constraint. Here the first order condition relating human capital investment and fertility choice is given by  $\frac{V'(h_{t+1})}{V(h_{t+1})}n_{t+1} = M\varepsilon$ , where  $M$  is some constant. If  $\frac{V'(h_{t+1})}{V(h_{t+1})}$  monotonically decreases, either  $n_{t+1}$  has to rise,  $h_{t+1}$  has to fall or both.

Two points deserve additional comment. First, it is important to remember that the key reason for these results is the lack of a market for children to buy education from their parents. Parents choose the educational level of their children to maximize their own utility and not their children's. So, even if children would prefer a higher human capital accumulation after mortality falls, parents do not necessarily share that preference. Second, these results explain why the fall in fertility is not associated with the change in mortality in the data. English data show a fall in mortality since at least the mid seventeenth century: life expectancy at birth rose from 32 to 42 years between 1740 and 1870 (Wrigley and Schofield (1981)). However, during that time, fertility increases (as the model predicts) and did not fall until 1870. This delay is implausible if the fall in mortality and the fall in fertility are closely related, even with substantial costs of adjustment as long learning processes.

A possible criticism of this experiment is that the reduction in mortality affects all ages

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<sup>21</sup>A monotonic change along all the value function is sufficient to reduce the elasticity of the children's value function with respect to changes in human capital.

while maybe the relevant change is only in child and infant mortality. In particular, if households care about the number of surviving children and they have a target number of survivors as in Eckstein, Mira and Wolpin (1998) or Ehrlich and Lui (1991), a reduction in child mortality can decrease total fertility.

Even if data is at odds with the hypothesis in the English case (since the fall in fertility precludes the fall in infant mortality), it is interesting to see the implications of the theory. The variation in infant and child mortality is implemented by changing the value of the age-specific parameters  $m_0$  and  $m_1$  by a 20% and keeping  $m_2$  and  $m_3$  constant. Again the change is perfectly foreseen by agents. The results are as strong as before: fertility increases to 34.5 and the death rate falls to 21.4. Indeed fertility goes up even more and mortality also falls more than before. Figure 5.5 plots the transition dynamics of the experiment.

These results are consistent with other findings in the literature. Szreter (1996) uses English micro data to compare the differences in fertility between before 1837 and 1911 and finds that the fall in infant mortality can only account for a fall in fertility of 0.036 children per woman. Wolpin (1984) estimates with a discrete fertility choice model and Malaysian data that an *increase* in the infant mortality risk by 0.05 would lead to a *reduction* in the number of births by 0.25.

## 6. What Matters

This section reviews how the results are affected by changes in parameters and in functional form choice. In addition, in this model the selection of moments to match can also play an potentially important role.

### 6.1. Changes in Parameters

For most parameters the effect of changes are predictable and usually small. The exception is the elasticity of the altruism function,  $\varepsilon$ . Changes in this parameter affect the curvature of the utility function with respect to variations in the number of children. For example, for  $\varepsilon = 0.5$  and keeping the value of  $m_1 = 1.04$ , fertility increases in the steady state to 38.3, a raise of a 16%. For  $\varepsilon = 0.3$  and again keeping  $m_1$  constant, the new fertility is 15.4, a fall of 54%. More interesting is the outcome of the experiments with different  $\varepsilon$ . For higher values of  $\varepsilon$ , the response of fertility to changes in mortality or in income is bigger. The intuition behind this result is simple. In the limit case where  $\varepsilon = 1$ , utility is linear in the number of children. The optimal choice is to consume only until the marginal utility of consumption is equal to the utility of children and, after that point, using all the resources to raise children. Any additional income of the household is then translated into additional children. If mortality falls, the raise in children utility raises moves the switching point from

consumption to children to the left and fertility increases. Similar conclusions, in the opposite direction, hold for a smaller  $\varepsilon$ . However, since  $\varepsilon$  was chosen to match the observed fertility rates, even after estimating a new optimal  $m_1$ , we cannot match the fertility rate in the data for these different values of  $\varepsilon$  and the performance of the model deteriorates.

There is an additional, hidden parameter, in the model: the utility of death, that was normalized to zero. Since mortality is in part endogenous, this may affect the behavior of agents. Sensitivity analysis clearly shows this is not the case. There are two reasons for that. First, the simultaneous estimation of  $m_1$  will change to match mortality in the data, undoing the effects of a different utility of death. Second, the changes in the probability of death are of second order in comparison with the first order effects on the marginal utility of consumption and household only marginally adjust their consumption to change mortality probabilities.

## 6.2. Changes in Functional Forms

The form of the altruism function has an impact on the results of the model. In particular, two cases deserve attention. First, an increasing elasticity,  $\varepsilon'(n) > 0$ . The first case, empirically not very plausible, does nothing but to reinforce the result of income increases or mortality reductions while reduces the effect of the change in capital productivity. The utility from children does not face diminishing results as quickly as with constant elasticity so parents have less of an incentive to reduce the rate of increment of fertility when external conditions improve. In the second case, a decreasing elasticity,  $\varepsilon'(n) < 0$ , reinforces the effect of the change of relative prices of skill *versus* unskilled labor: the last children would offer a very low marginal utility and then they will quickly substituted away in favor of more educated children. A falling elasticity can also generate a fall in mortality to trigger a fall in fertility.

