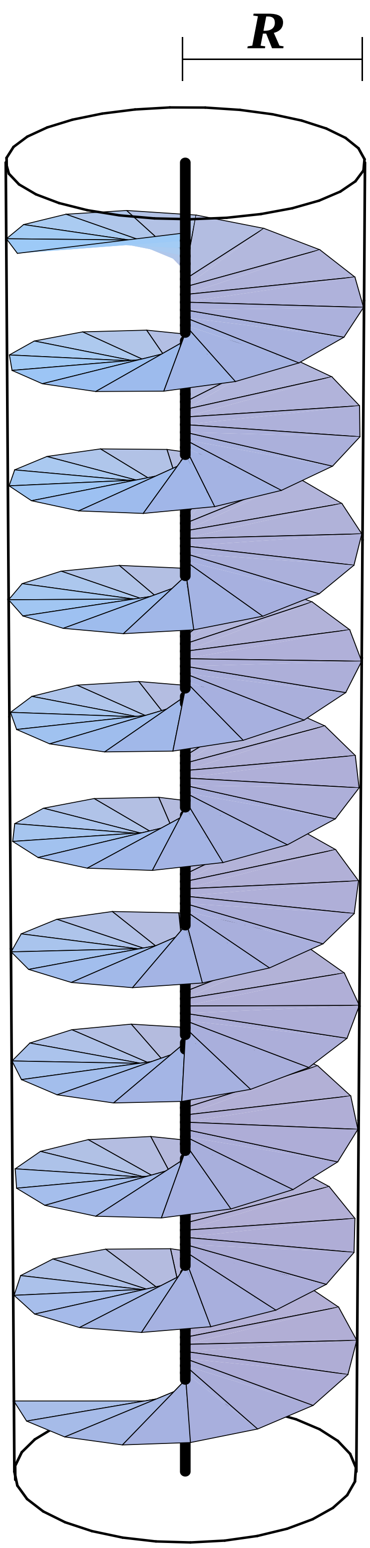
Measuring the Anchoring Strength of a Capillary using Topological Defects



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Sm A* in a capillary.
Chirality favors a screw dislocation down the center. Surface anchoring and an external magnetic field inhibit the formation of a dislocation. Measure the anchoring by decreasing the magnetic field until a defect pops in.

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see also cond–mat/9612169.

Free Energy to Quadratic Order

Bulk terms of the free energy:

$$\int \left\{ \frac{B}{2}(\nabla u + \delta\mathbf{n})^2 + \frac{K_2}{2}(\nabla \times \delta\mathbf{n})^2 - \frac{\chi}{2}(\mathbf{H}\cdot\mathbf{n})^2 \right\} d^3x$$

Surface terms of the free energy:

$$-K_2 q_0 \int_{\partial\Omega} \delta\mathbf{n}\cdot d\mathbf{s} + \frac{W}{2}\int_{\partial\Omega_+} \delta n^2 ds$$

- $\partial\Omega$ = total boundary, including defect boundary; $\partial_+\Omega$ = boundary with the capillary.
- Smectic order parameter: $\psi = |\psi| e^{i2\pi(z+u)/a}$; $\mathbf{n} = \hat{z}\sqrt{1-\delta n^2} + \delta\mathbf{n}$ with $\hat{z}\cdot\delta\mathbf{n} = 0$.

