Charge and Statistics of Dilute Laughlin Quasiparticles

C. L. Kane University of Pennsylvania

I. Introduction

• Laughlin Quasiparticle and Shot Noise

II. Transmission of Dilute Quasiparticles through a point contact

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• Three Terminal Geometry for preparing “dilute quasiparticle beam” (Moty Heiblum et al. ’02)

• Shot Noise Puzzle

• Luttinger Liquid Theory

III. Proposed measurement of fractional statistics: Telegraph Noise

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• Corbino Geometry (Moty Heiblum et al. ’03)

• Telegraph Noise and fractional statistics

• Luttinger Liquid Theory
Laughlin Quasiparticle for \( \nu = 1/m \)

- Energy Gap \( \sigma_{xx} = 0 \)
- Quantized Hall Conductance \( \sigma_{xy} = (1/m)e^2/h \)

Fractionally Charged Excitations

\[
\frac{dQ}{dt} = \sigma_{xy} \oint E \cdot d\ell = \sigma_{xy} \frac{d\Phi}{dt}
\]

\[Q = \sigma_{xy} (h/e) = e/m \equiv e^*\]
Fractional Statistics

Halperin 1984;
Arovas, Schrieffer, Wilczek 1984

• Phase $\Theta = \pi/m$ under interchange:

$$\Psi \rightarrow \Psi e^{i\Theta}$$

• Phase when one circles another:

$$\Psi \rightarrow \Psi e^{2i\Theta}$$

• Signature of topological order

Thouless 1989;
Wen, Niu 1990

m-fold topological degeneracy on torus
Measurement of Fractional Charge

1. Capacitive Measurement

Resonant Tunneling Through Anti-dot
Goldman & Su (Science, 1995)

\[ \Delta V_{BG} = e*/C \]
2. Shot Noise Measurement

Backscattering at Quantum Point Contact

Kane, Fisher (PRL 1994); De Picciotto et al. (Nature 1997); Glattli et al. (PRL 1997)

\[ \langle I^2 \rangle_{\omega \sim 0} = e I_t \]

\[ \langle I^2 \rangle_{\omega \sim 0} = e^* I_b \]

Strong Pinch Off

Electron Tunneling

Weak Pinch Off

Quasiparticle Tunneling
Three Terminal Configuration

Comforti et al. (Nature 2002)

- When QPC2 is open, a “Dilute Beam” of quasiparticles is isolated in lead 3.

- Transport of Dilute beam through QPC2 probes transmission of individual quasiparticles.
Transmission of Dilute Quasiparticles Through a Point Contact

Dilute: $q \sim 0.45\ e$

Not Dilute: $q \sim e$

Can Dilute Charge $e/3$ Quasiparticles Traverse a Nearly Opaque Barrier?

Comforti et al. (Nature 2002)
E THREE

e over three.

How can that be?
Luttinger Liquid Model: \( \nu = 1/m \)

\[
H = \frac{mv_F}{4\pi} \sum_{i=1}^{3} \int dx_i \left[ \partial_x \phi_i(x_i) \right]^2 + \nu_1 \cos(\phi_1 - \phi_2 - Vt) + \nu_2 \cos(\phi_2 - \phi_3)
\]

Analysis:

QPC1:
- Perturbative for small \( \nu_1 \) (Dilute Limit)

QPC2:
- Perturbative for small \( \nu_2 \)
- Perturbative for large \( \nu_2 \) (small \( t_2 \))
- Exact Solution for \( m=2 \) via fermionization
Perturbative Analysis

1. Small $t_2$ ($T \ll V$)

$$I(V) = eV_1^2 t_2^2 V^{2m+2/m-3}$$

$$S(V) = eV_1^2 t_2^2 V^{2m+2/m-3}$$

$$Q = e$$

2. Small $v_2$ ($T \ll V$)

$$I(V,T) = c_1 \frac{V_1^2}{V^{1-2/m}} \left[ 1 - c_2 \frac{V_2^2}{T^{2-2/m}} \right]$$

$$S(V,T) = e^* c_1 \frac{V_1^2}{V^{1-2/m}} \left[ 1 - c_3 \frac{V_2^2}{T^{2-2/m}} \right]$$

$$Q = e^* + c \frac{V_2^2}{T^{2-2/m}}$$

Perturbation theory diverges for $T=0$ even for fixed finite $V$. 

Exact Solution

For m=2 map problem to free fermions

1. Zero Temperature:

\[ I(V) = I_{in}(V) \left( 1 - \frac{2}{\pi} K \left( -\frac{V^2}{16v_2^4} \right) \right) \]

\[ S(V) = eI(V) \]

Q = e, independent of v_2, even when transmission through QPC2 is nearly perfect.