## 6.3. Which Moments to Match

The choice of matching moments deserves some comment. Basically two main alternatives exist to match the birth and death rates: matching the number of children per household and life expectancy or matching the age distribution of the population. The first alternative is not very rewarding. The existence of only four generations in life makes exact timing highly complicated unless arbitrary assumptions are included. The higher dimensionality of the second alternative and some data problems (stationarity and some missing observations) also makes matching the age distribution an inferior choice. Despite these problems a model with exogenous mortality chosen to match observed mortalities was computed without significant effect on the results.

## 7. Concluding Comments

Was Malthus right? In a sense, he was. He understood that technological improvements may result in increments in population and not in *per capita* consumption. From his point of view as an observer at the beginning of the industrial revolution history was on his side: once and again technological developments had only reverted in higher population and roughly constant per capita income levels. From the point of view of standard neoclassical theory he was also right: in a reasonably parametrized model as the one presented above, the income effect associated with increments in productivity dominates any offsetting substitution effect. How did then the world escape from this dreadful trap? What did Malthus miss? The proposed explanation is the increment in the returns to human capital caused by capital-skill complementarity. This paper shows how this mechanism can work in a neoclassical growth model. New technology embodied in new machines (or equivalently a falling relative price of capital) combined with different elasticities of substitution between inputs modifies relative factor prices during the process of economic growth, changing the relative opportunity cost of quality *versus* the quantity of children.

The main finding of the paper is that, using standard neoclassical theory, the movement in observed relative prices can account for more than 60% of the fall in fertility and over 50% in the increase in per capita income in England during the key years of the demographic transition. The model quantitatively shows how population and economic growth can move together or separately depending on the evolution of relative prices. Moreover the model is also consistent with other important observations, including a higher skill premium, bigger educational enrollments and changes in the composition of migration. In contrast, other alternatives proposed in the literature, as neutral technological change or the fall in mortality, have a difficult time to quantitatively deliver the desired results in the framework of our model.

Future research can apply this model to explain other empirical observations. Two of these observations are specially relevant. First, explaining the cross-country simultaneity of the fall of fertility: in a highly integrated world as the end of the nineteenth century Europe, international trade would equalize factor prices among countries with different levels of development and trigger the demographic transition in all of them, independently of income levels. The second observation is the evolution of the skill premium and fertility in the United States during the last century since falls in the skill premium were associated with increments in fertility (*i.e.* during the baby boom of the fifties) and increases in the skill premium with falls in fertility (*i.e.* during the eighties). Given the structure of the model, it may well also account for these two experiences that have been difficult to explain using standard theory.

## 8. Appendix

This appendix offers further details in several aspects of the paper. First, a population data section describes the data used. The next three sections explain some econometric aspects of the model. Finally, a computation section outlines the algorithm used to compute the model.

### 8.1. Population Data

The considerable effort undertaken by the *Cambridge Group for the History of Population and Social Structure* over the past quarter-century has resulted in two major works that present detailed information about fertility and mortality in England, Wrigley and Schofield (1981) and Wrigley *et al.* (1997). Both books try to use as thoroughly as possible the Anglican Parish data over the period prior to the institution of modern national data collection in England, *i.e.* 1541-1837. Indeed, in some dimensions as seasonality of data, there is now more knowledge about that era than about the subsequent years of data gathering by the Register-General. The main techniques used in these works have been inverse projection and family reconstitution, both of which give a similar picture of facts. After 1837, when a civil register was established, demographic data have been elaborated in England by the Register-General and the subsequent statistical agencies. Those data are reported in Chesnais (1992).

### 8.2. Test for Granger Causality

There is a substantial literature concerned with the relation between infant mortality and fertility (see Wolpin (1997) for a summary). A particular aspect of this literature links the fall in infant mortality with a fall in fertility: if parents have a *target* level of children surviving to adulthood, a fall in infant mortality reduces the number of children necessary to achieve that level. This behavior is known (Ben-Porath (1976)) as “hoarding” (for anticipated mortality) and “replacement” (for experienced mortality). However, a simple inspection of the English data suggest that the fall in mortality anticipated the fall in infant mortality (see Figure A1).

A statistical framework to evaluate this relation is the use of a *Granger Causality Test*. In this case the relevant series are the Gross Reproduction and Infant Mortality Rates for the period 1856-1921. Infant mortality is a good proxy for total child mortality since the first year of life concentrates most of the deaths and movements here are matched by movements in survival probabilities. The years selected include the fall in both levels and then encompass the relevant information. The use of longer samples will only add noise to the tests as neither series present a distinctive pattern before or after the chosen years.

First, an optimal length of the Bivariate VAR is chosen using the Schwarz criterion. Then, a Wald test on the null hypothesis of lack of causality of infant mortality on fertility is performed. Geweke, Meese and Dent (1983) suggest, based on analytical and Montecarlo evidence, that this Wald version of test performs well in small samples and that is superior to other alternatives. They also show that the Wald test has a higher value than the log-likelihood and Lagrange tests as in the general case of linear constraints. This implies that we are looking at the worst possible case for the null of lack of causality or the best possible case for existence of causality. The lack of rejection of the null is nevertheless straightforward

and confirms the intuition got from the graph of both series: fertility rates precedes the fall in infant mortality. This evidence is consistent with findings in Wolpin (1984) where the replacement levels are estimated to be small. Indeed, these results lead to the test of the alternative direction. Using the same test, granger-causality is only present over conventional levels in the one-lag case. There are several possible explanations for that result. For instance, fewer children can receive higher consumption per capita or higher parental attention. Also, a lower number of children can imply a lower probability of contagious diseases in the household. However even if this last result is not robust, the main point is clear: falls in infant mortality did not cause the fall in fertility.