We've assumed

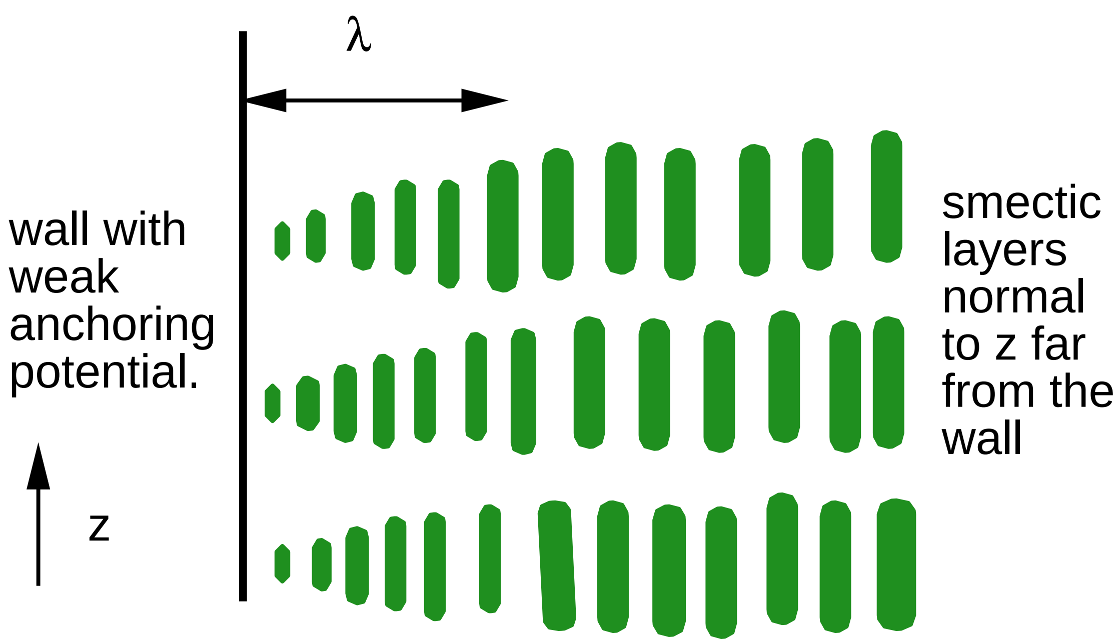
- z-independence,
- $\nabla_\perp \cdot \delta\mathbf{n} = 0$ since the strain due to the dislocation is purely transverse, and
- $K_{24}/R << W$. (K_{24} is saddle-splay.)

Penetration of Twist

Twist penetration depth $\lambda^2 \equiv K_2/B$. Coherence length for the smectic order is ξ; radius of defect core is approximately ξ. For type-II smectics $\lambda > \xi$.

- Twist can penetrate the smectic in the type-II regime

- λ is the distance away from the boundary at which the twist becomes small.



Screw Dislocation

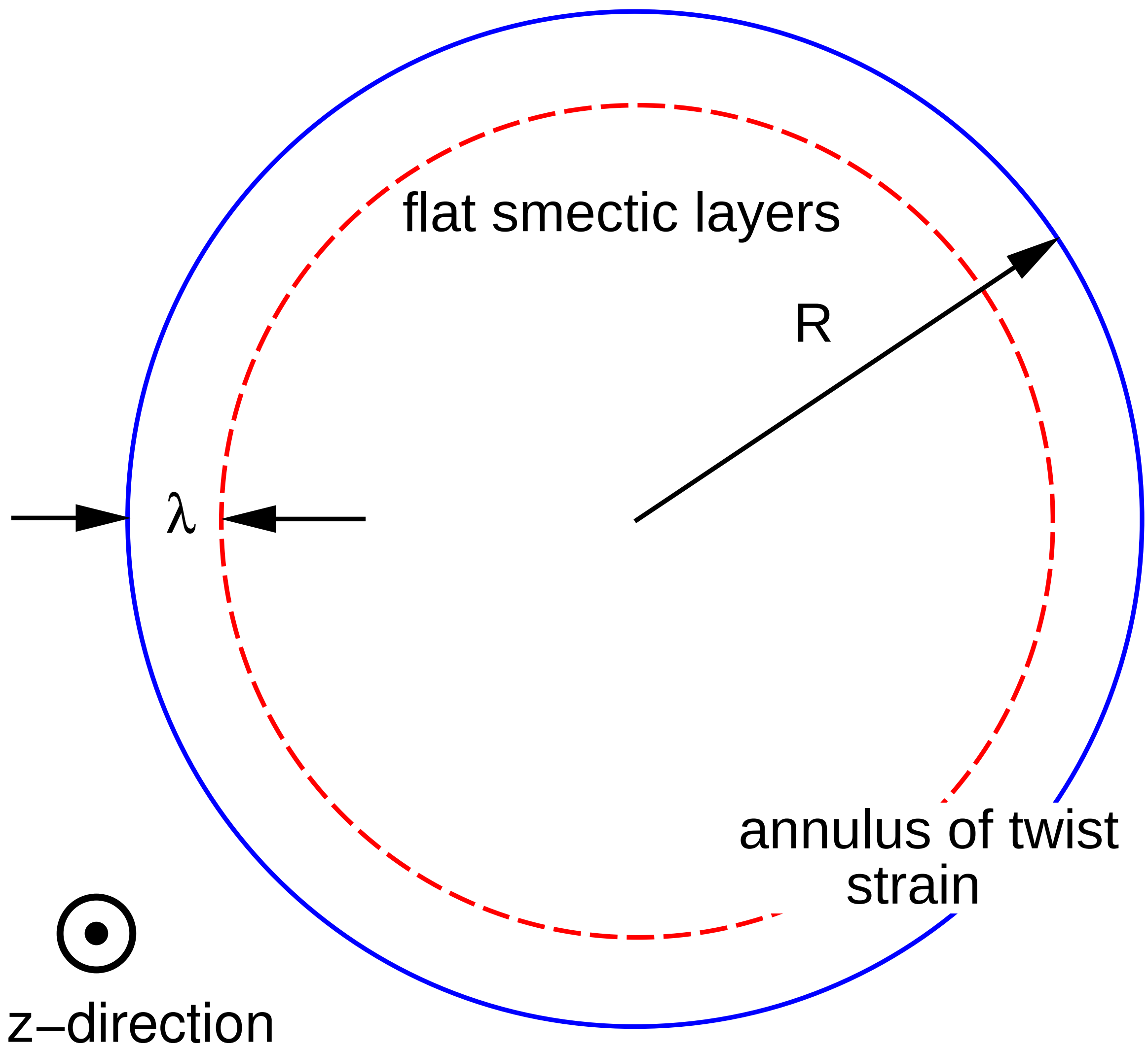
Condition for a screw dislocation:

$$\oint \nabla u \cdot d\mathbf{s} = am.$$

We can take $u = am\phi/2\pi$.

• Zero defect state has twist strain: chirality competes with anchoring and bulk elasticity leading to twist in annulus of thickness λ, as shown on the next page.

• Magnetic field along z favors small $\delta\mathbf{n}$, making the annulus thinner. Width is now given by the magnetic twist penetration depth $\lambda_H^2 = K_2/(B + \chi H^2)$.



Case of no defect.

Minimize the Energy

$$\tilde{F}/L \approx m^2 \left\{ E_{\text{core}} + \chi H^2 \frac{a^2 \lambda_H^2}{4\pi\lambda_0^2} \ln \frac{R}{\xi} + K_2 \frac{a^2\lambda_H^2}{4\pi\lambda_0^4} \ln \frac{\lambda_H}{\xi} \right\} + m \frac{K_2^2 a q_0 \lambda_H}{(W + \frac{K_2}{\lambda_H})\lambda_0^2}$$

Each term has a simple interpretation: the first three terms are the energy of a screw dislocation in a bulk sample subject to a magnetic field, and the last term is the usual chiral term of a bulk defect reduced by a factor involving the anchoring.

Note that the logarithm in the K_2 term is cut off by λ_H (like in a type-II superconductor), but the logarithm in the magnetic term is cut off by R (like in a superfluid).

Minimize over Defect Strength

No magnetic field.

- $q_0 = 0$: no defect.
- $q_0 > 0$: defect of strength $m = -1$.
- Critical chirality for large radius:

$$q_{0\text{c}} \approx q_{0\text{c,bulk}} \left(\frac{W \lambda_0}{K_2} + 1 \right) .$$

- Critical radius R_c is typically λ unless q_0 is very close to its critical value.

Defect in a Magnetic Field

When $R > R_c$, no defect forms for $H > H_c(R, W)$. The magnetic strain depends logarithmically on radius so H_c depends weakly on R. Below we've taken typical values for the parameters; $\overline{R} \equiv R/\lambda_0$, $\overline{H}^2 \equiv \chi H^2/B$, and $\overline{W} \equiv W\lambda_0/K_2$.