2. Finite Temperature:

Limits T→0 and v_2→0 do not commute:

\[ Q(T\to0,v_2=0) = e/m \]

\[ Q(T=0,v_2\to0) = e \]
A nasty integral

\[ \frac{8}{\pi^3} \int_R d^4 y d u \Theta(\{y_k\}, u) \sin \left( \frac{u}{2X^2} \right) e^{-2(y_{12}+y_{34})} \frac{y_1(y_3-u) + y_3(y_1-u)(y_2(y_4-u) + y_4(y_2-u))}{uy_{13}y_{24}} \frac{1}{|y_1(y_1-u)y_2(y_2-u)y_3(y_3-u)y_4(y_4-u)|^{1/2}} \]

with \( \Theta(\{y_k\}, u) = \begin{cases} 
  1 & \text{for } y_1 > u > y_2 > y_3 > 0 > y_4 \\
 -1 & \text{for } y_1 > y_2 > y_3 > u > 0 > y_4 \\
 -1 & \text{for } y_1 > u > 0 > y_2 > y_3 > y_4 \\
  0 & \text{otherwise.} 
\end{cases} \)

= 1 \hspace{1cm} (\text{independent of } X) \!
Interpretation: Andreev Reflection

Three possible scattering products for an incident quasiparticle

1. Transmission
   Probability $T$

2. Reflection
   Probability $R$

3. Andreev Reflection
   Probability $A$

Theory predicts $T=0$ at zero temperature
- Strong Pinch-off: $R \sim 1$, $0 < A \ll 1$
- Weak Pinch-off: $R = 1 - 1/m$, $A = 1/m$

Observe Andreev processes by measuring correlations between transmitted and reflected currents.
What about the experiments?

The measured charge does approach $e$ for small $t_2$ and low temperature, but “high” temperature data remains unexplained.

- Role of irrelevant operators, eg. $(d\phi/dx)^3$
- Role of smooth edges near point contact.
Fractional Statistics

- Phase $\Theta = \pi/m$ under interchange:
  \[ \Psi \rightarrow \Psi e^{i\Theta} \]

- Phase when one circles another:
  \[ \Psi \rightarrow \Psi e^{2i\Theta} \]

- Model: Bosons + Statistical Flux
  \[
  2\Theta = \frac{e^*}{\hbar} (h / e) \\
  = 2\pi / m
  \]
Measurement of Fractional Statistics

Quantum Interference Necessary To Measure Statistical Phase

Two Point Contact Interferometer

Chamon et al. (1997)

\[ I = I_0 + \Delta I \cos \left[ e^* \frac{\Phi}{h} + 2\Theta N_{qp}^{\text{enc}} \right] \]

- Net phase changes by \( 2\Theta = 2\pi/m \) when qp tunnels from outside ring to inside of ring.

- Equilibrium \( h/e^* \) oscillations eliminated by qp tunneling.
An Ohmic contact in the middle of a ring


A “Mach-Zehnder Interferometer”

[Diagram of a Mach-Zehnder Interferometer]

[Graph showing modulation gate voltage (V_{MG}) vs. time and magnetic field, with ν = 1]
Telegraph Noise in ring with inner contact:

A Direct Signature of Fractional Statistics

- Net phase changes by $2\Theta = 2\pi/m$ when qp passes from lead 1 or 2 to lead 3.
- m-state telegraph noise for $n = 1/m$
- Related to m-fold “topological degeneracy”
- $<I^2>_{\omega-0} = e^* \Delta I^2/I_3$
Chiral Luttinger Liquid Model

\[ H = \frac{mv_F}{4\pi} \sum_{i=1}^{3} dx_i \left[ \partial_x \phi_i(x_i) \right]^2 + \sum_{\alpha=1}^{3} U_\alpha \]

Tunneling at QPC\(\alpha\):

\[ U_\alpha = v_\alpha K_\alpha^+ e^{i(\Delta\phi_\alpha - e*V_\alpha t)} + v_\alpha^* K_\alpha^- e^{-i(\Delta\phi_\alpha - e*V_\alpha t)} \]

Low Frequency Noise:

\[ \langle I^2 \rangle_{\omega \to 0} = \frac{e^* \Delta I^2}{2I_3} \frac{\coth\left(e^*V_3 / 2T\right)}{1 + \sin^2 \Theta / \sinh^2\left(e^*V_3 / 2T\right)} \]

\[ \sim \frac{e^* \Delta I^2}{2I_3} \quad e^*V_3 >> kT \]

\[ \sim \frac{e^{*2} \Delta I^2}{4G_3 T \sin^2 \Theta} \quad e^*V_3 << kT \]
Conclusion

• Fractionally charged quasiparticles cannot traverse a nearly opaque barrier. At T=0 they cannot even get past a nearly perfectly transmitting barrier!

• Telegraph Noise is predicted for a ring with an inner contact. It is a direct consequence of a fractional statistical phase, and has a unique signature in the low frequency noise.

Questions

• Origin of observed suppression of transmitted charge for moderately weak tunneling and moderately low temperature?
  - Smooth edges?
  - Irrelevant operators?

• Observation of Andreev processes?

• Exact Solution for n=1/3?

• Telegraph Noise for Hierarchical Quantum Hall States?

• Effect of dephasing on edge state transport?