### 8.3. Life Tables

The transition function between periods in life has a flexible functional to accommodate different interactions of health technology and standard of living. In this appendix we describe how we find the values of  $m_i$ .

Wrigley *et al.* (1997) report mortality rates for different years and age groups. Table 6.19 in that book includes the decade averages of adult mortality between 25 and 84 years in five years bins (25-29, 30-34 and so on) for 1640-1809. Table 6.10 reports decade averages of child mortality of ages groups 1, 2, 3, 4, 5, 6-10 and 11-15 for 1580-1837, although for consistency with data from Table 6.19 only the subperiod 1640-1809 is considered. This child mortality rates however only refer to legitimate births. This exclusion of illegitimate births biases down slightly the rates but the difference was only important quantitatively in the first year of life. For this first year, illegitimacy ratio is kept track and an overall infant mortality rate is computed in Table 6.3. Averages over this years are computed and spline interpolation used to fill all the remaining years, taking a midpoint of the relevant bin of years as the interpolation point (*i.e.* for the group 25-29, 27 is used). For model consistency the survival probability is truncated at 82 years (the quantitative effects of this truncation are minimal as less than 1% of the population reached that age). This interpolation generates a life expectancy of 36.05 years, slightly lower than the observed average for that period (37.9).

With this yearly mortality rate, survival probabilities are computed and  $m_0$  and  $m_2$  found as functions of  $m_1$ . For  $m_3$  in general,  $c_{t+1}^t \neq c_{t+1}^{t-1}$  so it cannot be expressed directly as a function of  $m_1$ . Here the identification assumption is that  $c_{t+1}^t = c_{t+1}^{t-1}$ . This assumption is not very restrictive. Computationally it is the case that for most regions of parameters, in steady state  $c_{t+1}^t \simeq c_{t+1}^{t-1}$ . The intuition is simple. Households try to smooth consumption and, if in the first period they need to pay for children, in the second they basically save for retirement, so for both periods consumption is basically constant.

### 8.4. Unit Root Tests

This appendix further explains the stationarity tests. The procedure used follows Kwiatkowski *et al.* (1992). The values of the test are (with the 10% significance level in parenthesis) 0.2 (0.347) for the birth rate and 0.16 (0.346) for the death rate. These results strongly support the null hypothesis of stationarity and, with the non-significativity of the time trend, the existence of constant first moments.

Robustness is checked changing the burden of the proof with a test of the null of the

presence of a unit root. To minimize the well-known size and power small sample problems of unit root test against local alternatives, two different test are performed, an ADF test and the test proposed by Elliot, Rothemberg and Stock (1996). This second test has an asymptotic power function tangent to the power envelope at one point selected given the sample size and never falls below that envelope. That feature substantially improves the power and the small sample behavior of the test. The ADF test delivers quite strong evidence against the unit root hypothesis. The result of the second test are mixed. It is able to reject at 5 per cent significativity the null of unit root in fertility (but not at the 1 per cent level). However it is not able to reject the null of a unit root in the mortality rate. The reason probably is related with the higher variance induced in the series by the presence of a few outliers in the data or the fall in the rate toward the end of the sample.

The main conclusion is then that while there is no evidence in the data against stationarity there is an amount of it (specially in fertility) against it.

## 8.5. Computation

This appendix describes the computation of the model. The basic algorithm used to compute the steady state is:

- Discretize the individual state space by choosing a finite grid for assets,  $\tilde{h} = \{0, \dots, h_{\max}\}$ . Choices of  $h > h_{\max}$  can be computed using linear interpolation.
- Guess an initial distribution of agents  $\Psi_0(\cdot)$  and an equilibrium path of input prices  $w = \{r_t, w_t^u, w_t^s\}_{t=0}^{t=\max}$  for some  $t \max$ .
- Guess an initial value of  $\bar{\theta}(w)$ .
- Solve the problem of each generation with value function iteration.
- Compute the moments of the model given policy functions for some  $t$  big enough such that the initial  $\Psi_0(\cdot)$  is irrelevant.
- Update  $\bar{\theta}(w)$  until convergence.
- Check market clearing given policy functions and the measure  $\Psi_t(\cdot)$ .
- Update the price sequence using a Gauss-Seidel procedure and iterate until convergence.

The algorithm was repeated several times using different initial guess to check that the convergence to an equilibrium is global.

This algorithm is easily adapted to each of the three experiments:

- Take the solution of steady state and fed it into the experiment as the initial conditions.
- Compute, as in the first part, a new steady state corresponding to the new conditions.
- Guess a transition path of input prices  $w = \{r_t, w_t^u, w_t^s\}_{t=0}^{t=\max}$  for some  $t \max$ . This length is selected beginning with a high number is selected and it is recursively reduced until a point with further reductions change the behavior of the transition path.
- Compute the transition path iterating on prices until markets clear.

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Figure 2.1: Population and Income, England: 0-1999

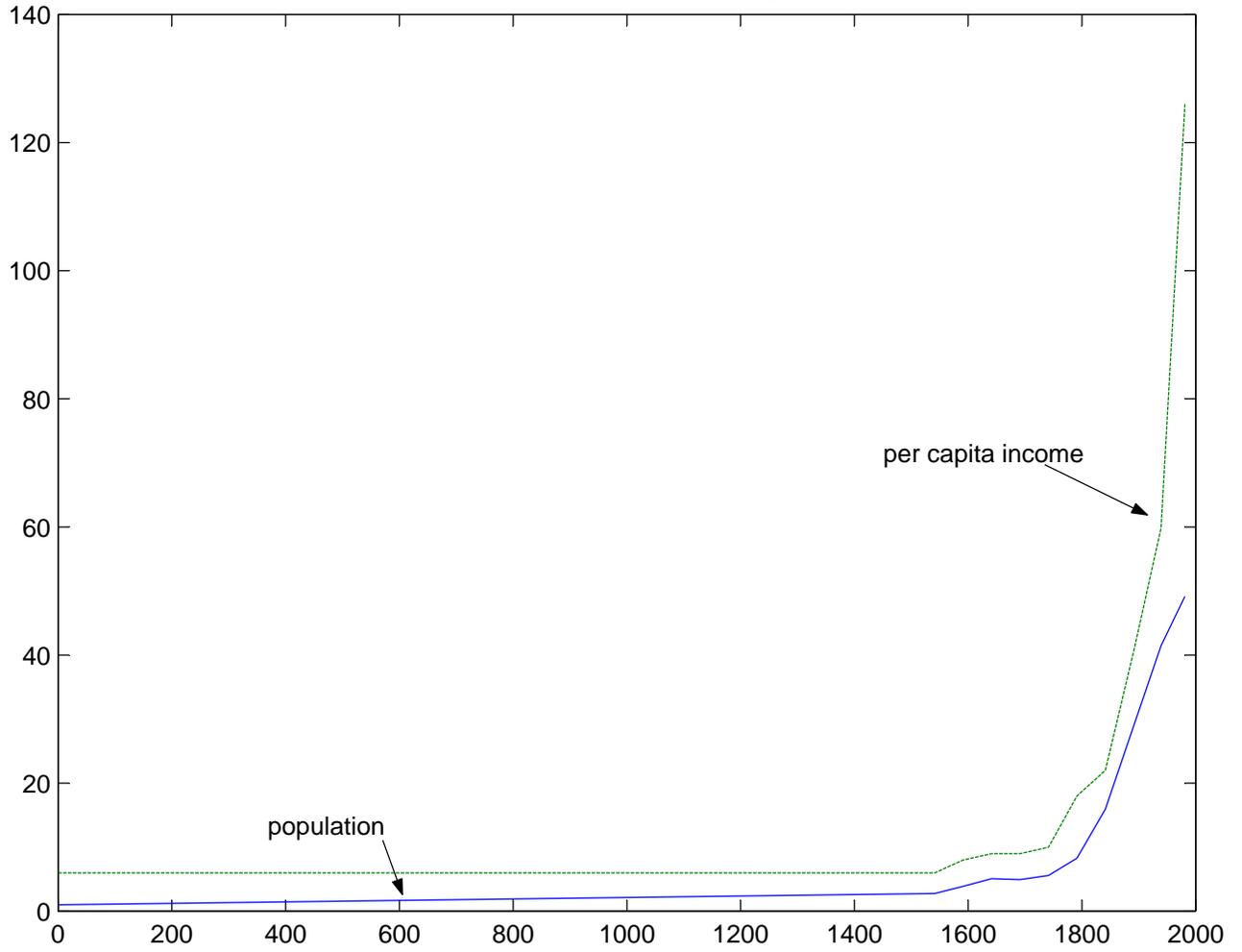


Figure 2.2: Birth and Death Rates in England: 1536-1981

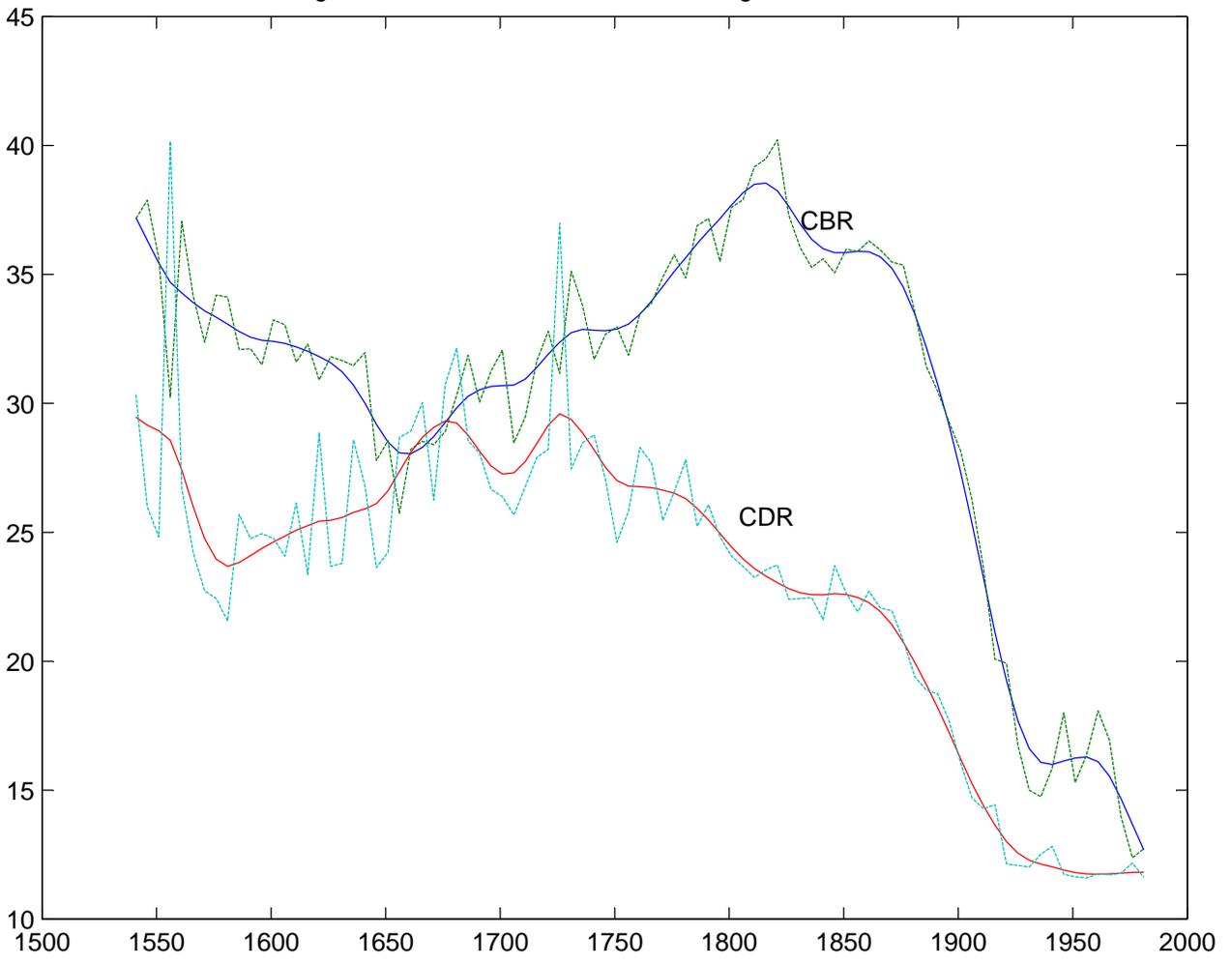


Figure 5.1: Transition Path for a Change in Income

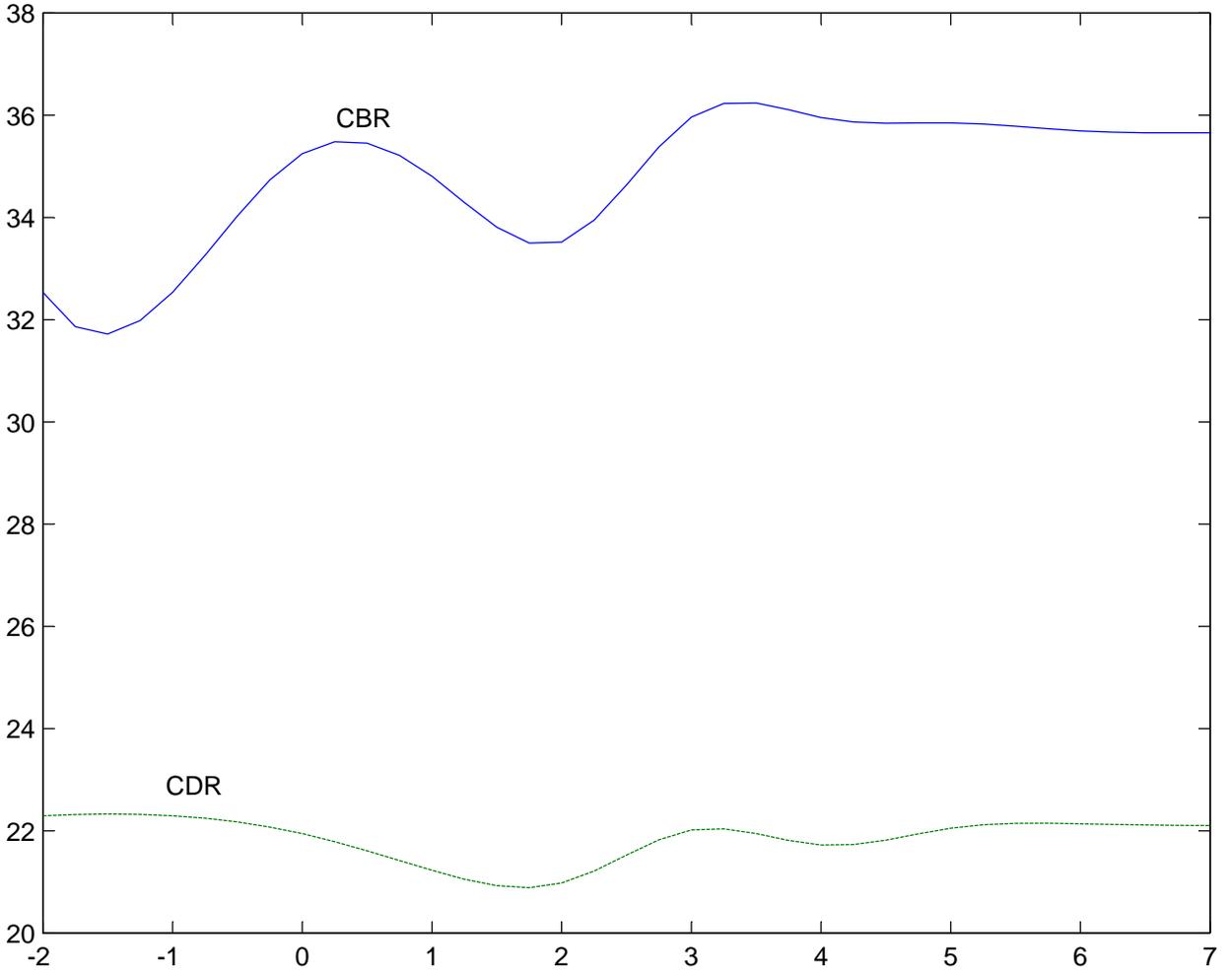


Figure 5.2: Transition Path for a Change in Capital Productivity

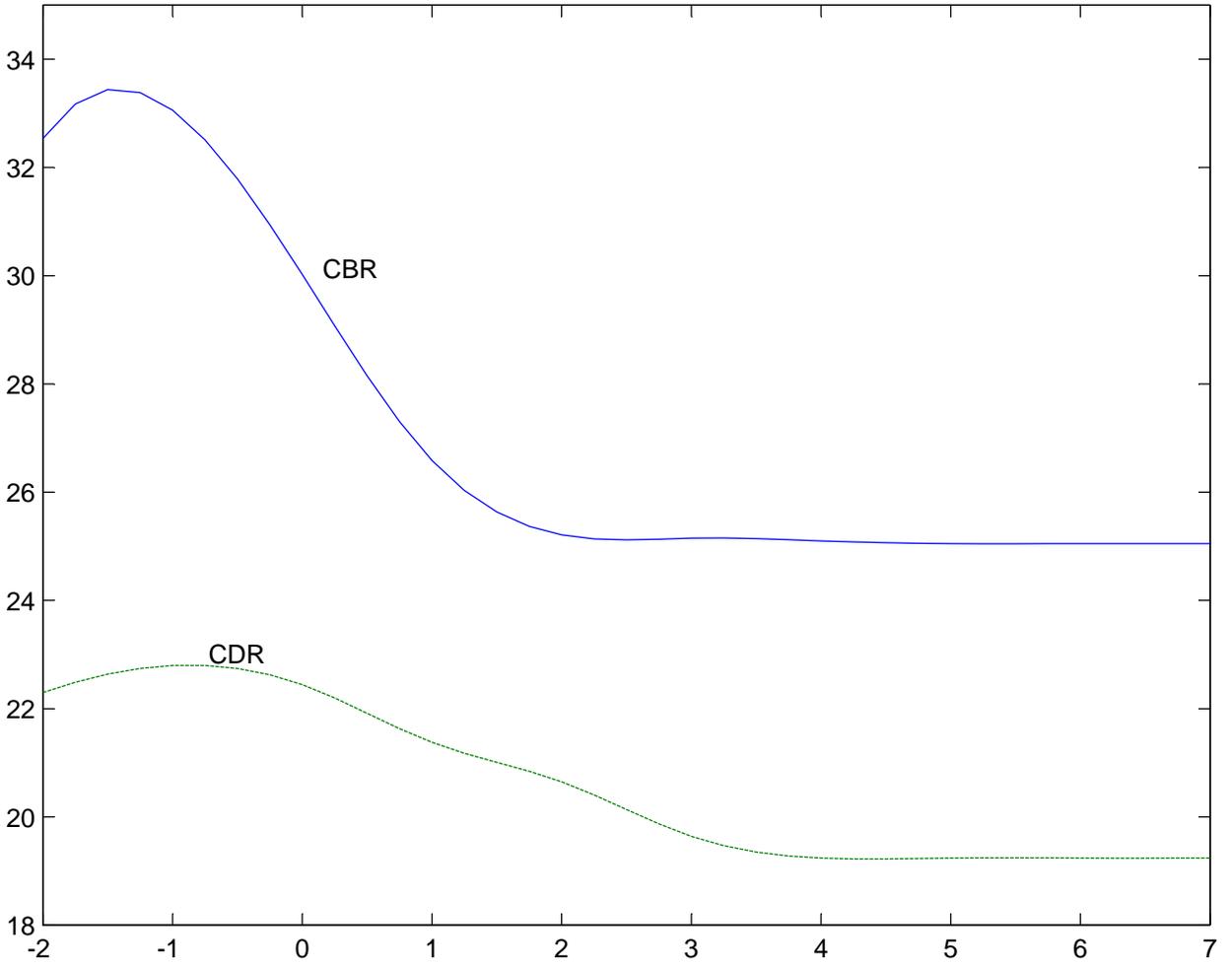


Figure 5.3: Fertility Rate and Average Years of Schooling

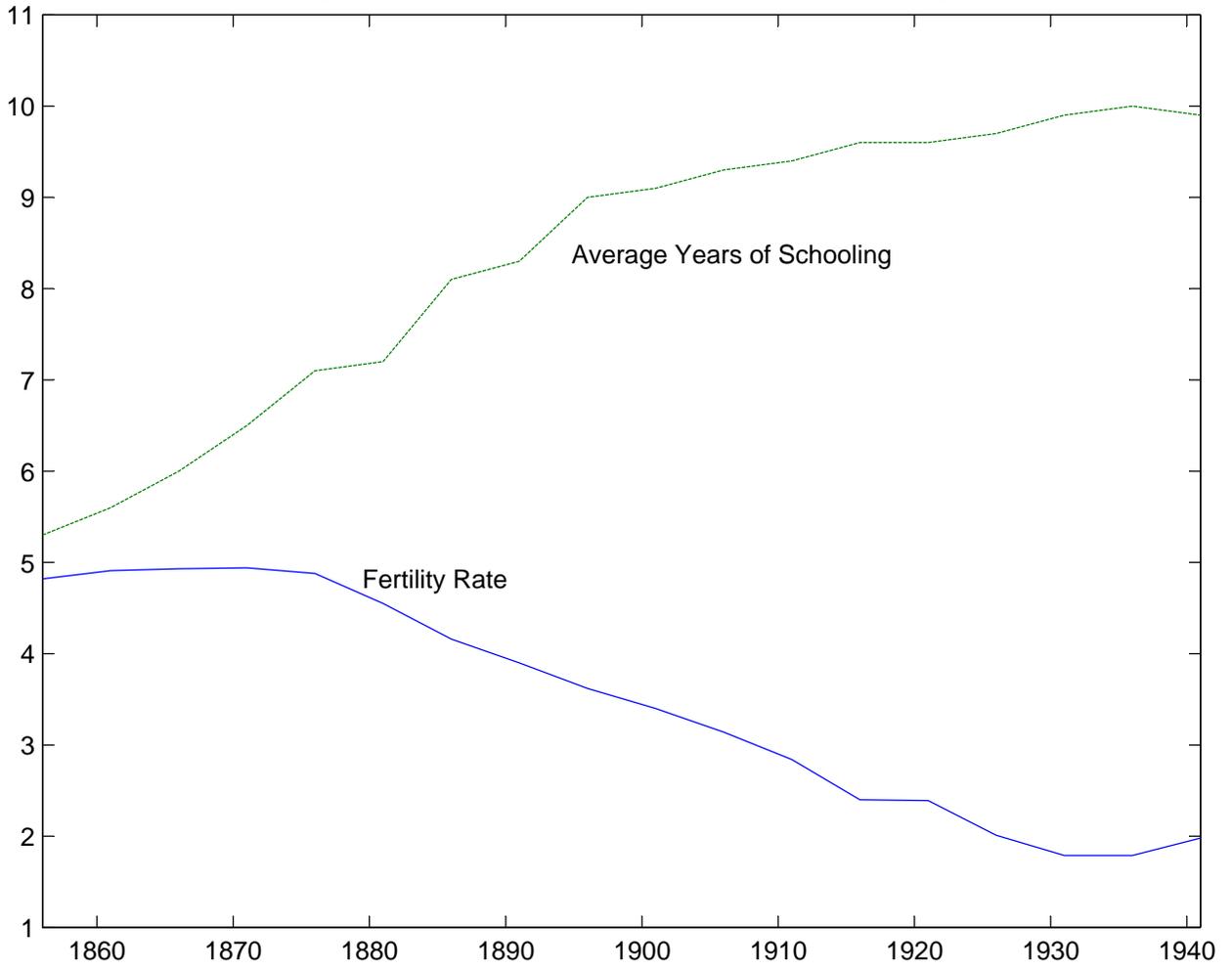


Figure 5.4: Transition Path for a Change in Medical Technology

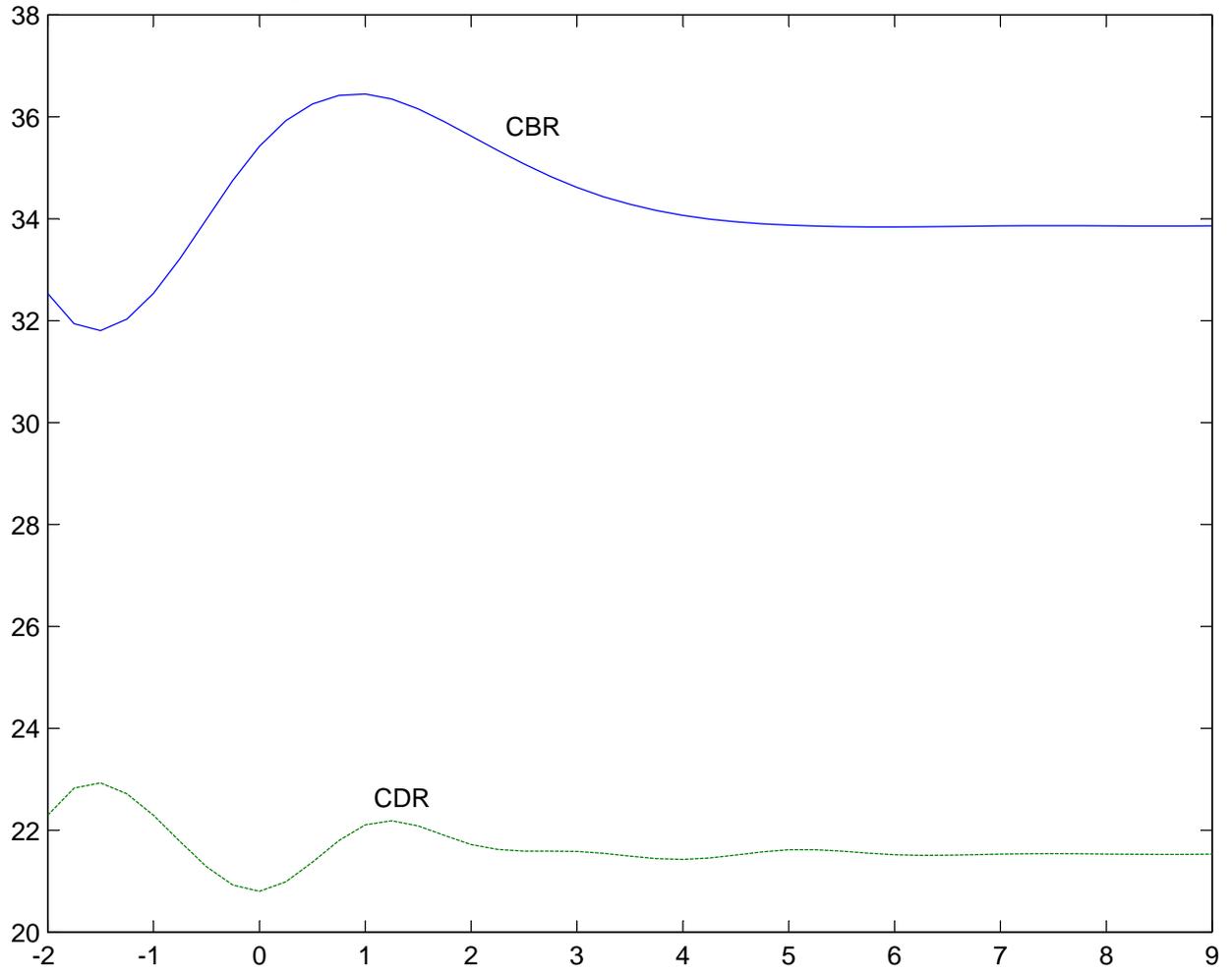


Figure 5.5: Transition Path for a Fall in Child Mortality

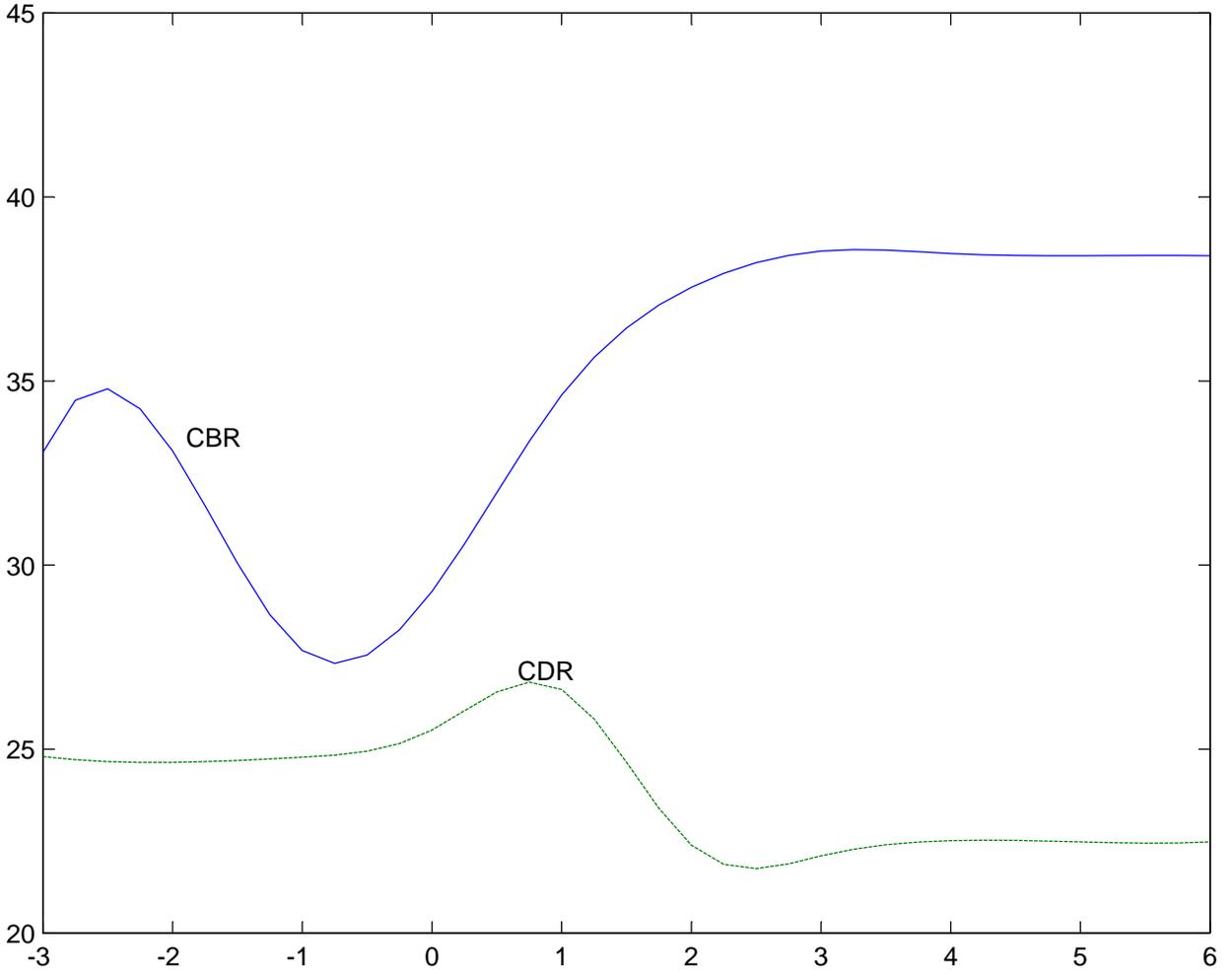


Figure A1: Gross Reproduction Rate vs. Infant Mortality, 1856=100

